
MODULE *Transaction*

INSTANCE *Naturals*

INSTANCE *FiniteSets*

INSTANCE *Sequences*

INSTANCE *TLC*

An empty constant

CONSTANT *Nil*

Transaction type constants

CONSTANTS

Change,

Rollback

$Type \triangleq \{Change, Rollback\}$

Proposal phase constants

CONSTANTS

Commit,

Apply

Status constants

CONSTANTS

Pending,

InProgress,

Complete,

Aborted,

Failed

$Status \triangleq \{Pending, InProgress, Complete, Aborted, Failed\}$

$Done \triangleq \{Complete, Aborted, Failed\}$

The set of all nodes

CONSTANT *Node*

$Empty \triangleq [p \in \{\} \mapsto Nil]$

Variables defined by other modules.

VARIABLES

proposal,

configuration

A transaction log. Transactions may either request a set of changes to a set of targets or rollback a prior change.

VARIABLE *transaction*

$$\begin{aligned}
TypeOK &\triangleq \\
&\forall i \in \text{DOMAIN } transaction : \\
&\quad \wedge transaction[i].type \in Type \\
&\quad \wedge transaction[i].index \in Nat \\
&\quad \wedge transaction[i].init \in Status \\
&\quad \wedge transaction[i].commit \in Status \\
&\quad \wedge transaction[i].apply \in Status \\
&\quad \wedge \forall p \in \text{DOMAIN } transaction[i].values : \\
&\quad \quad transaction[i].values[p] \neq Nil \Rightarrow transaction[i].values[p] \in STRING \\
Test &\triangleq \text{INSTANCE } Test \text{ WITH} \\
File &\quad \leftarrow \text{"Transaction.log"}, \\
CurrState &\leftarrow [\\
&\quad transactions \mapsto transaction, \\
&\quad proposals \mapsto proposal, \\
&\quad configuration \mapsto configuration], \\
SuccState &\leftarrow [\\
&\quad transactions \mapsto transaction', \\
&\quad proposals \mapsto proposal', \\
&\quad configuration \mapsto configuration']
\end{aligned}$$

This section models configuration changes and rollbacks. Changes are appended to the transaction log and processed asynchronously.

Add a set of changes 'c' to the transaction log

$$\begin{aligned}
RequestChange(p, v) &\triangleq \\
&\wedge transaction' = Append(transaction, [\\
&\quad type \mapsto Change, \\
&\quad index \mapsto 0, \\
&\quad values \mapsto (p :> v), \\
&\quad init \mapsto InProgress, \\
&\quad commit \mapsto Pending, \\
&\quad apply \mapsto Pending]) \\
&\wedge \text{UNCHANGED } \langle proposal, configuration \rangle
\end{aligned}$$

Add a rollback of transaction 't' to the transaction log

$$\begin{aligned}
RequestRollback(i) &\triangleq \\
&\wedge transaction' = Append(transaction, [\\
&\quad type \mapsto Rollback, \\
&\quad index \mapsto i, \\
&\quad values \mapsto Empty,
\end{aligned}$$

$\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 $\vee \wedge \text{transaction}[i].\text{commit} = \text{InProgress}$
 $\wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.phase} = \text{Commit}$
 If the change commit is still in the *Pending* state, set it to *InProgress*.
 $\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Pending}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![\text{transaction}[i].\text{index}].\text{change.state} = \text{InProgress},$
 $![\text{transaction}[i].\text{index}].\text{rollback.revision} = \text{configuration.committed.revision},$
 $![\text{transaction}[i].\text{index}].\text{rollback.values} = [$
 $p \in \text{DOMAIN } \text{proposal}[\text{transaction}[i].\text{index}].\text{change.values} \mapsto$
 IF $p \in \text{DOMAIN } \text{configuration.committed.values}$ THEN
 $\text{configuration.committed.values}[p]$
 ELSE
 $[\text{index} \mapsto 0, \text{value} \mapsto \text{Nil}]]]$
 $\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$
 If the change commit is *Complete*, mark the transaction *Complete*.
 $\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Complete}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{commit} = \text{Complete}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 If the change commit *Failed*, mark the transaction *Failed*.
 $\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Failed}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{commit} = \text{Failed}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$

