MODULE *Config*

INSTANCE *Naturals*

INSTANCE *FiniteSets*

INSTANCE *Sequences*

INSTANCE *TLC*

An empty constant
CONSTANT *Nil*

Transaction type constants
CONSTANTS
Change,
Rollback

Transaction isolation constants
CONSTANTS
ReadCommitted,
Serializable

Phase constants
CONSTANTS
Initialize,
Validate,
Abort,
Commit,
Apply

$Phase \triangleq$
{*Initialize*,
Validate,
Abort,
Commit,
Apply}

Status constants
CONSTANTS
InProgress,
Complete,
Failed

$State \triangleq$
{*InProgress*,
Complete,

Failed}

State constants

CONSTANTS
Pending,
Validated,
Committed,
Applied,
Aborted

Status $\triangleq$
{*Pending*,
Validated,
Committed,
Applied,
Aborted}

CONSTANTS
Valid,
Invalid

CONSTANTS
Success,
Failure

The set of all nodes

CONSTANT *Node*

Target is the set of all targets and their possible paths and values.

Example: *Target* $\triangleq$ [
*target*1 $\mapsto$ [*persistent* $\mapsto$ FALSE, *values* $\mapsto$ [
*path*1 $\mapsto$ {"*value*1", "*value*2"},
*path*2 $\mapsto$ {"*value*2", "*value*3"}]],
*target*2 $\mapsto$ [*persistent* $\mapsto$ TRUE, *values* $\mapsto$ [
*path*2 $\mapsto$ {"*value*3", "*value*4"},
*path*3 $\mapsto$ {"*value*4", "*value*5"}]]]

CONSTANT *Target*

Configuration update/rollback requests are tracked and processed through two data types. Transactions represent the lifecycle of a single configuration change request and are stored in an append-only log. Configurations represent the desired configuration of a *gNMI* target based on the aggregate of relevant changes in the Transaction log.

TYPE Type ::= *type* $\in$
{*Change*,
Rollback}

TYPE *Phase* ::= *phase* $\in$

```
  {Initialize,
   Validate,
   Abort,
   Commit,
   Apply}

TYPE State ::= state ∈
  {InProgress,
   Complete,
   Failed}

TYPE Status ::= status ∈
  {Pending,
   Validated,
   Committed,
   Applied,
   Aborted}

TYPE Isolation ::= isolation ∈
  {ReadCommitted,
   Serializable}

TYPE Transaction ≜ [
  type      ::= type ∈ Type,
  index     ::= index ∈ Nat,
  isolation ::= isolation ∈ Isolation
  values ::= [
    target ∈ SUBSET (DOMAIN Target) ↦ [ path ∈ SUBSET (DOMAIN Target[target].values) ↦
      [
        value ::= value ∈ STRING,
        delete ::= delete ∈ BOOLEAN ]]],
  rollback ::= index ∈ Nat,
  targets ::= targets ∈ SUBSET (DOMAIN Target)
  phase     ::= phase ∈ Phase,
  state     ::= state ∈ State,
  status ::= status ∈ Status]

TYPE Proposal ≜ [
  type     ::= type ∈ Type,
  txIndex ::= txIndex ∈ Nat,
  values ::= [ path ∈ SUBSET (DOMAIN Target[target].values) ↦ [
      value ::= value ∈ STRING,
      delete ::= delete ∈ BOOLEAN ]],
  rollback ::= index ∈ Nat,
  depIndex ::= depIndex ∈ Nat,
  rbIndex ::= rbIndex ∈ Nat,
  rbValues ::= [ path ∈ SUBSET (DOMAIN Target[target].values) ↦ [
      value ::= value ∈ STRING,
      delete ::= delete ∈ BOOLEAN ]],
  phase ::= phase ∈ Phase,
  state ::= state ∈ State]

TYPE Configuration ≜ [
```

id ::= $id \in$ STRING,
$target$ ::= $target \in$ STRING,
$values$ ::= [$path \in$ SUBSET (DOMAIN $Target[target]$) $\mapsto$ [
$value$ ::= $value \in$ STRING,
$index$ ::= $index \in Nat$,
$deleted$ ::= $delete \in$ BOOLEAN]],
$cfgIndex$ ::= $cfgIndex \in Nat$,
$cgfTerm$ ::= $cgfTerm \in Nat$,
$txIndex$ ::= $txIndex \in Nat$,
$cmtIndex$ ::= $cmtIndex \in Nat$,
$tgtIndex$ ::= $tgtIndex \in Nat$,
$tgtTerm$ ::= $tgtTerm \in Nat$,
$tgtValues$ ::= [$path \in$ SUBSET (DOMAIN $Target[target]$) $\mapsto$ [
$value$ ::= $value \in$ STRING,
$index$ ::= $index \in Nat$,
$deleted$ ::= $delete \in$ BOOLEAN]],
$state$ ::= $state \in State$]

A transaction log. Transactions may either request a set of changes to a set of targets or rollback a prior change.

VARIABLE *transaction*

A record of per-target proposals

VARIABLE *proposal*

A record of per-target configurations

VARIABLE *configuration*

A record of target states

VARIABLE *target*

A record of target masterships

VARIABLE *mastership*

$vars \triangleq \langle transaction, proposal, configuration, mastership, target \rangle$

This section models *mastership* for the configuration service.

