
MODULE *Config*

INSTANCE *Naturals*

INSTANCE *FiniteSets*

INSTANCE *Sequences*

INSTANCE *TLC*

An empty constant

CONSTANT *Nil*

Transaction type constants

CONSTANTS

Change,

Rollback

Transaction isolation constants

CONSTANTS

ReadCommitted,

Serializable

Phase constants

CONSTANTS

Initialize,

Validate,

Abort,

Commit,

Apply

Phase \triangleq

LET *phases* \triangleq \langle *Initialize,*

Validate,

Abort,

Commit,

Apply \rangle

IN $[p \in \{phases[i] : i \in \text{DOMAIN } phases\} \mapsto$

CHOOSE $i \in \text{DOMAIN } phases : phases[i] = p]$

Status constants

CONSTANTS

Pending,

Complete,

Failed

Status \triangleq

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LET statuses  $\triangleq$   $\langle$ Pending,
                  Complete,
                  Failed $\rangle$ 
IN  [s  $\in$  {statuses[i] : i  $\in$  DOMAIN statuses}  $\mapsto$ 
      CHOOSE i  $\in$  DOMAIN statuses : statuses[i] = s]

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CONSTANTS

Valid,
Invalid

CONSTANTS

Success,
Failure

The set of all nodes

CONSTANT *Node*

Target is the set of all targets and their possible paths and values.

Example: *Target* \triangleq [
 target1 \mapsto [*persistent* \mapsto FALSE, *values* \mapsto [
 path1 \mapsto {"*value1*", "*value2*"},
 path2 \mapsto {"*value2*", "*value3*" }]],
 target2 \mapsto [*persistent* \mapsto TRUE, *values* \mapsto [
 path2 \mapsto {"*value3*", "*value4*" },
 path3 \mapsto {"*value4*", "*value5*" }]]]

CONSTANT *Target*

Configuration update/rollback requests are tracked and processed through two data types. Transactions represent the lifecycle of a single configuration change request and are stored in an append-only log. Configurations represent the desired configuration of a *gNMI* target based on the aggregate of relevant changes in the Transaction log.

TYPE *Type* ::= *type* \in
 {*Change*,
 Rollback}

TYPE *Phase* ::= *phase* \in
 {*Initialize*,
 Validate,
 Abort,
 Commit,
 Apply}

TYPE *Status* ::= *status* \in
 {*Pending*,
 Initializing,
 Initialized,
 Validating,
 Validated,

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    Committing,
    Committed,
    Applying,
    Applied,
    Synchronizing,
    Synchronized,
    Persisted,
    Failed}

TYPE Transaction  $\triangleq$  [
  type      ::= type  $\in$  Type,
  index     ::= index  $\in$  Nat,
  isolation ::= isolation  $\in$  {IsolationDefault, IsolationSerializable}
  values ::= [
    target  $\in$  SUBSET (DOMAIN Target)  $\mapsto$  [ path  $\in$  SUBSET (DOMAIN Target[target].values)  $\mapsto$ 
    [
      value ::= value  $\in$  STRING,
      delete ::= delete  $\in$  BOOLEAN ]],
    rollback ::= index  $\in$  Nat,
    targets ::= targets  $\in$  SUBSET (DOMAIN Target)
    phase     ::= phase  $\in$  Phase]
    status ::= status  $\in$  Status]

TYPE Proposal  $\triangleq$  [
  type      ::= type  $\in$  Type,
  index     ::= index  $\in$  Nat,
  values     ::= [ path  $\in$  SUBSET (DOMAIN Target[target].values)  $\mapsto$  [
    value ::= value  $\in$  STRING,
    delete ::= delete  $\in$  BOOLEAN ]],
    rollback     ::= index  $\in$  Nat,
    dependencyIndex ::= dependencyIndex  $\in$  Nat,
    rollbackIndex ::= rollbackIndex  $\in$  Nat,
    rollbackValues ::= [ path  $\in$  SUBSET (DOMAIN Target[target].values)  $\mapsto$  [
      value ::= value  $\in$  STRING,
      delete ::= delete  $\in$  BOOLEAN ]],
    phase     ::= phase  $\in$  Phase]
    status     ::= status  $\in$  Status]

TYPE Configuration  $\triangleq$  [
  id      ::= id  $\in$  STRING,
  target  ::= target  $\in$  STRING,
  values  ::= [ path  $\in$  SUBSET (DOMAIN Target[target])  $\mapsto$  [
    value ::= value  $\in$  STRING,
    index ::= index  $\in$  Nat,
    deleted ::= delete  $\in$  BOOLEAN ]],
    configIndex ::= configIndex  $\in$  Nat,
    configTerm  ::= configTerm  $\in$  Nat,
    proposedIndex ::= proposedIndex  $\in$  Nat,
    committedIndex ::= committedIndex  $\in$  Nat,
    appliedIndex ::= appliedIndex  $\in$  Nat,
    appliedTerm  ::= appliedTerm  $\in$  Nat,

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$$\begin{aligned} appliedValues ::= & [\textit{path} \in \text{SUBSET } (\text{DOMAIN } \textit{Target}[\textit{target}]) \mapsto [\\ & \textit{value} ::= \textit{value} \in \text{STRING}, \\ & \textit{index} ::= \textit{index} \in \text{Nat}, \\ & \textit{deleted} ::= \textit{delete} \in \text{BOOLEAN}]], \\ & \textit{status} ::= \textit{status} \in \textit{Status}] \end{aligned}$$

A transaction log. Transactions may either request a set of changes to a set of targets or rollback a prior change.

