



LAB REPORT 3

[Document subtitle]



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INTRODUCTION:

Objective

The objective of the following experiment is to analyze the Behavior of Filters over a range of frequencies. We apply a sinusoid signal and record the Phase change and Amplitude over Input and Output and use it to derive the Bode and Nyquist plot.

Theory

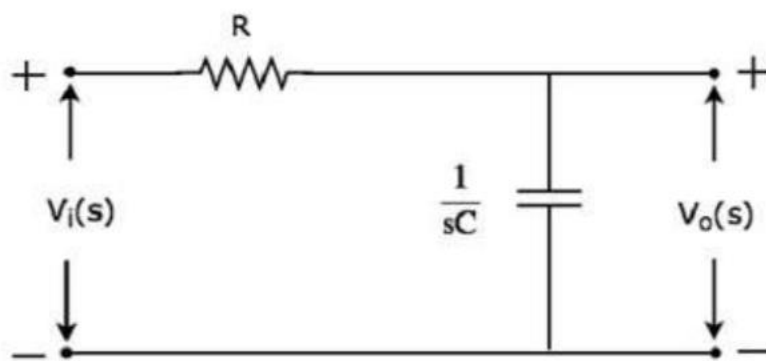
A filter is a network used to select a frequency or a range of frequencies of an input signal, while rejecting all other frequencies. There are several ways to construct these filters. One way is to use active components like transistors or operational amplifiers together with networks of resistors, capacitors, and inductors. Another way is to use digital signal processors together with analog to digital and digital to analog convertors. The simplest way is to use a passive network of resistors, capacitors, or inductors. There are four general types filters. High Pass, Low Pass, Band Pass, and Notch filters.

Filters are mainly classified into four types based on the band of frequencies that are allowing and / or the band of frequencies that are rejecting. Following are the types of filters.

Low Pass Filter

- High Pass Filter
- Band Pass Filter
- Band Stop Filter

Low Pass Filter Low pass filter as the name suggests, it allows (passes) only low frequency components. That means, it rejects (blocks) all other high frequency components. The s-domain circuit diagram (network) of Low Pass Filter is shown in the following figure.



It consists of two passive elements resistor and capacitor, which are connected in series. Input voltage is applied across this entire combination and the output is considered as the voltage across capacitor.

Here, $V_i(s)$ and $V_o(s)$ are the Laplace transforms of input voltage, $v_i(t)$ and output voltage, $v_o(t)$ respectively. The transfer function of the above network is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sCR}$$

$$\Rightarrow H(s) = \frac{1}{1 + sCR}$$

Substitute, $s=j\omega$ in the above equation.

$$H(j\omega) = \frac{1}{1+j\omega CR} \quad H(j\omega) = \frac{1}{1+j\omega CR}$$

Magnitude of transfer function is

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}} \quad |H(j\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

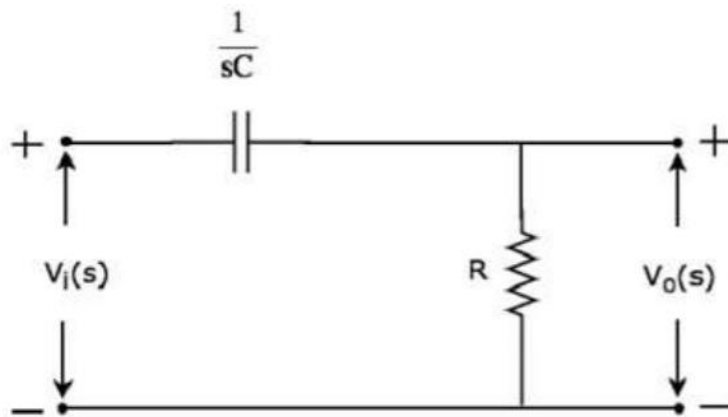
- At $\omega = 0$, the magnitude of transfer function is equal to 1.
- At $\omega = \frac{1}{CR}$, the magnitude of transfer function is equal to 0.707.
- At $\omega = \infty$, the magnitude of transfer function is equal to 0.

