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——— MODULE Config
INSTANCE Naturals
INSTANCE FiniteSets
{\tt INSTANCE}\ Sequences
LOCAL INSTANCE TLC
This section specifies constant parameters for the model.
CONSTANT None
Assume None \in \text{String}
CONSTANT Node
Assume \forall n \in Node : n \in String
CONSTANTS
   Change,
   Rollback
Event \triangleq \{Change, Rollback\}
Assume \forall e \in Event : e \in String
CONSTANTS
   Commit,
   Apply
Phase \triangleq \{Commit, Apply\}
\texttt{ASSUME} \ \forall \ p \in Phase : p \in \texttt{STRING}
CONSTANTS
   Pending,
   InProgress,
   Complete,
   Aborted,
   Failed
State \triangleq \{Pending, InProgress, Complete, Aborted, Failed\}
Working \triangleq \{Pending, InProgress\}
Finished \triangleq {Complete, Aborted, Failed}
Assume \forall s \in State : s \in String
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Constant Path

Assume \forall p \in Path : p \in String

Constant Value

Assume \forall v \in Value : v \in String

AllValues \triangleq Value \cup \{None\}

Constant NumProposals

Assume NumProposals \in Nat
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This section defines model state variables.
proposal \stackrel{\Delta}{=} [i \in 1.. Nat \mapsto [
     phase \mapsto Phase,
      change \mapsto [
        values \mapsto Change,
        commit \mapsto State,
        apply \mapsto State],
      rollback \mapsto [
        index\mapsto Nat,
        values \mapsto \mathit{Change},
        commit \mapsto State,
        apply \mapsto State]]]
configuration \stackrel{\Delta}{=} [
  committed \mapsto [
     index \mapsto Nat,
     values \mapsto Change],
   applied \mapsto [
     index \mapsto Nat,
     values \mapsto Change,
     term \mapsto Nat]]
mastership \stackrel{\Delta}{=} [
  master \mapsto \text{STRING},
  term \mapsto Nat,
  conn \mapsto Nat]
conn \; \stackrel{\Delta}{=} \; \; [ \; n \in \mathit{Node} \mapsto \; [ \;
     id \mapsto Nat,
     connected \mapsto \texttt{BOOLEAN} ]]
target \stackrel{\Delta}{=} [
  id \mapsto Nat,
  values \mapsto Change,
  running \mapsto \texttt{BOOLEAN}]
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VARIABLE proposal

