MODULE *Transaction*

INSTANCE *Naturals*

INSTANCE *FiniteSets*

INSTANCE *Sequences*

INSTANCE *TLC*

An empty constant
CONSTANT *Nil*

Transaction type constants
CONSTANTS
Change,
Rollback

Transaction isolation constants
CONSTANTS
ReadCommitted,
Serializable

Phase constants
CONSTANTS
Initialize,
Validate,
Abort,
Commit,
Apply

$Phase \triangleq$
{*Initialize*,
Validate,
Abort,
Commit,
Apply}

Status constants
CONSTANTS
InProgress,
Complete,
Failed

$State \triangleq$
{*InProgress*,
Complete,

Failed}

State constants
CONSTANTS
Pending,
Validated,
Committed,
Applied,
Aborted

Status $\triangleq$
{*Pending*,
Validated,
Committed,
Applied,
Aborted}

CONSTANTS
Valid,
Invalid

CONSTANTS
Success,
Failure

The set of all nodes
CONSTANT *Node*

Target is the set of all targets and their possible paths and values.

Example:
Target $\triangleq$
[*target*1 $\mapsto$
[persistent $\mapsto$ FALSE, *values* $\mapsto$ [
*path*1 $\mapsto$ {"*value*1", "*value*2"},
*path*2 $\mapsto$ {"*value*2", "*value*3"}]],
*target*2 $\mapsto$
[persistent $\mapsto$ TRUE, *values* $\mapsto$ [
*path*2 $\mapsto$ {"*value*3", "*value*4"},
*path*3 $\mapsto$ {"*value*4", "*value*5"}]]]
CONSTANT *Target*

Empty $\triangleq$ $[p \in \{\} \mapsto [value \mapsto Nil,\ delete \mapsto \text{FALSE}]]$

A transaction log. Transactions may either request a set
of changes to a set of targets or rollback a prior change.
VARIABLE *transaction*

A record of per-target proposals
VARIABLE *proposal*

A record of per-target configurations
VARIABLE *configuration*

A record of target states
VARIABLE *target*

A record of target masterships
VARIABLE *mastership*

This section models configuration changes and rollbacks. Changes are appended to the transaction log and processed asynchronously.

Add a set of changes 'c' to the transaction log

$$
\begin{array}{l}
\mathit{RequestChange}(i,\ c) \triangleq \\
\quad \land\ i = \mathit{Len}(\mathit{transaction}) + 1 \\
\quad \land\ \exists\, \mathit{isolation} \in \{\mathit{ReadCommitted},\ \mathit{Serializable}\} : \\
\qquad \land\ \mathit{transaction}' = \mathit{transaction} \mathbin{@@} (i :> [\mathit{type} \mapsto \mathit{Change}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{isolation} \mapsto \mathit{isolation}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{change} \mapsto c, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{targets} \mapsto \{\}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{phase} \mapsto \mathit{Initialize}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{state} \mapsto \mathit{InProgress}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{status} \mapsto \mathit{Pending}]) \\
\quad \land\ \text{UNCHANGED}\ \langle \mathit{proposal},\ \mathit{configuration},\ \mathit{mastership},\ \mathit{target} \rangle
\end{array}
$$

Add a rollback of transaction 't' to the transaction log

$$
\begin{array}{l}
\mathit{RequestRollback}(i,\ j) \triangleq \\
\quad \land\ i = \mathit{Len}(\mathit{transaction}) + 1 \\
\quad \land\ \exists\, \mathit{isolation} \in \{\mathit{ReadCommitted},\ \mathit{Serializable}\} : \\
\qquad \land\ \mathit{transaction}' = \mathit{transaction} \mathbin{@@} (i :> [\mathit{type} \mapsto \mathit{Rollback}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{isolation} \mapsto \mathit{isolation}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{rollback} \mapsto j, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{targets} \mapsto \{\}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{phase} \mapsto \mathit{Initialize}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{state} \mapsto \mathit{InProgress}, \\
\qquad\qquad\qquad\qquad\qquad\qquad \mathit{status} \mapsto \mathit{Pending}]) \\
\quad \land\ \text{UNCHANGED}\ \langle \mathit{proposal},\ \mathit{configuration},\ \mathit{mastership},\ \mathit{target} \rangle
\end{array}
$$

This section models the *Transaction* log reconciler.

