Lab Report

Natural Science Laboratory

Signal and System Lab

Experiment 2: RLC-Circuits - Transient Response

Fall Semester 2020

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Introduction:

The few main objectives of this experiment were:

• Study the transient response of the second-order systems, specifically RLC circuits.

• Learn about second-order system and its equation in general form.

• Learn the transient and steady-state response of second-order circuits with reference to Differential Equations.

• How to develop a second-order differential equation describing a series RLC circuit configuration.

• How the differential equation can be solved in order to have a complete solution consisting of the transient response and steady-state response. Second-order systems are very common in nature.

They are named second-order systems, as the highest power of derivative in the differential equation describing the system is two. In electrical engineering, circuits consisting of two energy storage elements, capacitors and inductors, for example RLC circuits, can be described as second-order electrical system. These circuits are frequently used to select or attenuate particular frequency ranges, as in tuning a radio or rejecting noise from the AC power lines. Circuits with two energy storage elements as the RLC circuits are described by a second-order ordinary D.E.

$$a_2 \frac{d^2y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = x(t)$$

where,

$y(t)$	is the response of the system to an applied input $x(t)$.
a_0, a_1 and a_2	are the system parameters.

But it is better to write above equation in the form of a linear constant coefficient nonhomogeneous differential equation. Therefore, the system parameters are:

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = K\omega_n^2 x(t)$$

$$\omega_n = \sqrt{\frac{a_0}{a_2}}$$

$$\zeta = \frac{a_1}{2\sqrt{a_0 a_2}}$$

$$K = \frac{1}{a_0}$$

where,

ω_n	is the natural frequency.
ζ	is the damping ratio.
K	is the gain of the system.

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$K = 1$$

Execution Transient response of RLC-Circuits

Instruments and tools used:

➢ Breadboard

➢ Function Generator

➢ R-Decade

➢ 10mH Inductor

➢ 6n8F Capacitor

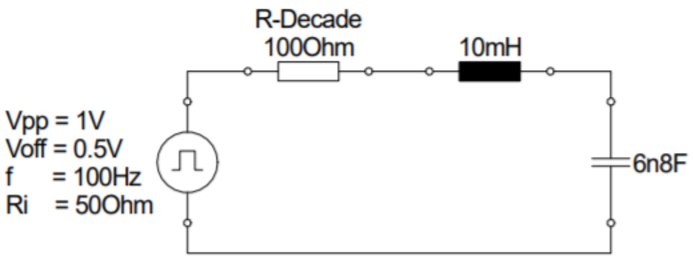
➢ Wires

➢ RLC Meter

➢ BNC cable

Problem: Design of an RLC circuit.

We setup the RLC circuit as shown below. The positive of signal generator was connected to decade, then to inductor, and then to capacitor and back to ground. All the components were connected in series.Similar to the diagram below:



1.

As we are using oscilloscope, first we calibrate oscilloscope to get a perfect square wave by using channel one. Then we connect oscilloscope and signal generator to check that the signal modulates between 0V and 1V.

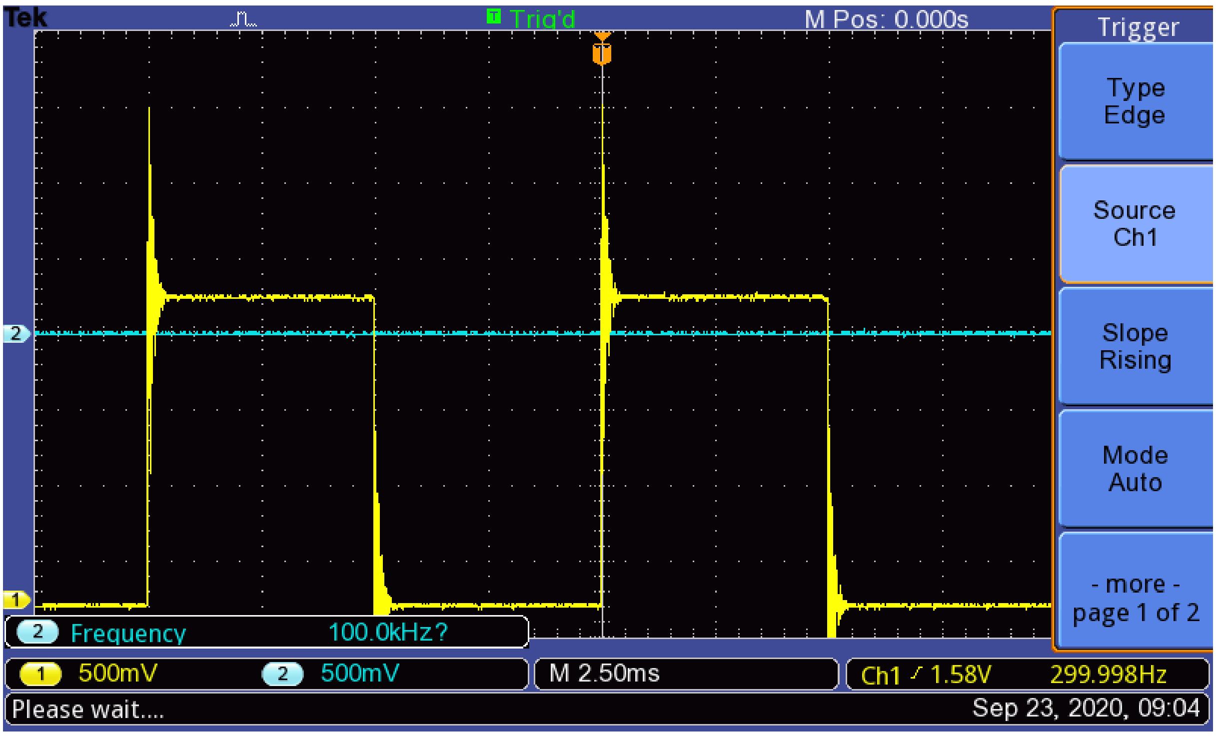
We use following settings for the generator:

