1. (13 marks) A study was made of 200 students to determine what cartoons they watch:

(i) 22 students don't watch any cartoons;

(ii) 73 students watch only "Sailor Moon";

(iii) 136 students watch "Sailor Moon";

(iv) 14 students watch only "Doraemon" and "Chibi Maruko-chan";

(v) 31 students watch only "Sailor Moon" and "Chibi Maruko-chan";

(vi) 63 students watch "Doraemon";

(vii) 135 students do not watch "Chibi Maruko-chan".

(a) (9 marks) Construct a Venn diagram for the above situation.

(b) (1 mark) Calculate the probability that a randomly chosen student watches all three cartoons.

(c) (3 marks) If a randomly chosen student watches "Doraemon", what is the probability that he/she also watches "Sailor Moon" but not "Chibi Maruko-chan"?

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2. (10 marks) A health insurance policy pays \$1,200 per day for the first 3 days of a hospitalization, \$600 per day for the next 3 days, and nothing thereafter. The probability that a hospitalization lasts exactly n days, denoted by $p(n)$, satisfies the relation

$$p(n+1) = \frac{3}{4}p(n), \qquad n = 1,2,3,\ldots.$$

(a) (3 marks) Determine the probability function $p(n)$.

(b) (7 marks) Calculate the expected value and standard deviation of the insurance payment per hospitalization.

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3. (10 marks) A company classifies injuries to its workers as minor if the worker does not have to take time off and severe if the worker has to take time off. The company has two plants, A and B. In plant A, 60% of the workers had no injuries, 30% had minor injuries, and 10% had severe injuries. In plant B, 50% had no injuries, 35% minor injuries, and 15% severe injuries. 70% of all workers work in plant A and 30% in plant B.

(a) (4 marks) What is the probability that a worker with a severe injury worked in plant A?

(b) (6 marks) What is the probability that a worker who had an injury worked in plant B and had a minor injury?

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4. (12 marks) Let $p(x,y) = (xy+y)/27$, for $x = 1,2,3$ and $y = 1,2$, be the joint probability distribution for the random variables X and Y.

(a) (3 marks) Construct a table of the joint probabilities of X and Y and calculate $P(X \geq 2 \mid Y = 1)$.

(b) (4 marks) Calculate $V(Y \mid X=2)$.

(c) (5 marks) Calculate $V(3Y-X)$.

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5. (15 marks) A company manufactures engines. Specifications require that the length of a certain rod in this engine be between 7.48 cm. and 7.52 cm. The lengths of the rods produced by their supplier have a normal distribution with a mean of 7.505 cm. and a standard deviation of 0.01 cm.

(a) (2 marks) Find the approximate 95^{th} percentile for the distribution of the length of the rods.

(b) (4 marks) What is the probability that a randomly selected rod meets these specifications?

(c) (3 marks) If a worker selects 4 of these rods at random, what is the probability that at least 3 of them meet these specifications?

(d) (6 marks) If a worker selects 100 of these rods at random, what is the approximate probability that at least 88 of them but no more than 98 of them meet these specifications?

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6. (13 marks) The loss amount X for an insurance policy has the following probability density function:

$$f(x) = \begin{cases} 0, & x < 0 \\ 1 - \frac{1}{2}x^2, & 0 \le x \le 1 \\ \frac{1}{6}\mathrm{e}^{1-x}, & x > 1 \end{cases}.$$

(a) (6 marks) Find the cumulative distribution function $F(x)$ for X.

(b) (3 marks) Calculate the probability that the loss is greater than 2, given that it is greater than 0.5.

(c) (4 marks) Calculate the expected loss amount.

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7. (11 marks) Let X be a random variable with the following moment generating function:

$$M_X(t) = \frac{\mathrm{e}^{\mu t}}{1 - b^2 t^2}, \qquad |t| < \frac{1}{b},$$

where b and μ are constants.

(a) (8 marks) Use the moment generating function to find the mean $E(X)$ and variance $V(X)$.

(b) (3 marks) Suppose Y is another random variable with moment generating function

$$M_Y(t) = \frac{p\mathrm{e}^t}{1 - (1-p)\mathrm{e}^t}, \qquad t < -\ln(1-p),$$

where $0 < p < 1$ is a constant. If X and Y are independent, find the moment generating function of $Z = 2X - 3Y + 4$.

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8. (16 marks) Let X and Y be two random variables with the following joint density function:

$$f(x,y) = \begin{cases} ax + 5y, & x^2 \le y \le x \\ 0, & \text{otherwise} \end{cases}.$$

(a) (4 marks) Find the value of a.

(b) (5 marks) Calculate $P(Y < 1/2)$.

(c) (7 marks) Find the marginal density function $f_Y(y)$, the conditional density function $f(x\,|\,y)$, and the conditional expected value $E(X\,|\,Y = y)$.

STANDARD NORMAL DISTRIBUTION TABLE

Entries represent $\Pr(Z \leq z)$. The value of z to the first decimal is given in the left column. The second decimal is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of $\Pr(Z \leq z)$							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
$\Pr(Z \leq z)$	0.800	0.850	0.900	0.950	0.975	0.990	0.995

— THE END —