VARIABLE *transaction*

A record of per-target proposals

VARIABLE *proposal*

A record of per-target configurations

VARIABLE *configuration*

A record of target states

VARIABLE *target*

A record of target masterhips

VARIABLE *mastership*

$\textit{vars} \triangleq \langle \textit{transaction}, \textit{proposal}, \textit{configuration}, \textit{mastership}, \textit{target} \rangle$

This section models *mastership* for the configuration service.

Mastership is used primarily to track the lifecycle of individual configuration targets and react to state changes on the southbound. Each target is assigned a master from the *Node* set, and masters can be unset when the target disconnects.

Set node *n* as the master for target *t*

$$\begin{aligned} \textit{SetMaster}(n, t) &\triangleq \\ &\wedge \textit{mastership}[t].\textit{master} \neq n \\ &\wedge \textit{mastership}' = [\textit{mastership} \text{ EXCEPT } ![t].\textit{term} = \textit{mastership}[t].\textit{term} + 1, \\ &\quad ![t].\textit{master} = n] \\ &\wedge \text{UNCHANGED } \langle \textit{transaction}, \textit{proposal}, \textit{configuration}, \textit{target} \rangle \end{aligned}$$

$\textit{UnsetMaster}(t) \triangleq$

$$\begin{aligned} &\wedge \textit{mastership}[t].\textit{master} \neq \textit{Nil} \\ &\wedge \textit{mastership}' = [\textit{mastership} \text{ EXCEPT } ![t].\textit{master} = \textit{Nil}] \\ &\wedge \text{UNCHANGED } \langle \textit{transaction}, \textit{proposal}, \textit{configuration}, \textit{target} \rangle \end{aligned}$$

This section models configuration changes and rollbacks. Changes are appended to the transaction log and processed asynchronously.

$\textit{Value}(s, t, p) \triangleq$

$$\begin{aligned} &\text{LET } \textit{value} \triangleq \text{CHOOSE } v \in s : v.\textit{target} = t \wedge v.\textit{path} = p \\ &\text{IN} \end{aligned}$$

$[value \mapsto value.value,$
 $delete \mapsto value.delete]$

$Paths(s, t) \triangleq$
 $[p \in \{v.path : v \in \{v \in s : v.target = t\}\} \mapsto Value(s, t, p)]$

$Changes(s) \triangleq$
 $[t \in \{v.target : v \in s\} \mapsto Paths(s, t)]$

$ValidValues(t, p) \triangleq$
 $UNION \{[value \mapsto v, delete \mapsto FALSE] : v \in Target[t].values[p], [value \mapsto Nil, delete \mapsto TRUE]\}$

$ValidPaths(t) \triangleq$
 $UNION \{\{v @ @ [path \mapsto p] : v \in ValidValues(t, p)\} : p \in DOMAIN Target[t].values\}$

$ValidTargets \triangleq$
 $UNION \{\{p @ @ [target \mapsto t] : p \in ValidPaths(t)\} : t \in DOMAIN Target\}$

The set of all valid sets of changes to all targets and their paths.

The set of possible changes is computed from the *Target* model value.

$ValidChanges \triangleq$
 $LET changeSets \triangleq \{s \in SUBSET ValidTargets :$
 $\quad \forall t \in DOMAIN Target :$
 $\quad \forall p \in DOMAIN Target[t].values :$
 $\quad \quad Cardinality(\{v \in s : v.target = t \wedge v.path = p\}) \leq 1\}$
 IN
 $\{Changes(s) : s \in changeSets\}$

The next available index in the transaction log.

This is computed as the max of the existing indexes in the log to
allow for changes to the log (*e.g.* log compaction) to be modeled.

$NextIndex \triangleq$
 $IF DOMAIN transaction = \{\} THEN$
 1
 $ELSE$
 $LET i \triangleq CHOOSE i \in DOMAIN transaction :$
 $\quad \forall j \in DOMAIN transaction : i \geq j$
 $IN i + 1$

