

EXTENDS

*Northbound*,  
*Transactions*,  
*Proposals*,  
*Configurations*,  
*Southbound*

INSTANCE *Naturals*

INSTANCE *FiniteSets*

INSTANCE *Sequences*

LOCAL INSTANCE *TLC*

*vars*  $\triangleq$   $\langle \text{transaction}, \text{proposal}, \text{configuration}, \text{mastership}, \text{target} \rangle$

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Formal specification, constraints, and theorems.

*Init*  $\triangleq$

$\wedge$  *InitTransaction*  
 $\wedge$  *InitProposal*  
 $\wedge$  *InitConfiguration*  
 $\wedge$  *InitNorthbound*  
 $\wedge$  *InitSouthbound*

*Next*  $\triangleq$

$\vee \wedge$  *NextTransaction*  
 $\wedge$  UNCHANGED  $\langle \text{configuration}, \text{target}, \text{mastership} \rangle$   
 $\vee \wedge$  *NextProposal*  
 $\wedge$  UNCHANGED  $\langle \text{transaction} \rangle$   
 $\vee \wedge$  *NextConfiguration*  
 $\wedge$  UNCHANGED  $\langle \text{transaction}, \text{proposal} \rangle$   
 $\vee \wedge$  *NextNorthbound*  
 $\wedge$  UNCHANGED  $\langle \text{proposal}, \text{configuration}, \text{target}, \text{mastership} \rangle$   
 $\vee \wedge$  *NextSouthbound*  
 $\wedge$  UNCHANGED  $\langle \text{transaction}, \text{proposal}, \text{configuration} \rangle$

*Spec*  $\triangleq$  *Init*  $\wedge$   $\Box[\text{Next}]_{\text{vars}} \wedge \text{WF}_{\text{vars}}(\text{Next})$

*Order*  $\triangleq$

$\forall t \in \text{DOMAIN } \text{proposal} :$   
 $\forall i \in \text{DOMAIN } \text{proposal}[t] :$   
 $\wedge \wedge \text{proposal}[t][i].\text{phase} = \text{ProposalCommit}$   
 $\wedge \text{proposal}[t][i].\text{state} = \text{ProposalInProgress}$

$$\begin{aligned}
& \Rightarrow \neg \exists j \in \text{DOMAIN } \text{proposal}[t] : \\
& \quad \wedge j > i \\
& \quad \wedge \text{proposal}[t][j].\text{phase} = \text{ProposalCommit} \\
& \quad \wedge \text{proposal}[t][j].\text{state} = \text{ProposalComplete} \\
& \wedge \wedge \text{proposal}[t][i].\text{phase} = \text{ProposalApply} \\
& \quad \wedge \text{proposal}[t][i].\text{state} = \text{ProposalInProgress} \\
& \Rightarrow \neg \exists j \in \text{DOMAIN } \text{proposal}[t] : \\
& \quad \wedge j > i \\
& \quad \wedge \text{proposal}[t][j].\text{phase} = \text{ProposalApply} \\
& \quad \wedge \text{proposal}[t][j].\text{state} = \text{ProposalComplete}
\end{aligned}$$

*Consistency*  $\triangleq$

$\forall t \in \text{DOMAIN } \text{proposal} :$

LET

Compute the transaction indexes that have been applied to the target

$$\begin{aligned}
\text{targetIndexes} & \triangleq \{i \in \text{DOMAIN } \text{transaction} : \\
& \quad \wedge i \in \text{DOMAIN } \text{proposal}[t] \\
& \quad \wedge \text{proposal}[t][i].\text{phase} = \text{ProposalApply} \\
& \quad \wedge \text{proposal}[t][i].\text{state} = \text{ProposalComplete} \\
& \quad \wedge t \in \text{DOMAIN } \text{transaction}[i].\text{targets} \\
& \quad \wedge \neg \exists j \in \text{DOMAIN } \text{transaction} : \\
& \quad \quad \wedge j > i \\
& \quad \quad \wedge \text{transaction}[j].\text{type} = \text{TransactionRollback} \\
& \quad \quad \wedge \text{transaction}[j].\text{rollback} = i \\
& \quad \quad \wedge \text{transaction}[j].\text{phase} = \text{TransactionApply} \\
& \quad \quad \wedge \text{transaction}[j].\text{state} = \text{TransactionComplete}\}
\end{aligned}$$

Compute the set of paths in the target that have been updated by transactions

$$\text{appliedPaths} \triangleq \text{UNION } \{\text{DOMAIN } \text{proposal}[t][i].\text{change.values} : i \in \text{targetIndexes}\}$$

Compute the highest index applied to the target for each path

$$\begin{aligned}
\text{pathIndexes} & \triangleq [p \in \text{appliedPaths} \mapsto \text{CHOOSE } i \in \text{targetIndexes} : \\
& \quad \forall j \in \text{targetIndexes} : \\
& \quad \quad \wedge i \geq j \\
& \quad \quad \wedge p \in \text{DOMAIN } \text{proposal}[t][i].\text{change.values}]
\end{aligned}$$

Compute the expected target configuration based on the last indexes applied to the target for each path.

$$\text{expectedConfig} \triangleq [p \in \text{DOMAIN } \text{pathIndexes} \mapsto \text{proposal}[t][\text{pathIndexes}[p]].\text{change.values}[p]]$$

IN

$$\text{target}[t] = \text{expectedConfig}$$

*Isolation*  $\triangleq$

$\forall i \in \text{DOMAIN } \text{transaction} :$

$$\begin{aligned}
& \wedge \wedge \text{transaction}[i].\text{phase} = \text{TransactionCommit} \\
& \quad \wedge \text{transaction}[i].\text{state} = \text{TransactionInProgress} \\
& \quad \wedge \text{transaction}[i].\text{isolation} = \text{Serializable} \\
& \Rightarrow \neg \exists j \in \text{DOMAIN } \text{transaction} :
\end{aligned}$$

$$\begin{aligned}
& \wedge j > i \\
& \wedge \text{DOMAIN } transaction[j].targets \cap \text{DOMAIN } transaction[i].targets \neq \{\} \\
& \wedge transaction[j].phase = \text{TransactionCommit} \\
& \wedge transaction[i].phase = \text{TransactionApply} \\
& \wedge transaction[i].state = \text{TransactionInProgress} \\
& \wedge transaction[i].isolation = \text{Serializable} \\
& \Rightarrow \neg \exists j \in \text{DOMAIN } transaction : \\
& \quad \wedge j > i \\
& \quad \wedge \text{DOMAIN } transaction[j].targets \cap \text{DOMAIN } transaction[i].targets \neq \{\} \\
& \quad \wedge transaction[j].phase = \text{TransactionApply}
\end{aligned}$$

$$Safety \triangleq \Box(\text{Order} \wedge \text{Consistency} \wedge \text{Isolation})$$

THEOREM  $Spec \Rightarrow Safety$

$$\begin{aligned}
Terminated(i) & \triangleq \\
& \wedge i \in \text{DOMAIN } transaction \\
& \wedge transaction[i].phase \in \{\text{TransactionApply}, \text{TransactionAbort}\} \\
& \wedge transaction[i].state = \text{TransactionComplete}
\end{aligned}$$

$$\begin{aligned}
Termination & \triangleq \\
& \forall i \in 1 \dots Len(transaction) : Terminated(i)
\end{aligned}$$

$$Liveness \triangleq \Diamond Termination$$

THEOREM  $Spec \Rightarrow Liveness$

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