
MODULE *Config*

INSTANCE *Naturals*
 INSTANCE *FiniteSets*
 INSTANCE *Sequences*
 LOCAL INSTANCE *TLC*

This section specifies constant parameters for the model.
 CONSTANT *None*
 ASSUME *None* ∈ STRING
 CONSTANT *Node*
 ASSUME $\forall n \in \text{Node} : n \in \text{STRING}$
 CONSTANTS
 Change,
 Rollback
Event $\triangleq \{ \text{Change}, \text{Rollback} \}$
 ASSUME $\forall e \in \text{Event} : e \in \text{STRING}$
 CONSTANTS
 Commit,
 Apply
Phase $\triangleq \{ \text{Commit}, \text{Apply} \}$
 ASSUME $\forall p \in \text{Phase} : p \in \text{STRING}$
 CONSTANTS
 Pending,
 InProgress,
 Complete,
 Aborted,
 Failed
State $\triangleq \{ \text{Pending}, \text{InProgress}, \text{Complete}, \text{Aborted}, \text{Failed} \}$
Done $\triangleq \{ \text{Complete}, \text{Aborted}, \text{Failed} \}$
 ASSUME $\forall s \in \text{State} : s \in \text{STRING}$
 CONSTANT *Path*

ASSUME $\forall p \in Path : p \in \text{STRING}$

CONSTANT *Value*

ASSUME $\forall v \in Value : v \in \text{STRING}$

CONSTANT *NumProposals*

ASSUME *NumProposals* $\in Nat$

This section defines model state variables.

proposal $\triangleq [i \in 1 \dots Nat \mapsto [$
 change $\mapsto [$
 values $\mapsto Change,$
 phase $\mapsto Phase,$
 state $\mapsto State],$
 rollback $\mapsto [$
 index $\mapsto Nat,$
 values $\mapsto Change,$
 phase $\mapsto Phase,$
 state $\mapsto State]]]$

configuration $\triangleq [$
 committed $\mapsto [$
 index $\mapsto Nat,$
 values $\mapsto Change],$
 applied $\mapsto [$
 index $\mapsto Nat,$
 values $\mapsto Change,$
 term $\mapsto Nat]]]$

mastership $\triangleq [$
 master $\mapsto \text{STRING},$
 term $\mapsto Nat,$
 conn $\mapsto Nat]$

conn $\triangleq [n \in Node \mapsto [$
 id $\mapsto Nat,$
 connected $\mapsto \text{BOOLEAN}]]$

target $\triangleq [$
 id $\mapsto Nat,$
 values $\mapsto Change,$
 running $\mapsto \text{BOOLEAN}]$

VARIABLE *proposal*

VARIABLE *configuration*

VARIABLE *mastership*

VARIABLE *conn*

VARIABLE *target*

VARIABLE *history*

$vars \triangleq \langle proposal, configuration, mastership, conn, target, history \rangle$

This section models configuration target.

$StartTarget \triangleq$

$\wedge \neg target.running$
 $\wedge target' = [target \text{ EXCEPT } !.id = target.id + 1,$
 $\quad \quad \quad !.running = TRUE]$
 $\wedge \text{UNCHANGED } \langle proposal, configuration, mastership, conn, history \rangle$

$StopTarget \triangleq$

$\wedge target.running$
 $\wedge target' = [target \text{ EXCEPT } !.running = FALSE,$
 $\quad \quad \quad !.values = [p \in \{\} \mapsto [value \mapsto None]]]$
 $\wedge conn' = [n \in Node \mapsto [conn[n] \text{ EXCEPT } !.connected = FALSE]]$
 $\wedge \text{UNCHANGED } \langle proposal, configuration, mastership, history \rangle$

This section models nodes connection to the configuration target.

