

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. -3/2

☐ C

B. 1

☐ D

C. 0

D. -2

Back

Next

Use Gram-Schmidt algorithm to convert the basis

$$\{[0 \ -3 \ 5]^T, [-2 \ -2 \ 2]^T, [-4 \ -3 \ 4]^T\}$$

into an orthogonal basis  $\{E_1, E_2, E_3\}$ .

Find the **first coordinate** of the vector  $E_2$ .

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. 11

☐ C

B. 12

☐ D

C. 10

D. 13

Back

Next

Let  $X, Y$  be two independent Gaussian distributions

$$p(x) \sim \mathcal{N}(2, 1)$$

$$p(y) \sim \mathcal{N}(1, 3).$$

What is the variance of  $X - 2Y$ ?

$$1 - 2 \cdot 3$$

$$\text{VAR} (\mu, \text{var})$$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☐ C

A. 1

☐ D

B. -3

☒ E

C. 0

D. 2

E. none of the other choices is true

Back

Next

Let  $A$  be a  $2 \times 2$  matrix. Given that 1 and 2 are eigenvalues of  $A$  with corresponding eigenvectors  $[1 \ 3]^T$  and  $[0 \ 1]^T$ .

Find the (1,2)- entry of the matrix  $A$ .

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. -1.852

☐ C

B. 1.54

☐ D

C. 0.795

D. 0.823

Back

Next

Let  $f(x,y) = x^2y + 3y$ . Applying the gradient descent algorithm with the step-size  $\gamma = 0.1$ , the **momentum**  $\alpha = 0.02$  and the initial point  $(x_0, y_0) = (1, -1)$ , find  $y_2$  of the point after 2<sup>nd</sup> iteration.

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. (iii)

☐ C

B. (iv)

☐ D

C. (ii)

D. (i)

Back

Next

Consider the function  $f(x, y) = e^{-x^2+y^2}$ . What is the Hessian matrix  $H(0, 0)$  of  $f$  at the point (0, 0)?

(i)  $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$

(ii)  $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

(iii)  $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(iv)  $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☒ A. (i)☐ C

B. (ii)

☐ D

C. (iv)

☐ E

D. (iii)

E. None of the others

Back

Next

Given that  $\lambda=1$  is an eigenvalue of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Find a set of basic eigenvectors corresponding to this eigenvalue  $\lambda=1$ .

(i)  $\{[0,0,1]^T\}$

(ii)  $\{[1,0,0]^T, [0,0,1]^T\}$

(iii)  $\{[1,0,0]^T\}$



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☐ C

A. (ii)

☐ D

B. (iii)

C. (i)

D. (iv)

Back

Next

The (2, 3)-entry of the Hessian matrix of

$$f(x, y, z) = 3xyz - x^2 + 3y^2 - 5yz$$

at  $(1, -1, 2)$  is

(i) -2                      (ii) -8

(iii) -5                    (iv) -3

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☐ C

A. 2 and -3

☐ D

B. 2 and 3

☐ E

C. -2 and 3

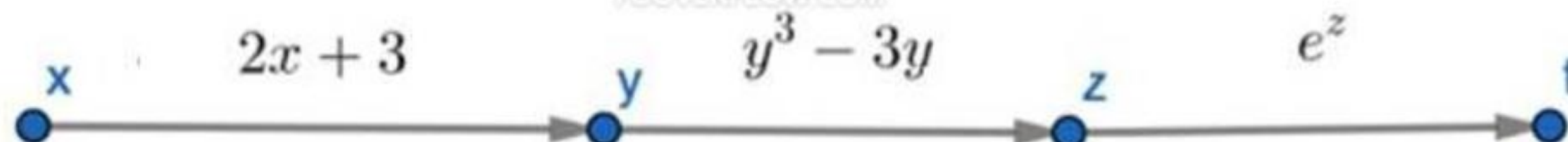
D. -2 and -3

E. -1 and -2

Back

Next

A computational graph is given as bellow.