Therefore, the magnitude of transfer function of Low pass filter will vary from 1 to 0 as ω varies from 0 to ∞ .

High Pass Filter

High pass filter as the name suggests, it allows (passes) only high frequency components. That means, it rejects (blocks) all low frequency components.

The s-domain circuit diagram (network) of High pass filter is shown in the following figure.



It consists of two passive elements capacitor and resistor, which are connected in series. Input voltage is applied across this entire combination and the output is considered as the voltage across resistor.

Here, $V_i(s)$ and $V_o(s)$ are the Laplace transforms of input voltage, $v_i(t)$ and output voltage, $v_o(t)$ respectively.

The transfer function of the above network is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{sC}} \quad H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{sC}}$$

$$\Rightarrow H(s) = \frac{sCR}{1 + sCR} \Rightarrow H(s) = \frac{sCR}{1 + sCR}$$

Substitute, $s=j\omega$ in the above equation.

$$H(j\omega) = \frac{j\omega CR}{1 + j\omega CR} \quad H(j\omega) = \frac{j\omega CR}{1 + j\omega CR}$$

Magnitude of transfer function is

$$|H(j\omega)| = \frac{\omega CR}{\sqrt{1+(\omega CR)^2}} \quad |H(j\omega)| = \frac{\omega CR}{\sqrt{1+(\omega CR)^2}}$$

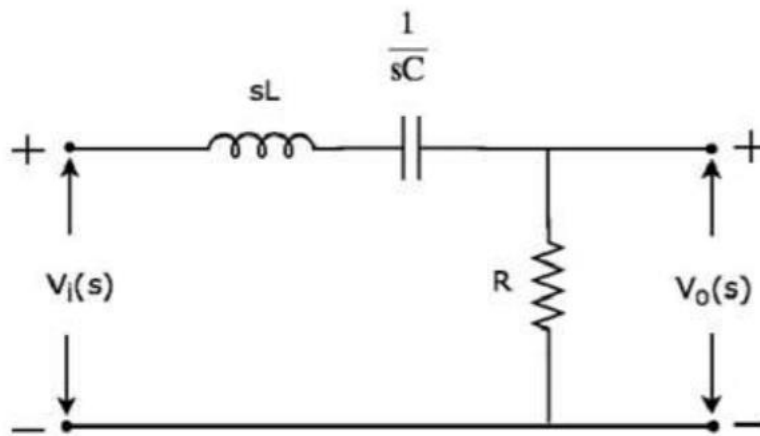
- At $\omega = 0$, the magnitude of transfer function is equal to 0.
- At $\omega = 1/CR$, the magnitude of transfer function is equal to 0.707.
- At $\omega = \infty$, the magnitude of transfer function is equal to 1.

Therefore, the magnitude of transfer function of High pass filter will vary from 0 to 1 as ω varies from 0 to ∞ .

Band Pass Filter

Band pass filter as the name suggests, it allows (passes) only one band of frequencies. In general, this frequency band lies in between low frequency range and high frequency range. That means, this filter rejects (blocks) both low and high frequency components.

The s-domain circuit diagram (network) of Band pass filter is shown in the following figure.



It consists of three passive elements inductor, capacitor and resistor, which are connected in series. Input voltage is applied across this entire combination and the output is considered as the voltage across resistor.

Here, $V_i(s)$ and $V_o(s)$ are the Laplace transforms of input voltage, $v_i(t)$ and output voltage, $v_o(t)$ respectively.