Function = Square

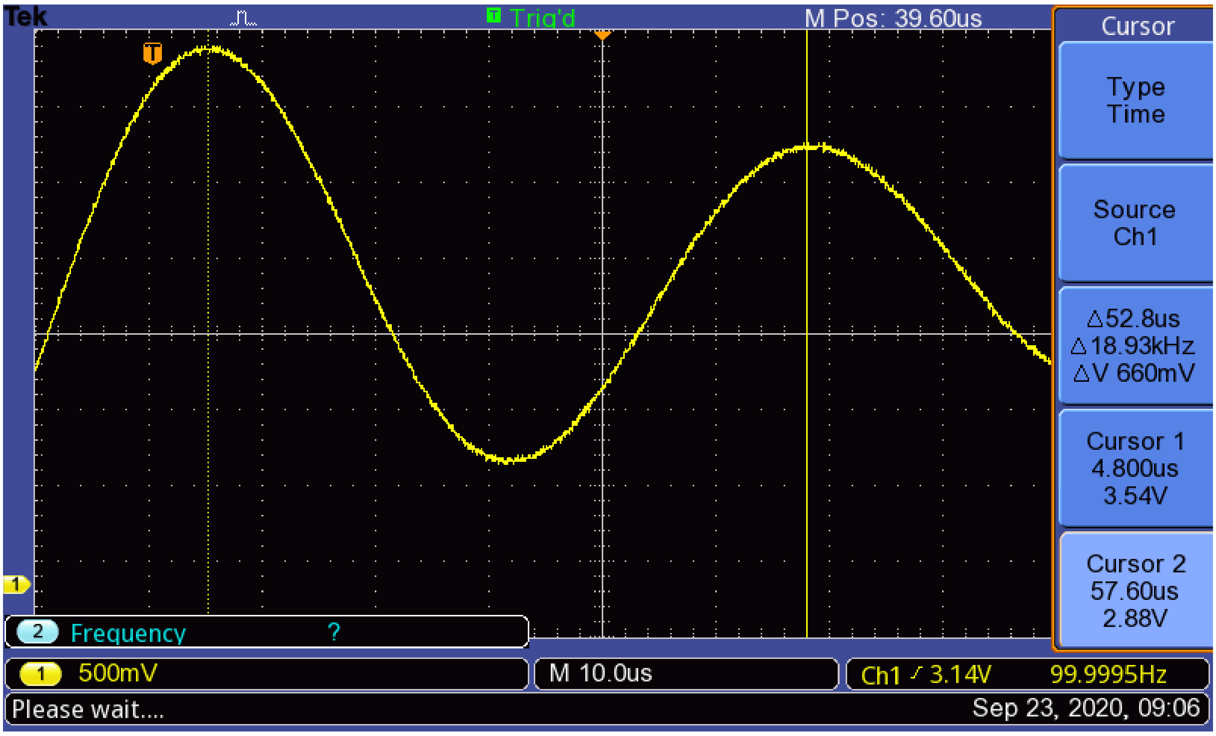
Frequency = 100 Hz

Amplitude = 0.5 V

Offset = 0.5 V



2. The damped frequency was measured using cursor i.e. fd = 18.76 kHz



3.

$$\omega_d = \omega_n\sqrt{1 - \xi^2}$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$R = 50\Omega$, $C = 6.8 \times 10^{-9}$, $L = 10 \times 10^{-3}$

