Jacobs University Bremen

Electrical Engineering 2

Lab Rotation 2

Spring 2020

Lab Experiment 3: The Wheatstone Bridge

Lab Report

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Place of execution: Research 1

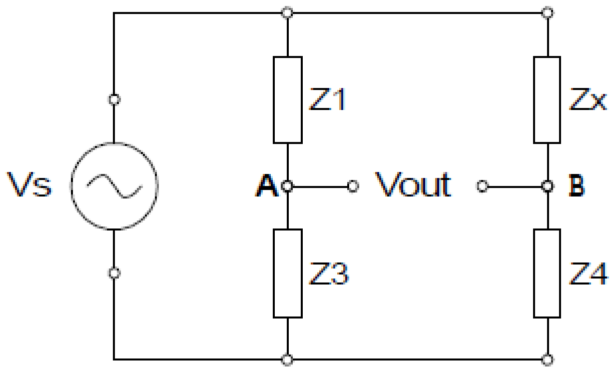
Introduction:

The purpose of this experiment was to demonstrate to students the general usage of the Wheatstone bridge to measure resistances or impedances. The experiment also helped to understand the difference between theory and applying the concepts in reality and understand how different errors in measurement may lead to a slightly different result.

Theory:

The Wheatstone Bridge is an electrical circuit that can be used to measure the resistance (or impedance) of an unknown component by balancing the resistance with the other side of the parallel branch. When the potential difference V_{out} is zero we find the resistance of the component Zx and Z_4 is equal to the resistance of Z_1 and Z_3.

The general circuit is as shown below:



From the circuit we can see that we have an equation that looks like:

$$\frac{z1}{z3} = \frac{zx}{z4} \leftrightarrow Z1 * Z4 = Zx * Z3$$

Another example of using the above circuit is when we have a component whose resistance changes with a varying property such as the temperature. In this case the change in V_{out} becomes a direct measurement for the varying quantity.

Considering cases where we have Impedances and phase shifts, it is necessary to balance both the amplitude and the phase shift as will be demonstrated in part 3 of the experiment below.

Execution:

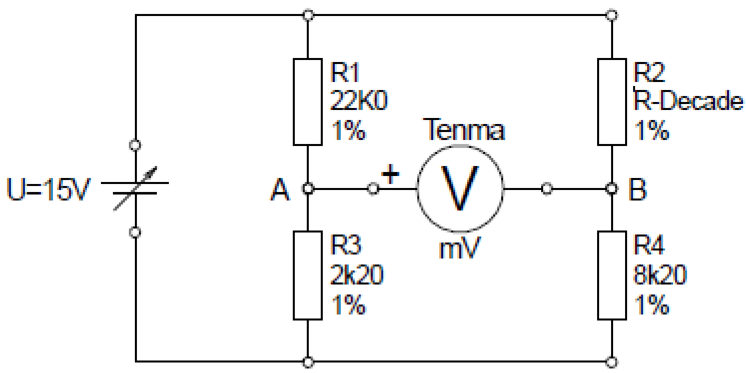
Part 1: Balanced DC Wheatstone bridge

Setup:

Tools Used:

- Power supply from the workbench
- Breadboard
- Resistor Decade
- Toolbox from the work bench
- TENMA multimeter

Using the tools provided the following circuit was setup exactly as shown:



The resistance of the R-Decade was changed until Uout was 0.8mV. The values that were asked to be recorded are in the table below:

Resistor	Resistance by the label (KΩ)	Resistance Elabo (Ω)	Range (KΩ)
R1	22	21996.2	200
R3	2.2	2194.31	20
R4	8.2	8190	20

