MODULE *Transaction*

INSTANCE *Naturals*

INSTANCE *FiniteSets*

INSTANCE *Sequences*

INSTANCE *TLC*

An empty constant
CONSTANT *Nil*

Transaction phase constants
CONSTANTS
Change,
Rollback

Proposal phase constants
CONSTANTS
Commit,
Apply

Status constants
CONSTANTS
Pending,
Complete,
Aborted,
Failed

$Status \triangleq \{Pending, Complete, Aborted, Failed\}$

The set of all nodes
CONSTANT *Node*

The set of possible paths and values
CONSTANT *Path*, *Value*

$Empty \triangleq [p \in \{\} \mapsto Nil]$

Variables defined by other modules.
VARIABLES
configuration,
mastership,
conn,
target

A transaction log. Transactions may either request a set
of changes to a set of targets or rollback a prior change.

VARIABLE *transaction*

A sequence of configuration changes used for model checking.

VARIABLE *history*

$$
\begin{array}{l}
\mathit{TypeOK} \triangleq \\
\quad \forall\, i \in \text{DOMAIN } \mathit{transaction} : \\
\qquad \land\ \mathit{transaction}[i].\mathit{type} \in \{\mathit{Change},\ \mathit{Rollback}\} \\
\qquad \land\ \mathit{transaction}[i].\mathit{index} \in \mathit{Nat} \\
\qquad \land\ \mathit{transaction}[i].\mathit{revision} \in \mathit{Nat} \\
\qquad \land\ \mathit{transaction}[i].\mathit{change}.\mathit{index} \in \mathit{Nat} \\
\qquad \land\ \mathit{transaction}[i].\mathit{change}.\mathit{revision} \in \mathit{Nat} \\
\qquad \land\ \forall\, p \in \text{DOMAIN } \mathit{transaction}[i].\mathit{change}.\mathit{values} : \\
\qquad\quad \mathit{transaction}[i].\mathit{change}.\mathit{values}[p] \neq \mathit{Nil} \Rightarrow \\
\qquad\qquad \mathit{transaction}[i].\mathit{change}.\mathit{values}[p] \in \text{STRING} \\
\qquad \land\ \mathit{transaction}[i].\mathit{rollback}.\mathit{index} \in \mathit{Nat} \\
\qquad \land\ \mathit{transaction}[i].\mathit{rollback}.\mathit{revision} \in \mathit{Nat} \\
\qquad \land\ \forall\, p \in \text{DOMAIN } \mathit{transaction}[i].\mathit{rollback}.\mathit{values} : \\
\qquad\quad \mathit{transaction}[i].\mathit{rollback}.\mathit{values}[p] \neq \mathit{Nil} \Rightarrow \\
\qquad\qquad \mathit{transaction}[i].\mathit{rollback}.\mathit{values}[p] \in \text{STRING} \\
\qquad \land\ \mathit{transaction}[i].\mathit{commit} \in \mathit{Status} \\
\qquad \land\ \mathit{transaction}[i].\mathit{apply} \in \mathit{Status}
\end{array}
$$

$$
\begin{array}{l}
\text{LOCAL } \mathit{State} \triangleq [\\
\quad \mathit{transactions} \mapsto [i \in \text{DOMAIN } \mathit{transaction} \mapsto \mathit{transaction}[i] \mathbin{@@} [\mathit{index} \mapsto i]], \\
\quad \mathit{configuration} \mapsto \mathit{configuration}]
\end{array}
$$

$$
\begin{array}{l}
\text{LOCAL } \mathit{Transitions} \triangleq \\
\quad \text{LET} \\
\qquad \mathit{transactions} \triangleq \{i \in \text{DOMAIN } \mathit{transaction}' : \\
\qquad\qquad\qquad\qquad i \in \text{DOMAIN } \mathit{transaction} \Rightarrow \mathit{transaction}'[i] \neq \mathit{transaction}[i]\} \\
\quad \text{IN} \\
\qquad [\mathit{transactions} \mapsto [i \in \mathit{transactions} \mapsto \mathit{transaction}'[i] \mathbin{@@} [\mathit{index} \mapsto i]]]
\end{array}
$$

$$
\begin{array}{l}
\mathit{Test} \triangleq \text{INSTANCE } \mathit{Test} \text{ WITH} \\
\quad \mathit{File} \leftarrow \text{“Transaction.log”}
\end{array}
$$