Putting values gives us:

$$\omega_d = \frac{1}{\sqrt{LC}}\sqrt{1 - (\frac{R}{2}\sqrt{\frac{C}{L}})^2}$$

$$\omega_d = 121.256k$$

$$f_d = 19.3khz$$

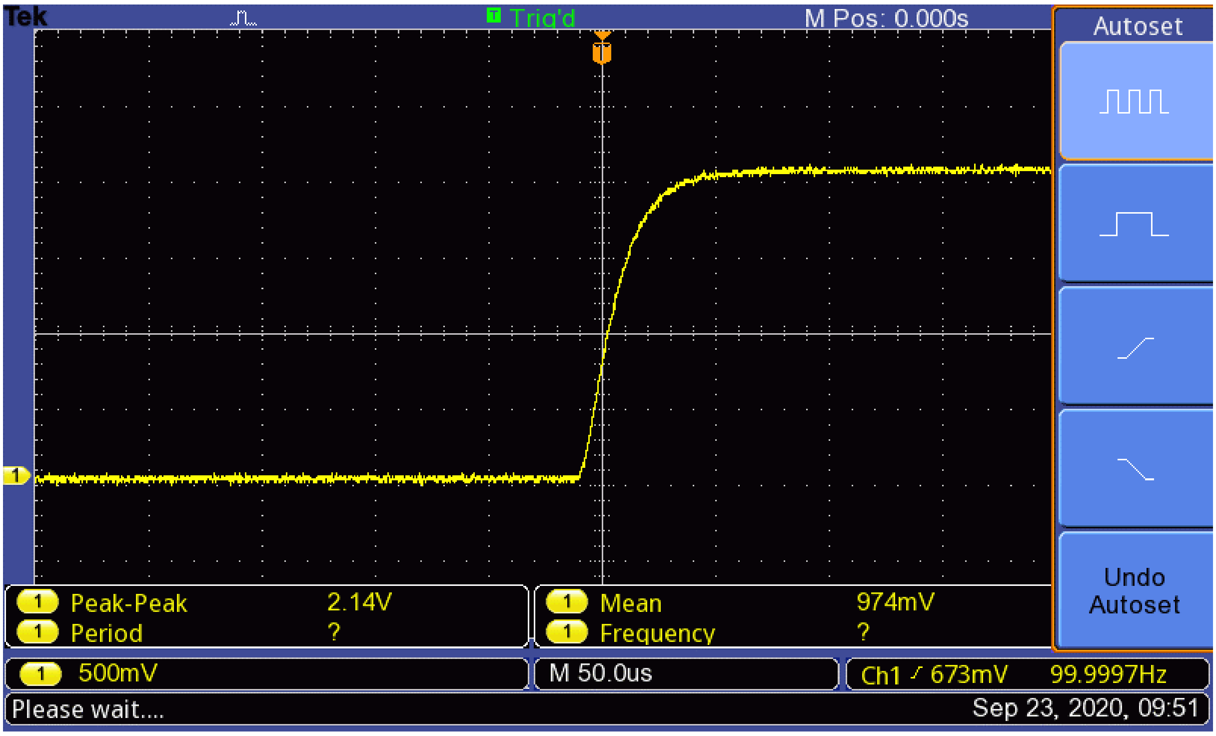
The frequency is very close to the obtained one, and the uncertainties and variation are due to wires and other components.

4. In order to have critically damped circuit, we put the damping ratio = 1. And then solve for R.

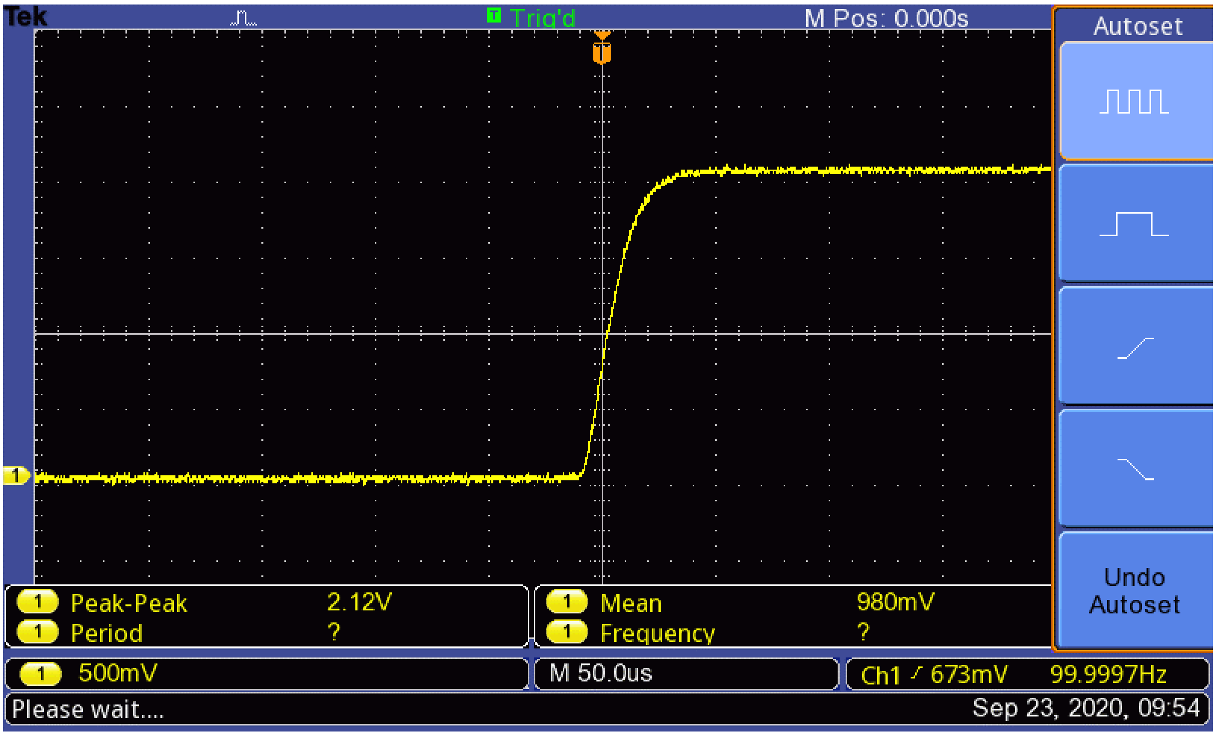
$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$R = 2 \times \sqrt{\frac{L}{C}}$$