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VARIABLE configuration
VARIABLE mastership
VARIABLE conn
Variable target
VARIABLE history
vars \triangleq \langle proposal, configuration, mastership, conn, target, history \rangle
This section models configuration target.
StartTarget \triangleq
   \land \neg target.running
   \wedge target' = [target \ EXCEPT \ !.id]
                                                 = target.id + 1,
                                     !.running = TRUE
   ∧ UNCHANGED ⟨proposal, configuration, mastership, conn, history⟩
StopTarget \triangleq
   \land target.running
   \land target' = [target \ EXCEPT \ !.running = FALSE,
                                     !.values = [p \in \{\} \mapsto [value \mapsto None]]]
   \land conn' = [n \in Node \mapsto [conn[n] \text{ EXCEPT } !.connected = \text{FALSE}]]
   \land UNCHANGED \langle proposal, configuration, mastership, history <math>\rangle
This section models nodes connection to the configuration target.
ConnectNode(n) \triangleq
   \land \neg conn[n].connected
   \land target.running
   \wedge conn' = [conn \ EXCEPT \ ![n].id]
                                                    = conn[n].id + 1,
                                   ![n].connected = TRUE]
   \land UNCHANGED \langle proposal, configuration, mastership, target, history <math>\rangle
DisconnectNode(n) \triangleq
   \land conn[n].connected
   \wedge conn' = [conn \ EXCEPT \ ![n].connected = FALSE]
   \land UNCHANGED \langle proposal, configuration, mastership, target, history <math>\rangle
This section models mastership reconciliation.
ReconcileMastership(n) \triangleq
   \land \lor \land conn[n].connected
          \land \ master ship.master = None
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\vee \wedge \neg conn[n].connected
          \land mastership.master = n
          \land mastership' = [mastership \ EXCEPT \ !.master = None]
   \land UNCHANGED \langle proposal, configuration, conn, target, history <math>\rangle
This section models configuration reconciliation.
ReconcileConfiguration(n) \stackrel{\Delta}{=}
   \land mastership.master = n
   \land \lor \land configuration.status \neq InProgress
          \land configuration.applied.term < mastership.term
          \land configuration' = [configuration EXCEPT !.status = InProgress]
          \land UNCHANGED \langle target \rangle
       \lor \land configuration.status = InProgress
          \land configuration.applied.term < mastership.term
          \land conn[n].connected
          \land \ target.running
          \land target' = [target \ EXCEPT \ !.values = configuration.applied.values]
          \land configuration' = [configuration EXCEPT !.applied.term = mastership.term,
                                                             !.applied.target = target.id,
                                                             !.status
                                                                                 = Complete
   \land UNCHANGED \langle proposal, mastership, conn, history \rangle
This section models proposal reconcilation.
CommitChange(n, i) \triangleq
   \land \lor \land proposal[i].change.commit = Pending
          \land proposal[i].rollback.commit = None
          \land \forall j \in \text{DOMAIN } proposal : j < i \Rightarrow
               \land proposal[j].change.commit \in Finished
               \land proposal[j].rollback.commit \notin Working
          \land proposal' = [proposal \ EXCEPT \ ![i].change.commit = InProgress]
          \land UNCHANGED \langle configuration, history \rangle
       \lor \land proposal[i].change.commit = InProgress
           Changes are validated during the commit phase. If a change fails validation,
           it will be marked failed before being applied to the configuration.
           If all the change values are valid, record the changes required to roll
           back the proposal and the index to which the rollback changes
           will roll back the configuration.
                                           \stackrel{\triangle}{=} configuration.committed.index
          \land \lor \text{LET } rollbackIndex
                                          \stackrel{\Delta}{=} [p \in \text{DOMAIN } proposal[i].change.values \mapsto
                     rollback Values
                                                 IF p \in DOMAIN configuration.committed.values THEN
                                                    configuration.committed.values[p]
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 $\land mastership' = [master \mapsto n, term \mapsto mastership.term + 1, conn \mapsto conn[n].id]$