Mastership is used primarily to track the lifecycle of individual configuration targets and react to state changes on the southbound. Each target is assigned a master from the *Node* set, and masters can be unset when the target disconnects.

Set node n as the master for target t

$SetMaster(n, t) \triangleq$
$\land\ mastership[t].master \neq n$
$\land\ mastership' = [mastership$ EXCEPT $![t].term = mastership[t].term + 1,$
$![t].master = n]$
$\land$ UNCHANGED $\langle transaction, proposal, configuration, target \rangle$

$UnsetMaster(t) \triangleq$

$\land$ $mastership[t].master \neq Nil$
$\land$ $mastership' = [mastership$ EXCEPT $![t].master = Nil]$
$\land$ UNCHANGED $\langle transaction, proposal, configuration, target\rangle$

This section models configuration changes and rollbacks. Changes are appended to the transaction log and processed asynchronously.

$Value(s, t, p) \triangleq$
  LET $value \triangleq$ CHOOSE $v \in s : v.target = t \land v.path = p$
  IN
    $[value \mapsto value.value,$
     $delete \mapsto value.delete]$

$Paths(s, t) \triangleq$
  $[p \in \{v.path : v \in \{v \in s : v.target = t\}\} \mapsto Value(s, t, p)]$

$Changes(s) \triangleq$
  $[t \in \{v.target : v \in s\} \mapsto Paths(s, t)]$

$ValidValues(t, p) \triangleq$
  UNION $\{\{[value \mapsto v, delete \mapsto$ FALSE$] : v \in Target[t].values[p]\}, \{[value \mapsto Nil, delete \mapsto$ TRUE$]\}\}$

$ValidPaths(t) \triangleq$
  UNION $\{\{v$ @@ $[path \mapsto p] : v \in ValidValues(t, p)\} : p \in$ DOMAIN $Target[t].values\}$

$ValidTargets \triangleq$
  UNION $\{\{p$ @@ $[target \mapsto t] : p \in ValidPaths(t)\} : t \in$ DOMAIN $Target\}$

The set of all valid sets of changes to all targets and their paths.
The set of possible changes is computed from the *Target* model value.

$ValidChanges \triangleq$
  LET $changeSets \triangleq \{s \in$ SUBSET $ValidTargets :$
                  $\forall t \in$ DOMAIN $Target :$
                    $\forall p \in$ DOMAIN $Target[t].values :$
                      $Cardinality(\{v \in s : v.target = t \land v.path = p\}) \leq 1\}$
  IN
    $\{Changes(s) : s \in changeSets\}$

The next available index in the transaction log.
This is computed as the max of the existing indexes in the log to allow for changes to the log (*e.g.* log compaction) to be modeled.

$NextIndex \triangleq$
  IF DOMAIN $transaction = \{\}$ THEN
    1
  ELSE
    LET $i \triangleq$ CHOOSE $i \in$ DOMAIN $transaction :$
         $\forall j \in$ DOMAIN $transaction : i \geq j$

IN $i + 1$

Add a set of changes 'c' to the transaction log

$RequestChange(c) \triangleq$
$\land \exists\, isolation \in \{ReadCommitted,\ Serializable\} :$
$\land\ transaction' = transaction\ @@\ (NextIndex :> [type \mapsto Change,$
$index \mapsto NextIndex,$
$isolation \mapsto isolation,$
$values \mapsto c,$
$targets \mapsto \{\},$
$phase \mapsto Initialize,$
$state \mapsto InProgress,$
$status \mapsto Pending])$
$\land$ UNCHANGED $\langle proposal,\ configuration,\ mastership,\ target\rangle$

Add a rollback of transaction 't' to the transaction log

$RequestRollback(t) \triangleq$
$\land \exists\, isolation \in \{ReadCommitted,\ Serializable\} :$
$\land\ transaction' = transaction\ @@\ (NextIndex :> [type \mapsto Rollback,$
$index \mapsto NextIndex,$
$isolation \mapsto isolation,$
$rollback \mapsto t,$
$targets \mapsto \{\},$
$phase \mapsto Initialize,$
$state \mapsto InProgress,$
$status \mapsto Pending])$
$\land$ UNCHANGED $\langle proposal,\ configuration,\ mastership,\ target\rangle$

This section models the Transaction log reconciler.

Transactions come in two flavors : − *Change* transactions contain a set of changes to be applied to a set of *targets* − *Rollback* transactions reference a prior change transaction to be reverted to the previous state

Transacations proceed through a series of phases:
* *Initialize* − create and link Proposals
* *Validate* − validate changes and rollbacks
* *Commit* − commit changes to Configurations
* *Apply* − commit changes to Targets