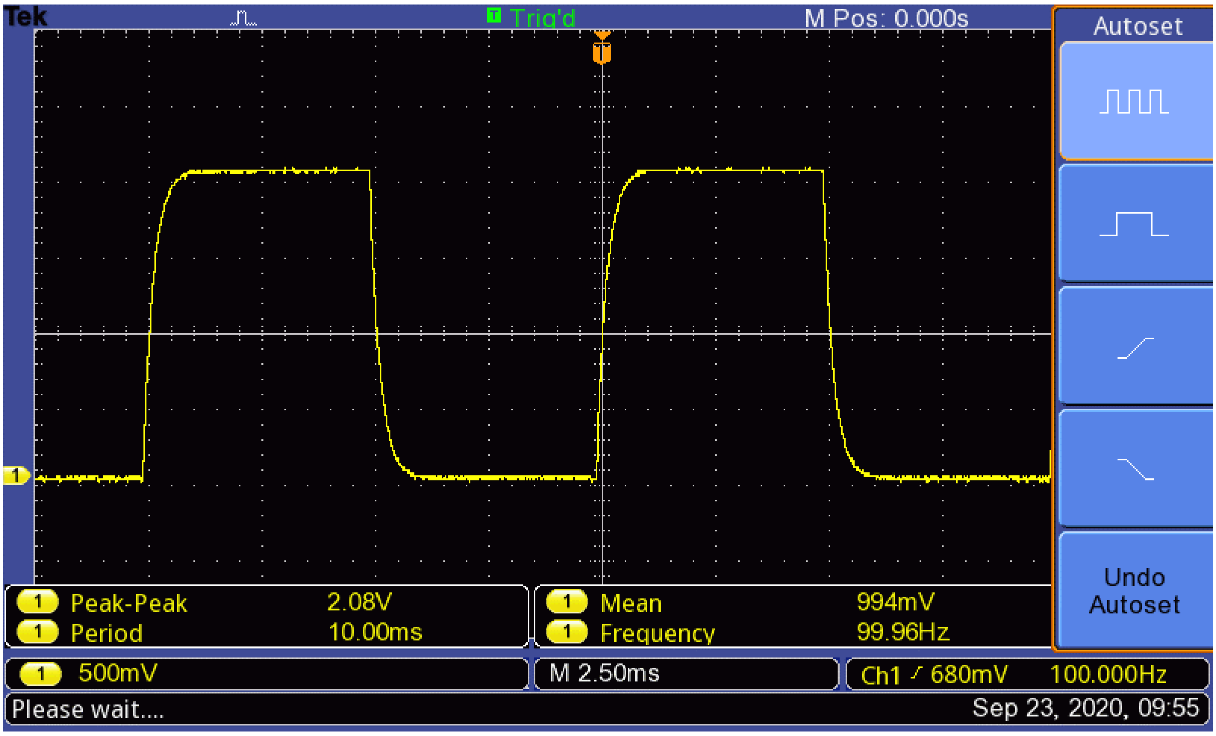
$$R = 2425.36\Omega$$



Rdecade= 2120 Ω



6. Setting the R-Decade to 30kOhm



Voltage across capacitor = 0.993V

Evaluation

1. The input voltage is the sum of the output voltage and voltage drops across the inductor and resistor

$$V_{in} = V_R + V_L + V_{out}$$

The current i is related to the current flowing through the capacitor

$$i = i_C = C\frac{dV_{out}}{dt}$$

The voltage drop across the resistor is given by

$$V_R = iR = RC\frac{dV_{out}}{dt}$$

The voltage drop across inductor is

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}(C\frac{dV_{out}}{dt}) = LC\frac{d^2V_{out}}{dt^2}$$

putting in first equation gives:

$$LC\frac{d^2V_{out}}{dt^2} + RC\frac{dV_{out}}{dt} + V_{out} = V_{in}$$

Next, we just plug in the values of R, C and L in the above equation;
R = 100 Ω,
L = 10mH,
C = 6.8 nF

The damping nature can be known after calculating the damping ratio using the following formula:

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\xi = \frac{100}{2} \times \sqrt{\frac{10 \times 10^{-3}}{6.8 \times 10^{-9}}} = 0.041$$

Since damping ratio is 0.041; which is 0 < ζ < 1, so we can say that the circuit has an under-damped nature.