Transactions come in two flavors: - *Change* transactions contain a set of changes to be applied to a set of targets - *Rollback* transactions reference a prior change transaction to be reverted to the previous state

Transacations proceed through a series of phases:
* *Initialize* - create and link Proposals
* *Validate* - validate changes and rollbacks
* *Commit* - commit changes to Configurations
* *Apply* - commit changes to Targets

Reconcile a transaction

$ReconcileTransaction(n,\ i) \triangleq$

Initialize is the only transaction phase that's globally serialized. While in the Initializing phase, the reconciler checks whether the prior transaction has been Initialized before creating Proposals in the *Initialize* phase. Once all of the transaction's proposals have been Initialized, the transaction will be marked Initialized. If any proposal is *Failed*, the transaction will be marked *Failed* as well.

$\land\ \lor\ \land\ transaction[i].phase = Initialize$
$\quad \land\ \lor\ \land\ transaction[i].state = InProgress$

All prior transaction must be initialized before proceeding to initialize this transaction.

$\quad \land \neg \exists\, j \in \text{DOMAIN}\ transaction :$
$\quad\quad \land\ j < i$
$\quad\quad \land\ transaction[j].phase = Initialize$
$\quad\quad \land\ transaction[j].state\ = InProgress$

If the transaction's targets are not yet set, create proposals and add targets to the transaction state.

$\quad \land\ \lor\ \land\ transaction[i].targets = \{\}$

If the transaction is a change, the targets are taken from the change values.

$\quad\quad \land\ \lor\ \land\ transaction[i].type = Change$
$\quad\quad\quad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].targets = \text{DOMAIN}\ transaction[i].change]$
$\quad\quad\quad \land\ proposal' = [t \in \text{DOMAIN}\ proposal \mapsto$
$\quad\quad\quad\quad \text{IF}\ t \in \text{DOMAIN}\ transaction[i].change\ \text{THEN}$
$\quad\quad\quad\quad\quad proposal[t]\ @@\ (i :> [type \mapsto Change,$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad change \mapsto$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad [index \mapsto i,$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\ values \mapsto transaction[i].change[t]],$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad rollback \mapsto$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad [index \mapsto 0,$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\ values \mapsto Empty],$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad dependency \mapsto [index \mapsto 0],$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad phase \mapsto Initialize,$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad state \mapsto InProgress])$
$\quad\quad\quad\quad \text{ELSE}$
$\quad\quad\quad\quad\quad proposal[t]]$

```
                If the transaction is a rollback, the targets affected are
                the targets of the change transaction being rolled back.
            ∨ ∧ transaction[i].type = Rollback
                    If the rollback index is a valid Change transaction,
                    initialize proposals for all of the Change targets.
                ∧ ∨ ∧ transaction[i].rollback ∈ DOMAIN transaction
                    ∧ transaction[transaction[i].rollback].type = Change
                    ∧ transaction′ = [transaction EXCEPT ![i].targets =
                                        DOMAIN transaction[transaction[i].rollback].change]
                    ∧ proposal′ = [t ∈ DOMAIN proposal ↦
                          IF t ∈ DOMAIN transaction[transaction[i].rollback].change THEN
                             proposal[t] @@ (i :> [type       ↦ Rollback,
                                                   change     ↦
                                                     [index   ↦ 0,
                                                      values  ↦ Empty],
                                                   rollback   ↦
                                                     [index   ↦ transaction[i].rollback,
                                                      values  ↦ Empty],
                                                   dependency ↦ [index ↦ 0],
                                                   phase      ↦ Initialize,
                                                   state      ↦ InProgress])
                          ELSE
                             proposal[t]]
                    If the rollback index is not a valid Change transaction
                    fail the Rollback transaction.
                ∨ ∧ ∨ ∧ transaction[i].rollback ∈ DOMAIN transaction
                      ∧ transaction[transaction[i].rollback].type = Rollback
                    ∨ transaction[i].rollback ∉ DOMAIN transaction
                    ∧ transaction′ = [transaction EXCEPT ![i].state = Failed]
                    ∧ UNCHANGED ⟨proposal⟩
            If the transaction's proposals have been initialized, check proposals
            for completion or failures.
        ∨ ∧ transaction[i].targets ≠ {}
                If all proposals have been Complete, mark the transaction Complete.
            ∧ ∨ ∧ ∀ t ∈ transaction[i].targets :
                      ∧ proposal[t][i].phase = Initialize
                      ∧ proposal[t][i].state  = Complete
                  ∧ transaction′ = [transaction EXCEPT ![i].state = Complete]
                  ∧ UNCHANGED ⟨proposal⟩
                If any proposal has been Failed, mark the transaction Failed.
              ∨ ∧ ∃ t ∈ transaction[i].targets :
                      ∧ proposal[t][i].phase  = Initialize
                      ∧ proposal[t][i].state = Failed
                  ∧ transaction′ = [transaction EXCEPT ![i].state = Failed]
                  ∧ UNCHANGED ⟨proposal⟩
```

