```
- Module Transaction -
INSTANCE Naturals
INSTANCE FiniteSets
Instance Sequences
INSTANCE TLC
 An empty constant
Constant Nil
 Transaction phase constants
CONSTANTS
   Change,
   Rollback
 Transaction phase constants
CONSTANTS
   Commit,
   Apply
 Status constants
CONSTANTS
   Pending,
   InProgress,
   Complete,
   Aborted,
   Canceled,
   Failed
Status \triangleq \{Pending, InProgress, Complete, Aborted, Canceled, Failed\}
Done \triangleq \{Complete, Aborted, Canceled, Failed\}
 The set of all nodes
CONSTANT Node
 The set of possible paths and values
CONSTANT Path, Value
Empty \stackrel{\triangle}{=} [p \in \{\} \mapsto Nil]
```

VARIABLES

```
configuration,
   mastership,
   conn.
   target
 A transaction log. Transactions may either request a set
 of changes to a set of targets or rollback a prior change.
Variable transactions
 A history of transaction change/rollback commit/apply events used for model checking.
Variable history
TypeOK \triangleq
   \forall i \in \text{DOMAIN } transactions:
      \land transactions[i].index \in Nat
      \land transactions[i].phase \in \{Change, Rollback\}
      \land transactions[i].change.commit \in Status
      \land transactions[i].change.apply \in Status
      \land \forall p \in DOMAIN \ transactions[i].change.values :
          transactions[i].change.values[p] \neq Nil \Rightarrow
              transactions[i].change.values[p] \in STRING
      \land transactions[i].rollback.commit \neq Nil \Rightarrow
            transactions[i].rollback.commit \in Status
      \land transactions[i].rollback.apply \neq Nil \Rightarrow
            transactions[i].rollback.apply \in Status
      \land \forall p \in DOMAIN \ transactions[i].rollback.values :
          transactions[i].rollback.values[p] \neq Nil \Rightarrow
              transactions[i].rollback.values[p] \in STRING
LOCAL State \triangleq [
   transactions \mapsto transactions,
   configuration \mapsto configuration,
   mastership
                   \mapsto mastership,
   conn
                    \mapsto conn,
   target
                    \mapsto target
LOCAL Transitions \triangleq
   LET
         indexes \stackrel{\triangle}{=} \{i \in \text{DOMAIN } transactions' : \}
                                 i \in \text{DOMAIN } transactions \Rightarrow transactions'[i] \neq transactions[i]
         [transactions \mapsto [i \in indexes \mapsto transactions'[i]]] @@
            (IF configuration' \neq configuration THEN [configuration \mapsto configuration'] ELSE Empty) @@
            (IF target' \neq target \text{ THEN } [target \mapsto target'] \text{ ELSE } Empty)
Test \stackrel{\triangle}{=} INSTANCE \ Test \ WITH
   File \leftarrow "Transaction.log"
```

```
Add a change for revision 'i' to the transaction log
AppendChange(i) \triangleq
   \wedge Len(transactions) = i - 1
   \land \exists p \in Path, v \in Value:
        LET transaction \triangleq [
                index
                           \mapsto Len(transactions) + 1,
                           \mapsto \mathit{Change},
                phase
                change \mapsto [
                    index \mapsto Len(transactions) + 1,
                    seqnum \mapsto 0,
                    values \mapsto (p :> v),
                    commit \mapsto Pending,
                    apply \mapsto Pending,
                rollback \mapsto [
                    index \mapsto 0,
                    segnum \mapsto 0,
                    values \mapsto Empty,
                    commit \mapsto Nil,
                    apply \mapsto Nil
              \land transactions' = Append(transactions, transaction)
   \land UNCHANGED \langle configuration, mastership, conn, target, history <math>\rangle
 Add a rollback of revision 'i' to the transaction log
RollbackChange(i) \triangleq
   \land \ i \in \texttt{DOMAIN} \ \textit{transactions}
   \land transactions[i].phase = Change
   \land transactions[i].change.commit = Complete
   \land transactions' = [transactions \ EXCEPT \ ![i].phase]
                                                                              = Rollback,
                                                     ![i].rollback.commit = Pending,
                                                     ![i].rollback.apply = Pending]
   \land UNCHANGED \langle configuration, mastership, conn, target, history <math>\rangle

    configuration

  - committed
    - transaction: the index of the last committed transaction
    - segnum: the sequence number for the last successfully completed transaction
    - target: the current target transaction
```

- -index: the highest committed index
- revision: the current committed index

^{*} If target = transaction then the current transaction is being changed

^{*} If target < transaction then the current transaction is being rolled back

```
CommitChange(n, i) \stackrel{\Delta}{=}
   \land \lor \land transactions[i].change.commit = Pending
         \land configuration.committed.maxIndex = i - 1
         \land \lor \land configuration.committed.target \neq i
              \land configuration.committed.index = configuration.committed.target
              \land configuration.committed.transaction \in Domain transactions \Rightarrow
                    \lor \land configuration.committed.target = configuration.committed.transaction
                       \land transactions[configuration.committed.transaction].change.commit \in Done
                    \lor \land configuration.committed.target < configuration.committed.transaction