For 0 < ζ < 1 the homogeneous solution of the second-order homogeneous differential equation exhibits a damped oscillatory behavior.

$$y(t) = \exp(-\zeta\omega_n t)(C_1 \cos(\omega_n \sqrt{(1-\zeta^2)}\, t) + C_2 \sin(\omega_n \sqrt{(1-\zeta^2)}\, t)) \quad +K$$

where,
C_1 and C_2 are unknown coefficients derived from the initial conditions.
On defining ω_d, the damped natural frequency is given as

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

Eq. (3.14) can be written as

$$y(t) = \exp(-\zeta\omega_n t)(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t))$$

Which can be rewritten as:

$$y(t) = \exp(-\zeta\omega_n t)(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t)) \; +K$$

For C1,t=0 because capacitor should be discharged:

$$Vc = 0V \rightarrow 0 = C1 + 0 + K \rightarrow C1 = -K = -1$$

The derivative of the equation above is put equal to zero to get C2:

$y'(t) = (-\zeta w_n)e^{(-\zeta w_n t)}\ (C_1 \cos(w_d t) + C_2 \sin(w_d t)) + e^{(-\zeta wnt)}\ (-w_d C_1 \sin(w_d t) + w_d C2 \cos(w_d t))$

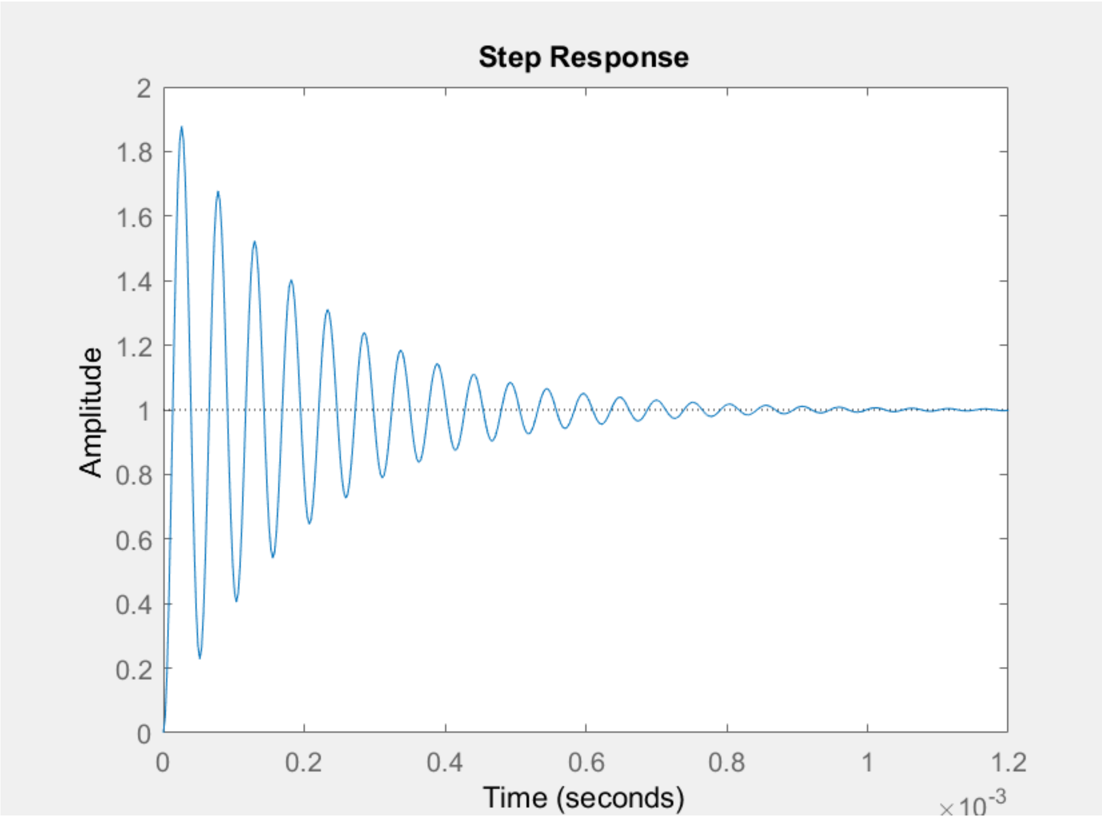
initial conditions: $Vc\ (0) = 0$, $i(0) = 0$, $\frac{\mathrm{d}V_c(0)}{\mathrm{d}(0)} = 0$, therefore all cases are at $t = 0$, and $w_d = w_n\sqrt{1 - \zeta^2}$.

$$y'(t=0) = (-\zeta w_n)(C_1) - w_d C_2 = (-\zeta w_n)(C_1) + (w_n\sqrt{1-\zeta^2})(C_2) = 0 \rightarrow$$
$$(-\zeta w_n)(C_1) = (w_n\sqrt{1-\zeta^2})(C_2) \rightarrow C_2 = \frac{(-\zeta)(C_1)}{\sqrt{1-\zeta^2}} = \frac{(-0.041)(-1)}{\sqrt{1-0.041^2}} = 0.04103$$

2. The Matlab script is as follows:

```
L=10e-3;
C=6.8e-9;
R=100;
Num=[1];
Den=[L*C R*C 1];
G=tf(Num,Den);
step(G);
```

The graph is:



3.