Once the transaction has been Initialized, proceed to the *Validate* phase.
If any of the transaction's proposals depend on a *Serializable* transaction,
verify the dependency has been *Validated* to preserve serializability before
moving the transaction to the *Validate* phase.

$$
\begin{array}{l}
\lor\ \land\ transaction[i].state = Complete \\
\quad \land\ \forall\, t \in transaction[i].targets : \\
\qquad \land\ proposal[t][i].dependency.index \in \text{DOMAIN } transaction \\
\qquad \land\ transaction[proposal[t][i].dependency.index].isolation = Serializable \\
\qquad \Rightarrow\ transaction[proposal[t][i].dependency.index].status \in \{\, Validated,\ Committed,\ Applied,\ A \\
\quad \land\ transaction' = [transaction \text{ EXCEPT } ![i].phase = Validate, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad ![i].state\ \ = InProgress] \\
\quad \land \text{ UNCHANGED } \langle proposal \rangle
\end{array}
$$

If the transaction failed initialization, proceed to the *Abort* phase
to ensure indexes are still updated for the target configurations.

$$
\begin{array}{l}
\lor\ \land\ transaction[i].state = Failed \\
\quad \land\ transaction' = [transaction \text{ EXCEPT } ![i].phase = Abort, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad ![i].state\ \ = InProgress] \\
\quad \land \text{ UNCHANGED } \langle proposal \rangle
\end{array}
$$

$$
\begin{array}{l}
\lor\ \land\ transaction[i].phase = Validate \\
\quad \land\ \lor\ \land\ transaction[i].state = InProgress \\
\qquad\qquad \text{Move the transaction's proposals to the Validating state} \\
\qquad\quad \land\ \lor\ \land\ \exists\, t \in transaction[i].targets : \\
\qquad\qquad\qquad\qquad \land\ proposal[t][i].phase \neq Validate \\
\qquad\qquad\qquad\qquad \land\ proposal' = [proposal \text{ EXCEPT } ![t] = \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad [proposal[t] \text{ EXCEPT } ![i].phase = Validate, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad ![i].state\ \ = InProgress]] \\
\qquad\qquad\quad \land \text{ UNCHANGED } \langle transaction \rangle \\
\qquad\qquad \text{If all proposals have been } \textit{Complete}\text{, mark the transaction } \textit{Complete}. \\
\qquad\qquad \lor\ \land\ \forall\, t \in transaction[i].targets : \\
\qquad\qquad\qquad\qquad \land\ proposal[t][i].phase\ \ = Validate \\
\qquad\qquad\qquad\qquad \land\ proposal[t][i].state = Complete \\
\qquad\qquad\quad \land\ transaction' = [transaction \text{ EXCEPT } ![i].state\ \ = Complete, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad ![i].status = Validated] \\
\qquad\qquad\quad \land \text{ UNCHANGED } \langle proposal \rangle \\
\qquad\qquad \text{If any proposal has been } \textit{Failed}\text{, mark the transaction } \textit{Failed}. \\
\qquad\qquad \lor\ \land\ \exists\, t \in transaction[i].targets : \\
\qquad\qquad\qquad\qquad \land\ proposal[t][i].phase\ \ = Validate \\
\qquad\qquad\qquad\qquad \land\ proposal[t][i].state = Failed \\
\qquad\qquad\quad \land\ transaction' = [transaction \text{ EXCEPT } ![i].state = Failed] \\
\qquad\qquad\quad \land \text{ UNCHANGED } \langle proposal \rangle
\end{array}
$$

Once the transaction has been *Validated*, proceed to the *Commit* phase.
If any of the transaction's proposals depend on a *Serializable* transaction,
verify the dependency has been *Committed* to preserve serializability before
moving the transaction to the *Commit* phase.

$$
\lor\ \land\ transaction[i].state = Complete
$$

$\land \forall\, t \in transaction[i].targets :$
$\land\ proposal[t][i].dependency.index \in \text{DOMAIN}\ transaction$
$\land\ transaction[proposal[t][i].dependency.index].isolation = Serializable$
$\Rightarrow\ transaction[proposal[t][i].dependency.index].status \in \{Committed,\ Applied,\ Aborted\}$
$\land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].phase = Commit,$
$![i].state\ = InProgress]$
$\land\ \text{UNCHANGED}\ \langle proposal \rangle$

If the transaction failed validation, proceed to the *Abort* phase
to ensure indexes are still updated for the target configurations.