Add a set of changes 'c' to the transaction log

$RequestChange(c) \triangleq$
 $\wedge \exists isolation \in \{ReadCommitted, Serializable\} :$
 $\quad \wedge transaction' = transaction @ @ (NextIndex :> [type \mapsto Change,$
 $\quad \quad \quad index \mapsto NextIndex,$
 $\quad \quad \quad isolation \mapsto isolation,$
 $\quad \quad \quad values \mapsto c,$
 $\quad \quad \quad targets \mapsto \{\},$

$$\begin{array}{l}
\text{phase} \mapsto \text{Initialize}, \\
\text{status} \mapsto \text{Pending}] \\
\wedge \text{UNCHANGED } \langle \text{proposal}, \text{configuration}, \text{mastership}, \text{target} \rangle \\
\text{Add a rollback of transaction 't' to the transaction log} \\
\text{RequestRollback}(t) \triangleq \\
\wedge \exists \text{isolation} \in \{\text{ReadCommitted}, \text{Serializable}\} : \\
\wedge \text{transaction}' = \text{transaction} @@ (\text{NextIndex} :> [\text{type} \mapsto \text{Rollback}, \\
\text{index} \mapsto \text{NextIndex}, \\
\text{isolation} \mapsto \text{isolation}, \\
\text{rollback} \mapsto t, \\
\text{targets} \mapsto \{\}, \\
\text{phase} \mapsto \text{Initialize}, \\
\text{status} \mapsto \text{Pending}] \\
\wedge \text{UNCHANGED } \langle \text{proposal}, \text{configuration}, \text{mastership}, \text{target} \rangle
\end{array}$$

This section models the Transaction log reconciler.

Transactions come in two flavors : – *Change* transactions contain a set of changes to be applied to a set of *targets* – *Rollback* transactions reference a prior change transaction to be reverted to the previous state

Transactions proceed through a series of phases:

- * *Initialize* – create and link Proposals
- * *Validate* – validate changes and rollbacks
- * *Commit* – commit changes to Configurations
- * *Apply* – commit changes to Targets

Reconcile a transaction

$\text{ReconcileTransaction}(n, i) \triangleq$

Initialize is the only transaction phase that's globally serialized.

While in the *Initializing* phase, the reconciler checks whether the prior transaction has been *Initialized* before creating Proposals in the *Initialize* phase. Once all of the transaction's proposals have been *Initialized*, the transaction will be marked *Initialized*. If any proposal is *Failed*, the transaction will be marked *Failed* as well.

$\wedge \vee \wedge \text{transaction}[i].\text{phase} = \text{Initialize}$
 $\wedge \vee \wedge \text{transaction}[i].\text{status} = \text{Pending}$

Serialize transaction initialization

$\wedge i - 1 \in \text{DOMAIN } \text{transaction} \Rightarrow$
 $\vee \text{Phase}[\text{transaction}[i - 1].\text{phase}] > \text{Phase}[\text{Initialize}]$
 $\vee \text{transaction}[i - 1].\text{status} \neq \text{Pending}$

If the transaction's targets are not yet set, create proposals and add targets to the transaction state.

$\wedge \vee \wedge \text{transaction}[i].\text{targets} = \{\}$

If the transaction is a change, the targets are taken from the change values.

$$\begin{aligned}
& \wedge \vee \wedge \text{transaction}[i].\text{type} = \text{Change} \\
& \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{targets} = \text{DOMAIN } \text{transaction}[i].\text{values}] \\
& \wedge \text{proposal}' = [t \in \text{DOMAIN } \text{proposal} \mapsto \\
& \quad \text{IF } t \in \text{DOMAIN } \text{transaction}[i].\text{values} \text{ THEN} \\
& \quad \quad \text{proposal}[t] @ @ (i :> [type \mapsto \text{Change}, \\
& \quad \quad \quad \text{index} \mapsto i, \\
& \quad \quad \quad \text{values} \mapsto \text{transaction}[i].\text{values}[t], \\
& \quad \quad \quad \text{dependencyIndex} \mapsto 0, \\
& \quad \quad \quad \text{rollbackIndex} \mapsto 0, \\
& \quad \quad \quad \text{rollbackValues} \mapsto \langle \rangle, \\
& \quad \quad \quad \text{phase} \mapsto \text{Initialize}, \\
& \quad \quad \quad \text{status} \mapsto \text{Pending}]) \\
& \quad \text{ELSE} \\
& \quad \quad \text{proposal}[t] \\
& \quad \text{If the transaction is a rollback, the targets affected are} \\
& \quad \text{the targets of the change transaction being rolled back.} \\
& \vee \wedge \text{transaction}[i].\text{type} = \text{Rollback} \\
& \wedge \vee \wedge \text{transaction}[i].\text{rollback} \in \text{DOMAIN } \text{transaction} \\
& \wedge \text{transaction}[\text{transaction}[i].\text{rollback}].\text{type} = \text{Change} \\
& \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{targets} = \\
& \quad \text{DOMAIN } \text{transaction}[\text{transaction}[i].\text{rollback}].\text{values}] \\
& \wedge \text{proposal}' = [t \in \text{DOMAIN } \text{proposal} \mapsto \\
& \quad \text{IF } t \in \text{DOMAIN } \text{transaction}[\text{transaction}[i].\text{rollback}].\text{values} \text{ THEN} \\
& \quad \quad \text{proposal}[t] @ @ (i :> [type \mapsto \text{Rollback}, \\
& \quad \quad \quad \text{index} \mapsto i, \\
& \quad \quad \quad \text{rollback} \mapsto \text{transaction}[i].\text{rollback}, \\
& \quad \quad \quad \text{dependencyIndex} \mapsto 0, \\
& \quad \quad \quad \text{rollbackIndex} \mapsto 0, \\
& \quad \quad \quad \text{rollbackValues} \mapsto \langle \rangle, \\
& \quad \quad \quad \text{phase} \mapsto \text{Initialize}, \\
& \quad \quad \quad \text{status} \mapsto \text{Pending}]) \\
& \quad \text{ELSE} \\
& \quad \quad \text{proposal}[t] \\
& \vee \wedge \vee \wedge \text{transaction}[i].\text{rollback} \in \text{DOMAIN } \text{transaction} \\
& \wedge \text{transaction}[\text{transaction}[i].\text{rollback}].\text{type} = \text{Rollback} \\
& \vee \text{transaction}[i].\text{rollback} \notin \text{DOMAIN } \text{transaction} \\
& \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{status} = \text{Failed}] \\
& \wedge \text{UNCHANGED } \langle \text{proposal} \rangle \\
& \vee \wedge \text{transaction}[i].\text{targets} \neq \{\} \\
& \quad \text{If all proposals have been Complete, mark the transaction Complete.} \\
& \wedge \vee \wedge \forall t \in \text{transaction}[i].\text{targets} : \\
& \quad \wedge \text{proposal}[t][i].\text{phase} = \text{Initialize} \\
& \quad \wedge \text{proposal}[t][i].\text{status} = \text{Complete} \\
& \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{status} = \text{Complete}] \\
& \wedge \text{UNCHANGED } \langle \text{proposal} \rangle
\end{aligned}$$