$ConnectNode(n) \triangleq$

$\wedge \neg conn[n].connected$
 $\wedge target.running$
 $\wedge conn' = [conn \text{ EXCEPT } ![n].id = conn[n].id + 1,$
 $\quad \quad \quad ![n].connected = TRUE]$
 $\wedge \text{UNCHANGED } \langle proposal, configuration, mastership, target, history \rangle$

$DisconnectNode(n) \triangleq$

$\wedge conn[n].connected$
 $\wedge conn' = [conn \text{ EXCEPT } ![n].connected = FALSE]$
 $\wedge \text{UNCHANGED } \langle proposal, configuration, mastership, target, history \rangle$

This section models *mastership* reconciliation.

$ReconcileMastership(n) \triangleq$

$\wedge \vee \wedge conn[n].connected$
 $\quad \wedge mastership.master = None$
 $\quad \wedge mastership' = [master \mapsto n, term \mapsto mastership.term + 1, conn \mapsto conn[n].id]$
 $\vee \wedge \neg conn[n].connected$
 $\quad \wedge mastership.master = n$

$$\begin{aligned} & \wedge \text{mastership}' = [\text{mastership} \text{ EXCEPT } !.\text{master} = \text{None}] \\ & \wedge \text{UNCHANGED } \langle \text{proposal}, \text{configuration}, \text{conn}, \text{target}, \text{history} \rangle \end{aligned}$$

This section models configuration reconciliation.

$$\begin{aligned} \text{ReconcileConfiguration}(n) & \triangleq \\ & \wedge \text{mastership}.\text{master} = n \\ & \wedge \vee \wedge \text{configuration}.\text{status} \neq \text{InProgress} \\ & \quad \wedge \text{configuration}.\text{applied}.\text{term} < \text{mastership}.\text{term} \\ & \quad \wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } !.\text{status} = \text{InProgress}] \\ & \quad \wedge \text{UNCHANGED } \langle \text{target} \rangle \\ & \vee \wedge \text{configuration}.\text{status} = \text{InProgress} \\ & \quad \wedge \text{configuration}.\text{applied}.\text{term} < \text{mastership}.\text{term} \\ & \quad \wedge \text{conn}[n].\text{connected} \\ & \quad \wedge \text{target}.\text{running} \\ & \quad \wedge \text{target}' = [\text{target} \text{ EXCEPT } !.\text{values} = \text{configuration}.\text{applied}.\text{values}] \\ & \quad \wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } !.\text{applied}.\text{term} = \text{mastership}.\text{term}, \\ & \quad \quad \quad !.\text{applied}.\text{target} = \text{target}.\text{id}, \\ & \quad \quad \quad !.\text{status} = \text{Complete}] \\ & \wedge \text{UNCHANGED } \langle \text{proposal}, \text{mastership}, \text{conn}, \text{history} \rangle \end{aligned}$$

This section models proposal reconciliation.

$$\begin{aligned} \text{CommitChange}(n, i) & \triangleq \\ & \wedge \text{proposal}[i].\text{change}.\text{phase} = \text{Commit} \\ & \wedge \vee \wedge \text{proposal}[i].\text{change}.\text{state} = \text{Pending} \\ & \quad \wedge \text{proposal}[i].\text{rollback}.\text{phase} = \text{None} \\ & \quad \wedge \forall j \in \text{DOMAIN } \text{proposal} : j < i \Rightarrow \\ & \quad \quad \wedge \vee \wedge \text{proposal}[j].\text{change}.\text{phase} = \text{Commit} \\ & \quad \quad \quad \wedge \text{proposal}[j].\text{change}.\text{state} \in \text{Done} \\ & \quad \quad \vee \text{proposal}[j].\text{change}.\text{phase} = \text{Apply} \\ & \quad \quad \wedge \text{proposal}[j].\text{rollback}.\text{phase} \neq \text{None} \Rightarrow \\ & \quad \quad \quad \text{proposal}[j].\text{rollback}.\text{phase} = \text{Apply} \\ & \quad \wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{change}.\text{state} = \text{InProgress}] \\ & \quad \wedge \text{UNCHANGED } \langle \text{configuration}, \text{history} \rangle \\ & \vee \wedge \text{proposal}[i].\text{change}.\text{state} = \text{InProgress} \\ & \quad \text{Changes are validated during the commit phase. If a change fails validation,} \\ & \quad \text{it will be marked failed before being applied to the configuration.} \\ & \quad \text{If all the change values are valid, record the changes required to roll} \\ & \quad \text{back the proposal and the index to which the rollback changes} \\ & \quad \text{will roll back the configuration.} \\ & \quad \wedge \vee \text{LET } \text{rollbackIndex} \triangleq \text{configuration}.\text{committed}.\text{index} \\ & \quad \quad \text{rollbackValues} \triangleq [p \in \text{DOMAIN } \text{proposal}[i].\text{change}.\text{values} \mapsto \\ & \quad \quad \quad \text{IF } p \in \text{DOMAIN } \text{configuration}.\text{committed}.\text{values} \text{ THEN} \end{aligned}$$