Reconcile a transaction

$ReconcileTransaction(n,\ i) \triangleq$

Initialize is the only transaction phase that's globally serialized. While in the *Initializing* phase, the reconciler checks whether the prior transaction has been *Initialized* before creating Proposals in the *Initialize* phase. Once all of the transaction's proposals have been *Initialized*, the transaction will be marked *Initialized*. If any

```
    proposal is Failed, the transaction will be marked Failed as well.
∧ ∨ ∧ transaction[i].phase = Initialize
    ∧ ∨ ∧ transaction[i].state = InProgress
        Serialize transaction initialization
        ∧ ¬∃ j ∈ DOMAIN transaction :
            ∧ j < i
            ∧ transaction[j].phase = Initialize
            ∧ transaction[j].state  = InProgress
        If the transaction's targets are not yet set, create proposals
        and add targets to the transaction state.
        ∧ ∨ ∧ transaction[i].targets = {}
            If the transaction is a change, the targets are taken
            from the change values.
          ∧ ∨ ∧ transaction[i].type = Change
              ∧ transaction' = [transaction EXCEPT ![i].targets = DOMAIN transaction[i].values]
              ∧ proposal' = [t ∈ DOMAIN proposal ↦
                    IF t ∈ DOMAIN transaction[i].values THEN
                       proposal[t] @@ (i :> [type      ↦ Change,
                                             index     ↦ i,
                                             values    ↦ transaction[i].values[t],
                                             depIndex  ↦ 0,
                                             rbIndex   ↦ 0,
                                             rbValues  ↦ ⟨⟩,
                                             phase     ↦ Initialize,
                                             state     ↦ InProgress])
                    ELSE
                       proposal[t]]
            If the transaction is a rollback, the targets affected are
            the targets of the change transaction being rolled back.
            ∨ ∧ transaction[i].type = Rollback
              ∧ ∨ ∧ transaction[i].rollback ∈ DOMAIN transaction
                  ∧ transaction[transaction[i].rollback].type = Change
                  ∧ transaction' = [transaction EXCEPT ![i].targets =
                                        DOMAIN transaction[transaction[i].rollback].values]
                  ∧ proposal' = [t ∈ DOMAIN proposal ↦
                        IF t ∈ DOMAIN transaction[transaction[i].rollback].values THEN
                           proposal[t] @@ (i :> [type      ↦ Rollback,
                                                 index     ↦ i,
                                                 rollback  ↦ transaction[i].rollback,
                                                 depIndex  ↦ 0,
                                                 rbIndex   ↦ 0,
                                                 rbValues  ↦ ⟨⟩,
                                                 phase     ↦ Initialize,
                                                 state     ↦ InProgress])
                        ELSE
```

$proposal[t]]$

$\lor \land \lor \land transaction[i].rollback \in \text{DOMAIN } transaction$

$\land transaction[transaction[i].rollback].type = Rollback$

$\lor transaction[i].rollback \notin \text{DOMAIN } transaction$

$\land transaction' = [transaction \text{ EXCEPT } ![i].state = Failed]$

$\land \text{UNCHANGED } \langle proposal \rangle$

$\lor \land transaction[i].targets \neq \{\}$

If all proposals have been *Complete*, mark the transaction *Complete*.

$\land \lor \land \forall t \in transaction[i].targets :$

$\land proposal[t][i].phase = Initialize$

$\land proposal[t][i].state = Complete$

$\land transaction' = [transaction \text{ EXCEPT } ![i].state = Complete]$

$\land \text{UNCHANGED } \langle proposal \rangle$

If any proposal has been *Failed*, mark the transaction *Failed*.

$\lor \land \exists t \in transaction[i].targets :$

$\land proposal[t][i].phase = Initialize$

$\land proposal[t][i].state = Failed$

$\land transaction' = [transaction \text{ EXCEPT } ![i].state = Failed]$

$\land \text{UNCHANGED } \langle proposal \rangle$

Once the transaction has been *Initialized*, proceed to the *Validate* phase.
If any of the transaction's proposals depend on a *Serializable* transaction,
verify the dependency has been *Validated* to preserve serializability before
moving the transaction to the *Validate* phase.

$\lor \land transaction[i].state = Complete$

$\land \forall t \in transaction[i].targets :$

$\land proposal[t][i].depIndex \in transaction$

$\land transaction[proposal[t][i].depIndex].isolation = Serializable$

$\Rightarrow transaction[proposal[t][i].depIndex].status \in \{Validated, Committed, Applied, Aborted\}$

$\land transaction' = [transaction \text{ EXCEPT } ![i].phase = Validate,$

$![i].state = InProgress]$

$\land \text{UNCHANGED } \langle proposal \rangle$

$\lor \land transaction[i].state = Failed$

$\land transaction' = [transaction \text{ EXCEPT } ![i].phase = Abort,$

$![i].state = InProgress]$

$\land \text{UNCHANGED } \langle proposal \rangle$

$\lor \land transaction[i].phase = Validate$

$\land \lor \land transaction[i].state = InProgress$

Move the transaction's proposals to the *Validating* state

$\land \lor \land \exists t \in transaction[i].targets :$

$\land proposal[t][i].phase \neq Validate$

$\land proposal' = [proposal \text{ EXCEPT } ![t] = [$

$proposal[t] \text{ EXCEPT } ![i].phase = Validate,$

$![i].state = InProgress]]$

$\land \text{UNCHANGED } \langle transaction \rangle$

If all proposals have been *Complete*, mark the transaction *Complete*.