$\text{ApplyChange}(n, i) \triangleq$

$\wedge \vee \wedge \text{transaction}[i].\text{apply} = \text{Pending}$
 If the commit phase was completed successfully, start the apply phase.
 $\wedge \vee \wedge \text{transaction}[i].\text{commit} = \text{Complete}$
 If the proposal is not in the apply phase, update the proposal phase.
 $\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.phase} \neq \text{Apply}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![\text{transaction}[i].\text{index}].\text{change.phase} = \text{Apply},$
 $![\text{transaction}[i].\text{index}].\text{change.state} = \text{Pending}]$
 $\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$
 If the proposal is in the apply phase and the previous transaction has completed the apply phase, start applying the change.
 $\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.phase} = \text{Apply}$
 $\wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Pending}$
 A transaction cannot be applied until the prior transaction has been applied.
 $\wedge \text{IsApplied}(i - 1)$
 If the prior change failed being applied, it must be rolled back before new changes can be applied.
 $\wedge \wedge \text{transaction}[i].\text{index} - 1 \in \text{DOMAIN } \text{proposal}$
 $\wedge \text{proposal}[\text{transaction}[i].\text{index} - 1].\text{change.phase} = \text{Apply}$
 $\wedge \text{proposal}[\text{transaction}[i].\text{index} - 1].\text{change.state} = \text{Failed}$
 $\Rightarrow \wedge \text{proposal}[\text{transaction}[i].\text{index} - 1].\text{rollback.phase} = \text{Apply}$

$$\begin{aligned}
& \wedge \text{proposal}[\text{transaction}[i].\text{index} - 1].\text{rollback.state} = \text{Complete} \\
& \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{InProgress}] \\
& \wedge \text{UNCHANGED } \langle \text{proposal} \rangle \\
& \text{If the commit phase was aborted or failed, abort the apply phase once the previous} \\
& \text{transaction has completed the apply phase.} \\
& \vee \wedge \text{transaction}[i].\text{commit} \in \{\text{Aborted}, \text{Failed}\} \\
& \quad \text{A transaction cannot be applied until the prior transaction has been applied.} \\
& \quad \wedge \text{IsApplied}(i - 1) \\
& \quad \text{If the prior change failed being applied, it must be rolled back before} \\
& \quad \text{new changes can be applied.} \\
& \quad \wedge \wedge \text{transaction}[i].\text{index} - 1 \in \text{DOMAIN } \text{proposal} \\
& \quad \wedge \text{proposal}[\text{transaction}[i].\text{index} - 1].\text{change.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[\text{transaction}[i].\text{index} - 1].\text{change.state} = \text{Failed} \\
& \quad \Rightarrow \wedge \text{proposal}[\text{transaction}[i].\text{index} - 1].\text{rollback.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[\text{transaction}[i].\text{index} - 1].\text{rollback.state} = \text{Complete} \\
& \quad \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{Aborted}] \\
& \quad \wedge \text{UNCHANGED } \langle \text{proposal} \rangle \\
& \vee \wedge \text{transaction}[i].\text{apply} = \text{InProgress} \\
& \quad \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.phase} = \text{Apply} \\
& \quad \text{If the change apply is still in the } \text{Pending} \text{ state, set it to } \text{InProgress}. \\
& \quad \wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Pending} \\
& \quad \wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![\text{transaction}[i].\text{index}].\text{change.state} = \text{InProgress}] \\
& \quad \wedge \text{UNCHANGED } \langle \text{transaction} \rangle \\
& \quad \text{If the change apply is } \text{Complete}, \text{ mark the transaction } \text{Complete}. \\
& \quad \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Complete} \\
& \quad \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{Complete}] \\
& \quad \wedge \text{UNCHANGED } \langle \text{proposal} \rangle \\
& \quad \text{If the change apply } \text{Failed}, \text{ mark the transaction } \text{Failed}. \\
& \quad \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Failed} \\
& \quad \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{Failed}] \\
& \quad \wedge \text{UNCHANGED } \langle \text{proposal} \rangle
\end{aligned}$$