$$\begin{aligned}
& \text{configuration.committed.values}[p] \\
& \text{ELSE} \\
& \quad [index \mapsto 0, value \mapsto \text{None}] \\
& \text{changeValues} \triangleq [p \in \text{DOMAIN } \text{proposal}[i].\text{change.values} \mapsto \\
& \quad \text{proposal}[i].\text{change.values}[p] @@ [index \mapsto i]] \\
\text{IN} \quad & \wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } !.\text{committed.index} = i, \\
& \quad !.\text{committed.values} = \text{changeValues}] \\
& \wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{change.values} = \text{changeValues}, \\
& \quad ![i].\text{rollback.values} = \text{rollbackValues}, \\
& \quad ![i].\text{change.phase} = \text{Apply}, \\
& \quad ![i].\text{change.state} = \text{Pending}] \\
& \wedge \text{history}' = \text{Append}(\text{history}, [\text{type} \mapsto \text{Change}, \text{phase} \mapsto \text{Commit}, \text{index} \mapsto i]) \\
& \vee \wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{change.state} = \text{Failed}] \\
& \wedge \text{UNCHANGED } \langle \text{configuration}, \text{history} \rangle \\
& \wedge \text{UNCHANGED } \langle \text{mastership}, \text{conn}, \text{target} \rangle \\
\text{ApplyChange}(n, i) \triangleq & \\
& \wedge \text{proposal}[i].\text{change.phase} = \text{Apply} \\
& \wedge \vee \wedge \text{proposal}[i].\text{change.state} = \text{Pending} \\
& \quad \wedge \text{proposal}[i].\text{rollback.phase} = \text{None} \\
& \wedge \forall j \in \text{DOMAIN } \text{proposal} : j < i \Rightarrow \\
& \quad \vee \wedge \text{proposal}[j].\text{change.phase} = \text{Commit} \\
& \quad \wedge \text{proposal}[j].\text{rollback.state} = \text{Complete} \\
& \quad \vee \wedge \text{proposal}[j].\text{change.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[j].\text{change.state} = \text{Complete} \\
& \quad \wedge \text{proposal}[j].\text{rollback.phase} = \text{None} \\
& \quad \vee \wedge \text{proposal}[j].\text{change.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[j].\text{change.state} = \text{Failed} \\
& \quad \wedge \text{proposal}[j].\text{rollback.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[j].\text{rollback.state} = \text{Complete} \\
& \wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{change.state} = \text{InProgress}] \\
& \wedge \text{UNCHANGED } \langle \text{configuration}, \text{target}, \text{history} \rangle \\
& \vee \wedge \text{proposal}[i].\text{change.state} = \text{InProgress} \\
& \quad \text{Verify the applied term is the current } \text{mastership} \text{ term to ensure the} \\
& \quad \text{configuration has been synchronized following restarts.} \\
& \wedge \text{configuration.applied.term} = \text{mastership.term} \\
& \quad \text{Verify the node's connection to the target.} \\
& \wedge \text{conn}[n].\text{connected} \\
& \wedge \text{mastership.conn} = \text{conn}[n].\text{id} \\
& \wedge \text{target.running} \\
& \quad \text{Model successful and failed target update requests.} \\
& \wedge \vee \wedge \text{target}' = [\text{target} \text{ EXCEPT } !.\text{values} = \text{proposal}[i].\text{change.values} @@ \text{target.values}] \\
& \quad \wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } !.\text{applied.index} = i, \\
& \quad \quad !.\text{applied.values} = \text{proposal}[i].\text{change.values} @@ \\
& \quad \quad \text{configuration.applied.values}]
\end{aligned}$$