                       \land transactions[configuration.committed.transaction].rollback.commit \in Done
              \land configuration' = [configuration \ EXCEPT \ !.committed.target = i]
              \wedge history' = Append(history, [
                                 type \mapsto Change,
                                 phase \mapsto Commit,
                                 index \mapsto i,
                                 status \mapsto InProgress)
              \land UNCHANGED \langle transactions \rangle
            \lor \land configuration.committed.target = i
              \land transactions' = [transactions \ EXCEPT \ ![i].change.commit = InProgress,
                                                            ![i].rollback.index
                                                                                    = configuration.committed.revision
                                                            ![i].rollback.values
                                                                                   =
                                                               p \in \text{DOMAIN } transactions[i].change.values \mapsto
                                                                 If p \in \text{DOMAIN} configuration.committed.values TH
                                                                    configuration.committed.values[p]
                                                                  ELSE Nil]]
              \land UNCHANGED \langle configuration, history \rangle
      \lor \land transactions[i].change.commit = InProgress
         \land \lor \land configuration.committed.index \neq i
              \land \lor \land configuration' = [configuration \ EXCEPT \ !.committed.transaction = i,
                                                                   !.committed.index
                                                                                                =i,
                                                                   !.committed.maxIndex
                                                                                                =i,
                                                                   !.committed.revision
                                                                   !.committed.segnum
                                                                                               = configuration.commit
                                                                   !.committed.values
                                                                                               = transactions[i].change
                                                                                                     configuration.comm
                    \land history' = Append(history, [
                                       type \mapsto Change,
                                      phase \mapsto Commit,
                                      index \mapsto i,
                                      status \mapsto Complete)
                 \vee \wedge configuration' = [configuration \ EXCEPT \ !.committed.transaction = i,
                                                                   !.committed.index
                                                                   !.committed.maxIndex = i
                    \land history' = Append(history, [
                                      type \mapsto Change,
```

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phase \mapsto Commit,
                                       index \mapsto i,
                                       status \mapsto Failed
               \land UNCHANGED \langle transactions \rangle
            \lor \land configuration.committed.index = i
              \land \ \lor \ \land \ configuration.committed.revision = i
                    \land transactions' = [transactions \ EXCEPT \ ![i].change.commit = Complete,
                                                                   ![i].change.seqnum = configuration.committed.seqr
                  \lor \land configuration.committed.revision \neq i
                     \land transactions' = [transactions \ EXCEPT \ ![i].change.commit = Failed,
                                                                   ![i].change.apply = Canceled]
               \land UNCHANGED \langle configuration, history \rangle
   \land UNCHANGED \langle mastership, conn, target \rangle
CommitRollback(n, i) \triangleq
   \land \lor \land transactions[i].rollback.commit = Pending
         \land \ configuration.committed.revision = i
         \land \lor \land configuration.committed.target = i
               \land configuration.committed.index = configuration.committed.target
               \land \lor \land configuration.committed.transaction = i
                     \land transactions[configuration.committed.transaction].change.commit = Complete
                  \lor \land configuration.committed.transaction > i
                     \land transactions[configuration.committed.transaction].rollback.commit = Complete
              \land configuration' = [configuration \ EXCEPT \ !.committed.target = transactions[i].rollback.index]
               \wedge history' = Append(history, [
                                  type \mapsto Rollback,
                                 phase \mapsto Commit,
                                 index \mapsto i,
                                 status \mapsto InProgress)
               \land UNCHANGED \langle transactions \rangle