$$\begin{aligned}
\text{ReconcileChange}(n, i) & \triangleq \\
& \wedge \text{transaction}[i].\text{type} = \text{Change} \\
& \wedge \vee \text{InitChange}(n, i) \\
& \quad \vee \text{CommitChange}(n, i) \\
& \quad \vee \text{ApplyChange}(n, i)
\end{aligned}$$

$$\begin{aligned}
\text{InitRollback}(n, i) & \triangleq \\
& \wedge \vee \wedge \text{transaction}[i].\text{init} = \text{InProgress} \\
& \quad \text{Rollbacks cannot be initialized until all prior transactions have been initialized.} \\
& \quad \wedge \text{IsInitialized}(i - 1) \\
& \quad \text{Rollback transactions must target valid proposal index.} \\
& \quad \wedge \vee \wedge \text{transaction}[i].\text{index} \in \text{DOMAIN } \text{proposal}
\end{aligned}$$

To roll back a transaction, all subsequent transactions must be rolled back first.
 Check whether the following proposal is being rolled back.
 $\wedge \vee \wedge \text{transaction}[i].\text{index} + 1 \in \text{DOMAIN } \text{proposal} \Rightarrow$
 $\text{proposal}[\text{transaction}[i].\text{index} + 1].\text{rollback.phase} \neq \text{Nil}$
 $\wedge \text{transaction}' = [\text{transaction EXCEPT } ![i].\text{init} = \text{Complete}]$
 If the subsequent proposal is not being rolled back, fail the rollback transaction.
 $\vee \wedge \text{transaction}[i].\text{index} + 1 \in \text{DOMAIN } \text{proposal}$
 $\wedge \text{proposal}[\text{transaction}[i].\text{index} + 1].\text{rollback.phase} = \text{Nil}$
 $\wedge \text{transaction}' = [\text{transaction EXCEPT } ![i].\text{init} = \text{Failed}]$
 If the rollback index is not a valid proposal index, fail the rollback request.
 $\vee \wedge \text{transaction}[i].\text{index} \notin \text{DOMAIN } \text{proposal}$
 $\wedge \text{transaction}' = [\text{transaction EXCEPT } ![i].\text{init} = \text{Failed}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$

$\text{CommitRollback}(n, i) \triangleq$

$\wedge \vee \wedge \text{transaction}[i].\text{commit} = \text{Pending}$

A transaction cannot be committed until the prior transaction has been committed.

In the case of rollbacks, we serialize all state changes to ensure consistency

when rolling back changes.

$\wedge \text{IsCommitted}(i - 1)$

If the transaction was initialized successfully, commit the rollback.

$\wedge \vee \wedge \text{transaction}[i].\text{init} = \text{Complete}$

If the target proposal is not yet being rolled back, transition the proposal.

$\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{rollback.phase} = \text{Nil}$

Update the proposal's rollback state based on its change state.

$\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.phase} = \text{Commit}$

If the target change is still pending, abort the change and rollback.

$\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Pending}$

$\wedge \text{proposal}' = [\text{proposal EXCEPT } ![transaction[i].index].\text{change.state} = \text{Aborted},$
 $![transaction[i].index].\text{rollback.phase} = \text{Commit}$
 $![transaction[i].index].\text{rollback.state} = \text{Aborted}]$

$\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$

If the target change is complete, start the rollback commit phase.