$\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{change.state} = \text{Complete}]$
 $\wedge \text{history}' = \text{Append}(\text{history}, [\text{type} \mapsto \text{Change}, \text{phase} \mapsto \text{Apply}, \text{index} \mapsto i])$
 If the proposal could not be applied, mark it failed but do not update the
 last applied index. The proposal must be rolled back before new proposals
 can be applied to the configuration/target.
 $\vee \wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{change.state} = \text{Failed}]$
 $\wedge \text{UNCHANGED } \langle \text{configuration}, \text{target}, \text{history} \rangle$
 $\wedge \text{UNCHANGED } \langle \text{mastership}, \text{conn} \rangle$

$\text{CommitRollback}(n, i) \triangleq$
 $\wedge \text{proposal}[i].\text{rollback.phase} = \text{Commit}$
 $\wedge \vee \wedge \text{proposal}[i].\text{rollback.state} = \text{Pending}$
 $\wedge \forall j \in \text{DOMAIN } \text{proposal} : j > i \Rightarrow$
 $\vee \wedge \text{proposal}[j].\text{rollback.phase} = \text{Commit}$
 $\wedge \text{proposal}[j].\text{rollback.state} \in \text{Done}$
 $\vee \text{proposal}[j].\text{rollback.phase} = \text{Apply}$
 $\wedge \vee \wedge \text{proposal}[i].\text{change.phase} = \text{Commit}$
 $\wedge \text{proposal}[i].\text{change.state} = \text{Pending}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{change.state} = \text{Aborted},$
 $\phantom{\wedge \text{proposal}' = [} ![i].\text{rollback.state} = \text{Complete}]$
 $\vee \wedge \text{proposal}[i].\text{change.phase} = \text{Commit}$
 $\wedge \text{proposal}[i].\text{change.state} \in \text{Done}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{rollback.state} = \text{Complete}]$
 $\vee \wedge \text{proposal}[i].\text{change.phase} = \text{Apply}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{rollback.state} = \text{InProgress}]$
 $\wedge \text{UNCHANGED } \langle \text{configuration}, \text{history} \rangle$
 $\vee \wedge \text{proposal}[i].\text{rollback.state} = \text{InProgress}$
 $\wedge \text{LET } \text{index} \triangleq \text{proposal}[i].\text{rollback.index}$
 $\phantom{\wedge \text{LET } } \text{values} \triangleq \text{proposal}[i].\text{rollback.values} @@ \text{configuration.committed.values}$
 $\text{IN } \wedge \text{configuration}' = [\text{configuration} \text{ EXCEPT } !.\text{committed.index} = \text{index},$
 $\phantom{\wedge \text{configuration}' = [} !.\text{committed.values} = \text{values}]$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{rollback.phase} = \text{Apply},$
 $\phantom{\wedge \text{proposal}' = [} ![i].\text{rollback.state} = \text{Pending}]$
 $\wedge \text{history}' = \text{Append}(\text{history}, [\text{type} \mapsto \text{Rollback}, \text{phase} \mapsto \text{Commit}, \text{index} \mapsto i])$
 $\wedge \text{UNCHANGED } \langle \text{mastership}, \text{conn}, \text{target} \rangle$