The transfer function of the above network is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + sL + \frac{1}{sC}} \Rightarrow H(s) = \frac{sCR}{s^2LC + sCR + 1}$$

Substitute $s = j\omega$ in the above equation.

$$H(j\omega) = \frac{j\omega CR}{1 - \omega^2 LC + j\omega CR}$$

Magnitude of transfer function is

$$|H(j\omega)| = \frac{\omega CR}{\sqrt{(1 - \omega^2 LC)^2 + (\omega CR)^2}}$$

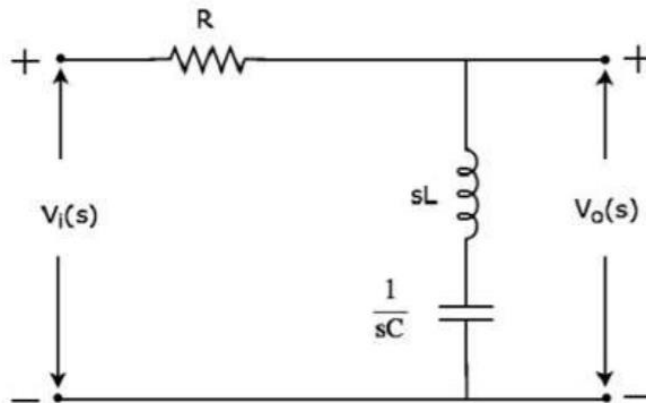
- At $\omega = 0$, the magnitude of transfer function is equal to 0.
- At $\omega = 1/LC$, the magnitude of transfer function is equal to 1.
- At $\omega = \infty$, the magnitude of transfer function is equal to 0.

Therefore, the magnitude of transfer function of Band pass filter will vary from 0 to 1 & 1 to 0 as ω varies from 0 to ∞ .

Band Stop Filter

Band stop filter as the name suggests, it rejects (blocks) only one band of frequencies. In general, this frequency band lies in between low frequency range and high frequency range. That means, this filter allows (passes) both low and high frequency components.

The s-domain (network) of circuit diagram and stop filter is shown in the following figure.



It consists of three passive elements resistor, inductor and capacitor, which are connected in series. Input voltage is applied across this entire combination and the output is considered as the voltage across the combination of inductor and capacitor.

Here, $V_i(s)$ and $V_o(s)$ are the Laplace transforms of input voltage, $v_i(t)$ and output voltage, $v_o(t)$ respectively.

The transfer function of the above network is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{sL}{sL + 1/sC} = \frac{s^2 LC}{s^2 LC + 1} \Rightarrow H(s) = \frac{s^2 LC}{s^2 LC + 1}$$

Substitute, $s = j\omega$ in the above equation.

$$H(j\omega) = \frac{1 - \omega^2 LC}{1 - \omega^2 LC + j\omega CR}$$

Magnitude of transfer function is

$$|H(j\omega)| = \frac{1 - \omega^2 LC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega CR)^2}}$$

- At $\omega = 0$, the magnitude of transfer function is equal to 1.
- At $\omega = 1/\sqrt{LC}$, the magnitude of transfer function is equal to 0.
- At $\omega = \infty$, the magnitude of transfer function is equal to 1.

Therefore, the magnitude of transfer function of Band stop filter will vary from 1 to 0 & 0 to 1 as ω varies from 0 to ∞ .

Cutoff frequency - The cutoff frequency or corner frequency is the frequency either above which or below which the power output of the filter is half the power of the passband, and since voltage is proportional to power P , V_{out} is $1/\sqrt{2}$ of the V_{out} in the passband. This happens to be close to -3 decibels, and the cutoff frequency is referred to as the -3dB point. **Center frequency** - A bandpass

circuit and a notch filter has two cutoff frequencies. Their geometric mean is the center frequency. The geometric mean of two numbers is: $f_{bw} = \sqrt{f_1 * f_2}$

Bandwidth - The bandwidth for a bandpass or notch filter is the difference between the upper and lower cutoff frequencies.

Time constant - In an RC circuit, the value of the time constant (in seconds) is equal to the product of the circuit resistance (in ohms) and the circuit capacitance (in farads), i.e. $\tau = RC$. It is the time required to charge the capacitor, through the resistor, to 63.2% (~ 63%) percent of full charge; or to discharge it to 36.8% (~ 37%) of its initial voltage. These values are derived from the mathematical constant e , specifically $1 - e^{-1}$ and e^{-1} respectively. e is the base of the natural logarithm.

