TD Analyse:

Définition :

$$U : \mathbb{N} \longrightarrow \mathbb{R}$$
$$n \longrightarrow U_n$$

- $U_n = \frac{1}{n}$, $n \in \mathbb{N}^*$, $U_1 = \frac{1}{1} = 1$.
- $U_n = 1 + \frac{n^2}{2}$, $n \in \mathbb{N}$, $U_0 = 1$.
- U_n majorée, $\exists M \in \mathbb{R}$, $U_n \leq M$, $\forall x \in \mathbb{N}$
- U_n minorée, $\exists m \in \mathbb{R}$, $U_n \geq m$, $\forall x \in \mathbb{N}$
- U_n bornée, $\exists M$, $m \in \mathbb{R}$, $U_n \geq m$, $\forall x \in \mathbb{N}$, $U_n = U_{n_0}$.
- U_n périodique $\exists k \in \mathbb{N}$ tq $U_{n+k} = U_n$, $\forall n \in \mathbb{N}$
- $U_n \nearrow$, $U_{n+1} \geq U_n$, U_{n+1}, Si $U_n \neq 0$, $\frac{U_{n+1}}{U_n} \geq 1$.
- $U_n \searrow$, $U_{n+1} \leq U_n$, Si $U_n \neq 0$, $\frac{U_{n+1}}{U_n} \leq 1$.
- U_n cte, $U_{n+1} = U_n$,

  Si $U_n \neq 0$, $\frac{U_{n+1}}{U_n} = 1$.

limit :

$$\lim_{+\infty} U_n = \ell \begin{cases} \in \mathbb{R} \\ +\infty \\ -\infty \end{cases}$$

$\ell = +\infty \Rightarrow \forall A > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0, U_n \geq A$.

$\ell = -\infty \Rightarrow \forall B > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0, U_n \leq B$.

$\ell \in \mathbb{R}$, $\lim_{+\infty} U_n = \ell$, $\Rightarrow \forall \varepsilon > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0$

$|U_n - \ell| < \varepsilon$

Suite Arith

$U_{n+1} = U_n + r$, $U_{n_0} \in \mathbb{R}$.

$U_n = (n - n_0) r + U_{n_0}$

$U_{n_0+1} = U_{n_0} + r$

$U_{n_0+2} = U_{n_0+1} + r = U_{n_0} + 2r$

$U_{n_0+3} = U_{n_0+2} + r = U_{n_0} + 3r$

$U_n = U_{n-1} + r = (n - n_0) r + U_{n_0}$

Sommation:

premier terme / dernier terme

$$\sum_{k=n_0}^{n} U_k = \frac{n \, de \, T \, (P^{ere} T + dT)}{2}$$

$$= (n - n_0 - 1) \left(\frac{U_{n_0} + U_n}{2} \right)$$

Suite geo

$U_{n+1} = q \, U_n$, $U_{n_0} \in \mathbb{R}$

$U_n = q^{n-n_0} U_{n_0}$, U_{n_0} "given"

$U_{n_0+1} = q U_{n_0}$

$U_{n_0+2} = q \, U_{n_0+1} = q^2 U_{n_0}$

$U_{n_0+3} = q U_{n_0+2} = q^3 U_{n_0}$

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$\downarrow$

$U_n = q U_{n-1} = q^{n-n_0} U_{n_0}$

Sommation:

$$\sum_{k=n_0}^{n} U_k = P^{ere} T \left(\frac{1 - (\text{raison})^{n \cdot de T}}{1 - \text{raison}} \right) \text{ Si } q \neq 1$$

$$= U_{n_0} \left(\frac{1 - q^{n-n_0+1}}{1 - q} \right)$$

Si $q = 1$:

$U_n = q^{n-n_0} U_{n_0}) = U_{n_0}$,

$$\sum_{k=n_0}^{n} 1 \; U_{n_0} = U_{n_0} \cdot \sum_{k=n_0}^{n} 1 .$$

$$\boxed{a^n = e^{Ln(a^n)} = e^{n Ln(a)}}$$