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ELSE
                                                   [index \mapsto 0, value \mapsto None]]
                    committedValues \stackrel{\triangle}{=} [p \in DOMAIN \ proposal[i].change.values \mapsto
                                               proposal[i].change.values[p]@@[index \mapsto i]]@@
                                                   configuration.committed.values
                     \land configuration' = [configuration \ EXCEPT \ !.committed.index = i,]
               ΙN
                                                                       !.committed.values = committedValues
                     \land proposal' = [proposal \ EXCEPT \ ![i].rollback.index = rollbackIndex,
                                                            ![i].rollback.values = rollbackValues,
                                                            ![i].change.commit = Complete]
                     \land history' = Append(history, [type \mapsto Change, phase \mapsto Commit, index \mapsto i])
            \lor \land proposal' = [proposal \ EXCEPT \ ![i].change.commit = Failed]
               \land UNCHANGED \langle configuration, history \rangle
   \land UNCHANGED \langle mastership, conn, target \rangle
ApplyChange(n, i) \triangleq
   \land \lor \land proposal[i].change.apply = Pending
         \land proposal[i].rollback.apply = None
         \land \lor \land proposal[i].change.commit = Complete
               \land \forall j \in \text{DOMAIN } proposal : j < i \Rightarrow
                    \lor \land proposal[j].change.apply = Complete
                       \land proposal[j].rollback.apply \notin Working
                    \lor \land proposal[j].change.apply = Failed
                       \land proposal[i].rollback.apply = Complete
               \land proposal' = [proposal \ EXCEPT \ ![i].change.apply = InProgress]
            \lor \land proposal[i].change.commit \in \{Aborted, Failed\}
               \land proposal' = [proposal \ EXCEPT \ ![i].change.apply = Aborted]
         \land UNCHANGED \langle configuration, target, history \rangle
      \lor \land proposal[i].change.apply = InProgress
          Verify the applied term is the current mastership term to ensure the
          configuration has been synchronized following restarts.
         \land configuration.applied.term = mastership.term
          Verify the node's connection to the target.
         \land conn[n].connected
         \land mastership.conn = conn[n].id
         \land target.running
          Model successful and failed target update requests.
         \land \lor \land \text{LET } values \stackrel{\triangle}{=} [p \in \text{DOMAIN } proposal[i].change.values \mapsto
                                       proposal[i].change.values[p] @@[index \mapsto i]]
                        \land target' = [target \ EXCEPT \ !.values = values @@ target.values]
                        \land configuration' = [configuration \ EXCEPT \ !.applied.index = i,
                                                                          !.applied.values = values @@
                                                                             configuration.applied.values
                        \land proposal' = [proposal \ EXCEPT \ ![i].change.apply = Complete]
                        \land history' = Append(history, [type \mapsto Change, phase \mapsto Apply, index \mapsto i])
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\lor \land proposal' = [proposal \ EXCEPT \ ![i].change.apply = Failed]
                \land UNCHANGED \langle configuration, target, history \rangle
   \land UNCHANGED \langle mastership, conn \rangle
CommitRollback(n, i) \triangleq
   \land \lor \land proposal[i].rollback.commit = Pending
         \land \forall j \in DOMAIN \ proposal: j > i \land proposal[j].phase \neq None \Rightarrow
              proposal[j].rollback.commit \in Finished
         \land \ \lor \ \land \ proposal[i].change.commit = Pending
               \land proposal' = [proposal \ EXCEPT \ ![i].change.commit = Aborted,
                                                       ![i].rollback.commit = Complete]
             \lor \land proposal[i].change.commit \in Finished
                \land proposal' = [proposal \ EXCEPT \ ![i].rollback.commit = InProgress]
         \land UNCHANGED \langle configuration, history \rangle
       \lor \land proposal[i].rollback.commit = InProgress
         \land LET index \triangleq proposal[i].rollback.index
                  values \triangleq proposal[i].rollback.values @@ configuration.committed.values
                  \land configuration' = [configuration EXCEPT !.committed.index = index,
            IN