$\text{ApplyRollback}(n, i) \triangleq$
 $\wedge \text{proposal}[i].\text{rollback.phase} = \text{Apply}$
 $\wedge \vee \wedge \text{proposal}[i].\text{rollback.state} = \text{Pending}$
 $\wedge \forall j \in \text{DOMAIN } \text{proposal} : j > i \Rightarrow$
 $\wedge \text{proposal}[j].\text{rollback.phase} = \text{Apply}$
 $\wedge \text{proposal}[j].\text{rollback.state} \in \text{Done}$
 $\wedge \vee \wedge \text{proposal}[i].\text{change.phase} = \text{Apply}$
 $\wedge \text{proposal}[i].\text{change.state} = \text{Pending}$
 $\wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{change.state} = \text{Aborted},$

$$\begin{aligned}
& \wedge \text{proposal}[i].\text{rollback.phase} = \text{None} \\
& \wedge \text{proposal}' = [\text{proposal} \text{ EXCEPT } ![i].\text{rollback.phase} = \text{Commit}, \\
& \quad \quad \quad ![i].\text{rollback.state} = \text{Pending}] \\
& \wedge \text{UNCHANGED } \langle \text{configuration}, \text{mastership}, \text{conn}, \text{target}, \text{history} \rangle
\end{aligned}$$

Formal specification, constraints, and theorems.

$$\begin{aligned}
\text{Init} & \triangleq \\
& \wedge \text{proposal} = [\\
& \quad i \in 1 \dots \text{NumProposals} \mapsto [\\
& \quad \quad \text{change} \mapsto [\\
& \quad \quad \quad \text{values} \mapsto [p \in \{\} \mapsto [\text{index} \mapsto 0, \text{value} \mapsto \text{None}]], \\
& \quad \quad \quad \text{phase} \mapsto \text{None}, \\
& \quad \quad \quad \text{state} \mapsto \text{None}], \\
& \quad \quad \text{rollback} \mapsto [\\
& \quad \quad \quad \text{index} \mapsto 0, \\
& \quad \quad \quad \text{values} \mapsto [p \in \{\} \mapsto [\text{index} \mapsto 0, \text{value} \mapsto \text{None}]], \\
& \quad \quad \quad \text{phase} \mapsto \text{None}, \\
& \quad \quad \quad \text{state} \mapsto \text{None}]] \\
& \wedge \text{configuration} = [\\
& \quad \text{committed} \mapsto [\\
& \quad \quad \text{index} \mapsto 0, \\
& \quad \quad \text{values} \mapsto [p \in \{\} \mapsto [\text{index} \mapsto 0, \text{value} \mapsto \text{None}]]], \\
& \quad \text{applied} \mapsto [\\
& \quad \quad \text{index} \mapsto 0, \\
& \quad \quad \text{term} \mapsto 0, \\
& \quad \quad \text{target} \mapsto 0, \\
& \quad \quad \text{values} \mapsto [p \in \{\} \mapsto [\text{index} \mapsto 0, \text{value} \mapsto \text{None}]]], \\
& \quad \text{status} \mapsto \text{Pending}] \\
& \wedge \text{mastership} = [\text{master} \mapsto \text{None}, \text{term} \mapsto 0, \text{conn} \mapsto 0] \\
& \wedge \text{conn} = [n \in \text{Node} \mapsto [\text{id} \mapsto 0, \text{connected} \mapsto \text{FALSE}]] \\
& \wedge \text{target} = [\\
& \quad \text{id} \mapsto 0, \\
& \quad \text{values} \mapsto [p \in \{\} \mapsto [\text{index} \mapsto 0, \text{value} \mapsto \text{None}]], \\
& \quad \text{running} \mapsto \text{FALSE}] \\
& \wedge \text{history} = \langle \rangle \\
\text{Next} & \triangleq \\
& \vee \exists i \in 1 \dots \text{NumProposals} : \\
& \quad \vee \text{ProposeChange}(i) \\
& \quad \vee \text{ProposeRollback}(i) \\
& \vee \exists n \in \text{Node}, i \in \text{DOMAIN } \text{proposal} : \text{ReconcileProposal}(n, i) \\
& \vee \exists n \in \text{Node} : \text{ReconcileConfiguration}(n) \\
& \vee \exists n \in \text{Node} : \text{ReconcileMastership}(n) \\
& \vee \exists n \in \text{Node} :
\end{aligned}$$