$\lor\ \land\ transaction[i].state = Failed$
$\land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].phase = Abort,$
$![i].state\ = InProgress]$
$\land\ \text{UNCHANGED}\ \langle proposal \rangle$

$\lor\ \land\ transaction[i].phase = Commit$
$\land\ \lor\ \land\ transaction[i].state = InProgress$

Move the transaction's proposals to the Committing state

$\land\ \lor\ \land \exists\, t \in transaction[i].targets :$
$\land\ proposal[t][i].phase \neq Commit$
$\land\ proposal' = [proposal\ \text{EXCEPT}\ ![t] =$
$[proposal[t]\ \text{EXCEPT}\ ![i].phase = Commit,$
$![i].state\ = InProgress]]$
$\land\ \text{UNCHANGED}\ \langle transaction \rangle$

If all proposals have been *Complete*, mark the transaction *Complete*.

$\lor\ \land \forall\, t \in transaction[i].targets :$
$\land\ proposal[t][i].phase\ = Commit$
$\land\ proposal[t][i].state = Complete$
$\land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].state\ = Complete,$
$![i].status = Committed]$
$\land\ \text{UNCHANGED}\ \langle proposal \rangle$

Once the transaction has been *Committed*, proceed to the *Apply* phase.
If any of the transaction's proposals depend on a *Serializable* transaction,
verify the dependency has been *Applied* to preserve serializability before
moving the transaction to the *Apply* phase.

$\lor\ \land\ transaction[i].state = Complete$
$\land \forall\, t \in transaction[i].targets :$
$\land\ proposal[t][i].dependency.index \in \text{DOMAIN}\ transaction$
$\land\ transaction[proposal[t][i].dependency.index].isolation = Serializable$
$\Rightarrow\ transaction[proposal[t][i].dependency.index].status \in \{Applied,\ Aborted\}$
$\land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].phase = Apply,$
$![i].state\ = InProgress]$
$\land\ \text{UNCHANGED}\ \langle proposal \rangle$

$\lor\ \land\ transaction[i].phase = Apply$
$\land\ transaction[i].state\ = InProgress$

Move the transaction's proposals to the Applying state

$\land\ \lor\ \land \exists\, t \in transaction[i].targets :$

$\land\ proposal[t][i].phase \neq Apply$
$\land\ proposal' = [proposal \text{ EXCEPT } ![t] =$
$[proposal[t] \text{ EXCEPT } ![i].phase = Apply,$
$![i].state = InProgress]]$
$\land$ UNCHANGED $\langle transaction \rangle$

If all proposals have been *Complete*, mark the transaction *Complete*.

$\lor\ \land \forall\, t \in transaction[i].targets :$
$\land\ proposal[t][i].phase = Apply$
$\land\ proposal[t][i].state = Complete$
$\land\ transaction' = [transaction \text{ EXCEPT } ![i].state = Complete,$
$![i].status = Applied]$
$\land$ UNCHANGED $\langle proposal \rangle$

If any proposal has been *Failed*, mark the transaction *Failed*.

$\lor\ \land \exists\, t \in transaction[i].targets :$
$\land\ proposal[t][i].phase = Apply$
$\land\ proposal[t][i].state = Failed$
$\land\ transaction' = [transaction \text{ EXCEPT } ![i].state = Failed]$
$\land$ UNCHANGED $\langle proposal \rangle$

The Aborting state is used to clean up transactions that have failed during the Initializing or Validating phases.

$\lor\ \land\ transaction[i].phase = Abort$
$\land\ transaction[i].state = InProgress$

Move the transaction's proposals to the Aborting state

$\land\ \lor\ \land \exists\, t \in transaction[i].targets :$
$\land\ proposal[t][i].phase \neq Abort$
$\land\ proposal' = [proposal \text{ EXCEPT } ![t] =$
$[proposal[t] \text{ EXCEPT } ![i].phase = Abort,$
$![i].state = InProgress]]$
$\land$ UNCHANGED $\langle transaction \rangle$

If all proposals have been *Complete*, mark the transaction *Complete*.

$\lor\ \land \forall\, t \in transaction[i].targets :$
$\land\ proposal[t][i].phase = Abort$
$\land\ proposal[t][i].state = Complete$
$\land\ transaction' = [transaction \text{ EXCEPT } ![i].state = Complete,$
$![i].status = Aborted]$
$\land$ UNCHANGED $\langle proposal \rangle$

$\land$ UNCHANGED $\langle configuration, mastership, target \rangle$