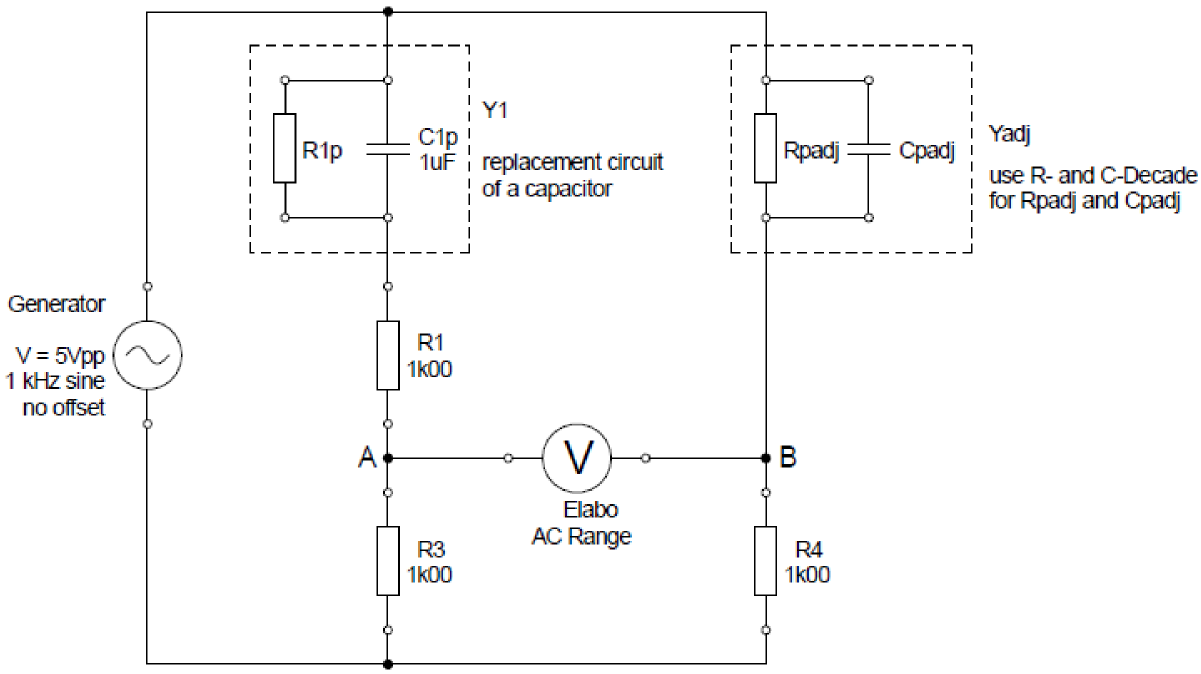
Part 3: Balanced AC Wheatstone bridge

Setup:

Tools Used:

- Power supply from the workbench
- Breadboard
- Resistor Decade
- Capacitor Decade
- Toolbox from the work bench
- TENMA multimeter
- Oscilloscope
- AC Signal Generator

Using the tools provided the following circuit was setup exactly as shown:



The Generator was set to 5V peak to peak with a frequency of 1KHz and no offset.

To continue the experiment we needed the exact values of C1p and R1p to calculate Rpadj and Cpadj. C1p and R1p were measured using the RLC meter and were obtained as follows:

R1p (KΩ)	C1p(μF)
1.0160	19.98

Since R3=R4, we have an equation that looks like:

$$Zadj = 1000 + \frac{1}{j * 2\pi * 1000 * 1.039 * 10E - 6} = 1000 - 158.5366502j$$

$$Yadj = \frac{1}{Zadj} = 9.754823539E - 4 + 1.546497047E - 4j$$

$$Rpadj = \frac{1}{Ry} = 1025.133869\Omega$$

$$Cpadj = \frac{Iy}{1000 * 2\pi} = 2.461326495E - 8 = 24.613nF$$

With the values for Rapdj and Cpadj calculated, the values were set so as to make Vab as close as possible to 0. In the end Vab was recorded as 4.97mV.

The values for Rpadj and Cpadj were recorded in the table below:

	Decade value	RLC Value
Rpadj(Ω)	1010	1009.9
Cpadj(nF)	20	20.172

Evaluation:

Part 1: Balanced DC Wheatstone bridge

Derive a formula for R_2. Use KVL.
Using KVL:

-V1+V2-Vab=0

-V3+Vab+V4=0

Since Vab=0, The equations become:

V2=V1 and V4=V3

Divide equation 1 by 2 and substitute values for V1, V2, V3 and V4 to get:

$$\frac{R1}{R3} = \frac{R2}{R4}$$

Solve for R2 to get:

$$R2 = \frac{R4*R1}{R3}$$

Calculate R_2 twice, first with the nominal resistor values and second with the measured values from R_1, R_3, and R_4.

Nominal Value:

$$R2 = \frac{22000 * 8200}{2200} = 82000\Omega$$

Measured Values:

$$R2 = \frac{21996.1 * 8190}{2194.31} = 82096\Omega$$

Calculate the maximum relative error of the found R_2 values for the following cases:
- value read from the resistor decade box
The relative error for the decade box is 1%(from error manual on the coarse website)
- directly measured with the Elabo multimeter
Error = $\frac{0.06}{1000} * 81.1k + \frac{0.01}{100} * 200000 = 68.66\Omega$
Relative error = $\frac{68.66}{82096} * 100 = 0.08363\%$
- calculated from the nominal resistor values
To get the following errors we need to solve the following equation:

$$\mathrm{dR2} = \frac{\delta R2}{\delta R1} dR1 + \frac{\delta R2}{\delta R3} dR3 + \frac{\delta R2}{\delta R4} dR4$$