$$
\begin{array}{l}
\lor\ \land\ \forall\, t \in transaction[i].targets : \\
\qquad \land\ proposal[t][i].phase\ \ = Validate \\
\qquad \land\ proposal[t][i].state = Complete \\
\quad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].state\ \ = Complete, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ![i].status = Validated] \\
\quad \land\ \text{UNCHANGED}\ \langle proposal \rangle \\
\text{If any proposal has been } \textit{Failed}\text{, mark the transaction } \textit{Failed}\text{.} \\
\lor\ \land\ \exists\, t \in transaction[i].targets : \\
\qquad \land\ proposal[t][i].phase\ \ = Validate \\
\qquad \land\ proposal[t][i].state = Failed \\
\quad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].state = Failed] \\
\quad \land\ \text{UNCHANGED}\ \langle proposal \rangle \\
\text{Once the transaction has been } \textit{Validated}\text{, proceed to the } \textit{Commit} \text{ phase.} \\
\text{If any of the transaction's proposals depend on a } \textit{Serializable} \text{ transaction,} \\
\text{verify the dependency has been } \textit{Committed} \text{ to preserve serializability before} \\
\text{moving the transaction to the } \textit{Commit} \text{ phase.} \\
\lor\ \land\ transaction[i].state = Complete \\
\quad \land\ \forall\, t \in transaction[i].targets : \\
\qquad \land\ proposal[t][i].depIndex \in transaction \\
\qquad \land\ transaction[proposal[t][i].depIndex].isolation = Serializable \\
\qquad \Rightarrow\ transaction[proposal[t][i].depIndex].status \in \{Committed,\ Applied,\ Aborted\} \\
\quad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].phase = Commit, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ![i].state\ \ = InProgress] \\
\quad \land\ \text{UNCHANGED}\ \langle proposal \rangle \\
\lor\ \land\ transaction[i].state = Failed \\
\quad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].phase = Abort, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ![i].state\ \ = InProgress] \\
\quad \land\ \text{UNCHANGED}\ \langle proposal \rangle \\
\lor\ \land\ transaction[i].phase = Commit \\
\quad \land\ \lor\ \land\ transaction[i].state = InProgress \\
\text{Move the transaction's proposals to the } \textit{Committing} \text{ state} \\
\qquad \land\ \lor\ \land\ \exists\, t \in transaction[i].targets : \\
\qquad\qquad \land\ proposal[t][i].phase \neq Validate \\
\qquad\qquad \land\ proposal' = [proposal\ \text{EXCEPT}\ ![t] = [\\
\qquad\qquad\qquad\qquad proposal[t]\ \text{EXCEPT}\ ![i].phase = Commit, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ![i].state\ \ = InProgress]] \\
\qquad\qquad \land\ \text{UNCHANGED}\ \langle transaction \rangle \\
\text{If all proposals have been } \textit{Complete}\text{, mark the transaction } \textit{Complete}\text{.} \\
\qquad \lor\ \land\ \forall\, t \in transaction[i].targets : \\
\qquad\qquad \land\ proposal[t][i].phase\ \ = Commit \\
\qquad\qquad \land\ proposal[t][i].state = Complete \\
\qquad \land\ transaction' = [transaction\ \text{EXCEPT}\ ![i].state\ \ = Complete, \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ![i].status = Committed] \\
\qquad \land\ \text{UNCHANGED}\ \langle proposal \rangle \\
\text{Once the transaction has been } \textit{Committed}\text{, proceed to the } \textit{Apply} \text{ phase.}
\end{array}
$$

```
        If any of the transaction's proposals depend on a Serializable transaction,
        verify the dependency has been Applied to preserve serializability before
        moving the transaction to the Apply phase.
      ∨ ∧ transaction[i].state = Complete
        ∧ ∀ t ∈ transaction[i].targets :
            ∧ proposal[t][i].depIndex ∈ transaction
            ∧ transaction[proposal[t][i].depIndex].isolation = Serializable
            ⇒ transaction[proposal[t][i].depIndex].status ∈ {Applied, Aborted}
        ∧ transaction' = [transaction EXCEPT ![i].phase = Apply,
                                             ![i].state  = InProgress]
        ∧ UNCHANGED ⟨proposal⟩
    ∨ ∧ transaction[i].phase = Apply
      ∧ transaction[i].state  = InProgress
        Move the transaction's proposals to the Applying state
      ∧ ∨ ∧ ∃ t ∈ transaction[i].targets :
            ∧ proposal[t][i].phase ≠ Validate
            ∧ proposal' = [proposal EXCEPT ![t] = [
                              proposal[t] EXCEPT ![i].phase = Apply,
                                                 ![i].state  = InProgress]]
        ∧ UNCHANGED ⟨transaction⟩
        If all proposals have been Complete, mark the transaction Complete.
      ∨ ∧ ∀ t ∈ transaction[i].targets :
            ∧ proposal[t][i].phase  = Apply
            ∧ proposal[t][i].state = Complete
        ∧ transaction' = [transaction EXCEPT ![i].state  = Complete,
                                             ![i].status = Applied]
        ∧ UNCHANGED ⟨proposal⟩
        If any proposal has been Failed, mark the transaction Failed.
      ∨ ∧ ∃ t ∈ transaction[i].targets :
            ∧ proposal[t][i].phase  = Apply
            ∧ proposal[t][i].state = Failed
        ∧ transaction' = [transaction EXCEPT ![i].state = Failed]
        ∧ UNCHANGED ⟨proposal⟩
    The Aborting state is used to clean up transactions that have failed during
    the Initializing or Validating phases.
    ∨ ∧ transaction[i].phase = Abort
      ∧ transaction[i].state  = InProgress
        Move the transaction's proposals to the Aborting state
      ∧ ∨ ∧ ∃ t ∈ transaction[i].targets :
            ∧ proposal[t][i].phase ≠ Validate
            ∧ proposal' = [proposal EXCEPT ![t] = [
                              proposal[t] EXCEPT ![i].phase = Abort,