[illegible]

$\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{status} = \text{Failed}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 Once the transaction has been *Validated*, proceed to the *Commit* phase.
 If any of the transaction's proposals depend on a *Serializable* transaction,
 verify the dependency has been *Committed* to preserve serializability before
 moving the transaction to the *Commit* phase.
 $\vee \wedge \text{transaction}[i].\text{status} = \text{Complete}$
 $\wedge \forall t \in \text{transaction}[i].\text{targets} :$
 $\wedge \text{proposal}[t][i].\text{dependencyIndex} > 0 \Rightarrow$
 $\vee \text{transaction}[\text{proposal}[t][i].\text{dependencyIndex}].\text{isolation} \neq \text{Serializable}$
 $\vee \text{Phase}[\text{transaction}[\text{proposal}[t][i].\text{dependencyIndex}].\text{phase}] > \text{Phase}[\text{Commit}]$
 $\vee \wedge \text{transaction}[\text{proposal}[t][i].\text{dependencyIndex}].\text{phase} = \text{Commit}$
 $\wedge \text{transaction}[\text{proposal}[t][i].\text{dependencyIndex}].\text{status} \in \{\text{Complete}, \text{Failed}\}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{phase} = \text{Commit},$
 $![i].\text{status} = \text{Pending}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 $\vee \wedge \text{transaction}[i].\text{status} = \text{Failed}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{phase} = \text{Abort},$
 $![i].\text{status} = \text{Pending}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 $\vee \wedge \text{transaction}[i].\text{phase} = \text{Commit}$
 $\wedge \vee \wedge \text{transaction}[i].\text{status} = \text{Pending}$
 Move the transaction's proposals to the *Committing* state
 $\wedge \vee \wedge \exists t \in \text{transaction}[i].\text{targets} : \text{proposal}[t][i].\text{phase} \neq \text{Commit}$
 $\wedge \text{proposal}' = [t \in \text{DOMAIN } \text{proposal} \mapsto$
 $\text{IF } t \in \text{transaction}[i].\text{targets} \text{ THEN}$
 $[\text{proposal}[t] \text{ EXCEPT } ![i].\text{phase} = \text{Commit},$
 $![i].\text{status} = \text{Pending}]$
 ELSE
 $\text{proposal}[t]]$
 $\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$
 If all proposals have been *Complete*, mark the transaction *Complete*.
 $\vee \wedge \forall t \in \text{transaction}[i].\text{targets} :$
 $\wedge \text{proposal}[t][i].\text{phase} = \text{Commit}$
 $\wedge \text{proposal}[t][i].\text{status} = \text{Complete}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{status} = \text{Complete}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 Once the transaction has been *Committed*, proceed to the *Apply* phase.
 If any of the transaction's proposals depend on a *Serializable* transaction,
 verify the dependency has been *Applied* to preserve serializability before
 moving the transaction to the *Apply* phase.
 $\vee \wedge \text{transaction}[i].\text{status} = \text{Complete}$
 $\wedge \forall t \in \text{transaction}[i].\text{targets} :$
 $\wedge \text{proposal}[t][i].\text{dependencyIndex} > 0 \Rightarrow$
 $\vee \text{transaction}[\text{proposal}[t][i].\text{dependencyIndex}].\text{isolation} \neq \text{Serializable}$

$\wedge \text{proposal}[t][i].\text{status} = \text{Complete}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{status} = \text{Complete}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 $\wedge \text{UNCHANGED } \langle \text{configuration}, \text{mastership}, \text{target} \rangle$

Reconcile a proposal

$\text{ReconcileProposal}(n, t, i) \triangleq$
 $\wedge \vee \wedge \text{proposal}[t][i].\text{phase} = \text{Initialize}$
 $\wedge \text{proposal}[t][i].\text{status} = \text{Pending}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [$
 $\quad \text{proposal}[t] \text{ EXCEPT } ![i] = [$
 $\quad \quad \text{status} \mapsto \text{Complete},$
 $\quad \quad \text{dependencyIndex} \mapsto \text{configuration}[t].\text{proposedIndex}] @@ \text{proposal}[t][i]]$
 $\wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } ![t].\text{proposedIndex} = i]$
 $\wedge \text{UNCHANGED } \langle \text{target} \rangle$

While in the *Validate* phase, validate the proposed changes.