Angular frequency ω - Is a scalar measure of rotation rate. One revolution is equal to $2\pi f = 2\pi T$.

The Bode plot describes the output response of a frequency-dependent system for a normalized input. It is often used in signal processing to show the transfer function of a system. It consists of two graphs. The magnitude and the phase plot. A Nyquist plot is a parametric plot of a frequency response. In Cartesian coordinates, the real part of the transfer function is plotted on the X axis. The imaginary part is plotted on the Y axis. The frequency is swept as a parameter, resulting in a plot per frequency.

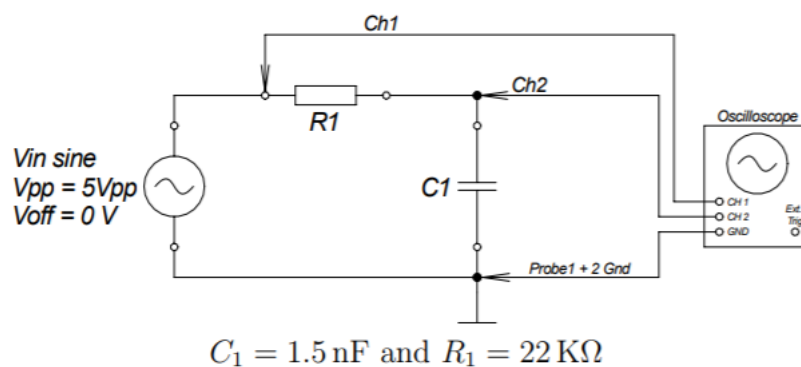
APPARATUS:

1. Oscilloscope
2. Multimeter
3. BNC cable
4. BNC-T-connector
5. BNC-Banana Connector
6. lab wires
7. Bread Board
8. Signal Generator
9. Capacitor ($1.5\text{nF} \times 1$)
10. Resistor ($1 \times 22\text{K}\Omega$)
11. Tenma Multi-meter
12. RLC meter

EXPERIMENT:

Part 1: Lo-Pass

PREPARATION & EXECUTION

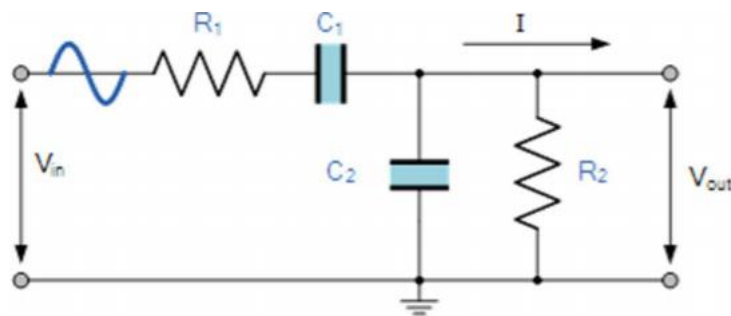


Connect the generator via the BNC-to-Kleps cable. Use Ch1 of the oscilloscope for the input signal and Ch2 for the output signal. Use voltage probes for both channels. Select an appropriate setting for the attenuation! Vary the frequency of the generator from 50 Hz to 100 kHz. Use 1, 2, 5 steps (e.g. 50 Hz, 100 Hz, 200 Hz, 500 Hz, 1 kHz...). Measure and record the 10 input and output amplitudes and the phase shift between the signals. Hint: Be careful!!! Phase shift may be positive or negative.

RC-Low Pass measured					
f/Hz	R		C		f-3db/[Hz]
	22000		1.5E-09		4822.9
	Uin/V	Uout/V	A /dB	phi/ms	phi/deg
50	10.14	9.93			-0.6
100	10.14	9.93			-1.2
200	10.14	9.92			-2.4
500	10.14	9.85			-5.8
1000	10.12	9.67			-11.7
2000	10.14	9.11			-22.6
5000	10.14	6.77			-45.5
10000	10.13	4.29			-61.6
20000	10.12	2.35			-73.4
50000	10.14	977E-3			-79.2
100000	10.12	495E-3			-81.4

Part 2: RC Band Pass

Preparation & Execution



The task is to determine the properties of a Band-Pass filter and the characteristics over a frequency range. The result should be displayed as Bode and Nyquist plot.