                                                 ![i].state  = InProgress]]
        ∧ UNCHANGED ⟨transaction⟩
        If all proposals have been Complete, mark the transaction Complete.
```

```
        ∨ ∧ ∀ t ∈ transaction[i].targets :
              ∧ proposal[t][i].phase  = Abort
              ∧ proposal[t][i].state = Complete
          ∧ transaction' = [transaction EXCEPT ![i].state  = Complete,
                                               ![i].status = Aborted]
          ∧ UNCHANGED ⟨proposal⟩
    ∧ UNCHANGED ⟨configuration, mastership, target⟩

Reconcile a proposal
ReconcileProposal(n, t, i) ≜
    ∧ ∨ ∧ proposal[t][i].phase = Initialize
        ∧ proposal[t][i].state  = InProgress
        ∧ proposal' = [proposal EXCEPT ![t] = [
              proposal[t] EXCEPT ![i] = [
                 state      ↦ Complete,
                 depIndex ↦ configuration[t].txIndex] @@ proposal[t][i]]]
        ∧ configuration' = [configuration EXCEPT ![t].txIndex = i]
        ∧ UNCHANGED ⟨target⟩
      While in the Validate phase, validate the proposed changes.
      If validation is successful, the proposal also records the changes
      required to roll back the proposal and the index to which to roll back.
      ∨ ∧ proposal[t][i].phase = Validate
        ∧ proposal[t][i].state  = InProgress
        ∧ configuration[t].cmtIndex = proposal[t][i].depIndex
            For Change proposals validate the set of requested changes.
        ∧ ∨ ∧ proposal[t][i].type = Change
            ∧ LET rbIndex ≜ configuration[t].cfgIndex
                  rbValues ≜ [p ∈ DOMAIN proposal[t][i].values ↦
                                IF p ∈ DOMAIN configuration[t].values THEN
                                   configuration[t].values[p]
                                ELSE
                                  [value  ↦ Nil,
                                   delete ↦ TRUE]]
              Model validation successes and failures with Valid and Invalid results.
              IN ∃ r ∈ {Valid, Invalid} :
                   If the Change is Valid, record the changes required to roll
                   back the proposal and the index to which the rollback changes
                   will roll back the configuration.
                   ∨ ∧ r = Valid
                     ∧ proposal' = [proposal EXCEPT ![t] = [
                                       proposal[t] EXCEPT ![i].rbIndex  = rbIndex,
                                                          ![i].rbValues = rbValues,
                                                          ![i].state    = Complete]]
                   ∨ ∧ r = Invalid
                     ∧ proposal' = [proposal EXCEPT ![t] = [
```

```
                                                    proposal[t] EXCEPT ![i].state = Failed]]
        For Rollback proposals, validate the rollback changes which are
        proposal being rolled back.
      ∨ ∧ proposal[t][i].type = Rollback
            Rollbacks can only be performed on Change type proposals.
          ∧ ∨ ∧ proposal[t][proposal[t][i].rollback].type = Change
                  Only roll back the change if it's the lastest change made
                  to the configuration based on the configuration index.
              ∧ ∨ ∧ configuration[t].cfgIndex = proposal[t][i].rollback
                  ∧ LET rbIndex  ≜ proposal[t][proposal[t][i].rollback].rbIndex
                        rbValues ≜ proposal[t][proposal[t][i].rollback].rbValues
                    IN  ∃ r ∈ {Valid, Invalid} :
                          If the Rollback is Valid, record the changes required to
                          roll back the target proposal and the index to which the
                          configuration is being rolled back.
                        ∨ ∧ r = Valid
                          ∧ proposal' = [proposal EXCEPT ![t] = [
                                proposal[t] EXCEPT ![i].rbIndex  = rbIndex,
                                                   ![i].rbValues = rbValues,
                                                   ![i].state    = Complete]]
                        ∨ ∧ r = Invalid
                          ∧ proposal' = [proposal EXCEPT ![t] = [
                                                  proposal[t] EXCEPT ![i].state = Failed]]
                  If the Rollback target is not the most recent change to the configuration,
                  fail validation for the proposal.
                ∨ ∧ configuration[t].cfgIndex ≠ proposal[t][i].rollback
                  ∧ proposal' = [proposal EXCEPT ![t] = [proposal[t] EXCEPT ![i].state = Failed]]
            If a Rollback proposal is attempting to roll back another Rollback,
            fail validation for the proposal.
          ∨ ∧ proposal[t][proposal[t][i].rollback].type = Rollback
            ∧ proposal' = [proposal EXCEPT ![t] = [
                  proposal[t] EXCEPT ![i].state = Failed]]
      ∧ UNCHANGED ⟨configuration, target⟩
  While in the Commit state, commit the proposed changes to the configuration.
∨ ∧ proposal[t][i].phase = Commit
  ∧ proposal[t][i].state  = InProgress
    Only commit the proposal if the prior proposal has already been committed.
  ∧ configuration[t].cmtIndex = proposal[t][i].depIndex
      If the proposal is a change, commit the change values and set the configuration
      index to the proposal index.
  ∧ ∨ ∧ proposal[t][i].type = Change
      ∧ configuration' = [configuration EXCEPT ![t].values   = proposal[t][i].values,
                                               ![t].cfgIndex = i,
                                               ![t].cmtIndex = i]
      If the proposal is a rollback, commit the rollback values and index. This
```