Find all values of  $x$  such that the derivative of  $t$  with re



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☒ A. (iii)☐ C

B. (iv)

☐ D

C. (i)

D. (ii)

Back

Next

Compute the length of the vector  $\mathbf{x} = [2 \ -1]^T$  using the inner product

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{A} \mathbf{y},$$

in which  $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ .

- (i) 4
- (ii)  $\sqrt{17}$
- (iii)  $\sqrt{13}$
- (iv)  $\sqrt{15}$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. (ii)

☐ C

B. (iv)

☐ D

C. (iii)

D. (i)

Back

Next

Consider the following bivariate distribution  $f(x, y)$  of two random variables  $X$  and  $Y$

$$f(x, y) = \begin{cases} 1.5x + 0.5y & \text{if } 0 \leq x, y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability density function  $f(x)$  of  $X$ .

(i)  $f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

(ii)  $f(x) = \begin{cases} 0.5x + 0.75 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

(iii)  $f(x) = \begin{cases} x + 0.5 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

0,375

## Answer

☐ A☐ B☐ C☐ D☐ E

(Choose 1 answer)

(See picture)

A. 0.25

B. None of the other choices is correct

C. 0.5

D. 0.75

E. 0.125

Back

Next

Let  $X, Y$  be independent continuous random variables having uniform distributions over  $[0, 2]$ .

The probability  $P(0 \leq X \leq 1, 0 \leq Y \leq 1)$  is





Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☐ C☐ D

A. (i)

B. (iv)

C. (ii)

D. (iii)

Back

Next

The second order Taylor polynomial of

$$f(x, y) = \sin(2xy) + \cos y$$

about  $(0, 0)$  is:

(i)  $1 - xy + y^2$

(ii)  $1 + xy - y^2$

(iii)  $1 - 2xy + 0.5y^2$

(iv)  $1 + 2xy - 0.5y^2$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☐ C☐ D

A. (i)

B. (ii)

C. (iv)

D. (iii)

Back

Next

Let  $A$  be an invertible square matrix of size  $2 \times 2$ . Let 1 and 2 be the two singular values of  $A$ . Choose the correct answer :

- (i) 1 and  $1/2$  are two singular values of matrix  $A^{-1}$
- (ii) 1 and  $1/2$  are two singular values of matrix  $A^T$
- (iii) -1 and -2 are two singular values of matrix  $A^{-1}$
- (iv) 1 and  $\sqrt{2}$  are two singular values of matrix  $A^T$



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. (i)

☐ C

B. (ii)

☐ D

C. (iv)

D. (iii)

Back

Next

Find the dual optimization problem to the following linear programming

$$\min 2x_1 - 3x_2$$

$$\text{subject to } x_1 + x_2 \leq 1$$

$$3x_1 - 4x_2 \leq -2$$

$$x_2 \leq -1$$

$$\begin{aligned} \text{(i) } \max & \quad -\lambda_1 + 2\lambda_2 + \lambda_3 \\ \text{subject to } & \quad \lambda_1 + 3\lambda_2 = -2 \\ & \quad \lambda_1 - 4\lambda_2 + \lambda_3 = 3 \\ & \quad \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \text{(ii) } \max & \quad -\lambda_1 + 2\lambda_2 + \lambda_3 \\ \text{subject to } & \quad \lambda_1 + 3\lambda_2 = 2 \\ & \quad \lambda_1 - 4\lambda_2 + \lambda_3 = -3 \\ & \quad \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \text{(iii) } \max & \quad \lambda_1 + 2\lambda_2 - \lambda_3 \\ \text{subject to } & \quad \lambda_1 + 3\lambda_2 = -2 \\ & \quad \lambda_1 - 4\lambda_2 + \lambda_3 = 3 \\ & \quad \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \text{(iv) } \max & \quad \lambda_1 + 2\lambda_2 - \lambda_3 \\ \text{subject to } & \quad \lambda_1 + 3\lambda_2 = 2 \\ & \quad \lambda_1 - 4\lambda_2 + \lambda_3 = -3 \\ & \quad \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0 \end{aligned}$$



Answer

(Choose 1 answer)

- ☐ A (See picture)
- ☐ B
- ☒ A. (i)
- ☐ C
- ☐ D
- B. (iii)
- C. (iv)
- D. (i)

Back

Next

The first order Taylor polynomial of  $f(x, y) = (x^2 + 2xy)^{-2}$  at  $(1, 1)$  is of the form  $a + b(x - 1) + c(y - 1)$ .