Use one 1.5nF, 8K2 and one 100nF, 10k0 RC combination to build a band pass. Determine from the cutoff frequencies which RC combination has to be used as high pass and which as low pass! Connect the signal generator via the BNC-toKleps cable to the input. Use the oscilloscope to measure input and output signal.

$f_{-3dB} = 1 / 2\pi RC$ Hi-pass Combination:

R1 and C1

$$f_{-3dB} = 1 / (2\pi \times 100 \times 10^{-9} \times 10 \times 10^3)$$

) = 159.155 Hz R1 = $10 \times 10^3 \Omega$, C1 = $100 \times 10^{-9} F$ Lo-pass Combination:

R2 and C2

$$f_{-3dB} = 1 / (2\pi \times 1.5 \times 10^{-9} \times 8.2 \times 10^3)$$

) = 12939.426 Hz R2 = $8.2 \times 10^3 \Omega$, C2 = $1.5 \times 10^{-9} F$ Use a Sine signal with 5Vpp amplitude, no offset.

First check if the circuit behaves

like you expect from the calculation of the cutoff frequencies! Then vary the

frequency of the generator from 50Hz to 100kHz. Use 1, 2, 5 steps (e.g. 50Hz, 100Hz, 200Hz, 500Hz, 1kHz...) Measure and record the input and output amplitude and the phase shift between the signals. The phase shift goes in the negative range as well.

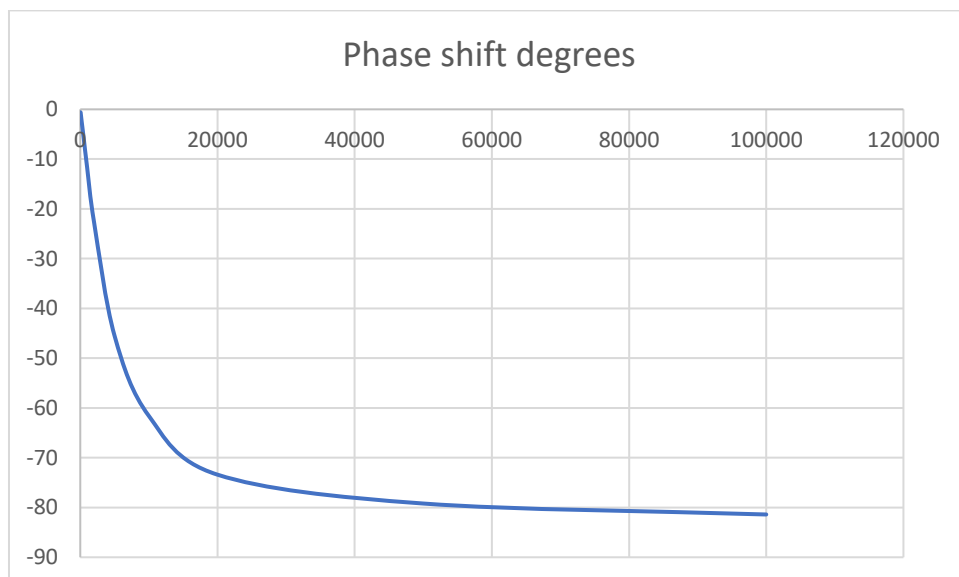
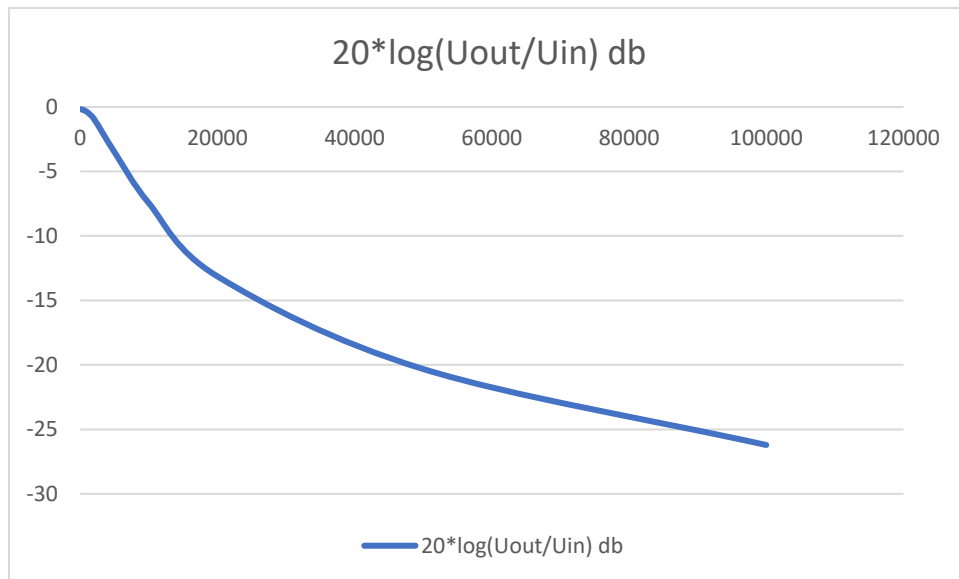
Band Pass Filter measured					
	$R_{hi} = 10000\Omega$	$C_{hi} = 100nF$	159.1549		
	$R_{lo} = 8200\Omega$	$C_{lo} = 1.5nF$	12939.43		
f/[Hz]	Uin/[V]	Uout/[V]	A /[dB]	phi/[ms]	phi/[deg]
10	10.100	597.2E-3			85.7
20	10.100	1.2E+0			82.1
50	10.100	2.9E+0			72.0
100	10.100	5.141			58.0
159	10.100	6.812			45.1
200	10.100	7.573			38.4
500	10.100	9.363			16.1
1000	10.100	9.712			5.3
1465	10.000	9.744			0.4
2000	10.000	9.716			-3.7
5000	10.100	9.242			-18.2
10000	10.100	7.948			-34.8
12939	10.000	7.184			-42.3
20000	9.990	5.625			-54.3
50000	10.100	2.617			-73.4
100000	10.100	1.351			-80.6