If validation is successful, the proposal also records the changes

required to roll back the proposal and the index to which to roll back.

$\vee \wedge \text{proposal}[t][i].\text{phase} = \text{Validate}$
 $\wedge \text{proposal}[t][i].\text{status} = \text{Pending}$
 $\wedge \text{configuration}[t].\text{committedIndex} = \text{proposal}[t][i].\text{dependencyIndex}$

For *Change* proposals validate the set of requested changes.

$\wedge \vee \wedge \text{proposal}[t][i].\text{type} = \text{Change}$
 $\wedge \text{LET } \text{rollbackIndex} \triangleq \text{configuration}[t].\text{configIndex}$
 $\quad \text{rollbackValues} \triangleq [p \in \text{DOMAIN } \text{proposal}[t][i].\text{values} \mapsto$
 $\quad \quad \text{IF } p \in \text{DOMAIN } \text{configuration}[t].\text{values} \text{ THEN}$
 $\quad \quad \quad \text{configuration}[t].\text{values}[p]$
 $\quad \quad \text{ELSE}$
 $\quad \quad \quad [value \mapsto \text{Nil},$
 $\quad \quad \quad \text{delete} \mapsto \text{TRUE}]$

Model validation successes and failures with *Valid* and *Invalid* results.

IN $\exists r \in \{ \text{Valid}, \text{Invalid} \} :$

If the *Change* is *Valid*, record the changes required to roll
 back the proposal and the index to which the rollback changes
 will roll back the configuration.

$\vee \wedge r = \text{Valid}$

$\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [$
 $\quad \text{proposal}[t] \text{ EXCEPT } ![i].\text{rollbackIndex} = \text{rollbackIndex},$
 $\quad \quad \quad ![i].\text{rollbackValues} = \text{rollbackValues},$
 $\quad \quad \quad ![i].\text{status} = \text{Complete}]$

$\vee \wedge r = \text{Invalid}$

$\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [$
 $\quad \text{proposal}[t] \text{ EXCEPT } ![i].\text{status} = \text{Failed}]$

For *Rollback* proposals, validate the rollback changes which are
 proposal being rolled back.

$\vee \wedge \text{proposal}[t][i].\text{type} = \text{Rollback}$
 Rollbacks can only be performed on *Change* type proposals.
 $\wedge \vee \wedge \text{proposal}[t][\text{proposal}[t][i].\text{rollback}].\text{type} = \text{Change}$
 Only roll back the change if it's the latest change made
 to the configuration based on the configuration index.
 $\wedge \vee \wedge \text{configuration}[t].\text{configIndex} = \text{proposal}[t][i].\text{rollback}$
 $\wedge \text{LET } \text{rollbackIndex} \triangleq \text{proposal}[t][\text{proposal}[t][i].\text{rollback}].\text{rollbackIndex}$
 $\text{rollbackValues} \triangleq \text{proposal}[t][\text{proposal}[t][i].\text{rollback}].\text{rollbackValues}$
 IN $\exists r \in \{ \text{Valid}, \text{Invalid} \} :$
 If the *Rollback* is *Valid*, record the changes required to
 roll back the target proposal and the index to which the
 configuration is being rolled back.
 $\vee \wedge r = \text{Valid}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [$
 $\text{proposal}[t] \text{ EXCEPT } ![i].\text{rollbackIndex} = \text{rollbackIndex},$
 $![i].\text{rollbackValues} = \text{rollbackValues},$
 $![i].\text{status} = \text{Complete}]$
 $\vee \wedge r = \text{Invalid}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [$
 $\text{proposal}[t] \text{ EXCEPT } ![i].\text{status} = \text{Failed}]$
 If the *Rollback* target is not the most recent change to the configuration,
 fail validation for the proposal.
 $\vee \wedge \text{configuration}[t].\text{configIndex} \neq \text{proposal}[t][i].\text{rollback}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [\text{proposal}[t] \text{ EXCEPT } ![i].\text{status} = \text{Failed}]$
 If a *Rollback* proposal is attempting to roll back another *Rollback*,
 fail validation for the proposal.
 $\vee \wedge \text{proposal}[t][\text{proposal}[t][i].\text{rollback}].\text{type} = \text{Rollback}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [$
 $\text{proposal}[t] \text{ EXCEPT } ![i].\text{status} = \text{Failed}]$
 $\wedge \text{UNCHANGED } \langle \text{configuration}, \text{target} \rangle$
 While in the *Commit* state, commit the proposed changes to the configuration.
 $\vee \wedge \text{proposal}[t][i].\text{phase} = \text{Commit}$
 $\wedge \text{proposal}[t][i].\text{status} = \text{Pending}$
 Only commit the proposal if the prior proposal has already been committed.
 $\wedge \text{configuration}[t].\text{committedIndex} = \text{proposal}[t][i].\text{dependencyIndex}$
 If the proposal is a change, commit the change values and set the configuration
 index to the proposal index.
 $\wedge \vee \wedge \text{proposal}[t][i].\text{type} = \text{Change}$
 $\wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } ![t].\text{values} = \text{proposal}[t][i].\text{values},$
 $![t].\text{configIndex} = i,$
 $![t].\text{committedIndex} = i]$
 If the proposal is a rollback, commit the rollback values and index. This
 will cause the configuration index to be reverted to the index prior to
 the transaction/proposal being rolled back.
 $\vee \wedge \text{proposal}[t][i].\text{type} = \text{Rollback}$

