TD Serie :

ex 1 :

1) $f_1(x) = \dfrac{x}{\sqrt{1+x^2} - \sqrt{1+x}}$

$\Rightarrow \sqrt{1+x^2} - \sqrt{1+x} \neq 0$.

x should not be 0 or 1

$\Rightarrow x$ should not be $< -1 \Leftarrow$

$\Leftrightarrow Df_1 = \{]-1 ; +\infty[\setminus (0, 1) \}$

$(Df_1 = \{]-1 ; 0 [\cup]0 ; 1 [\cup]1 ; +\infty [\setminus (0, 1) \}$

2) $f_2(x) = \ln(1+x^3) + x - 1$.

$Df_2 = \{]-1 ; +\infty [\}$

3) $f_3(x) = \dfrac{\ln(x)}{E(x^2) - 4} \neq 0$.

$\Leftrightarrow x$ doit être $\neq \bar{2}$.

$\Leftrightarrow Df_3 = \{]0 ; +\infty[\setminus (2, -2) \}$

$$f_4(x) = \sqrt{|x+2| - |2x-4|}$$

$$\Leftrightarrow |x+2| - |2x-4| \geq 0$$

$$\Leftrightarrow |x+2| = |2x-4|$$

$$\Leftrightarrow x+2 = 2x-4$$

$$\Leftrightarrow x = 2x - 4 - 2$$

$$\Leftrightarrow x = 2x - 6$$

$$\lim_{0} \frac{x - \ln(1+x)}{x}$$

$$\Leftrightarrow x\left(\frac{1 - \ln(1+x)}{x}\right)$$

Les primitives :

I - $$\boxed{1)\ x^n = \frac{x^{n+1}}{n+1}.}$$

ex :

- $f(x) = 5x^4$, $F(x) = \frac{5x^5}{5} = x^5$.) To verify the process.
- $f(x) = x^3 - 2x$,

  $F(x) = \frac{x^4}{4} - x^2 + C$.
- $f(x) = 5x - 3 = \frac{5}{2}x^2 - 3x$.

II $$\boxed{\frac{1}{x^2} = \frac{-1}{x}}$$

1) $f(x) = \frac{-3}{x^2}$, $F(x) = \frac{3}{x}$

2) $f(x) = \frac{5}{x^2} \Rightarrow F(x) = \left(5 \cdot \frac{1}{x^2}\right)$prim,

$$F(x) = -\frac{5}{x}$$

3) $f(x) = 4x - \frac{1}{x^2} \Rightarrow F(x) = \frac{4x^2}{2} + \frac{1}{x}$

$$= 2x^2 + \frac{1}{x}$$

III) $\frac{1}{x} = \ln(x)$ $\quad e^x = e^x$

1) $f(x) = \frac{-2}{x} = -2\,\frac{1}{x}$, $F(x) = -2\ln(x)$.

2) $f(x) = 4e^x$, $F(x) = 4e^x$

3) $f(x) = \frac{3}{x} + \frac{1}{x^2} = 3 \cdot \frac{1}{x} + \frac{1}{x^2}$.

$\Rightarrow F(x) = 3\ln(x) - \frac{1}{x}$

IV) $u' e^u = e^u$

1) $f(x) = x e^{x^2} = \frac{1}{2} 2x\, e^{x^2}$, $F(x) = \frac{e^{x^2}}{2}$

2) $f(x) = e^{4x+1} = \frac{1}{4} \cdot 4\ e^{4x+1} = \frac{1}{4} e^{4x+1}$

V)

Fonction	Une primitive.
$u'u^n$; $n \neq -1$	$\frac{1}{n+1}\, u^{n+1}$
$\frac{u'}{2\sqrt{u}}$	$\sqrt{u}$
$\frac{u'}{u}$	$\ln(u)$
$u'e^u$	e^u
$u'\cos(u)$	$\sin(u)$
$u'\sin(u)$	$-\cos(u)$

1) $f(x) = \dfrac{x}{\sqrt{x^2+1}}$

$(x^2+1)' = 2x$; so let's add 2.

$$\frac{2x}{2\sqrt{x^2+1}} = f(x).$$

$$F(x) = \sqrt{x^2+1}$$

sin

cos.

2) $f(x) = \cos(2x) - 3\sin(3x-1)$

$\cos(2x) \longrightarrow -\sin(2x)$

$\sin(3x-1) \Rightarrow \cos(3x-1)$

$$= -\frac{\sin(2x)}{2} + \cos(3x-1)$$

Methode 2 :

$f(x) = \cos(2x) - 3\sin(3x-1).$

$= \frac{1}{2} \cdot 2\cos(2x) - 3\sin(3x-1)$

$\Rightarrow F(x) = \frac{1}{2}\sin(2x) + \cos(3x-1)$

3) $f(x) = \dfrac{3x}{x^2+2} = \dfrac{3}{2} \cdot \dfrac{2x}{x^2+2} \Rightarrow F(x)\ \dfrac{3}{2}\ \ln(x^2+2)$

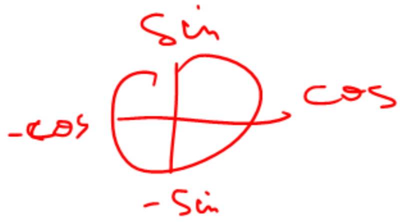
$u = x^2+1,\ u' = 2x$

Intégrale :

I) Intégration par partie :

Soit la formule suivante :

$$A = \int \underbrace{x}_{v} \underbrace{\sin(x)}_{u'} \, dx$$



$v = x^{\,a}$ $\qquad$ $v' = 1^{\,b}$.

$u' = \sin\overset{d}{(x)}$ $\qquad$ $u = -\cos^{\,c}$.

$$A = a \cdot c - \int b \cdot c \, dx.$$

$$\Rightarrow A = -x\cos(x) - \int -\cos(x) \, dx.$$

$$= -x\cos(x) + \int \cos(x) \, dx.$$

$$= -x\cos(x) + \sin(x) + C$$

$$\boxed{C \Rightarrow \text{constante}}$$

$$f(x) = \frac{x}{\sqrt{x^2+1}}$$

Soit $g(x) = x^2+1$, $g'(x) = 2x$ (équilibre).

$$f(x) = \frac{2x}{2\sqrt{x^2+1}} = \sqrt{x^2+1}.$$

continue Later.

Integrale avec changement de variable.