will cause the configuration index to be reverted to the index prior to
the transaction/proposal being rolled back.

$\lor \land proposal[t][i].type = Rollback$
$\quad \land configuration' = [configuration \text{ EXCEPT } ![t].values = proposal[t][i].rbValues,$
$\qquad ![t].cfgIndex = proposal[t][i].rbIndex,$
$\qquad ![t].cmtIndex = i]$
$\land proposal' = [proposal \text{ EXCEPT } ![t] = [proposal[t] \text{ EXCEPT } ![i].state = Complete]]$
$\land \text{UNCHANGED } \langle target \rangle$

While in the *Apply* phase, apply the proposed changes to the target.

$\lor \land proposal[t][i].phase = Apply$
$\land proposal[t][i].state = InProgress$
$\land configuration[t].tgtIndex = proposal[t][i].depIndex$
$\land configuration[t].tgtTerm = mastership[t].term$
$\land mastership[t].master = n$

Model successful and failed target update requests.

$\land \exists\, r \in \{Success,\ Failure\} :$
$\quad \lor \land r = Success$

If the proposal is a change, apply the change values to the target
and update the configuration's applied index and values.

$\qquad \land \lor \land proposal[t][i].type = Change$
$\qquad\quad \land target' = [target \text{ EXCEPT } ![t] = proposal[t][i].values \mathbin{@@} target[t]]$
$\qquad\quad \land configuration' = [configuration \text{ EXCEPT}$
$\qquad\qquad ![t].tgtIndex = i,$
$\qquad\qquad ![t].tgtValues = proposal[t][i].values \mathbin{@@} configuration[t].tgtValues]$

If the proposal is a rollback, apply the rollback values and update the
configuration's applied values with the rolled back values.

$\qquad\quad \lor \land proposal[t][i].type = Rollback$
$\qquad\quad \land target' = [target \text{ EXCEPT } ![t] = proposal[t][i].rbValues \mathbin{@@} target[t]]$
$\qquad\quad \land configuration' = [configuration \text{ EXCEPT}$
$\qquad\qquad ![t].tgtIndex = i,$
$\qquad\qquad ![t].tgtValues = proposal[t][i].rbValues \mathbin{@@} configuration[t].tgtValues]$
$\qquad \land proposal' = [proposal \text{ EXCEPT } ![t] = [proposal[t] \text{ EXCEPT } ![i].state = Complete]]$

If the proposal could not be applied, update the configuration's applied index
and mark the proposal *Failed*.

$\quad \lor \land r = Failure$
$\qquad \land configuration' = [configuration \text{ EXCEPT } ![t].tgtIndex = i]$
$\qquad \land proposal' = [proposal \text{ EXCEPT } ![t] = [proposal[t] \text{ EXCEPT } ![i].state = Failed]]$
$\qquad \land \text{UNCHANGED } \langle target \rangle$

$\lor \land proposal[t][i].phase = Abort$
$\land proposal[t][i].state = InProgress$

The *cmtIndex* will always be greater than or equal to the *tgtIndex*.
If only the *cmtIndex* matches the proposal's *depIndex*, update
the *cmtIndex* to enable commits of later proposals, but do not
mark the *Abort* phase *Complete* until the *tgtIndex* has been incremented.

$\land \lor \land configuration[t].cmtIndex = proposal[t][i].depIndex$

$\land$ $configuration' = [configuration$ EXCEPT $![t].cmtIndex = i]$
$\land$ UNCHANGED $\langle proposal \rangle$

If the configuration's *tgtIndex* matches the proposal's *depIndex*, update the *tgtIndex* and mark the proposal *Complete* for the *Abort* phase.

$\lor \land configuration[t].cmtIndex \geq i$
$\land configuration[t].tgtIndex = proposal[t][i].depIndex$
$\land configuration' = [configuration$ EXCEPT $![t].tgtIndex = i]$
$\land proposal' = [proposal$ EXCEPT $![t] = [proposal[t]$ EXCEPT $![i].state = Complete]]$

If both the configuration's *cmtIndex* and *tgtIndex* match the proposal's *depIndex*, update the *cmtIndex* and *tgtIndex* and mark the proposal *Complete* for the *Abort* phase.