$\wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } ![t].\text{values} = \text{proposal}[t][i].\text{rollbackValues},$
 $![t].\text{configIndex} = \text{proposal}[t][i].\text{rollbackIndex},$
 $![t].\text{committedIndex} = i]$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [\text{proposal}[t] \text{ EXCEPT } ![i].\text{status} = \text{Complete}]]$
 $\wedge \text{UNCHANGED } \langle \text{target} \rangle$
 While in the *Apply* phase, apply the proposed changes to the target.
 $\vee \wedge \text{proposal}[t][i].\text{phase} = \text{Apply}$
 $\wedge \text{proposal}[t][i].\text{status} = \text{Pending}$
 $\wedge \text{configuration}[t].\text{appliedIndex} = \text{proposal}[t][i].\text{dependencyIndex}$
 $\wedge \text{configuration}[t].\text{appliedTerm} = \text{mastership}[t].\text{term}$
 $\wedge \text{mastership}[t].\text{master} = n$
 Model successful and failed target update requests.
 $\wedge \exists r \in \{\text{Success}, \text{Failure}\} :$
 $\vee \wedge r = \text{Success}$
 If the proposal is a change, apply the change values to the target
 and update the configuration's applied index and values.
 $\wedge \vee \wedge \text{proposal}[t][i].\text{type} = \text{Change}$
 $\wedge \text{target}' = [\text{target} \text{ EXCEPT } ![t] = \text{proposal}[t][i].\text{values} @@ \text{target}[t]]$
 $\wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT }$
 $![t].\text{appliedIndex} = i,$
 $![t].\text{appliedValues} = \text{proposal}[t][i].\text{values} @@ \text{configuration}[t].\text{appliedValues}]$
 If the proposal is a rollback, apply the rollback values and update the
 configuration's applied values with the rolled back values.
 $\vee \wedge \text{proposal}[t][i].\text{type} = \text{Rollback}$
 $\wedge \text{target}' = [\text{target} \text{ EXCEPT } ![t] = \text{proposal}[t][i].\text{rollbackValues} @@ \text{target}[t]]$
 $\wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT }$
 $![t].\text{appliedIndex} = i,$
 $![t].\text{appliedValues} = \text{proposal}[t][i].\text{rollbackValues} @@ \text{configuration}[t].\text{appliedValues}]$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [\text{proposal}[t] \text{ EXCEPT } ![i].\text{status} = \text{Complete}]]$
 If the proposal could not be applied, update the configuration's applied index
 and mark the proposal *Failed*.
 $\vee \wedge r = \text{Failure}$
 $\wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } ![t].\text{appliedIndex} = i]$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![t] = [\text{proposal}[t] \text{ EXCEPT } ![i].\text{status} = \text{Failed}]]$
 $\wedge \text{UNCHANGED } \langle \text{target} \rangle$
 $\vee \wedge \text{proposal}[t][i].\text{phase} = \text{Abort}$
 $\wedge \text{proposal}[t][i].\text{status} = \text{Pending}$
 The *committedIndex* will always be greater than or equal to the *appliedIndex*.
 If only the *committedIndex* matches the proposal's *dependencyIndex*, update
 the *committedIndex* to enable commits of later proposals, but do not
 mark the *Abort* phase *Complete* until the *appliedIndex* has been incremented.
 $\wedge \vee \wedge \text{configuration}[t].\text{committedIndex} = \text{proposal}[t][i].\text{dependencyIndex}$
 $\wedge \text{configuration}[t].\text{appliedIndex} \neq \text{proposal}[t][i].\text{dependencyIndex}$
 $\wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } ![t].\text{committedIndex} = i]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$