The value of  $b + c$  is:

- |       |     |      |      |
|-------|-----|------|------|
| (i)   | 4/9 | (ii) | -4/9 |
| (iii) | 2/9 | (iv) | -2/9 |

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☐ C☒ D

A. (iv)

B. (iii)

C. (i)

D. (ii)

Back

Next

All nonzero singular values of  $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix}$  are:

(i) 3 and 1

(ii)  $\sqrt{3}$  and 1

(iii) 2 and 1

(iv)  $\sqrt{2}$  and 1

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☐ C

A. (iv)

☒ D

B. (iii)

C. (i)

D. (ii)

Back

Next

All nonzero singular values of  $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix}$  are:

(i) 3 and 1

(ii)  $\sqrt{3}$  and 1

(iii) 2 and 1

(iv)  $\sqrt{2}$  and 1



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. 20

☒ C

B. -64

☐ D

C. -32

D. 74

Back

Next

Find  $\frac{\partial z}{\partial x}(4, -4)$  for  $z = (4x + 6y)^2$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B☐ C☐ D☐ E

A. 0.6

B. 0.8

C. 0.7

D. None of the other choices is correct

E. 0.5

Back

Next

Consider the following bivariate distribution  $p(x, y)$  of two discrete random variables  $X$  and  $Y$ .

		Y		
		1	2	3
X	-1	0.2	0.3	0.1
	1	0.15	0.15	0.1

Compute  $P(X \leq 1, Y \leq 2)$ .

$$0.2 + 0.3 + 0.15 + 0.15$$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. (ii)

☐ C

B. (iii)

☐ D

C. (iv)

D. (i)

Back

Next

Let  $W = [X \ Y]^T$  be a 2-dimensional random variable with normal distribution  $W = N(\mu, \Sigma)$ , in which

$$\mu = [0 \ 1]^T, \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.$$

Find the formula for the probability density function  $p(x,y)$  of  $W$ .

(i)  $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{(x-1)^2}{2} - \frac{y^2}{4}}$

(ii)  $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{(x-1)^2}{4} - \frac{y^2}{2}}$

(iii)  $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{x^2}{4} - \frac{(y-1)^2}{2}}$

(iv)  $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{x^2}{2} - \frac{(y-1)^2}{4}}$



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. Only g

☐ C

B. Neither f nor g

☒ D

C. Both f and g

D. Only f

Back

Next

Which the following functions are convex on  $\mathbb{R}^2$ ?

$$f(x, y) = x^2 + 2y^2$$

$$g(x, y) = x^2 + xy + y^2.$$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. -0.00694

☐ C

B. 0.0017

☐ D

C. -0.0017

D. 0.00694

Back

Next

Let  $p(x,y) = y-x$  with  $0 \leq x \leq 1$ ,  $1 \leq y \leq 2$  be the joint probability function of some continuous random variables  $X$  and  $Y$ . Given the following marginal probability functions of  $X$  and  $Y$ .

$$p(x) = 1.5-x, \quad 0 \leq x \leq 1$$

$$p(y) = y-0.5, \quad 1 \leq y \leq 2$$

Find the covariance of  $X$  and  $Y$ .