$\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Complete}$

$\wedge \text{proposal}' = [\text{proposal EXCEPT } ![transaction[i].index].\text{rollback.phase} = \text{Commit},$
 $![transaction[i].index].\text{rollback.state} = \text{Pending}]$

$\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$

If the target change failed commit, complete the rollback commit.

$\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} \in \{\text{Aborted}, \text{Failed}\}$

$\wedge \text{transaction}' = [\text{transaction EXCEPT } ![i].\text{commit} = \text{Complete}]$

$\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$

If the target change is in the *Apply* phase, commit the rollback.

$\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.phase} = \text{Apply}$

$\wedge \text{proposal}' = [\text{proposal EXCEPT } ![transaction[i].index].\text{rollback.phase} = \text{Commit},$

$![transaction[i].index].rollback.state = Pending]$

$\wedge \text{UNCHANGED } \langle transaction \rangle$

If the target rollback is being committed, transition the underlying proposal.

$\vee \wedge proposal[transaction[i].index].rollback.phase = Commit$

If the target proposal is being rolled back, begin the rollback commit once the prior transaction has completed the commit phase.

$\wedge \vee \wedge proposal[transaction[i].index].rollback.state = Pending$
 $\wedge transaction' = [transaction \text{ EXCEPT } ![i].commit = InProgress]$
 $\wedge \text{UNCHANGED } \langle proposal \rangle$

If the target rollback was aborted, abort the transaction rollback once the prior transaction has completed the commit phase.

$\vee \wedge proposal[transaction[i].index].rollback.state = Aborted$
 $\wedge transaction' = [transaction \text{ EXCEPT } ![i].commit = Aborted]$
 $\wedge \text{UNCHANGED } \langle proposal \rangle$

If the transaction failed initialization, abort the commit phase.

$\vee \wedge transaction[i].init \in \{Aborted, Failed\}$
 $\wedge transaction' = [transaction \text{ EXCEPT } ![i].commit = Aborted]$
 $\wedge \text{UNCHANGED } \langle proposal \rangle$

$\vee \wedge transaction[i].commit = InProgress$
 $\wedge proposal[transaction[i].index].rollback.phase = Commit$

If the rollback commit is still in the *Pending* state, set it to *InProgress*.

$\wedge \vee \wedge proposal[transaction[i].index].rollback.state = Pending$
 $\wedge proposal' = [proposal \text{ EXCEPT } ![transaction[i].index].rollback.state = InProgress]$
 $\wedge \text{UNCHANGED } \langle transaction \rangle$

If the rollback commit is *Complete*, mark the transaction *Complete*.

$\vee \wedge proposal[transaction[i].index].rollback.state = Complete$
 $\wedge transaction' = [transaction \text{ EXCEPT } ![i].commit = Complete]$
 $\wedge \text{UNCHANGED } \langle proposal \rangle$

If the rollback commit *Failed*, mark the transaction *Failed*.

$\vee \wedge proposal[transaction[i].index].rollback.state = Failed$
 $\wedge transaction' = [transaction \text{ EXCEPT } ![i].commit = Failed]$
 $\wedge \text{UNCHANGED } \langle proposal \rangle$

$ApplyRollback(n, i) \triangleq$

$\wedge \vee \wedge transaction[i].apply = Pending$

A transaction cannot be applied until the prior transaction has been applied.

In the case of rollbacks, we serialize all state changes to ensure consistency when rolling back changes.

$\wedge IsApplied(i - 1)$

If the commit phase was completed successfully, start the apply phase.

$\wedge \vee \wedge transaction[i].commit = Complete$

If the target rollback is not yet being applied, transition the rollback.

$\wedge \vee \wedge proposal[transaction[i].index].rollback.phase = Commit$

Update the proposal's rollback state based on its change state.

