Equivalence of Two Finite Automata

In this chapter, we will discuss the concept of the equivalence of two finite automata. The primary goal is to determine whether two given finite automata are the same or equivalent. By "same" or "equivalent", we mean that the two automata perform the same function, implying that they accept the same languages.

To identify whether two automata are equivalent, certain steps need to be followed. We will outline these steps and illustrate the process with examples.

Steps to Identify Equivalence

Let us see the steps involves to check whether two automata are equivalent.

Step 1: Transition Analysis for State Pairs

For any pair of states q_i and q_j in the two automata, the transition for input $A \in \Sigma$ is defined by q_A q_B, where the transition from q_i on A goes to q_A, and the transition from q_j on A goes to q_B.

The two automata are not equivalent if for a pair q_A q_B, one is an intermediate state and the other is a final state.

This means that we need to —

- Select pairs of states, one from each automaton.
- Determine the transitions of these pairs on a given input A.
- Check whether the resulting states q_A, and q_B, are both final states or both intermediate states.

If q_A is an intermediate state (non-final) and q_B is a final state, then the two automata are not equivalent.

Step 2: Initial and Final State Consistency

In the next step, consider the initial state is a final state in one automaton, then in the second automaton, the initial state must also be a final state for them to be equivalent.

This means that, in the first automaton, if the initial and final states are the same, then in the second automaton, the initial and final states should also be the same.

Example to Illustrate Equivalence

Now let us see how we can apply these steps to check whether the following two automata are same or not. Consider these two automata below.

Machine A

Machine B
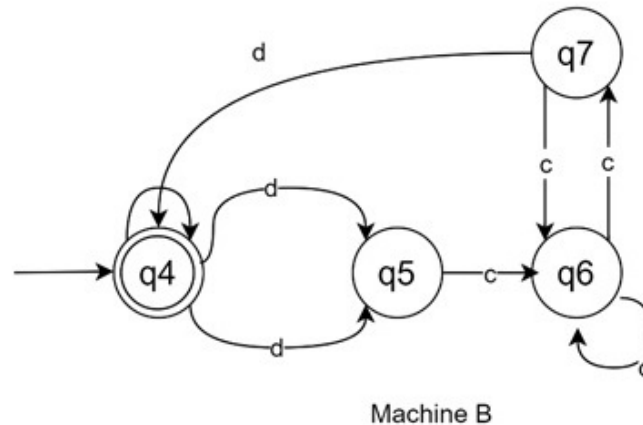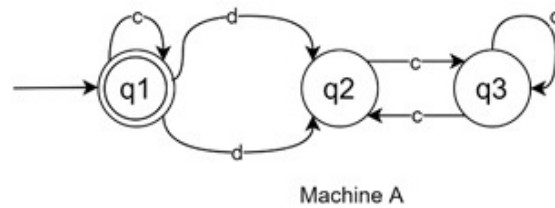
Check the **initial and final states**,

- **Automaton A** − Initial state q1, Final state q1.
- **Automaton B** − Initial state q4, Final state q4.

Since both automata have the same initial and final states, this condition is satisfied.

State Transition Pairs

Make pairs of states and check their transitions on inputs 'c' and 'd'.

Now let us see **pairwise**.

Pair q1 and q4

- For input **c**, q1 goes to q1 (final state)
- For input **c**, q4 goes to q4 (final state)
- For input **d**, q1 goes to q2 (intermediate state)
- For input **d**, q4 goes to q5 (intermediate state)

Since both transitions result in pairs of the same kind (either both final or both intermediate), they are equivalent so far.

Pair q2 and q5

- For input **c**, q2 goes to q3 (intermediate state)
- For input **c**, q5 goes to q7 (intermediate state)
- For input **d**, q2 goes to q1 (intermediate state)
- For input **d**, q5 goes to q6 (intermediate state)

Since both transitions result in pairs of the same kind (either both final or both intermediate), they are equivalent.

We will use the **analysis table** for a clear analysis of the above discussion.

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Analysis Table

We will make a table to compare the state pairs for inputs **c** and **d**.

State Pair	Input C	Input D
(q1, q4)	(q1, q4) [both are final]	(q2, q5) [both are intermediate]
(q2, q5)	(q3, q6) [both are intermediate]	(q1, q4) [both are final]
(q3, q6)	(q2, q7) [both are intermediate]	(q3, q6) [both are intermediate]
(q2, q7)	(q3, q6) [both are intermediate]	(q1, q4) [both are final]

If the candidates of the pair are of same type, we proceed with the table. If one is intermediate and the other is final, then we do not need to process further they will not be equivalent.

Conclusion

In this chapter, we explained the steps to check whether two finite automata are the same or equivalent or not. Initially, we checked through pairs and then using a table which will be easier to write. By following the given steps, it is easy to check the equivalence in less time.

The main criteria involve analyzing state transitions and ensuring consistency in initial and final states. In the provided examples, automata A and B were found equivalent due to the match of final and intermediate states. This procedure can be applied to any finite automata to determine their equivalence.