Answer

☐ A☐ B☐ C☐ D

(Choose 1 answer)

(See picture)

A. (i)

B. (iv)

C. (ii)

D. (iii)

Back

Next

Consider the following bivariate distribution  $p(x, y)$  of two random variables  $X$  and  $Y$

$$p(x, y) = \begin{cases} xy, & \text{if } 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

The probability  $P(0 < X < 1, 0 \leq Y \leq 0.5)$  is

(i) 0.05

(ii) 0.0625

(iii) 0.075

(iv) 0.0825



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. -2

☐ CB.  $2/3$ ☐ D

C. 2

☐ ED.  $-4/3$ 

E. 1

Back

Next

Given a dataset consisting of the following vectors

$$\mathbf{x}_1 = [1 \ 0 \ 2]^T, \quad \mathbf{x}_2 = [0 \ 2 \ -1]^T, \quad \mathbf{x}_3 = [2 \ -2 \ 2]^T.$$

Let  $S = [s_{ij}]$  be the covariance matrix of this dataset. Find  $s_{33}$ .

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. -2

☐ CB.  $2/3$ ☐ D

C. 2

☐ ED.  $-4/3$ 

E. 1

Back

Next

Given a dataset consisting of the following vectors

$$\mathbf{x}_1 = [1 \ 0 \ 2]^T, \quad \mathbf{x}_2 = [0 \ 2 \ -1]^T, \quad \mathbf{x}_3 = [2 \ -2 \ 2]^T.$$

Let  $S = [s_{ij}]$  be the covariance matrix of this dataset. Find  $s_{33}$ .

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. -1 and 30

☐ C

B. -5 and -11

☐ D

C. 5 and 1

☐ E

D. 11 and -30

E. 1 and 11

Back

Next

Suppose that the characteristic polynomial of a matrix  $A$  is

$$p_A(\lambda) = \lambda^4 + \lambda^3 - 11\lambda^2 - 5\lambda + 30.$$

The trace and the determinant of  $A$  respectively are:



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. 3.231

☐ C

B. 3.196

☐ D

C. 3.017

D. 3.162

Back

Next

Let  $\hat{A}_1$  be the rank-1 approximation of the matrix  $A$  given by:

$$A = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix}$$

What is the spectral norm of  $A - \hat{A}_1$ ?

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. (iv)

☐ C

B. (ii)

☐ D

C. (iii)

D. (i)

Back

Next

For the following functions, calculate  $\frac{df}{dt}$  in matrix form:

$$f(x) = f(x_1, x_2) = x_1^4 \sin(x_2)$$

$$x_1 = 2t$$

$$x_2 = 1 + t^2$$

(i)  $\begin{bmatrix} 4x_1^3 \sin(x_2) \\ x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2 & 2t \end{bmatrix}$

(ii)  $\begin{bmatrix} 4x_1^3 \sin(x_2) & x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2 \\ 2t \end{bmatrix}$

(iii)  $\begin{bmatrix} 4x_1^3 \sin(x_2) \\ x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2t & 1 + t^2 \end{bmatrix}$

(iv)  $\begin{bmatrix} 4x_1^3 \sin(x_2) & x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2t \\ 1 + t^2 \end{bmatrix}$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. (iv)

☐ C

B. (ii)

☐ D

C. (iii)

D. (i)

Back

Next

For the following functions, calculate  $\frac{df}{dt}$  in matrix form:

$$f(x) = f(x_1, x_2) = x_1^4 \sin(x_2)$$

$$x_1 = 2t$$

$$x_2 = 1 + t^2$$

(i)  $\begin{bmatrix} 4x_1^3 \sin(x_2) \\ x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2 & 2t \end{bmatrix}$

(ii)  $\begin{bmatrix} 4x_1^3 \sin(x_2) & x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2 \\ 2t \end{bmatrix}$

(iii)  $\begin{bmatrix} 4x_1^3 \sin(x_2) \\ x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2t & 1 + t^2 \end{bmatrix}$

(iv)  $\begin{bmatrix} 4x_1^3 \sin(x_2) & x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2t \\ 1 + t^2 \end{bmatrix}$



Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. 0.23

☐ C

B. 0.07

☐ D

C. 0.16

D. 0.45

Back

Next

Consider the problem of minimizing  $f(x, y) = x^2 + xy + 2y^2$  using the gradient descent method. Starting from the point  $(x_0, y_0) = (1, 1)$  and step-size  $\gamma = 0.1$ . Find  $y_2$ .