            \lor \land configuration.committed.revision = i
               \land configuration.committed.target = transactions[i].rollback.index
               \land transactions' = [transactions \ EXCEPT \ ![i].rollback.commit = InProgress]
               \land UNCHANGED \langle configuration, history \rangle
      \lor \land transactions[i].rollback.commit = InProgress
         \land \lor \land configuration.committed.revision = i
               \land configuration' = [configuration \ EXCEPT \ !.committed.transaction = i,
                                                               !.committed.index
                                                               !.committed.seq num
                                                                                           = configuration.committed.set
                                                               !.committed.revision
                                                                                           = transactions[i].rollback.ind
                                                               !.committed.values
                                                                                           = transactions[i].rollback.value
                                                                                                 configuration.committed
              \land history' = Append(history, [
                                 type \mapsto Rollback,
```

 $phase \mapsto Commit$,

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index \mapsto i,
                                  status \mapsto Complete)
               \land UNCHANGED \langle transactions \rangle
            \lor \land configuration.committed.revision = transactions[i].rollback.index
               \land transactions' = [transactions \ EXCEPT \ ![i].rollback.commit = Complete,
                                                              ![i].rollback.seqnum = configuration.committed.seqnum]
               \land UNCHANGED \langle configuration, history \rangle
   \land UNCHANGED \langle mastership, conn, target \rangle
-\ configuration

    applied

    - transaction: the index of the last applied transaction
    - sequence number
    - target: the current target revision (index)
    - revision: the last successfully applied revision
* If target = transaction then the current transaction is being changed
* If target < transaction then the current transaction is being rolled back
ApplyChange(n, i) \stackrel{\triangle}{=}
   \land transactions[i].change.commit = Complete
   \land \lor \land transactions[i].change.apply = Pending
         \land \lor \land configuration.applied.seqnum = transactions[i].change.seqnum - 1
               \land \lor \land configuration.applied.target \neq i
                     \land configuration.applied.transaction \in DOMAIN transactions \Rightarrow
                           \lor \land configuration.applied.target = configuration.applied.transaction
                               \land transactions[configuration.applied.transaction].change.apply \in Done
                           \lor \land configuration.applied.target < configuration.applied.transaction
                               \land transactions[configuration.applied.transaction].rollback.apply \in Done
                     \land \lor \land configuration.applied.revision = transactions[i].rollback.index
                           \land configuration' = [configuration \ EXCEPT \ !.applied.target = i,
                                                                            !.applied.index = i
                           \wedge history' = Append(history, [
                                              type \mapsto Change,
                                              phase \mapsto Apply,
                                              index \mapsto i,
                                              status \mapsto InProgress])
                        \lor \land \lor transactions[i].phase = Rollback
                              \lor configuration.applied.revision < transactions[i].rollback.index
                           \land configuration' = [configuration \ EXCEPT \ !.applied.target]
                                                                            !.applied.transaction = i,
                                                                            !.applied.seqnum
                                                                                                     = transactions[i].chang
                           \land history' = Append(history, [
                                              type \mapsto Change,
                                              phase \mapsto Apply,
                                              index \mapsto i,
```

 $status \mapsto Aborted$)

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\land UNCHANGED \langle transactions \rangle
           \lor \land configuration.applied.target = i
               \land transactions' = [transactions \ EXCEPT \ ![i].change.apply = InProgress]
               \land UNCHANGED \langle configuration, history \rangle
      \lor \land configuration.applied.seqnum = transactions[i].change.seqnum
         \land transactions' = [transactions \ EXCEPT \ ![i].change.apply = Aborted]
         \land UNCHANGED \langle configuration, history \rangle
  \land UNCHANGED \langle target \rangle
\lor \land transactions[i].change.apply = InProgress
      If the change has not yet been applied, attempt to apply it.