$\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.phase} = \text{Apply}$
 If the target change is still pending, abort the change and rollback.
 $\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Pending}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![\text{transaction}[i].\text{index}].\text{change.state} = \text{Aborted},$
 $![\text{transaction}[i].\text{index}].\text{rollback.phase} = \text{Apply},$
 $![\text{transaction}[i].\text{index}].\text{rollback.state} = \text{Aborted}]$
 $\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$
 If the target change is complete, start the rollback apply phase.
 $\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} = \text{Complete}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![\text{transaction}[i].\text{index}].\text{rollback.phase} = \text{Apply},$
 $![\text{transaction}[i].\text{index}].\text{rollback.state} = \text{Pending}]$
 $\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$
 If the target change failed apply, complete the rollback apply.
 $\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.state} \in \{ \text{Aborted}, \text{Failed} \}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{Complete}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 If the target change is in the *Commit* phase, abort the change and rollback.
 $\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{change.phase} = \text{Commit}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![\text{transaction}[i].\text{index}].\text{change.state} = \text{Aborted},$
 $![\text{transaction}[i].\text{index}].\text{rollback.phase} = \text{Apply},$
 $![\text{transaction}[i].\text{index}].\text{rollback.state} = \text{Aborted}]$
 $\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$
 If the target rollback is being applied, transition the underlying proposal.
 $\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{rollback.phase} = \text{Apply}$
 If the target proposal is being rolled back, begin the rollback apply
 once the prior transaction has completed the apply phase.
 $\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{rollback.state} = \text{Pending}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{InProgress}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 If the target rollback was aborted, abort the transaction rollback
 once the prior transaction has completed the apply phase.
 $\vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{rollback.state} = \text{Aborted}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{Aborted}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 If the transaction failed initialization, abort the apply phase.
 $\vee \wedge \text{transaction}[i].\text{init} \in \{ \text{Aborted}, \text{Failed} \}$
 $\wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{Aborted}]$
 $\wedge \text{UNCHANGED } \langle \text{proposal} \rangle$
 $\vee \wedge \text{transaction}[i].\text{apply} = \text{InProgress}$
 $\wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{rollback.phase} = \text{Apply}$
 If the rollback apply is still in the *Pending* state, set it to *InProgress*.
 $\wedge \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{rollback.state} = \text{Pending}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![\text{transaction}[i].\text{index}].\text{rollback.state} = \text{InProgress}]$
 $\wedge \text{UNCHANGED } \langle \text{transaction} \rangle$
 If the rollback apply is *Complete*, mark the transaction *Complete*.

$$\begin{aligned}
& \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{rollback.state} = \text{Complete} \\
& \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{Complete}] \\
& \wedge \text{UNCHANGED } \langle \text{proposal} \rangle \\
& \text{If the rollback apply } \text{Failed}, \text{ mark the transaction } \text{Failed}. \\
& \vee \wedge \text{proposal}[\text{transaction}[i].\text{index}].\text{rollback.state} = \text{Failed} \\
& \wedge \text{transaction}' = [\text{transaction} \text{ EXCEPT } ![i].\text{apply} = \text{Failed}] \\
& \wedge \text{UNCHANGED } \langle \text{proposal} \rangle
\end{aligned}$$

$$\begin{aligned}
& \text{ReconcileRollback}(n, i) \triangleq \\
& \wedge \text{transaction}[i].\text{type} = \text{Rollback} \\
& \wedge \vee \text{InitRollback}(n, i) \\
& \vee \text{CommitRollback}(n, i) \\
& \vee \text{ApplyRollback}(n, i)
\end{aligned}$$

$$\begin{aligned}
& \text{Reconcile a transaction} \\
& \text{ReconcileTransaction}(n, i) \triangleq \\
& \wedge i \in \text{DOMAIN } \text{transaction} \\
& \wedge \vee \text{ReconcileChange}(n, i) \\
& \vee \text{ReconcileRollback}(n, i) \\
& \wedge \text{UNCHANGED } \langle \text{configuration} \rangle
\end{aligned}$$