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. (ii)

☐ C

B. (iii)

☐ D

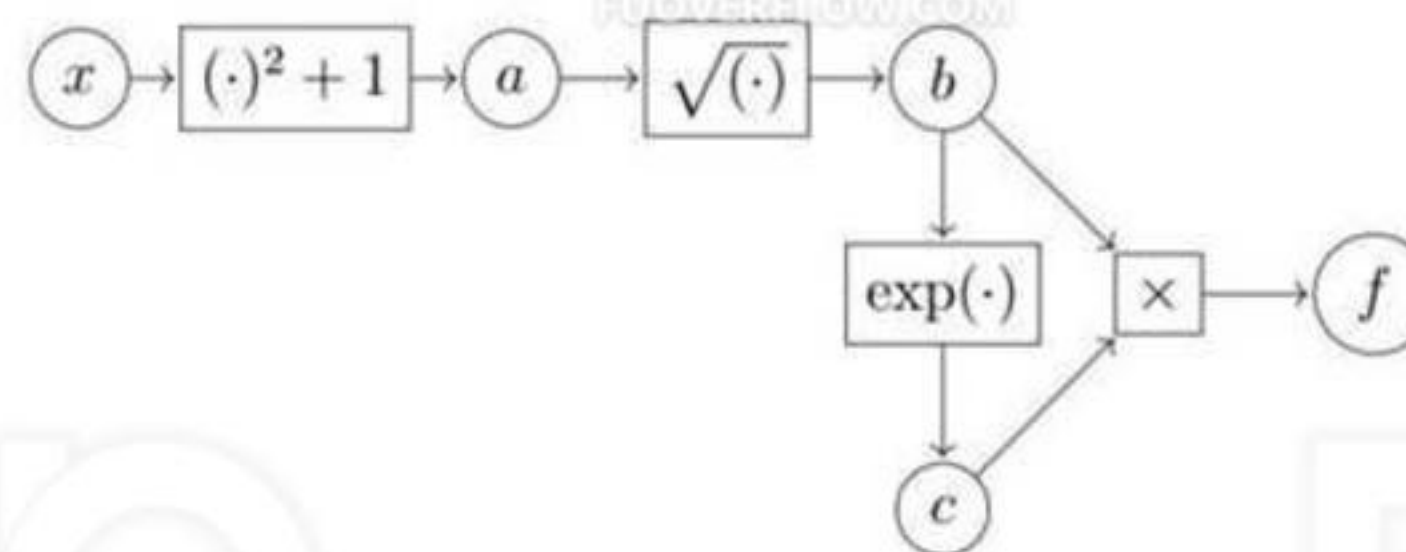
C. (iv)

D. (i)

Back

Next

Consider a computation graph with inputs  $x$ , function value  $f$  and intermediate variables  $a, b, c$ . Find the formula of  $f$ .



(i)  $(x^2 + 1) \exp(x^2 + 1)$

(ii)  $(x^2 + 1) \exp(\sqrt{x^2 + 1})$

(iii)  $\sqrt{x^2 + 1} \exp(x^2 + 1)$

(iv)  $\sqrt{x^2 + 1} \exp(\sqrt{x^2 + 1})$

Answer

(Choose 1 answer)

☐ A

(See picture)

☐ B

A. Both x and y

☐ C

B. x only

☐ D

C. y only

D. None of x or y

Back

Next

Let  $U = \text{span}\{[1 \ 1 \ -2]^T, [1 \ 0 \ 1]^T\}$ .  
Which of the following vectors are in  $U^\perp$ ?

$$\mathbf{x} = [1 \ -3 \ -1]^T \quad \mathbf{y} = [0 \ 0 \ -1]^T$$