  \land \ \lor \ \land \ configuration.applied.seqnum \neq transactions[i].change.seqnum
         \land \lor \land transactions[i].phase = Change
               \land \ configuration.state = Complete
               \land configuration.term = mastership.term
               \land conn[n].id = mastership.conn
               \land conn[n].connected
               \land target.running
               \land \lor \land target' = [target \ EXCEPT \ !.values = transactions[i].change.values @@ target.values]
                     \land configuration' = [configuration \ EXCEPT \ !.applied.transaction = i,
                                                                       !.applied.seqnum
                                                                                                = transactions[i].change
                                                                       !.applied.revision
                                                                                                =i,
                                                                                                = transactions[i].change
                                                                       !.applied.values
                                                                                                       configuration.app
                     \wedge history' = Append(history, [
                                        type \mapsto Change,
                                        phase \mapsto Apply,
                                        index \mapsto i,
                                        status \mapsto Complete])
                  \vee \wedge configuration' = [configuration \ EXCEPT \ !.applied.transaction = i,
                                                                       !.applied.seqnum \\
                                                                                                = transactions[i].change
                     \wedge history' = Append(history, [
                                        type \mapsto Change,
                                        phase \mapsto Apply,
                                        index \mapsto i,
                                        status \mapsto Failed)
                     \land UNCHANGED \langle target \rangle
            \lor \land transactions[i].phase = Rollback
               \land configuration' = [configuration \ EXCEPT \ !.applied.transaction = i,
                                                                 !.applied.seqnum
                                                                                          = transactions[i].change.seq
               \wedge history' = Append(history, [
                                  type \quad \mapsto \mathit{Change},
                                  phase \mapsto Apply,
                                  index \mapsto i,
                                  status \mapsto Failed)
```

 \land UNCHANGED $\langle target \rangle$

```
\land UNCHANGED \langle transactions \rangle
             If the change has been applied, update the transaction status.
            \lor \land configuration.applied.seqnum = transactions[i].change.seqnum
               \land \lor \land configuration.applied.revision = i
                     \land transactions' = [transactions \ EXCEPT \ ![i].change.apply = Complete]
                  \lor \land configuration.applied.revision = transactions[i].rollback.index
                     \land transactions' = [transactions \ EXCEPT \ ![i].change.apply = Failed]
               \land UNCHANGED \langle configuration, target, history \rangle
   \land UNCHANGED \langle mastership, conn \rangle
ApplyRollback(n, i) \stackrel{\Delta}{=}
   \land transactions[i].rollback.commit = Complete
   \land \lor \land transactions[i].rollback.apply = Pending
         \land \lor \land configuration.applied.seqnum = transactions[i].rollback.seqnum - 1
               \land \lor \land configuration.applied.target \neq transactions[i].rollback.index
                     \land \lor \land configuration.applied.transaction = i
                           \land transactions[configuration.applied.transaction].change.apply \in Done
                        \lor \land configuration.applied.transaction > i
                           \land transactions[configuration.applied.transaction].rollback.apply \in Done
                     \land configuration' = [configuration \ EXCEPT \ !.applied.target = transactions[i].rollback.index]
                     \land history' = Append(history, [
                                        type \quad \mapsto Rollback,
                                        phase \mapsto Apply,
                                        index \mapsto i,
                                        status \mapsto InProgress)
                     \land UNCHANGED \langle transactions \rangle
                  \lor \land configuration.applied.target = transactions[i].rollback.index
                     \land transactions' = [transactions \ EXCEPT \ ![i].rollback.apply = InProgress]
                     \land UNCHANGED \langle configuration, history \rangle
         \land UNCHANGED \langle target \rangle
      \lor \land transactions[i].rollback.apply = InProgress
             If this transaction has not yet been applied, attempt to apply it.
         \land \lor \land configuration.applied.seqnum \neq transactions[i].rollback.seqnum
               \land configuration.state = Complete
               \land configuration.term = mastership.term
               \wedge conn[n].id = mastership.conn
               \land conn[n].connected
               \land target.running
               \land target' = [target \ Except \ !.values = transactions[i].rollback.values@@target.values]
               \land configuration' = [configuration EXCEPT !.applied.transaction = i,
                                                                !.applied.seqnum
                                                                                         = transactions[i].rollback.seqnum
                                                                !.applied.revision
                                                                                         = transactions[i].rollback.index,
                                                                !.applied.values
                                                                                         = transactions[i].rollback.values
                                                                                                configuration. applied. values
```

 $\land history' = Append(history, [$

```
type \mapsto Rollback,
                                      phase \mapsto Apply,
                                      index \mapsto i,
                                      status \mapsto Complete)
                 \land UNCHANGED \langle transactions \rangle
                  If the change has been applied, update the transaction status.