This section models configuration changes and rollbacks. Changes are appended to the transaction log and processed asynchronously.

$$
\begin{array}{l}
\text{LOCAL } \mathit{Transaction}(i) \triangleq \\
\quad \text{IF } i \in \text{DOMAIN } \mathit{transaction} \text{ THEN} \\
\qquad \mathit{transaction}[i] \\
\quad \text{ELSE } [
\end{array}
$$

$index \mapsto i,$
$revision \mapsto 0,$
$change \mapsto [$
$index \mapsto 0,$
$revision \mapsto 0],$
$rollback \mapsto [$
$index \mapsto 0,$
$revision \mapsto 0],$
$commit \mapsto Nil,$
$apply \mapsto Nil]$

LOCAL $LastTransaction \triangleq Transaction(Len(transaction))$

CHANGE [*index* = 1, *revision* = 1, *change* = (*index* = 1, *revision* = 1), *rollback* = (*index* = 0, *revision* = 0)] ← − *Change* revision 1
CHANGE [*index* = 2, *revision* = 2, *change* = (*index* = 2, *revision* = 2), *rollback* = (*index* = 1, *revision* = 1)]
CHANGE [*index* = 3, *revision* = 3, *change* = (*index* = 3, *revision* = 3), *rollback* = (*index* = 2, *revision* = 2)]
ROLLBACK [*index* = 4, *revision* = 3, *change* = (*index* = 2, *revision* = 2), *rollback* = (*index* = 3, *revision* = 3)] ← − Roll back revision 3 at index 3, leading to revision 2
ROLLBACK [*index* = 5, *revision* = 3, *change* = (*index* = 1, *revision* = 1), *rollback* = (*index* = 2, *revision* = 2)]
CHANGE [*index* = 6, *revision* = 4, *change* = (*index* = 6, *revision* = 4), *rollback* = (*index* = 1, *revision* = 1)]
CHANGE [*index* = 7, *revision* = 5, *change* = (*index* = 7, *revision* = 5), *rollback* = (*index* = 6, *revision* = 4)]
ROLLBACK [*index* = 8, *revision* = 5, *change* = (*index* = 6, *revision* = 4), *rollback* = (*index* = 7, *revision* = 5)] ← − Roll back revision 5 at index 7, leading to revision 4
ROLLBACK [*index* = 9, *revision* = 5, *change* = (*index* = 1, *revision* = 1), *rollback* = (*index* = 6, *revision* = 4)] ← − Roll back revision 4 at index 6, leading to revision 1 CHANGE [*index* = 10, *revision* = 6, *change* = (*index* = 10, *revision* = 6), *rollback* = (*index* = 1, *revision* = 1)]

Add a change for revision 'i' to the transaction log

$$
\begin{array}{l}
AppendChange(i) \triangleq \\
\quad \land LastTransaction.revision = i - 1 \\
\quad \land Len(transaction) > 0 \Rightarrow transaction[Len(transaction)].commit = Complete \\
\quad \land \exists\, p \in Path,\ v \in Value : \\
\qquad \land transaction' = Append(transaction, [\\
\qquad\qquad type \mapsto Change, \\
\qquad\qquad index \mapsto Len(transaction) + 1, \\
\qquad\qquad revision \mapsto i, \\
\qquad\qquad change \mapsto [\\
\qquad\qquad\quad index \mapsto Len(transaction) + 1, \\
\qquad\qquad\quad revision \mapsto i, \\
\qquad\qquad\quad values \mapsto (p :> v)], \\
\qquad\qquad rollback \mapsto [\\
\qquad\qquad\quad index \mapsto LastTransaction.change.index,
\end{array}
$$