When ζ=1 R=2425.36Ω

$$y(t) = C_1 \exp(-\zeta\omega_n t) + C_2 t \exp(-\zeta\omega_n t) + K$$

When we input the first initial condition for t = 0 at which point the capacitor is discharged

$y(t) = K + (C_1 + C_2 t)e^{-\omega_n t} \Rightarrow y(t=0) = C_1(1) + 0 + 1 \Rightarrow C_1 = -1\,V$

Now the second initial conditions are when $i_c = C\frac{dv(0)}{d(0)} = 0$ hence t = 0

$$y'(t) = C[(-\omega_n)(C_1 - \omega_n C_2 t + C_2)e^{-\omega_n t}]$$, where C is capacitance

$$y'(t=0) = i_c = C\frac{dv(0)}{d(0)} = 0 = C[(-\omega_n)(C_1 - \omega_n C_2(0) + C_2)e^{-\omega_n(0)}] \Rightarrow$$

$$y'(t=0) = [(-\omega_n)(-1 - 0t + C_2)(1)] = 0 = \omega_n + C_2 \Rightarrow C_2 = -\omega_n$$

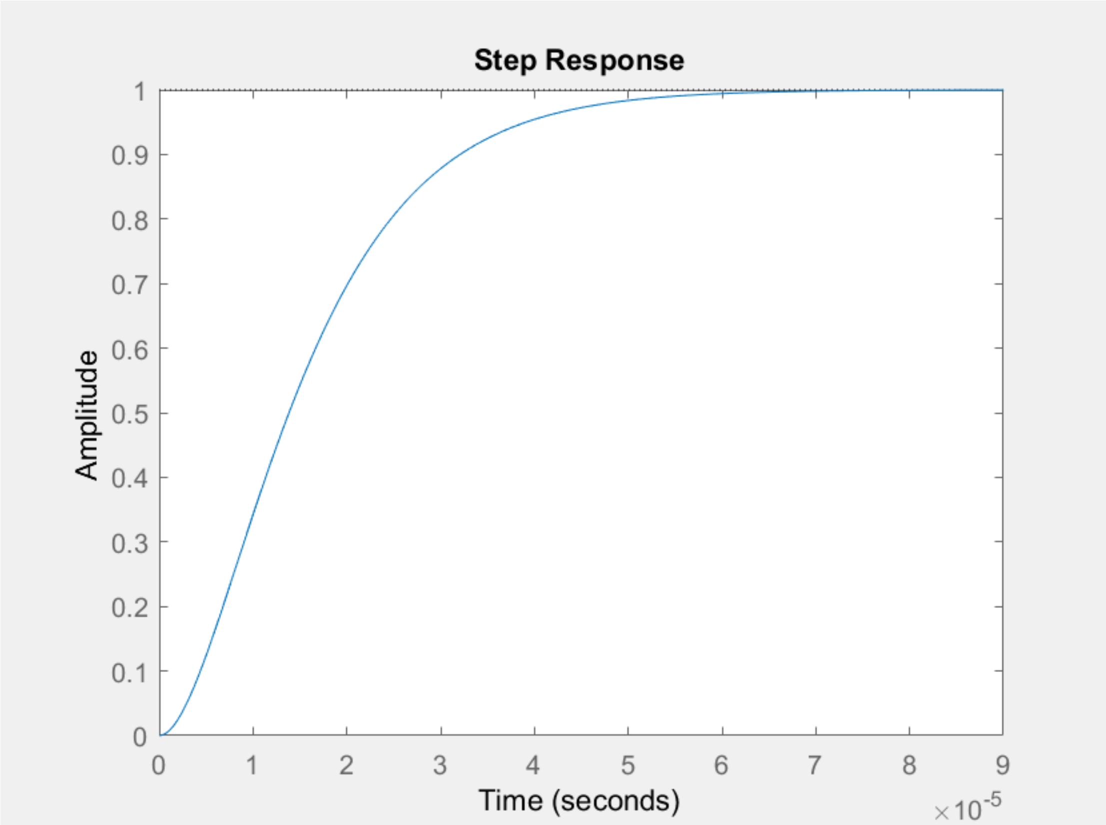
=-121267.8

MATLAB script

```
L=10e-3;
C=6.8e-9;
R=2425;
Num=[1];
Den=[L*C R*C 1];
G=tf(Num,Den);
```

step(G);

Graph



4.