                                                                     !.committed.values = values
                  \land proposal' = [proposal \ EXCEPT \ ![i].rollback.commit = Complete]
                  \land history' = Append(history, [type \mapsto Rollback, phase \mapsto Commit, index \mapsto i])
   \land UNCHANGED \langle mastership, conn, target \rangle
ApplyRollback(n, i) \triangleq
   \land \lor \land proposal[i].rollback.apply = Pending
         \land proposal[i].rollback.commit = Complete
         \land \forall j \in \text{DOMAIN } proposal: j > i \land proposal[j].phase \neq None \Rightarrow
              proposal[j].rollback.apply \in Finished
         \land \lor \land proposal[i].change.apply = Pending
                \land proposal' = [proposal \ EXCEPT \ ![i].change.apply = Aborted,
                                                       ![i].rollback.apply = Complete]
             \lor \land proposal[i].change.apply \in Finished
                \land proposal' = [proposal \ EXCEPT \ ![i].rollback.apply = InProgress]
         \land UNCHANGED \langle configuration, target, history \rangle
       \lor \land proposal[i].rollback.apply = InProgress
          Verify the applied term is the current mastership term to ensure the
          configuration has been synchronized following restarts.
         \land configuration.applied.term = mastership.term
          Verify the node's connection to the target.
         \land conn[n].connected
         \land target.running
         \land target' = [target \ EXCEPT \ !.values = proposal[i].rollback.values @@ target.values]
         \land LET index \stackrel{\triangle}{=} proposal[i].rollback.index
                  values \stackrel{\triangle}{=} proposal[i].rollback.values @@ configuration.applied.values
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\land configuration' = [configuration \ EXCEPT \ !.applied.index = index,]
                                                                  !.applied.values = values
                \land proposal' = [proposal \ EXCEPT \ ![i].rollback.apply = Complete]
                \land history' = Append(history, [type \mapsto Rollback, phase \mapsto Apply, index \mapsto i])
   \land UNCHANGED \langle mastership, conn \rangle
ReconcileProposal(n, i) \triangleq
   \land mastership.master = n
   \land \lor CommitChange(n, i)
       \vee ApplyChange(n, i)
      \vee CommitRollback(n, i)
       \vee ApplyRollback(n, i)
   \land UNCHANGED \langle mastership, conn \rangle
This section models changes to the proposal queue.
 Propose change at index 'i'
ProposeChange(i) \triangleq
   \land proposal[i].phase = None
   \land i-1 \in \text{DOMAIN } proposal \Rightarrow proposal[i-1].phase \neq None
   \land \exists p \in Path, v \in AllValues:
        \land proposal' = [proposal \ EXCEPT \ ![i].phase]
                                                                       = Change,
                                               ![i].change.values = (p:>[value \mapsto v]),
                                               ![i].change.commit = Pending,
                                               ![i].change.apply = Pending
   \land UNCHANGED \langle configuration, mastership, conn, target, history <math>\rangle
 Rollback proposed change at index 'i'
ProposeRollback(i) \triangleq
   \land proposal[i].phase = Change
   \land proposal' = [proposal \ EXCEPT \ ![i].phase]
                                                                  = Rollback,
                                          ![i].rollback.commit = Pending,
                                          ![i].rollback.apply = Pending]
   ∧ UNCHANGED ⟨configuration, mastership, conn, target, history⟩
Formal specification, constraints, and theorems.
Init \triangleq
    \land proposal = [
         i \in 1 \dots NumProposals \mapsto [
                    \mapsto None,
           phase
            change \mapsto [
               values \mapsto [p \in \{\} \mapsto [index \mapsto 0, value \mapsto None]],
               commit \mapsto None,
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apply \mapsto None,
              rollback \mapsto [
                  index \mapsto 0,
                  values \mapsto [p \in \{\} \mapsto [index \mapsto 0, value \mapsto None]],
                  commit \mapsto None,
                  apply \mapsto None
    \land configuration = [
           committed \mapsto [
               index \mapsto 0,
               values \mapsto [p \in \{\} \mapsto [index \mapsto 0, value \mapsto None]]],
           applied \mapsto \lceil
               index \mapsto 0,
               term \mapsto 0,
               target \mapsto 0,
               values \mapsto [p \in \{\} \mapsto [index \mapsto 0, value \mapsto None]]],