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad revision \mapsto LastTransaction.change.revision, \\
\qquad\qquad\qquad\qquad values \mapsto \text{IF } p \in \text{DOMAIN } configuration.committed.values \text{ THEN} \\
\qquad\qquad\qquad\qquad\qquad (p :> configuration.committed.values[p]) \\
\qquad\qquad\qquad\qquad\quad \text{ELSE} \\
\qquad\qquad\qquad\qquad\qquad (p :> Nil)], \\
\qquad\qquad\qquad\qquad commit \mapsto Pending, \\
\qquad\qquad\qquad\qquad apply \mapsto Pending]) \\
\quad \land \text{UNCHANGED } \langle configuration, mastership, conn, target, history \rangle
\end{array}
$$

Add a rollback of revision 'i' to the transaction log

$$
\begin{array}{l}
RollbackChange(i) \triangleq \\
\quad \land LastTransaction.change.revision = i \\
\quad \land Len(transaction) > 0 \Rightarrow transaction[Len(transaction)].commit = Complete \\
\quad \land transaction' = Append(transaction, [\\
\qquad\qquad\qquad type \mapsto Rollback, \\
\qquad\qquad\qquad index \mapsto Len(transaction) + 1, \\
\qquad\qquad\qquad revision \mapsto LastTransaction.revision, \\
\qquad\qquad\qquad change \mapsto [\\
\qquad\qquad\qquad\quad index \mapsto transaction[LastTransaction.change.index].rollback.index, \\
\qquad\qquad\qquad\quad revision \mapsto transaction[LastTransaction.change.index].rollback.revision, \\
\qquad\qquad\qquad\quad values \mapsto transaction[LastTransaction.change.index].rollback.values], \\
\qquad\qquad\qquad rollback \mapsto [\\
\qquad\qquad\qquad\quad index \mapsto LastTransaction.change.index, \\
\qquad\qquad\qquad\quad revision \mapsto i, \\
\qquad\qquad\qquad\quad values \mapsto Empty], \\
\qquad\qquad\qquad commit \mapsto Pending, \\
\qquad\qquad\qquad apply \mapsto Pending]) \\
\quad \land \text{UNCHANGED } \langle configuration, mastership, conn, target, history \rangle
\end{array}
$$

$$
\begin{array}{l}
CommitChange(n, i) \triangleq \\
\quad \land transaction[i].commit = Pending \\
\quad \land i - 1 \in \text{DOMAIN } transaction \Rightarrow \\
\qquad\quad transaction[i-1].commit \neq Pending \\
\quad \land configuration' = [configuration \text{ EXCEPT } !.committed.index = transaction[i].change.index, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad !.committed.revision = transaction[i].change.revision, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad !.committed.values = transaction[i].change.values \;@@ \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad configuration.committed.values] \\
\quad \land transaction' = [transaction \text{ EXCEPT } ![i].commit = Complete] \\
\quad \land history' = Append(history, [\\
\qquad\qquad\qquad type \mapsto Change, \\
\qquad\qquad\qquad phase \mapsto Commit, \\
\qquad\qquad\qquad revision \mapsto transaction[i].change.revision]) \\
\quad \land \text{UNCHANGED } \langle target \rangle
\end{array}
$$

$$
\begin{array}{l}
ApplyChange(n,\ i)\ \triangleq \\
\quad \land\ transaction[i].apply = Pending \\
\quad \land\ transaction[i].commit = Complete \\
\quad \land\ i-1 \in \text{DOMAIN}\ transaction \Rightarrow \\
\qquad\quad transaction[i-1].apply \neq Pending \\
\quad \land\ \lor\ \land\ i-1 \in \text{DOMAIN}\ transaction \Rightarrow \\
\qquad\qquad\quad transaction[i-1].apply = Complete \\
\qquad \land\ configuration.state = Complete \\
\qquad \land\ configuration.term = mastership.term \\
\qquad \land\ conn[n].id = mastership.conn \\
\qquad \land\ conn[n].connected \\
\qquad \land\ target.running \\
\qquad\quad \text{Apply to the target successfully.} \\
\qquad \land\ \lor\ \land\ target' = [target\ \text{EXCEPT}\ !.values = transaction[i].change.values\ @@\ target.values] \\
\qquad\qquad\quad \land\ configuration' = [configuration\ \text{EXCEPT}\ !.applied.index\ \ = transaction[i].change.index, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad !.applied.revision\ = transaction[i].change.revision, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad !.applied.values\ \ = transaction[i].change.values\ @@ \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad configuration.applied.values] \\
\qquad\qquad\quad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].apply = Complete] \\
\qquad\qquad\quad \land\ history' = Append(history,\ [\\
\qquad\qquad\qquad\qquad\qquad type\ \ \mapsto Change, \\
\qquad\qquad\qquad\qquad\qquad phase\ \ \mapsto Apply, \\
\qquad\qquad\qquad\qquad\qquad revision \mapsto transaction[i].change.revision]) \\
\qquad\qquad\quad \text{Apply to the target failed.} \\
\qquad\qquad \lor\ \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].apply = Failed] \\
\qquad\qquad\quad \land\ \text{UNCHANGED}\ \langle configuration,\ target,\ history\rangle \\
\qquad \lor\ \land\ i-1 \in \text{DOMAIN}\ transaction \\
\qquad\quad \land\ transaction[i-1].apply \in \{Aborted,\ Failed\} \\
\qquad\quad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].apply = Aborted] \\
\qquad\quad \land\ \text{UNCHANGED}\ \langle configuration,\ target,\ history\rangle
\end{array}
$$