EVALUATION:

PART 1:

Draw the Bode magnitude and phase plot from the values you measured.

Frequency /hz	$20 \cdot \log(U_{out}/U_{in})$ db	Phase shift degrees
50	-0.182	-0.6
100	-0.182	-1.2
200	-0.191	-2.4
500	-0.252	-5.8
1000	-0.395	-11.7
2000	-0.930	-22.6
5000	-3.509	-45.5
10000	-7.463	-61.6
20000	-13.138	-73.4
50000	-20.323	-79.2
100000	-26.212	-81.4



- Draw the Bode magnitude and phase plot from the formulas given in the theory section together with the measured graphs. As variable use the same frequencies you set in the experiment. Compare!

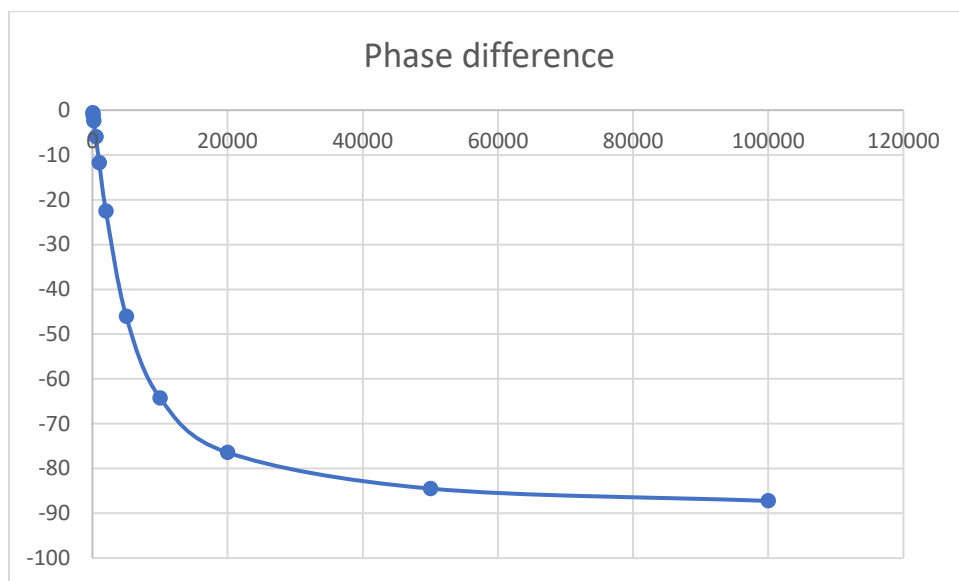
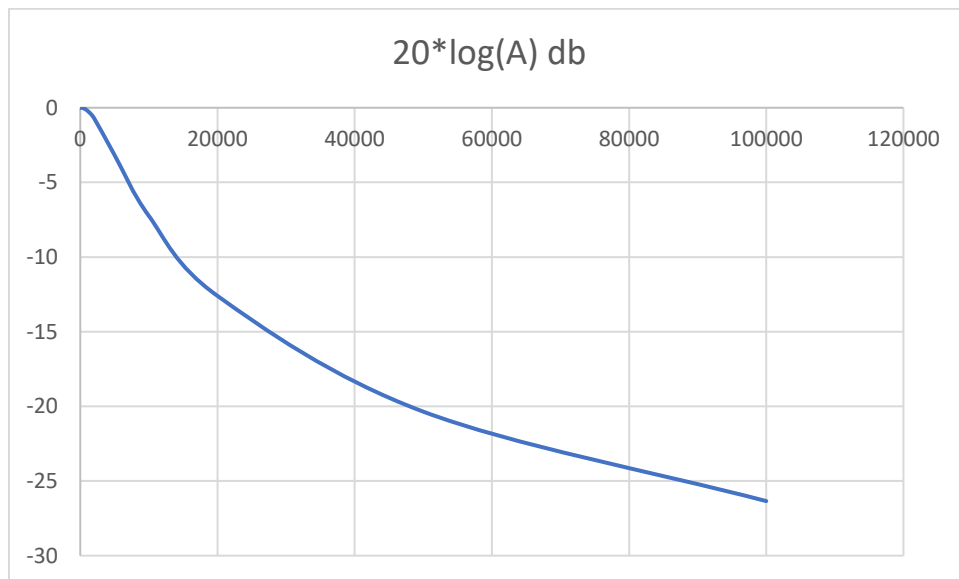
In theory, $\omega = 2\pi f$

$$|A| = 1/\sqrt{1 + (\omega * 22000 * 0.0000000015)^2}$$

$$\varphi = -\tan^{-1}((\omega * 22000 * 0.0000000015))$$

Frequency /hz	ω	A	20*log(A) db	Phase difference
50	314.16	0.9999463	-0.0005	-0.594
100	628.32	0.9997851	-0.002	-1.188
200	1256.63	0.9991413	-0.007	-2.375
500	3141.59	0.9946689	-0.046	-5.919
1000	6283.19	0.9791732	-0.183	-11.714
2000	12566.37	0.9237238	-0.689	-22.523
5000	31415.92	0.6942437	-3.169	-46.033
10000	62831.85	0.4344051	-7.242	-64.253

20000	125663.71	0.2344254	-12.5999	-76.442
50000	314159.27	0.0960119	-20.3534	-84.490
100000	628318.53	0.0481728	-26.3439	-87.2388



- Calculate the -3dB frequency from the components given. Read it from the diagram with the measured values and compare!

$$f_{-3dB} = 1/2\pi RC = 109 / (2\pi \times 22000 \times 1.5) = 4822.9 \text{ Hz}$$