After derivating:

$$\Delta R2 = \Delta R4 \left(\frac{R1}{R3} + \frac{R1*R4}{R3^2}\right) + \Delta R1 \frac{R4}{R3}$$

Rearrange the equation to get:

$$\frac{\Delta R2}{R2} = \frac{\Delta R4}{R4} + \frac{\Delta R3}{R3} + \frac{\Delta R1}{R3}$$

Given the tolerance of all the resistors is 1%, The nominal error is 1+1+1=3%
- calculated from the measured resistor values

$$\Delta R1 = \frac{0.06}{100} * 21996.1 + \frac{0.01}{100} * 200000 = 33.198\Omega$$

$$\Delta R3 = \frac{0.06}{100} * 2194.31 + \frac{0.01}{100} * 20000 = 3.32\Omega$$
$$\Delta R4 = \frac{0.06}{100} * 8190 + \frac{0.01}{100} * 20000 = 6.914\Omega$$
$$\frac{\Delta R2}{R2} * 100 = \left(\frac{6.914}{8190} + \frac{3.32}{2194.31} + \frac{33.198}{21996.1}\right) * 100 = 0.3866\%$$

Use the tolerances of the resistors (+-1%) and/or the tolerance of the multimeter (see Manual). For the last two cases use partial differentiation to derive a formula for the maximum relative error. Show the complete way how you got it!

Shown Above!!

Make a table and show all measured and calculated values for R_2 together with its errors. Which value is the best one? What are the error sources for the different methods?

The table is as Follows:

Method	Value of R2(Ω)	Error
Nominal	82000	3%
Measured	82138	1.75%
Elabo	82150	0.08%

As can be seen from the Table the Value we get from the Elabo Multimeter is the most accurate value since it has the lowest percentage error.

The different sources of errors may include:

- Human Error
- Not taking the resistance of the wires and components used into account.
- Instrument error
- Inconsistent values for the resistors used
- Resistor decade not accurate enough
- Error propagation

As you can see from the values our bridge has not the best accuracy in this 'contest'!! What should be improved to make it the best way to measure the resistance?

To improve the accuracy of the experiment, we could use a more accurate multimeter, use resistors with better tolerance or take the resistance of the wires into consideration and add them to our calculations.

Part 2: Un-Balanced DC Wheatstone bridge

Calculate R_{PT1000} for the two cases. Use the given component values and the measured voltages!

$$Rt = \frac{R4}{\frac{R3}{R3 + R1} + \frac{Uout}{Us}} - R4$$

When Us =1,

$$Rt = \frac{1000}{\frac{1000}{1000 + 1000} + \frac{0.02304}{1.0415}} - 1000 = 915.26\Omega$$

When Us=10

$$Rt = \frac{1000}{\frac{1000}{1000+1000} + \frac{0.2496}{10.088}} - 1000 = 905.697\Omega$$

Convert the values to temperature.

Given $Rt = Ro * (1 + \propto \Delta T)$

We re arrange the equation to get:

$$T = \frac{\frac{Rt}{Ro} - 1}{\propto}$$

For Us=1 we get:

$$T = \frac{\frac{Rt}{Ro} - 1}{\propto} = 22.599°\text{C}$$

For Us=10 we get:

$$T = \frac{\frac{Rt}{Ro} - 1}{\propto} = 24.844°\text{C}$$

What are the error sources for the calculated temperature value?

The errors maybe due to heating of the PTC due to the voltage or an instrument error for the Multimeter.

With higher supply voltage the sensor is heated because of power dissipation.
What is the additional temperature at the sensor for the used supply voltages?
The maximum self-heating coefficient in air for the sensor is E = 0.2C/mW.
If you look at the calculated temperatures from the question before, what is the conclusion?

The Power in the PTC is given by $P = R * I^2$

The current is given by $I = \frac{Us}{Rt+1000}$

For Us=1 we have:

$$I = \frac{Us}{1016+1000} = 4.9603 * 10^{-4}$$

$$P = 1016 * (4.9603 * 10^{-4})^2 = 2.4998 * 10^{-4}W$$

The temperature increase due to self-heating is

$$0.24998 * 0.2 = 0.049997°\text{C}$$

For Us=10 we have:

$$I = \frac{10.088}{1016+1000} = 5.00397 * 10^{-3}$$

$$P = 1016 * (5.00397 * 10^{-3})^2 = 0.025440W$$

The temperature increase due to self-heating is

$$25.4 * 0.2 = 5.08°\text{C}$$

By looking at the calculations it can be seen that the effect of self-heating is higher for increased values of the source voltage hence for increased voltage the value of the temperature is not accurate.