$$\begin{aligned}
& \vee \text{ConnectNode}(n) \\
& \vee \text{DisconnectNode}(n) \\
& \vee \text{StartTarget} \\
& \vee \text{StopTarget} \\
\text{Spec} & \triangleq \\
& \wedge \text{Init} \\
& \wedge \Box[\text{Next}]_{\text{vars}} \\
& \wedge \forall i \in 1 \dots \text{NumProposals} : \text{WF}_{\text{vars}}(\text{ProposeChange}(i) \vee \text{ProposeRollback}(i)) \\
& \wedge \forall n \in \text{Node}, i \in 1 \dots \text{NumProposals} : \text{WF}_{\text{vars}}(\text{ReconcileProposal}(n, i)) \\
& \wedge \forall n \in \text{Node} : \text{WF}_{\langle \text{configuration}, \text{mastership}, \text{conn}, \text{target} \rangle}(\text{ReconcileConfiguration}(n)) \\
& \wedge \forall n \in \text{Node} : \text{WF}_{\langle \text{mastership}, \text{conn}, \text{target} \rangle}(\text{ReconcileMastership}(n)) \\
& \wedge \forall n \in \text{Node} : \text{WF}_{\langle \text{conn}, \text{target} \rangle}(\text{ConnectNode}(n) \vee \text{DisconnectNode}(n)) \\
& \wedge \text{WF}_{\langle \text{target} \rangle}(\text{StartTarget}) \\
& \wedge \text{WF}_{\langle \text{target} \rangle}(\text{StopTarget}) \\
\text{IsOrderedChange}(p, i) & \triangleq \\
& \wedge \text{history}[i].\text{type} = \text{Change} \\
& \wedge \text{history}[i].\text{phase} = p \\
& \wedge \neg \exists j \in \text{DOMAIN } \text{history} : \\
& \quad \wedge j < i \\
& \quad \wedge \text{history}[j].\text{type} = \text{Change} \\
& \quad \wedge \text{history}[j].\text{phase} = p \\
& \quad \wedge \text{history}[j].\text{index} \geq \text{history}[i].\text{index} \\
\text{IsOrderedRollback}(p, i) & \triangleq \\
& \wedge \text{history}[i].\text{type} = \text{Rollback} \\
& \wedge \text{history}[i].\text{phase} = p \\
& \wedge \neg \exists j \in \text{DOMAIN } \text{history} : \\
& \quad \wedge j < i \\
& \quad \wedge \text{history}[j].\text{type} = \text{Change} \\
& \quad \wedge \text{history}[j].\text{phase} = p \\
& \quad \wedge \text{history}[j].\text{index} > \text{history}[i].\text{index} \\
& \wedge \neg \exists k \in \text{DOMAIN } \text{history} : \\
& \quad \wedge k > j \\
& \quad \wedge k < i \\
& \quad \wedge \text{history}[k].\text{type} = \text{Rollback} \\
& \quad \wedge \text{history}[k].\text{phase} = p \\
& \quad \wedge \text{history}[k].\text{index} = \text{history}[j].\text{index} \\
\text{Order} & \triangleq \\
& \wedge \forall i \in \text{DOMAIN } \text{history} : \\
& \quad \vee \text{IsOrderedChange}(\text{Commit}, i) \\
& \quad \vee \text{IsOrderedChange}(\text{Apply}, i) \\
& \quad \vee \text{IsOrderedRollback}(\text{Commit}, i) \\
& \quad \vee \text{IsOrderedRollback}(\text{Apply}, i)
\end{aligned}$$