The experimental result obtained in the lab for the resistance was 2120 Ω. While, from the calculations the value of resistance was calculated out to be: 2425 Ω. These two values deviate from one another. The possible origin for the deviation are:

i. Coils of Inductor:

→ We expect that inductor has some impedance. But also it is realized that the inductor in made by wrapping up with coils. The coils used in making of inductor also possess some resistance, it is not taken into consideration. The difference in the experimental and theoretical value of resistance might be due to the contribution by the resistance of the coil in the inductor.

ii. Error in values of Capacitance and Inductance:

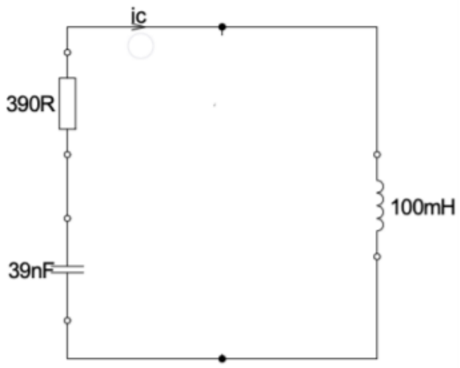
→ The value of capacitance and inductance used in the experiment is with 10% of its specified value. So, in the experiment the value for the respective parameters are not accurate enough. Hence, it contributes for the deviation between the measured and calculated values.

iii. Difference R-decade value :

→ The value of R-decade which we set manually is within certain percentage range of accuracy. So it is not accurate to estimate the exact value of resistance. Hence, it creates difference in the value between experimental and manipulated value of resistance.

5.

At time t> 0,
The equivalent circuit looks like:



Then,

$$v_s = v_c + v_l + v_r$$
$$0 = v_c + v_l + 390$$
$$0 = \frac{i_c}{C} + 390\frac{di_c}{dt} + L\frac{d^2 i_c}{dt^2}$$

$$\frac{d^2 i_c}{dt^2} + \frac{390}{L}\frac{di_c}{dt} + \frac{i_c}{LC} = 0$$

We check damping using ζ

$$\zeta = \frac{390}{2}\sqrt{\frac{39 * 10^{-9}}{100 * 10^{-3}}} \Rightarrow 0.12178$$

Since $\zeta = 0.12178 < 1$ hence it is underdamped

$$\omega_n = \frac{1}{\sqrt{100 * 10^{-3} * 39 * 10^{-9}}} \Rightarrow 16012.8$$

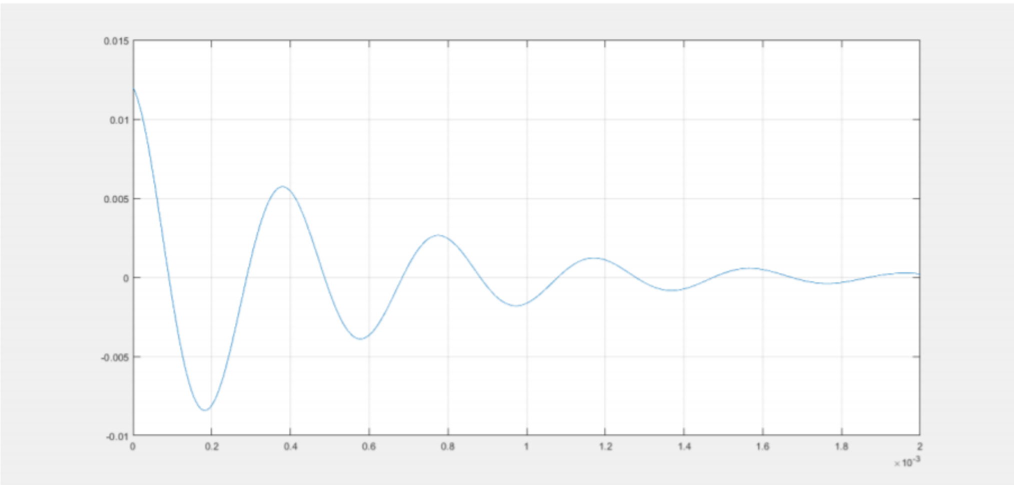
$$\omega_d = 16012.8\sqrt{1-(0.12178)^2} \Rightarrow 15893.64$$

$$f_d = \frac{15893.64}{2\pi} \Rightarrow 2.529\ kHz$$

$$f_d = 2.529\ kHz$$

$$i_c = exp(-\zeta\omega_n t)\,(C_1 cos\,(\omega_d t) + C_2 sin\,(\omega_d t))$$

$$i_l(0) = \frac{V_{in}}{R} = \frac{12}{1K} \Rightarrow 12mA$$
$$C_1 = 12mA$$
$$i'_c(t) = (-\zeta\omega_n)exp(-\zeta\omega_n t)(C_1 cos(\omega_d t) + C_2 sin(\omega_d t))$$
$$+ exp(-\zeta\omega_n t)(C_1(-\omega_d)sin(\omega_d t) + (\omega_d)C_2 cos(\omega_d t))$$
$$i'_c(t=0) = (-\zeta\omega_n)exp(-\zeta\omega_n(0))(C_1 cos(\omega_d(0)) + C_2 sin(\omega_d(0)))$$
$$+ exp(-\zeta\omega_n(0))(C_1(-\omega_d)sin(\omega_d(0)) + (\omega_d)C_2 cos(\omega_d(0)))$$
$$i'_c(t=0) = (-\zeta\omega_n)(1)(C_1(1) + C_2(0)) + (1)(C_1(-\omega_d)(0) + (\omega_d)C_2(1))$$
$$0 = (-\zeta\omega_n)(C_1) + ((\omega_d)(C_2)) \Rightarrow C_2 = \frac{(\zeta\omega_n)(C_1)}{(\omega_d)}$$
$$C_2 = \frac{(\zeta\omega_n)(C_1)}{\omega_n\sqrt{1-\zeta^2}} \Rightarrow C_2 = C_1\frac{\zeta\omega_n}{\omega_n\sqrt{1-\zeta^2}} \text{, now } C_1 = 12mA$$
$$C_2 = 12 * \frac{0.12178}{\sqrt{1-(0.12178)^2}}$$
$$C_2 = 1.472\ mA$$



