TD Analyse :

Définition :

$$u : \mathbb{N} \longrightarrow \mathbb{R}$$
$$n \longrightarrow u_n$$

- $u_n = \frac{1}{n}$, $n \in \mathbb{N}^*$, $u_1 = \frac{1}{1} = 1$.
- $u_n = 1 + \frac{n^2}{2}$, $n \in \mathbb{N}$, $u_0 = 1$.
- u_n majorée, $\exists M \in \mathbb{R}$, $u_n \leq M$, $\forall x \in \mathbb{N}$
- u_n minorée, $\exists m \in \mathbb{R}$, $u_n \geq m$, $\forall x \in \mathbb{N}$
- u_n bornée, $\exists M$, $m \in \mathbb{R}$, $u_n \geq m$, $\forall x \in \mathbb{N}$, $u_n = u_{n_0}$.
- u_n périodique $\exists k \in \mathbb{N}$ tq $u_{n+k} = u_n$, $\forall n \in \mathbb{N}$
- $u_n \nearrow$, $u_{n+1} \geq u_n$, u_{n+1}, Si $u_n \neq 0$, $\frac{u_{n+1}}{u_n} \geq 1$.
- $u_n \searrow$, $u_{n+1} \leq u_n$, Si $u_n \neq 0$, $\frac{u_{n+1}}{u_n} \leq 1$.
- u_n cte, $u_{n+1} = u_n$,

  Si $u_n \neq 0$, $\frac{u_{n+1}}{u_n} = 1$.

limit :

$$\lim_{+\infty} u_n = \ell \begin{cases} \in \mathbb{R} \\ +\infty \\ -\infty \end{cases}$$

$\ell = +\infty \Rightarrow \forall A > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0, u_n \geq A$.

$\ell = -\infty \Rightarrow \forall B > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0, u_n \leq B$.

$\ell \in \mathbb{R}$, $\lim_{+\infty} u_n = \ell$, $\Rightarrow \forall \varepsilon > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0$

$|u_n - \ell| < \varepsilon$

Suite Arith

$U_{n+1} = U_n + r$, $U_{n_0} \in \mathbb{R}$.

$U_n = (n - n_0)r + U_{n_0}$

$U_{n_0+1} = U_{n_0} + r$

$U_{n_0+2} = U_{n_0+1} + r = U_{n_0} + 2r$

$U_{n_0+3} = U_{n_0+2} + r = U_{n_0} + 3r$

$U_n = U_{n-1} + r = (n - n_0)r + U_{n_0}$

Sommation :

premier terme, dernière terme

$$\sum_{k=n_0}^{n} U_k = \frac{n\text{deT}\,(P^{\text{ère}}T + dT)}{2}$$

$$= (n - n_0 - 1)\left(\frac{U_{n_0} + U_n}{2}\right)$$

Suite geo

$U_{n+1} = q\,U_n$, $U_{n_0} \in \mathbb{R}$

$U_n = q^{n-n_0}\,U_{n_0}$, U_{n_0} "given"

$U_{n_0+1} = qU_{n_0}$

$U_{n_0+2} = q\,U_{n_0+1} = q^2\,U_{n_0}$

$U_{n_0+3} = qU_{n_0+2} = q^3\,U_{n_0}$

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$\downarrow$

$U_n = qU_{n-1} = q^{n-n_0}\,U_{n_0}$

Sommation :

$$\sum_{k=n_0}^{n} U_k = P^{\text{ère}}T\left(\frac{1-(\text{raison})^{n\text{.deT}}}{1-\text{raison}}\right) \text{ Si } q \neq 1$$

$$= U_{n_0}\left(\frac{1-q^{n-n_0+1}}{1-q}\right)$$

Si $q = 1$:

$U_n = q^{n-n_0}\,U_{n_0}) = U_{n_0}$,

$$\sum_{k=n_0}^{n} 1\ U_{n_0} = U_{n_0} \cdot \sum_{k=n_0}^{n} 1.$$

$$\boxed{a^n = e^{\ln(a^n)} = e^{n\ln(a)}}$$

$$\boxed{(a+b)\cdot(a-b) = a^2 - b^2}$$

Serie 1 :

ex 1 :

Limits :

1) $U_n = \left(1+\frac{1}{n}\right)^n = e^{\ln\left(1+\frac{1}{n}\right)} = e^{0 \cdot \ln\left(\frac{1}{n}\right)} = e^0 = 1.$

2) $U_n = \sqrt{n^2+n+1} - \sqrt{n^2-n+1}$

$(a - b) \times (a + b) = a^2 - b^2$

$$\lim_{+\infty} U_n = \lim_{+\infty} \frac{\left(\sqrt{n^2+n+1} - \sqrt{n^2-n+1}\right)\left(\sqrt{n^2+n+1} + \sqrt{n^2-n+1}\right)}{\left(\sqrt{n^2+n+1} + \sqrt{n^2-n+1}\right)}$$

$$= \lim_{+\infty} \frac{2n}{\left(\sqrt{n^2+n+1} + \sqrt{n^2-n+1}\right)}$$

$$= \lim_{+\infty} \frac{2n}{\left(\sqrt{n^2\left(1+\frac{1}{n}+\frac{1}{n^2}\right)}\right) + \sqrt{n^2\left(1-\frac{1}{n}+\frac{1}{n^2}\right)}}$$

$$= \lim_{+\infty} \frac{2n}{\left(n\underbrace{\sqrt{1+\underbrace{\tfrac{1}{n}}_{0}+\underbrace{\tfrac{1}{n^2}}_{0}}}_{\sqrt{1}}\right) + \left(n\underbrace{\sqrt{1-\underbrace{\tfrac{1}{n}}_{0}+\underbrace{\tfrac{1}{n^2}}_{0}}}_{\sqrt{1}}\right)}$$

$$= \lim_{+\infty} \frac{2n}{2n} = 1$$

3) $U_n = \frac{4^n - (-2)^n}{4^n + (-2)^n}$; $\lim_{+\infty} U_n = \frac{(2^{2n}) - (-2)^n}{2^{2n} + (-2)^n}$

$= \lim_{+\infty}$