              \lor \land configuration.applied.seqnum = transactions[i].rollback.seqnum
                 \land configuration.applied.revision = transactions[i].rollback.index
                 \land transactions' = [transactions \ EXCEPT \ ![i].rollback.apply = Complete]
                 \land UNCHANGED \langle configuration, target, history \rangle
   \land UNCHANGED \langle mastership, conn \rangle
ReconcileTransaction(n, i) \triangleq
   \land i \in \text{DOMAIN} \ transactions
   \land mastership.master = n
   \land \lor CommitChange(n, i)
       \vee ApplyChange(n, i)
       \vee CommitRollback(n, i)
       \vee ApplyRollback(n, i)
CommitChange(n, i) \stackrel{\Delta}{=}
  \land \lor \land transactions[i].change.commit = Pending
      \land \forall j \in \text{DOMAIN } transactions : j < i \Rightarrow
           \land transactions[j].change.commit \in Done
           \land transactions[j].rollback.commit \neq InProgress
      \land transactions' = [transactions \ EXCEPT \ ![i].change.commit = InProgress,
                                    ![i].rollback.values = [
                                      p \in \text{DOMAIN} \ transactions[i].change.values \mapsto
                                        IF p \in \text{DOMAIN} configuration.committed.values Then
                                           configuration.committed.value[p]
                                         ELSE Nil]]
      ∧ UNCHANGED ⟨configuration, history⟩
    \lor \ \land \ transactions[i].change.commit = InProgress
      \* Changes are validated during the commit phase. If a change fails validation,
      \* it will be marked failed before being applied to the configuration.
      \* If all the change values are valid, record the changes required to roll
      \* back the transactions and the index to which the rollback changes
      \* will roll back the configuration.
                                                       \land \lor \land configuration' =
                                                                                            [configuration
       \texttt{EXCEPT} \ !.committed.values = transactions[i].change.values @@
                                                            configuration.committed.values
           \land transactions' = [transactions \ EXCEPT \ ![i].change.commit = Complete]
           \land history' = Append(history, [type \mapsto Change, phase \mapsto Commit, index \mapsto i])
         \lor \land transactions' = [transactions \ EXCEPT \ ![i].change.commit = Failed]
           \land \ \mathtt{UNCHANGED} \ \langle \mathit{configuration}, \ \mathit{history} \rangle
  \land UNCHANGED \langle mastership, conn, target \rangle
CommitRollback(n, i) \stackrel{\Delta}{=}
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\land \lor \land transactions[i].rollback.commit = Pending
      \land \forall j \in \text{DOMAIN } transactions :
           \wedge j > i
           \land transactions[j].phase \neq Nil
           \land \ transactions[j].change.commit \neq Pending
           \Rightarrow transactions[j].rollback.commit = Complete
      \land transactions' = [transactions \ Except \ ![i].rollback.commit = InProgress]
       ∧ UNCHANGED ⟨configuration, history⟩
    \lor \land transactions[i].rollback.commit = InProgress
         \land configuration'
                                                 [configuration
                                                                      EXCEPT !.committed.values
                                                         transactions[i].rollback.values @@ configuration.committed.values]\\
      \land transactions' = [transactions \ Except \ ![i].rollback.commit = Complete]
       \land history' = Append(history, [type \mapsto Rollback, phase \mapsto Commit, index \mapsto i])
  ∧ UNCHANGED ⟨mastership, conn, target⟩
ApplyChange(n, i) \stackrel{\Delta}{=}
  \land \ \lor \ \land \ transactions[i].change.apply = Pending
      \land \lor \land transactions[i].change.commit = Complete
           \land \forall j \in \text{DOMAIN } transactions : j < i \Rightarrow
                \lor \land transactions[j].change.apply = Complete
                  \land transactions[j].rollback.apply \neq InProgress
                \lor \land transactions[j].change.apply = Failed
                  \land transactions[j].rollback.apply = Complete
           \land i-1 \in \text{DOMAIN} \ transactions \land transactions[i-1]. change.apply = Failed \Rightarrow
               transactions[i-1].rollback.apply = Complete
           \land transactions' = [transactions \ EXCEPT \ ![i].change.apply = InProgress]
         \lor \land transactions[i].change.commit = Failed
           \land transactions' = [transactions \ EXCEPT \ ![i].change.apply = Aborted]
      ∧ UNCHANGED ⟨configuration, target, history⟩
     \lor \land transactions[i].change.apply = InProgress
      \ Verify the applied term is the current mastership term to ensure the
      \* configuration has been synchronized following restarts.
