TD Serie :

ex 1 :

1) $f_1(x) = \dfrac{x}{\sqrt{1+x^2} - \sqrt{1+x}}$

$\Rightarrow \sqrt{1+x^2} - \sqrt{1+x} \neq 0$.

x should not be 0 or 1

=> x should not be < -1 <=>

<=> $Df_1 = \{]-1 ; +\infty[\setminus (0,1) \}$

$\left(Df_1 = \{]-1 ; 0 [\cup]0 ; 1 [\cup]1 ; +\infty[\setminus (0,1) \} \right.$

2) $f_2(x) = \ln(1+x^3) + x - 1$.

$Df_2 = \{]-1 ; +\infty[\}$

3) $f_3(x) = \dfrac{\ln(x)}{E(x^2) - 4}) \leftarrow 0$.

<=> x doit être $\neq 2$.

<=> $Df_3 = \{]0 ; +\infty[\setminus (2, -2) \}$

$$f_4(x) = \sqrt{|x+2| - |2x-4|}$$

$$\Leftrightarrow |x+2| - |2x-4| \geq 0 .$$

$$\Leftrightarrow |x+2| = |2x-4|$$

$$\Leftrightarrow x+2 = 2x-4$$

$$\Leftrightarrow x = 2x-4-2$$

$$\Leftrightarrow x = 2x-6$$

$$\lim_{0} \frac{x - \ln(1+x)}{x}$$

$$\Leftrightarrow x\left(\frac{1 - \ln(1+x)}{x}\right)$$

Les primitives :

I - $$\boxed{(1)\ x^n = \frac{x^{n+1}}{n+1}.}$$

ex :

- $f(x) = 5x^4$, $F(x) = \frac{5x^5}{5} = x^5$. To verify the process.
- $f(x) = x^3 - 2x$,

  $F(x) = \frac{x^4}{4} - x^2 + C$.
- $f(x) = 5x - 3 = \frac{5}{2}x^2 - 3x$.

II $$\boxed{\frac{1}{x^2} = \frac{-1}{x}}$$

1) $f(x) = \frac{-3}{x^2}$, $F(x) = \frac{3}{x}$

2) $f(x) = \frac{5}{x^2} \Rightarrow F(x) = \left(5 \cdot \frac{1}{x^2}\right)$prim,

$$F(x) = -\frac{5}{x}$$

3) $f(x) = 4x - \frac{1}{x^2} \Rightarrow F(x) = \frac{4x^2}{2} + \frac{1}{x}$

$$= 2x^2 + \frac{1}{x}$$

III)

$\frac{1}{x} = \ln(x)$ $\qquad$ $e^x = e^x$

1) $f(x) = \frac{-2}{x} = -2\,\frac{1}{x}$, $F(x) = -2\ln(x)$.

2) $f(x) = 4e^x$, $F(x) = 4e^x$

3) $f(x) = \frac{3}{x} + \frac{1}{x^2} = 3 \cdot \frac{1}{x} + \frac{1}{x^2}$.

$\Rightarrow F(x) = 3\ln(x) - \frac{1}{x}$

IV)

$u' e^u = e^u$

1) $f(x) = x e^{x^2} = \frac{1}{2} 2x\, e^{x^2}$, $F(x) = \frac{e^{x^2}}{2}$

2) $f(x) = e^{4x+1} = \frac{1}{4} \cdot 4\; e^{4x+1} = \frac{1}{4} e^{4x+1}$

V)

Fonction	Une primitive.
$u'u^n \,;\, n \neq -1$	$\frac{1}{n+1} u^{n+1}$
$\frac{u'}{2\sqrt{u}}$	$\sqrt{u}$
$\frac{u'}{u}$	$\ln(u)$
$u'e^u$	e^u
$u'\cos(u)$	$\sin(u)$
$u'\sin(u)$	$-\cos(u)$

1) $f(x) = \frac{x}{\sqrt{x^2+1}}$

$(x^2+1)' = 2x$; so let's add 2.

$$\frac{2x}{2\sqrt{x^2+1}} = f(x).$$

$$F(x) = \sqrt{x^2+1}$$

sin

cos.

2) $f(x) = \cos(2x) - 3\sin(3x-1)$

$\cos(2x) \longrightarrow -\sin(2x)$

$\sin(3x-1) \Rightarrow \cos(3x-1)$

$$= -\frac{\sin(2x)}{2} + \cos(3x-1)$$

Methode 2:

$f(x) = \cos(2x) - 3\sin(3x-1).$

$= \frac{1}{2}\cdot 2\cos(2x) - 3\sin(3x-1)$

$\Rightarrow F(x) = \frac{1}{2}\sin(2x) + \cos(3x-1)$

3) $f(x) = \frac{3x}{x^2+2} = \frac{3}{2}\cdot\frac{2x}{x^2+2} \Rightarrow F(x)\ \frac{3}{2}\ln(x^2+2)$

$u = x^2+1,\ u' = 2x$

Intégrale :

I) Intégration par partie :

Soit la formule suivante :

$$A = \int \underbrace{x}_{v} \; \underbrace{\sin(x)}_{u'} \, dx$$

$v = \overset{a}{x}$ $\qquad$ $v' = \overset{b}{1}$.

$u' = \sin \overset{d}{(x)}$ $\qquad$ $u = -\overset{c}{\cos}$.

$$A = a \cdot c - \int b \cdot c \, dx.$$

$$\Rightarrow A = -x\cos(x) - \int -\cos(x) \, dx.$$

$$= -x\cos(x) + \int \cos(x) \, dx.$$

$$= -x\cos(x) + \sin(x) + C$$

$$\boxed{C \Rightarrow \text{constante}}$$

$$f(x) = \frac{x}{\sqrt{x^2+1}}$$

Serie Analyse n°3 :

Ex 1 :

1) $\int (at+B)^n \, dt$, $a \neq 0$.

$$= \frac{(at+B)^{n+1}}{n+1} + C.$$

2) $\int \sin^3(t) \cos(t) \, dt$.

$$= \frac{1}{4} \sin^4(t) + C$$

3) $\int \sin(3t) \, dt = -\frac{1}{3} \sin(3t) + C$

4) $\int (1+\cos(t))^2 \, dt = \frac{1}{3} (1+\cos(t))^3 + C$

5) $\int \frac{dt}{1+e^t}$ what ?

6) $\int t^2 e^{2t^3+1} \, dt$.

integration directe :

$$= \frac{\cancel{t^3}}{3} \cdot \frac{e^{2t^3+1}}{2\cancel{t^3}} = \frac{e^{2t^3}+1}{6}$$

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integration par partie :

$u = e^{2t^3+1}$, $u' = 6t^2 e^{2t^3+1}$.

$v' = \frac{1}{3} 3t^2$, $u = t^3$

$\Rightarrow t^3 e^{2t^3+1} - 6t^6 e^{2t^3+1}$