$\lor \land configuration[t].cmtIndex = proposal[t][i].depIndex$
$\land configuration[t].tgtIndex = proposal[t][i].depIndex$
$\land configuration' = [configuration$ EXCEPT $![t].cmtIndex = i,$
$![t].tgtIndex = i]$
$\land proposal' = [proposal$ EXCEPT $![t] = [proposal[t]$ EXCEPT $![i].state = Complete]]$

$\land$ UNCHANGED $\langle target \rangle$

$\land$ UNCHANGED $\langle transaction, mastership \rangle$

This section models the Configuration reconciler.

$ReconcileConfiguration(n, t) \triangleq$

$\land \lor \land Target[t].persistent$
$\land configuration[t].state \neq Complete$
$\land configuration' = [configuration$ EXCEPT $![t].state = Complete]$
$\land$ UNCHANGED $\langle target \rangle$
$\lor \land \neg Target[t].persistent$
$\land \lor mastership[t].term > configuration[t].cfgTerm$
$\lor \land mastership[t].term = configuration[t].cfgTerm$
$\land mastership[t].master = Nil$
$\land configuration' = [configuration$ EXCEPT $![t].cfgTerm = mastership[t].term,$
$![t].state = InProgress]$
$\land$ UNCHANGED $\langle target \rangle$
$\lor \land configuration[t].state = InProgress$
$\land mastership[t].term = configuration[t].cfgTerm$
$\land mastership[t].master = n$
$\land target' = [target$ EXCEPT $![t] = configuration[t].tgtValues]$
$\land configuration' = [configuration$ EXCEPT $![t].tgtTerm = mastership[t].term,$
$![t].state = Complete]$
$\land$ UNCHANGED $\langle proposal, transaction, mastership \rangle$

Formal specification, constraints, and theorems.

$$
\begin{array}{l}
\mathit{Init} \triangleq \\
\quad \land\ \mathit{transaction} = \langle\rangle \\
\quad \land\ \mathit{proposal} = [t \in \text{DOMAIN } \mathit{Target} \mapsto \\
\qquad\qquad [p \in \{\} \mapsto [\mathit{phase} \mapsto \mathit{Initialize}, \\
\qquad\qquad\qquad\qquad \mathit{state} \ \mapsto \mathit{InProgress}]]] \\
\quad \land\ \mathit{configuration} = [t \in \text{DOMAIN } \mathit{Target} \mapsto \\
\qquad\qquad\qquad [\mathit{target} \mapsto t, \\
\qquad\qquad\qquad\ \mathit{state} \ \mapsto \mathit{InProgress}, \\
\qquad\qquad\qquad\ \mathit{values} \mapsto \\
\qquad\qquad\qquad\quad [\mathit{path} \in \{\} \ \mapsto \\
\qquad\qquad\qquad\qquad [\mathit{path} \ \mapsto \mathit{path}, \\
\qquad\qquad\qquad\qquad\ \mathit{value} \ \mapsto \mathit{Nil}, \\
\qquad\qquad\qquad\qquad\ \mathit{index} \ \mapsto 0, \\
\qquad\qquad\qquad\qquad\ \mathit{deleted} \mapsto \text{FALSE}]], \\
\qquad\qquad\qquad\ \mathit{cfgIndex} \ \mapsto 0, \\
\qquad\qquad\qquad\ \mathit{cfgTerm} \ \mapsto 0, \\
\qquad\qquad\qquad\ \mathit{txIndex} \ \mapsto 0, \\
\qquad\qquad\qquad\ \mathit{cmtIndex} \mapsto 0, \\
\qquad\qquad\qquad\ \mathit{tgtIndex} \ \mapsto 0, \\
\qquad\qquad\qquad\ \mathit{tgtTerm} \ \mapsto 0, \\
\qquad\qquad\qquad\ \mathit{tgtValues} \mapsto \\
\qquad\qquad\qquad\quad [\mathit{path} \in \{\} \ \mapsto \\
\qquad\qquad\qquad\qquad [\mathit{path} \ \mapsto \mathit{path}, \\
\qquad\qquad\qquad\qquad\ \mathit{value} \ \mapsto \mathit{Nil}, \\
\qquad\qquad\qquad\qquad\ \mathit{index} \ \mapsto 0, \\
\qquad\qquad\qquad\qquad\ \mathit{deleted} \mapsto \text{FALSE}]]]] \\
\quad \land\ \mathit{target} = [t \in \text{DOMAIN } \mathit{Target} \mapsto \\
\qquad\qquad [\mathit{path} \in \{\} \mapsto \\
\qquad\qquad\quad [\mathit{value} \mapsto \mathit{Nil}]]] \\
\quad \land\ \mathit{mastership} = [t \in \text{DOMAIN } \mathit{Target} \mapsto [\mathit{master} \mapsto \mathit{Nil},\ \mathit{term} \mapsto 0]]
\end{array}
$$