      \land \ configuration.applied.term = mastership.term
      \* Verify the node's connection to the target.
      \land conn[n].connected
       \land mastership.conn = conn[n].id
       \land target.running
       \ * Model successful and failed target update requests.
      \land \lor \land target' = [target \ Except \ !.values = transactions[i].change.values@@target.values]
              \land configuration'
                                                     [configuration
                                                                         EXCEPT !.applied.values
                                                            transactions[i].change.values @@ configuration.applied.values]
           \land transactions' = [transactions \ EXCEPT \ ![i].change.apply = Complete]
           \land history' = Append(history, [type \mapsto Change, phase \mapsto Apply, index \mapsto i])
         \vee \wedge transactions' = [transactions \ EXCEPT \ ![i].change.apply = Failed]
           ∧ UNCHANGED ⟨configuration, target, history⟩
  \land UNCHANGED \langle mastership, conn \rangle
ApplyRollback(n, i) \stackrel{\Delta}{=}
  \land \lor \land transactions[i].rollback.apply = Pending
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\land transactions[i].rollback.commit = Complete
       \land \forall j \in \text{DOMAIN } transactions :
            \wedge j > i
            \land transactions[j].phase \neq Nil
            \land transactions[j].change.apply \neq Pending
            \Rightarrow transactions[j].rollback.apply \in Done
       \land \lor \land transactions[i].change.apply = Pending
            \land transactions' = [transactions \ EXCEPT \ ![i].change.apply = Aborted,
                                             ![i].rollback.apply = Complete]
          \lor \ \land \ transactions[i].change.apply \in Done
            \land transactions' = [transactions \ EXCEPT \ ![i].rollback.apply = InProgress]
       \land UNCHANGED \langle configuration, target, history \rangle
     \lor \land transactions[i].rollback.apply = InProgress
       \* Verify the applied term is the current mastership term to ensure the
       \* configuration has been synchronized following restarts.
       \land configuration.applied.term = mastership.term
       \* Verify the node's connection to the target.
       \land conn[n].connected
       \land target.running
       \land \textit{target'} = [\texttt{target} \; \texttt{Except} \; !.\textit{values} = \textit{transactions}[i].\textit{rollback}.\textit{values} @@ \textit{target}.\textit{values}]
                                                                              EXCEPT !.applied.values
                                                       [configuration
          \land configuration'
                                                            transactions[i].rollback.values @@ configuration.applied.values]
       \land transactions' = [transactions \ EXCEPT \ ![i].rollback.apply = Complete]
       \land history' = Append(history, [type \mapsto Rollback, phase \mapsto Apply, index \mapsto i])
  \land UNCHANGED \langle mastership, conn \rangle
ReconcileChange(n, i) \stackrel{\triangle}{=}
  \land transactions[i].phase = Change
  \land \lor CommitChange(n, i)
     \vee ApplyChange(n, i)
ReconcileRollback(n, i) \stackrel{\Delta}{=}
  \land \ transactions[i].phase = Rollback
  \land \lor CommitRollback(n, i)
     \vee ApplyRollback(n, i)
Reconcile Transaction(n, i) \stackrel{\Delta}{=}
  \land i \in \text{domain} \ transactions
  \land \ master ship.master = n
  \land \lor ReconcileChange(n, i)
     \vee ReconcileRollback(n, i)
TODO:
```

Change:

- if configuration.committed.index = i-1 and associated transaction change commit is done
- increment configuration.committed.index
- change transaction commit status to InProgress and store rollback revision and values
- if configuration.committed.index = i then validate and commit change
- increment configuration.committed.revision to i if valid

```
Rollback:
  - if configuration.committed.revision = i