$$\begin{aligned}
& [path \in \{\} \mapsto \\
& \quad [path \mapsto path, \\
& \quad \quad value \mapsto Nil, \\
& \quad \quad index \mapsto 0, \\
& \quad \quad deleted \mapsto FALSE]], \\
& configIndex \mapsto 0, \\
& configTerm \mapsto 0, \\
& proposedIndex \mapsto 0, \\
& committedIndex \mapsto 0, \\
& appliedIndex \mapsto 0, \\
& appliedTerm \mapsto 0, \\
& appliedValues \mapsto \\
& \quad [path \in \{\} \mapsto \\
& \quad \quad [path \mapsto path, \\
& \quad \quad \quad value \mapsto Nil, \\
& \quad \quad \quad index \mapsto 0, \\
& \quad \quad \quad deleted \mapsto FALSE]]]] \\
\wedge target = [t \in \text{DOMAIN } Target \mapsto \\
& \quad [path \in \{\} \mapsto \\
& \quad \quad [value \mapsto Nil]]] \\
\wedge mastership = [t \in \text{DOMAIN } Target \mapsto [master \mapsto Nil, term \mapsto 0]] \\
Next \triangleq & \\
& \vee \exists c \in ValidChanges : \\
& \quad RequestChange(c) \\
& \vee \exists t \in \text{DOMAIN } transaction : \\
& \quad RequestRollback(t) \\
& \vee \exists n \in Node : \\
& \quad \exists t \in \text{DOMAIN } Target : \\
& \quad \quad SetMaster(n, t) \\
& \quad \vee \exists t \in \text{DOMAIN } Target : \\
& \quad \quad UnsetMaster(t) \\
& \vee \exists n \in Node : \\
& \quad \exists t \in \text{DOMAIN } transaction : \\
& \quad \quad ReconcileTransaction(n, t) \\
& \vee \exists n \in Node : \\
& \quad \exists t \in \text{DOMAIN } proposal : \\
& \quad \quad \exists i \in \text{DOMAIN } proposal[t] : \\
& \quad \quad \quad ReconcileProposal(n, t, i) \\
& \vee \exists n \in Node : \\
& \quad \exists c \in \text{DOMAIN } configuration : \\
& \quad \quad ReconcileConfiguration(n, c) \\
Spec \triangleq & Init \wedge \Box [Next]_{vars} \\
Order \triangleq &
\end{aligned}$$

$$\begin{aligned}
& \wedge \forall i, j \in \text{DOMAIN } \textit{transaction} : \\
& \quad \vee j \leq i \\
& \quad \vee \text{Phase}[\textit{transaction}[i].\textit{phase}] \geq \text{Phase}[\textit{transaction}[j].\textit{phase}] \\
& \quad \vee \textit{transaction}[i].\textit{targets} \cap \textit{transaction}[j].\textit{targets} = \{\} \\
& \quad \vee \textit{transaction}[j].\textit{status} \in \{\textit{Pending}, \textit{Failed}\} \\
& \wedge \forall t \in \text{DOMAIN } \textit{proposal} : \\
& \quad \forall i, j \in \text{DOMAIN } \textit{proposal}[t] : \\
& \quad \quad \vee j \leq i \\
& \quad \quad \vee \text{Phase}[\textit{proposal}[t][i].\textit{phase}] \geq \text{Phase}[\textit{proposal}[t][j].\textit{phase}] \\
& \quad \quad \vee \textit{proposal}[t][j].\textit{status} \in \{\textit{Pending}, \textit{Failed}\}
\end{aligned}$$

Consistency \triangleq

$\forall t \in \text{DOMAIN } \textit{target} :$

LET

Compute the transaction indexes that have been applied to the target

$$\begin{aligned}
\textit{appliedIndexes} & \triangleq \{i \in \text{DOMAIN } \textit{transaction} : \\
& \quad \wedge \textit{transaction}[i].\textit{type} = \textit{Change} \\
& \quad \wedge i \in \text{DOMAIN } \textit{proposal}[t] \\
& \quad \wedge \textit{proposal}[t][i].\textit{phase} = \textit{Apply} \\
& \quad \wedge \textit{proposal}[t][i].\textit{status} = \textit{Complete} \\
& \quad \wedge t \in \text{DOMAIN } \textit{transaction}[i].\textit{values} \\
& \quad \wedge \neg \exists j \in \text{DOMAIN } \textit{transaction} : \\
& \quad \quad \wedge j > i \\
& \quad \quad \wedge \textit{transaction}[j].\textit{type} = \textit{Rollback} \\
& \quad \quad \wedge \textit{transaction}[j].\textit{rollback} = i \\
& \quad \quad \wedge \textit{transaction}[j].\textit{phase} = \textit{Apply} \\
& \quad \quad \wedge \textit{transaction}[j].\textit{status} = \textit{Complete}\}
\end{aligned}$$

Compute the set of paths in the target that have been updated by transactions

$$\textit{appliedPaths} \triangleq \text{UNION } \{\text{DOMAIN } \textit{transaction}[i].\textit{values}[t] : i \in \textit{appliedIndexes}\}$$

Compute the highest index applied to the target for each path

$$\begin{aligned}
\textit{pathIndexes} & \triangleq [p \in \textit{appliedPaths} \mapsto \text{CHOOSE } i \in \textit{appliedIndexes} : \\
& \quad \forall j \in \textit{appliedIndexes} : \\
& \quad \quad \wedge i \geq j \\
& \quad \quad \wedge p \in \text{DOMAIN } \textit{transaction}[i].\textit{values}[t]]
\end{aligned}$$

