TD Analyse:

Definition :

$u : \mathbb{N} \longrightarrow \mathbb{R}$

$n \longrightarrow u_n$

- $u_n = \frac{1}{n}$, $n \in \mathbb{N}^*$, $u_1 = \frac{1}{1} = 1$.
- $u_n = 1 + \frac{n^2}{2}$, $n \in \mathbb{N}$, $u_0 = 1$.
- u_n majorée, $\exists M \in \mathbb{R}$, $u_n \leq M$, $\forall x \in \mathbb{N}$
- u_n minorée, $\exists m \in \mathbb{R}$, $u_n \geq m$, $\forall x \in \mathbb{N}$
- u_n bornée, $\exists M, m \in \mathbb{R}$, $u_n \geq m$, $\forall x \in \mathbb{N}$, $u_n = u_{n_0}$.
- u_n périodique $\exists k \in \mathbb{N}$ tq $u_{n+k} = u_n$, $\forall n \in \mathbb{N}$
- $u_n \nearrow$, $u_{n+1} \geq u_n$, u_{n+1}, Si $u_n \neq 0$, $\frac{u_{n+1}}{u_n} \geq 1$.
- $u_n \searrow$, $u_{n+1} \leq u_n$, Si $u_n \neq 0$, $\frac{u_{n+1}}{u_n} \leq 1$.
- u_n cte, $u_{n+1} = u_n$,

  Si $u_n \neq 0$, $\frac{u_{n+1}}{u_n} = 1$.

limit :

$\lim\limits_{+\infty} u_n = l$ $\begin{cases} \in \mathbb{R} \\ \to +\infty \\ \to -\infty \end{cases}$

$l = +\infty \Rightarrow \forall A > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0, u_n \geq A$.

$l = -\infty \Rightarrow \forall B > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0, u_n \leq B$.

$l \in \mathbb{R}$, $\lim\limits_{+\infty} u_n = l$, $\Rightarrow \forall \varepsilon > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0$

$|u_n - l| < \varepsilon$

Suite Arith

$U_{n+1} = U_n + r$, $U_{n_0} \in \mathbb{R}$.

$U_n = (n - n_0)r + U_{n_0}$

$U_{n_0+1} = U_{n_0} + r$

$U_{n_0+2} = U_{n_0+1} + r = U_{n_0} + 2r$

$U_{n_0+3} = U_{n_0+2} + r = U_{n_0} + 3r$

$U_n = U_{n-1} + r = (n - n_0)r + U_{n_0}$

Sommation:

premier terme, dernière terme

$$\sum_{k=n_0}^{n} U_k = \frac{\text{nde}T\,(P^{ère}T + dT)}{2}$$

$$= (n - n_0 - 1)\left(\frac{U_{n_0} + U_n}{2}\right)$$

Suite geo

$U_{n+1} = q\,U_n$, $U_{n_0} \in \mathbb{R}$

$U_n = q^{n-n_0}\,U_{n_0}$, U_{n_0} "given"

$U_{n_0+1} = qU_{n_0}$

$U_{n_0+2} = q\,U_{n_0+1} = q^2\,U_{n_0}$

$U_{n_0+3} = qU_{n_0+2} = q^3\,U_{n_0}$