  - set configuration.committed.index to i and change rollback commit status to InProgress
  - change configuration.committed.values to rollback values and revision to rollback revision
  - if configuration.committed.index = i + 1 and associated transaction change commit failed or
  rollback is committed
  - or if configuration.committed.index = i and transaction change is committed
  - \ \ \text{if} \ \textit{transactions} [\textit{configuration.committed.index}]. \textit{commit} \ i
  - decrement configuration.committed.index
TRANSACTION 1 [index = 1, phase = Change, change = (commit = Pending, apply =
Pending), rollback = (commit = Pending, apply = Pending)
CHANGE
commit = Pending
  \lor \land transactions[configuration.committed.index].phase = Change
    \land transactions[configuration.committed.index].change.commit \in Done
  \lor \land transactions[configuration.committed.index].phase = Rollback
    \land transactions[configuration.committed.index].rollback.commit \in Done
  else \ if \ transactions[configuration.committed.index].phase = Rollback
apply = Pending
ROLLBACK
configuration \stackrel{\Delta}{=} [
          \mapsto 0,
  index
  version \mapsto 0, \setminus* transaction version number
  committed \mapsto \ [
    index \mapsto 0, \* committed transaction number
    version \mapsto 0, \setminus* committed serial number
    revision \mapsto 0, \ \ \ \  committed revision number
    values \mapsto [\ldots]],
  applied \mapsto [
    index \mapsto 0, \ \  applied transaction number
    version \mapsto 0, \* applied serial number
    revision \mapsto 0, \ \ * applied revision number
    values \mapsto [\ldots]]
 TODO: Serialize transactions by managing the next transaction for changes and the previous transaction for rollbacks
 When change is committed, set next transaction's change commit to InProgress
  WHen rollback is committed, set previous transaction's rollback commit to InProgress
 CHANGE [index = 1, change = (prev = 0), rollback = ()]
    configuration.transaction = 1 \\
    configuration.revision = 1 \\
 CHANGE [index = 1, change = (prev = 0), rollback = ()]
 CHANGE [index = 2, change = (prev = 1), rollback = ()]
    configuration.transaction = 2 \\
```

```
configuration.revision = 2
 CHANGE [index = 1, change = (prev = 0), rollback = ()]
 CHANGE [index = 2, change = (prev = 1), rollback = ()]
 CHANGE [index = 3, change = (prev = 2), rollback = ()]
    configuration.transaction = 3
    configuration.revision = 3
 CHANGE [index = 1, change = (prev = 0), rollback = ()]
 CHANGE [index = 2, change = (prev = 1), rollback = ()]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)]
    configuration.transaction = 3 \\
    configuration.revision = 2
 CHANGE [index = 1, change = (prev = 0), rollback = ()]
ROLLBACK [index = 2, change = (prev = 1), rollback = (prev = 3)]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)]
    configuration.transaction = 2
    configuration.revision = 1
 CHANGE [index = 1, change = (prev = 0), rollback = ()]
{\it ROLLBACK} \ [index=2, \ change=(prev=1), \ rollback=(prev=3)]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)] CHANGE [index = 4,
 change = (prev = 2), rollback = ()]
    configuration.transaction = 4
    configuration.revision = 4
 CHANGE [index = 1, change = (prev = 0), rollback = ()]
ROLLBACK [index = 2, change = (prev = 1), rollback = (prev = 3)]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)]
 CHANGE [index = 4, change = (prev = 2), rollback = ()]
 CHANGE [index = 5, change = (prev = 4), rollback = ()]
    configuration.transaction = 5
    configuration.revision = 5 \\
 CHANGE \ [index = 1, \ change = (prev = 0), \ rollback = ()]
ROLLBACK [index = 2, change = (prev = 1), rollback = (prev = 3)]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)]
ROLLBACK [index = 4, change = (prev = 2), rollback = ()] CHANGE [index = 5, change =
 (prev = 4), rollback = ()
    configuration.transaction = 5 \\
    configuration.revision = 5
 CHANGE [index = 1, change = (prev = 0), rollback = ()]
ROLLBACK [index = 2, change = (prev = 1), rollback = (prev = 3)]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)]
