QUESTIONS

1. The random variable $X_1, X_2, \dots, X_{2500}$ are independent and exponentially distributed with $\lambda = 5$. Calculate the probability that the sum of these random variables less is less than or equal to 550.

(7 marks)

2. Let X be the random variable whose density function is given by

$$f(x) = \begin{cases} cxe^{-0.0016x^2}, & \text{for } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of c if $f(x)$ is a valid density function.

(b) Find $P(x > 25)$.

[Hint 1: Using change of variables in the integration by letting "$t = 0.0016x^2$". Then $dt = 0.0032xdx$.]

[Hint 2: Consider (3.19) in lecture note: $\lim_{x\to\infty} x^n e^{-an} = 0$, for $a > 0$.]

(14 marks)

3. (a) Define $z_{0.32}$ which $P(Z \leq z_{0.32}) = 0.32$. Find $z_{0.32}$.

(b) If a normal distribution with mean μ and variance $\sigma^2 > 0$ has 32th percentile equal to 10σ, find μ in term of σ.

(10 marks)

4. Given the random variable X parameters μ and σ, and moment generating function for the normal distribution

$$M_X(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}.$$

Using the moment generating function to find the expected value and variance of X.

(Please provide your steps.)

(10 marks)

5. Find the integral of following function.

$$\int_0^1 1.2x - 2.5e^{-3x}\, dx$$

(4 marks)

6. The joint probability function of X and Y is shown in the following table.

Joint probability function		y			
		1	3	5	$p_X(x)$
x	1	0.3	0.1	0.1	e
	4	a	0.1	0.3	f
	$p_{Y(y)}$	b	c	d	

(a) Find the values of a, b, c, d, e, f.

(b) Find $P(X < Y)$. Please show your steps.

(5 marks)