Until now we completely neglected the influence of the voltmeter which measures U_{OUT} (also for Part 1)!

a) What was the internal resistance of the voltmeter measuring U_{OUT} in the experiment?

The internal resistance is 1GΩ(as written in the manual.

b) What happens with U_{OUT} if you use a voltmeter with only 10k internal resistance? Derive a formula for $U_{OUT} = f(R_{multi})$. (R_{multi} is the internal resistance of the voltmeter). Use 1V for U_S and a R_{Pt} which corresponds to 20_C. What is the maximum relative error with the low input impedance voltmeter?

(Hint : use the Thevenin equivalent circuit!)

$$Uout = \frac{Rmulti}{Rmulti + Rth} * Uth$$

Where Uth $= \left(\frac{R3}{R3+R1} + \frac{R4}{R4+Rptc}\right) * Us$ and $Rth = \frac{R1*R3}{R1+R3} + \frac{R4*Rptc}{R4+Rptc}$

Rth=1016 Ω

Uth=0.01853635 V

For Rmulti=10000Ω, $Uout = \frac{10000}{10000+1016} * 0.01853635 = 0.016822879$

Error=$\frac{Uth-Uout}{Uth} * 100 = 9.24\%$

c) Was the accuracy good enough in our experiment? What is the conclusion for measuring the output voltage of a bridge?

The accuracy was not good enough for our experiment. The conclusion from the result is that the higher the voltmeter impedance the more accurate the result.

Part 3: Balanced AC Wheatstone bridge

Why do we need two components to balance the bridge? Explain!

When Measuring the potential difference across two points in ac we always look at both the amplitude and the phase difference between the two points. This is the same for an AC Wheatstone Bridge. Both the phase and the amplitude need to be taken into consideration when we have complex impedances as capacitors and inductors affect the phase by either increasing it or decreasing it. By not taking the phase into consideration, we will definitely be unable to balance the bridge.

If not already done show the complete calculations how you get the values for R_{padj} and C_{padj} in the preparation.

Shown In Execution!!

Calculate the complex conductance Y_1! Use the measured values from R_1, R_3, R_4, and Y_{adj} . Determine R_{1P} and C_{1P} .

$$\boldsymbol{Zadj = \frac{1}{\frac{1}{998.5} + j * 2\pi * 1000 * 20 * 10E-6} = 983.02325 - 123.34505j}$$

$$\boldsymbol{Yadj = \frac{1}{Zadj} = 1.0015E-3 + 1.2566E-4j}$$

R1=998.5Ω
R3=1003.6Ω
R4=1002.4Ω

For Z1, we have $Z1 = \frac{R3}{R4} * Zadj = 984.20 - 123.49271j$
Using Z1 in the equation $Y1 = \frac{1}{Z1 - R1}$, we get $Y1 = \frac{1}{984.20 - 123.49271j - 998.5} = 9.252669E - 4 + 7.9905E - 4j$
Using the imaginary part of Y1 we get $C1 = \frac{7.9905 * 10^{-4}}{2\pi * 1000} = 0.12717\mu F$
Using the real part of Y1 we get $\frac{1}{9.252669 * 10^{-4}} = 1080.7\Omega$

Compare the calculated R_{1P} and C_{1P} values with the measured ones from the RLC-Meter. Discuss the errors (no calculation)!

The values are presented in the table below:

Component	Measured	calculated
R1p(Ω)	35943	1080.7
C1p(μF)	1.0039	0.12717

There is a huge difference in the calculated and the measured values. The errors may have been due to instrument errors for example when measuring the resistance of the resistors in use or when using the RLC meter. Human errors may also have been a prime factor as the resistors R1, R2 and R3 may not have been placed in the right position

What is the influence of the voltmeter and the oscilloscope to the circuit?

The Oscilloscope and the voltmeter impacted the Impedance of the circuit though not greatly.

Conclusion:

After the experiment we realized the fundamental properties of a Wheatstone Bridge circuit and found how it can be used to measure the resistance of an unknown component or how the Un-balanced version can be used in circuits to build temperature dependent circuits e.t.c. We also saw how an AC version of the Wheatstone Bridge works and how to balance the AC version by considering both the phase and the magnitude of the impedance on both sides of the bridge. As always, the measured values were found to be different than the theoretical values due to a number of conditions such as the instrument, the environment or even human errors hence it never hurts to be mindful of the possible errors involved when performing the experiments and taking measurements.