$$
\begin{array}{l}
\mathit{Next} \triangleq \\
\quad \lor\ \exists\, c \in \mathit{ValidChanges} : \\
\qquad \mathit{RequestChange}(c) \\
\quad \lor\ \exists\, t \in \text{DOMAIN } \mathit{transaction} : \\
\qquad \mathit{RequestRollback}(t) \\
\quad \lor\ \exists\, n \in \mathit{Node} : \\
\qquad \exists\, t \in \text{DOMAIN } \mathit{Target} : \\
\qquad\quad \mathit{SetMaster}(n,\ t) \\
\quad \lor\ \exists\, t \in \text{DOMAIN } \mathit{Target} : \\
\qquad \mathit{UnsetMaster}(t) \\
\quad \lor\ \exists\, n \in \mathit{Node} : \\
\qquad \exists\, t \in \text{DOMAIN } \mathit{transaction} : \\
\qquad\quad \mathit{ReconcileTransaction}(n,\ t) \\
\quad \lor\ \exists\, n \in \mathit{Node} :
\end{array}
$$

```
        ∃ t ∈ DOMAIN proposal :
          ∃ i ∈ DOMAIN proposal[t] :
              ReconcileProposal(n, t, i)
    ∨ ∃ n ∈ Node :
        ∃ c ∈ DOMAIN configuration :
            ReconcileConfiguration(n, c)

Spec ≜ Init ∧ □[Next]_vars

Order ≜
  ∀ t ∈ DOMAIN proposal :
    ∀ i ∈ DOMAIN proposal[t] :
      ∧ ∧ proposal[t][i].phase = Commit
        ∧ proposal[t][i].state  = InProgress
        ⇒ ¬∃ j ∈ DOMAIN proposal[t] :
              ∧ j > i
              ∧ proposal[t][j].phase = Commit
              ∧ proposal[t][j].state  = Complete
      ∧ ∧ proposal[t][i].phase = Apply
        ∧ proposal[t][i].state  = InProgress
        ⇒ ¬∃ j ∈ DOMAIN proposal[t] :
              ∧ j > i
              ∧ proposal[t][j].phase = Apply
              ∧ proposal[t][j].state  = Complete

Consistency ≜
  ∀ t ∈ DOMAIN target :
    LET
        Compute the transaction indexes that have been applied to the target
      tgtIndexes ≜ {i ∈ DOMAIN transaction :
                      ∧ transaction[i].type = Change
                      ∧ i ∈ DOMAIN proposal[t]
                      ∧ proposal[t][i].phase = Apply
                      ∧ proposal[t][i].state  = Complete
                      ∧ t ∈ DOMAIN transaction[i].values
                      ∧ ¬∃ j ∈ DOMAIN transaction :
                            ∧ j > i
                            ∧ transaction[j].type = Rollback
                            ∧ transaction[j].rollback = i
                            ∧ transaction[j].phase = Apply
                            ∧ transaction[j].state  = Complete}
        Compute the set of paths in the target that have been updated by transactions
      appliedPaths ≜ UNION {DOMAIN transaction[i].values[t] : i ∈ tgtIndexes}
        Compute the highest index applied to the target for each path
      pathIndexes ≜ [p ∈ appliedPaths ↦ CHOOSE i ∈ tgtIndexes :
                        ∀ j ∈ tgtIndexes :
```

$\land\ i \geq j$
$\land\ p \in \text{DOMAIN}\ transaction[i].values[t]]$

Compute the expected target configuration based on the last indexes applied to the target for each path.

$expectedConfig \triangleq [p \in \text{DOMAIN}\ pathIndexes \mapsto transaction[pathIndexes[p]].values[t][p]]$

IN

$target[t] = expectedConfig$

$Isolation \triangleq$
$\forall\, i \in \text{DOMAIN}\ transaction :$
$\land\ \land\ transaction[i].phase = Commit$
$\land\ transaction[i].isolation = Serializable$
$\Rightarrow \neg\exists\, j \in \text{DOMAIN}\ transaction :$
$\land\ j > i$
$\land\ transaction[j].targets \cap transaction[i].targets \neq \{\}$
$\land\ transaction[j].phase = Commit$
$\land\ \land\ transaction[i].phase = Apply$
$\land\ transaction[i].isolation = Serializable$
$\Rightarrow \neg\exists\, j \in \text{DOMAIN}\ transaction :$
$\land\ j > i$
$\land\ transaction[j].targets \cap transaction[i].targets \neq \{\}$
$\land\ transaction[j].phase = Apply$

$Safety \triangleq \Box(Order \land Consistency \land Isolation)$

THEOREM $Spec \Rightarrow Safety$

$Completion \triangleq$
$\forall\, i \in \text{DOMAIN}\ transaction :$
$\land\ transaction[i].phase \in \{Apply,\ Abort\}$
$\land\ transaction[i].state\ = Complete$

$Liveness \triangleq \Diamond Completion$

THEOREM $Spec \Rightarrow Liveness$

Type assumptions.

ASSUME $Nil \in \text{STRING}$

ASSUME $\forall\, phase \in Phase : phase \in \text{STRING}$

ASSUME $\forall\, state\ \in State\ : state\ \in \text{STRING}$

ASSUME $\forall\, status \in Status : status \in \text{STRING}$

ASSUME $\land\ IsFiniteSet(Node)$
$\land\ \forall\, n \in Node :$

$\land\ n \notin$ DOMAIN $Target$
$\land\ n \in$ STRING

ASSUME $\land \forall\, t \in$ DOMAIN $Target$:
$\land\ t \notin Node$
$\land\ t \in$ STRING
$\land\ Target[t].persistent \in$ BOOLEAN
$\land \forall\, p \in$ DOMAIN $Target[t].values$:
$IsFiniteSet(\, Target[t].values[p])$

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