ROLLBACK [index = 4, change = (prev = 2), rollback = (prev = 5)]
ROLLBACK [index = 5, change = (prev = 4), rollback = (prev = 5)]
    configuration.transaction = 5
    configuration.revision = 4
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configuration.transaction = 4
          configuration.revision = 1
   CHANGE \ [index = 1, \ change = (prev = 0), \ rollback = ()]
ROLLBACK [index = 2, change = (prev = 1), rollback = (prev = 3)]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)]
ROLLBACK [index = 4, change = (prev = 2), rollback = (prev = 5)]
{\tt ROLLBACK} \ [index = 5, \ change = (prev = 4), \ rollback = (prev = 5)]
   CHANGE [index = 6, change = (prev = 2, commit = Complete, apply = Failed), rollback = ()]
   CHANGE [index = 7, change = (prev = 6, commit = Complete, apply = Aborted), rollback =
   CHANGE [index = 8, change = (prev = 7, commit = Complete, apply = Aborted), rollback =
    ()]
          configuration.transaction = 6
          configuration.revision = 6\\
          configuration.transaction = 7 \\
          configuration.revision = 7
          configuration.transaction = 8
          configuration.revision = 8
   CHANGE [index = 1, change = (prev = 0), rollback = ()]
ROLLBACK [index = 2, change = (prev = 1), rollback = (prev = 3)]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)]
ROLLBACK [index = 4, change = (prev = 2), rollback = (prev = 5)]
ROLLBACK [index = 5, change = (prev = 4), rollback = (prev = 5)]
   CHANGE [index = 6, change = (prev = 2, commit = Complete, apply = Failed), rollback = ()]
   CHANGE [index = 7, change = (prev = 6, commit = Complete, apply = Aborted), rollback =
ROLLBACK [index = 8, change = (prev = 7, commit = Complete, apply = Aborted), rollback =
(prev = 8)
          configuration.transaction = 8
          configuration.revision = 7
   CHANGE [index = 1, change = (prev = 0), rollback = ()]
{\tt ROLLBACK} \; [index = 2, \; change = (prev = 1), \; rollback = (prev = 3)]
{\it ROLLBACK \ [index = 3, \ change = (prev = 2), \ rollback = (prev = 3)]}
ROLLBACK [index = 4, change = (prev = 2), rollback = (prev = 5)]
ROLLBACK [index = 5, change = (prev = 4), rollback = (prev = 5)] CHANGE [index = 6,
   change = (prev = 2, commit = Complete, apply = Failed), rollback = ()]
ROLLBACK [index = 7, change = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 6, commit = Complete, apply = Aborted), rollback = (prev = 
ROLLBACK [index = 8, change = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Aborted), rollback = (prev = 7, commit = Complete, apply = Complete, apply = (prev = 7, commit = Complete, apply = Complete, apply = (prev = 7, commit = Complete, apply = Complete, apply = (prev = 7, commit = Complete, apply = Commit = Complete, apply = (prev = 7, commit = Complete, apply = Complete, apply = (prev = 7, commit = Complete, apply = Complete, apply = (prev = 7, commit = Complete, apply = Complete, apply = (prev = 7, commit = Complete, apply = Complete, apply = (prev = 7, commit = Complete, apply = Complete, apply = (prev = 7, commit = Complete, 
(prev = 8)
          configuration.transaction = 7
          configuration.revision = 6
   CHANGE [index = 1, change = (prev = 0), rollback = ()]
 ROLLBACK [index = 2, change = (prev = 1), rollback = (prev = 3)]
ROLLBACK [index = 3, change = (prev = 2), rollback = (prev = 3)]
ROLLBACK [index = 4, change = (prev = 2), rollback = (prev = 5)]
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\begin{aligned} & \text{ROLLBACK} \ [index = 5, \ change = (prev = 4), \ rollback = (prev = 5)] \\ & \text{ROLLBACK} \ [index = 6, \ change = (prev = 2, \ commit = Complete, \ apply = Failed), \ rollback = (prev = 7)] \\ & \text{ROLLBACK} \ [index = 7, \ change = (prev = 6, \ commit = Complete, \ apply = Aborted), \ rollback = (prev = 8)] \\ & \text{ROLLBACK} \ [index = 8, \ change = (prev = 7, \ commit = Complete, \ apply = Aborted), \ rollback = (prev = 8)] \ \text{CHANGE} \ [index = 9, \ change = (prev = 6), \ rollback = ()] \\ & configuration.transaction = 6 \\ & configuration.revision = 1 \end{aligned}
```