$$\begin{aligned}
& \wedge \forall i \in \text{DOMAIN } \text{proposal} : \\
& \quad \wedge \text{proposal}[i].\text{change.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[i].\text{change.state} \notin \{\text{Pending}, \text{Aborted}\} \\
& \Rightarrow \forall j \in \text{DOMAIN } \text{proposal} : \\
& \quad \wedge j < i \\
& \quad \wedge \text{proposal}[j].\text{change.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[j].\text{change.state} = \text{Failed} \\
& \quad \Rightarrow \wedge \text{proposal}[j].\text{rollback.phase} = \text{Apply} \\
& \quad \quad \wedge \text{proposal}[j].\text{rollback.state} = \text{Complete}
\end{aligned}$$

$$\begin{aligned}
\text{Consistency} & \triangleq \\
& \wedge \text{target.running} \\
& \wedge \text{configuration.status} = \text{Complete} \\
& \wedge \text{configuration.applied.target} = \text{target.id} \\
& \Rightarrow \forall i \in \text{DOMAIN } \text{proposal} : \\
& \quad \wedge \text{proposal}[i].\text{change.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[i].\text{change.state} = \text{Complete} \\
& \quad \wedge \text{proposal}[i].\text{rollback.state} \neq \text{Complete} \\
& \Rightarrow \forall p \in \text{DOMAIN } \text{proposal}[i].\text{change.values} : \\
& \quad \wedge \neg \exists j \in \text{DOMAIN } \text{proposal} : \\
& \quad \quad \wedge j > i \\
& \quad \quad \wedge \text{proposal}[i].\text{change.phase} = \text{Apply} \\
& \quad \quad \wedge \text{proposal}[i].\text{change.state} = \text{Complete} \\
& \quad \quad \wedge \text{proposal}[i].\text{rollback.state} \neq \text{Complete} \\
& \Rightarrow \wedge p \in \text{DOMAIN } \text{target.values} \\
& \quad \wedge \text{target.values}[p].\text{value} = \text{proposal}[i].\text{change.values}[p].\text{value} \\
& \quad \wedge \text{target.values}[p].\text{index} = \text{proposal}[i].\text{change.values}[p].\text{index}
\end{aligned}$$

$$\text{Safety} \triangleq \Box(\text{Order} \wedge \text{Consistency})$$

THEOREM $\text{Spec} \Rightarrow \text{Safety}$

$$\begin{aligned}
\text{Termination} & \triangleq \\
& \forall i \in 1 \dots \text{NumProposals} : \\
& \quad \wedge \text{proposal}[i].\text{change.phase} = \text{Commit} \leadsto \\
& \quad \quad \vee \wedge \text{proposal}[i].\text{change.phase} = \text{Commit} \\
& \quad \quad \quad \wedge \text{proposal}[i].\text{change.state} \in \text{Done} \\
& \quad \quad \vee \text{proposal}[i].\text{change.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[i].\text{change.phase} = \text{Apply} \leadsto \\
& \quad \quad \wedge \text{proposal}[i].\text{change.state} \in \text{Done} \\
& \quad \wedge \text{proposal}[i].\text{rollback.phase} = \text{Commit} \leadsto \\
& \quad \quad \vee \wedge \text{proposal}[i].\text{rollback.phase} = \text{Commit} \\
& \quad \quad \quad \wedge \text{proposal}[i].\text{rollback.state} \in \text{Done} \\
& \quad \quad \vee \text{proposal}[i].\text{rollback.phase} = \text{Apply} \\
& \quad \wedge \text{proposal}[i].\text{rollback.phase} = \text{Apply} \leadsto \\
& \quad \quad \wedge \text{proposal}[i].\text{rollback.state} \in \text{Done}
\end{aligned}$$

$$\begin{aligned}
& \wedge \vee \wedge \text{proposal}[i].\text{change.phase} = \text{Commit} \\
& \quad \wedge \text{proposal}[i].\text{change.state} \in \text{Done} \\
& \vee \text{proposal}[i].\text{change.phase} = \text{Apply}
\end{aligned}$$

$$\text{Liveness} \triangleq \text{Termination}$$

THEOREM $\text{Spec} \Rightarrow \text{Liveness}$

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