           status \mapsto Pending
    \land mastership = [master \mapsto None, term \mapsto 0, conn \mapsto 0]
    \land conn = [n \in Node \mapsto [id \mapsto 0, connected \mapsto FALSE]]
    \land target = [
           id
           values \mapsto [p \in \{\} \mapsto [index \mapsto 0, value \mapsto None]],
           running \mapsto FALSE
    \wedge history = \langle \rangle
Next \triangleq
    \vee \exists i \in 1 ... NumProposals :
          \vee ProposeChange(i)
          \lor ProposeRollback(i)
    \vee \exists n \in Node, i \in DOMAIN \ proposal : ReconcileProposal(n, i)
    \vee \exists n \in Node : ReconcileConfiguration(n)
    \vee \exists n \in Node : ReconcileMastership(n)
    \vee \exists n \in Node:
        \vee ConnectNode(n)
        \vee DisconnectNode(n)
    \vee StartTarget
    \vee Stop Target
Spec \triangleq
    \wedge Init
    \wedge \Box [Next]_{vars}
    \land \forall i \in 1 ... NumProposals : WF_{vars}(ProposeChange(i) \lor ProposeRollback(i))
    \land \forall n \in Node, i \in 1.. NumProposals : WF_{vars}(ReconcileProposal(n, i))
    \land \forall n \in Node : WF_{(configuration, mastership, conn, target)}(ReconcileConfiguration(n))
    \land \forall \, n \in \mathit{Node} : \mathrm{WF}_{\langle \mathit{mastership}, \, \mathit{conn}, \, \mathit{target} \rangle}(\mathit{ReconcileMastership}(n))
    \land \forall n \in Node : WF_{\langle conn, target \rangle}(ConnectNode(n) \lor DisconnectNode(n))
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\wedge \operatorname{WF}_{\langle target \rangle}(StartTarget)
    \wedge \operatorname{WF}_{\langle target \rangle}(StopTarget)
IsOrderedChange(p, i) \stackrel{\Delta}{=}
    \land history[i].type = Change
    \land history[i].phase = p
    \wedge \neg \exists j \in \text{DOMAIN } history :
            \wedge i < i
            \land history[j].type = Change
            \land history[j].phase = p
            \land \ history[j].index \ge history[i].index
IsOrderedRollback(p, i) \stackrel{\triangle}{=}
    \land history[i].type = Rollback
    \land history[i].phase = p
    \wedge \neg \exists j \in \text{DOMAIN } history :
            \wedge i < i
            \land history[j].type = Change
            \land history[j].phase = p
            \land history[j].index > history[i].index
            \land \neg \exists k \in \text{DOMAIN } history :
                    \wedge k > j
                    \wedge k < i
                    \land history[k].type = Rollback
                    \land history[k].phase = p
                    \land history[k].index = history[j].index
Order \triangleq
    \land \forall i \in \text{DOMAIN } history:
        \vee IsOrderedChange(Commit, i)
        \vee IsOrderedChange(Apply, i)
        \vee IsOrderedRollback(Commit, i)
        \lor IsOrderedRollback(Apply, i)
    \land \forall i \in \text{DOMAIN } proposal :
           \land proposal[i].change.apply = Failed
           \land proposal[i].rollback.apply \neq Complete
           \Rightarrow \forall j \in \text{domain } proposal : j > i \Rightarrow
                 proposal[j].change.apply \in \{None, Pending, Aborted\}
Consistency \triangleq
    \land target.running
    \land \ configuration.status = Complete
    \land configuration.applied.target = target.id
    \Rightarrow \forall i \in \text{DOMAIN } proposal :
           \land proposal[i].change.apply = Complete
           \land proposal[i].rollback.apply \neq Complete
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\Rightarrow \forall p \in \text{DOMAIN } proposal[i].change.values :
                 \land \neg \exists j \in \text{DOMAIN } proposal :
                        \wedge j > i
                        \land proposal[j].change.apply = Complete
                        \land proposal[j].rollback.apply \neq Complete
                 \Rightarrow \land p \in \text{domain } target.values
                     \land target.values[p].value = proposal[i].change.values[p].value
                     \land target.values[p].index = i
Safety \triangleq \Box(Order \land Consistency)
THEOREM Spec \Rightarrow Safety
Termination \triangleq
   \forall i \in 1 ... NumProposals:
      \land proposal[i].change.commit = Pending \leadsto
            proposal[i].change.commit \in Finished
      \land proposal[i].change.apply = Pending \leadsto
            proposal[i].change.apply \in Finished
      \land proposal[i].rollback.commit = Pending \leadsto
            proposal[i].rollback.commit \in Finished
      \land proposal[i].rollback.apply = Pending \leadsto
            proposal[i].rollback.apply \in Finished
Liveness \triangleq Termination
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Theorem $Spec \Rightarrow Liveness$

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