$$
\begin{array}{l}
ReconcileChange(n,\ i)\ \triangleq \\
\quad \land\ transaction[i].type = Change \\
\quad \land\ \lor\ CommitChange(n,\ i) \\
\qquad \lor\ ApplyChange(n,\ i)
\end{array}
$$

$$
\begin{array}{l}
CommitRollback(n,\ i)\ \triangleq \\
\quad \land\ transaction[i].commit = Pending \\
\quad \land\ i-1 \in \text{DOMAIN}\ transaction \Rightarrow \\
\qquad\quad transaction[i-1].commit \neq Pending \\
\quad \land\ configuration' = [configuration\ \text{EXCEPT}\ !.committed.index\ \ = transaction[i].change.index, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad !.committed.revision\ = transaction[i].change.revision, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad !.committed.values\ \ = transaction[i].change.values\ @@ \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad configuration.committed.values] \\
\quad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].commit = Complete]
\end{array}
$$

```
    ∧ history′ = Append(history, [
                        type     ↦ Rollback,
                        phase    ↦ Commit,
                        revision ↦ transaction[i].rollback.revision])
    ∧ UNCHANGED ⟨target⟩

ApplyRollback(n, i) ≜
    ∧ transaction[i].apply = Pending
    ∧ transaction[i].commit = Complete
    ∧ i − 1 ∈ DOMAIN transaction ⇒
          transaction[i − 1].apply ≠ Pending
    ∧ ∨ ∧ transaction[transaction[i].rollback.index].apply ∈ {Complete, Failed}
        ∧ configuration.state = Complete
        ∧ configuration.term = mastership.term
        ∧ conn[n].id = mastership.conn
        ∧ conn[n].connected
        ∧ target.running
        ∧ target′ = [target EXCEPT !.values = transaction[i].change.values @@ target.values]
        ∧ configuration′ = [configuration EXCEPT !.applied.index    = transaction[i].change.index,
                                                 !.applied.revision = transaction[i].change.revision,
                                                 !.applied.values   = transaction[i].change.values @@
                                                                      configuration.applied.values]
        ∧ transaction′ = [transaction EXCEPT ![i].apply = Complete]
        ∧ history′ = Append(history, [
                            type     ↦ Rollback,
                            phase    ↦ Apply,
                            revision ↦ transaction[i].rollback.revision])
      ∨ ∧ transaction[transaction[i].rollback.index].apply = Aborted
        ∧ transaction′ = [transaction EXCEPT ![i].apply = Aborted]
        ∧ UNCHANGED ⟨configuration, target, history⟩

ReconcileRollback(n, i) ≜
    ∧ transaction[i].type = Rollback
    ∧ ∨ CommitRollback(n, i)
      ∨ ApplyRollback(n, i)

ReconcileTransaction(n, i) ≜
    ∧ i ∈ DOMAIN transaction
    ∧ ∨ ReconcileChange(n, i)
      ∨ ReconcileRollback(n, i)
    ∧ UNCHANGED ⟨mastership, conn⟩
```