Conclusion:

Throughout this experiment, we observed the transient response of a series RLC circuit. We studied all the cases for circuits i.e underdamped, critically damped and over damped states. The values from the oscilloscope were compared with calculated values which lead to realization of errors in our experiments and the cause of them. They were varied because of impedance, capacitive reactance, inductive reactance and the phase shift angle between voltage and current. We found the unknown resistance of 305ohm at resonant frequencies given of capacitance and inductance using the oscilloscope. Due to loose connections, methodical errors were occurred. We also learnt how to plot in MATLAB and work with transfer functions.

References

- Signals and Systems Lab Manual
- Gen ECE and Signals and Systems Notes
- Differentiation Calculator

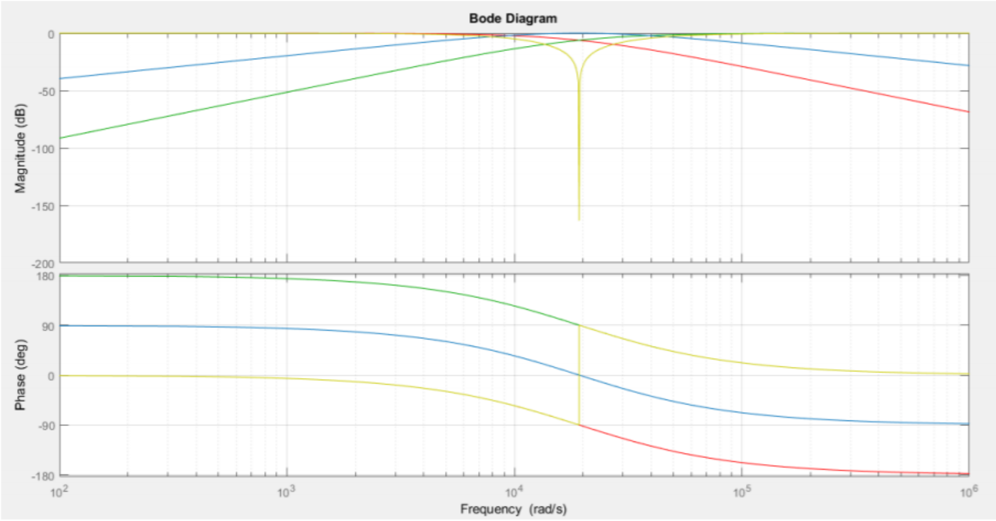
Prelab

4.4 Prelab RLC Circuits - Frequency response

4.4.1 Problem: RLC resonator 1 and 2)

```
L=10*10^-3;
V=5;
R=390;
C=270*10^-9;

s=tf('s');
```

```
%for the capacitor
T1=(1/(s*C))/(R+(s*L)+(1/(C*s)));
bode(T1, 'r')
grid on
hold on

% for the inductor
T2=(s*L)/(R+(s*L)+(1/(C*s)));
bode(T2, 'g')
grid on
hold on

%for the resistor:
T3=R/(R+(s*L)+(1/(s*C)));
bode(T3)
grid on
hold on

%both capacitor and inductor
T4=((1/(s*C))+(s*L))/(R+(s*L)+(1/(C*s)));
bode(T4, 'y')
grid on
hold on
```




```
%From the phase plot, the bandwidth can be extracted by taking the difference of the phase between +45 at pi/4 and -45 at -pi/4)
 fhigh=4.69e+04;
 flow=7.89e+03;
 Bandwidth = fhigh - flow %% approx frequency at 0 degrees
 fc= 1.93e+04;
 Q= fc/Bandwidth
```

```
Bandwidth =

       39010


Q =

    0.4947
```

3)
Resistor: (blue) band-pass filter.

Capacitor: (red) Low-pass filter.
Inductor: (green) High-pass filter.
Capacitor/Inductor: (yellow)
The frequencies mostly all pass through without being altered or changed, however the some are attenuated at a specific range such that it is a very low level. It will be the opposite of a band-pass filter which in the first part was our initial case. Finally, we will come up with a conclusion that the filter is band-pass filter.

For other Electrical component combinations, its behavior in terms of filter properties are shown in the table below:

Components(Output Transfer Function at:)	Filter Type
Resistor	Band Pass Filter
Capacitor	Low Pass Filter
Inductor	High Pass Filter
Inductor and Capacitor	Band Stop / Notch Filter