Compute the expected target configuration based on the last indexes applied to the target for each path.

$$\textit{expectedConfig} \triangleq [p \in \text{DOMAIN } \textit{pathIndexes} \mapsto \textit{transaction}[\textit{pathIndexes}[p]].\textit{values}[t][p]]$$

IN

$$\textit{target}[t] = \textit{expectedConfig}$$

Isolation \triangleq

$$\begin{aligned}
& \forall i, j \in \text{DOMAIN } \textit{transaction} : \\
& \quad \vee j \leq i \\
& \quad \vee \textit{transaction}[i].\textit{targets} \cap \textit{transaction}[j].\textit{targets} = \{\} \\
& \quad \vee \textit{transaction}[i].\textit{isolation} \neq \textit{Serializable}
\end{aligned}$$

$$\begin{aligned}
& \vee \wedge \vee \wedge \text{transaction}[i].\text{phase} \in \{\text{Commit}, \text{Abort}\} \\
& \quad \wedge \text{transaction}[i].\text{status} \in \{\text{Complete}, \text{Failed}\} \\
& \quad \vee \text{Phase}[\text{transaction}[i].\text{phase}] > \text{Phase}[\text{Commit}] \\
& \quad \vee \text{Phase}[\text{transaction}[j].\text{phase}] < \text{Phase}[\text{Commit}] \\
& \wedge \vee \wedge \text{transaction}[i].\text{phase} \in \{\text{Apply}, \text{Abort}\} \\
& \quad \wedge \text{transaction}[i].\text{status} \in \{\text{Complete}, \text{Failed}\} \\
& \quad \vee \text{Phase}[\text{transaction}[j].\text{phase}] < \text{Phase}[\text{Apply}] \\
& \vee \text{transaction}[j].\text{status} = \text{Failed}
\end{aligned}$$

THEOREM *Safety* $\triangleq \text{Spec} \Rightarrow \Box(\text{Order} \wedge \text{Consistency} \wedge \text{Isolation})$

Completion \triangleq

$$\begin{aligned}
& \wedge \forall i \in \text{DOMAIN } \text{transaction} : \\
& \quad \wedge \text{transaction}[i].\text{phase} = \text{Commit} \\
& \quad \wedge \text{transaction}[i].\text{status} \in \{\text{Complete}, \text{Failed}\} \\
& \wedge \forall i \in \text{DOMAIN } \text{transaction} : \\
& \quad \wedge \text{transaction}[i].\text{phase} = \text{Apply} \\
& \quad \wedge \text{transaction}[i].\text{status} \in \{\text{Complete}, \text{Failed}\} \\
& \wedge \forall t \in \text{DOMAIN } \text{proposal} : \\
& \quad \forall i \in \text{DOMAIN } \text{proposal}[t] : \\
& \quad \quad \wedge \text{proposal}[t][i].\text{phase} = \text{Commit} \\
& \quad \quad \wedge \text{proposal}[t][i].\text{status} \in \{\text{Complete}, \text{Failed}\} \\
& \wedge \forall t \in \text{DOMAIN } \text{proposal} : \\
& \quad \forall i \in \text{DOMAIN } \text{proposal}[t] : \\
& \quad \quad \wedge \text{proposal}[t][i].\text{phase} = \text{Apply} \\
& \quad \quad \wedge \text{proposal}[t][i].\text{status} \in \{\text{Complete}, \text{Failed}\}
\end{aligned}$$

THEOREM *Liveness* $\triangleq \text{Spec} \Rightarrow \Diamond \text{Completion}$

Type assumptions.

ASSUME $\text{Nil} \in \text{STRING}$

ASSUME $\forall \text{phase} \in \text{DOMAIN } \text{Phase} : \text{phase} \in \text{STRING}$

ASSUME $\forall \text{status} \in \text{DOMAIN } \text{Status} : \text{status} \in \text{STRING}$

ASSUME $\wedge \text{IsFiniteSet}(\text{Node})$

$$\begin{aligned}
& \wedge \forall n \in \text{Node} : \\
& \quad \wedge n \notin \text{DOMAIN } \text{Target} \\
& \quad \wedge n \in \text{STRING}
\end{aligned}$$

ASSUME $\wedge \forall t \in \text{DOMAIN } \text{Target} :$

$$\begin{aligned}
& \quad \wedge t \notin \text{Node} \\
& \quad \wedge t \in \text{STRING} \\
& \quad \wedge \text{Target}[t].\text{persistent} \in \text{BOOLEAN} \\
& \quad \wedge \forall p \in \text{DOMAIN } \text{Target}[t].\text{values} :
\end{aligned}$$

IsFiniteSet(Target[t].values[p])

* Modification History
* Last modified *Mon Feb 07 14:53:44 PST 2022* by *jordanhalterman*
* Created *Wed Sep 22 13:22:32 PDT 2021* by *jordanhalterman*