

Answer

(Choose 1 answer)

- A (See picture)
- B A. -3/2
- C B. 1
- D C. 0
- D. -2

[Back](#)[Next](#)

Use Gram-Schmidt algorithm to convert the basis

$$\{[0 \ -3 \ 5]^T, [-2 \ -2 \ 2]^T, [-4 \ -3 \ 4]^T\}$$

into an orthogonal basis $\{E_1, E_2, E_3\}$.

Find the first coordinate of the vector E_2 .

Answer

(Choose 1 answer)

- A (See picture)
- B A. 11
- C B. 12
- D C. 10
- D D. 13

[Back](#)[Next](#)

Let X, Y be two independent Gaussian distributions

$$\begin{aligned} p(x) &\sim \mathcal{N}(2, 1) \\ p(y) &\sim \mathcal{N}(1, 3). \end{aligned}$$

What is the variance of $\underline{X - 2Y}$?

$$1 - 2 \cdot 3$$

Var (μ, var)

Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. 1
- D B. -3
- C. 0
- E D. 2
- E. none of the other choices is true

[Back](#)[Next](#)

Let A be a 2×2 matrix. Given that 1 and 2 are eigenvalues of A with corresponding eigenvectors $[1 \ 3]^T$ and $[0 \ 1]^T$.

Find the (1,2)- entry of the matrix A .

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Answer

(Choose 1 answer)

- A (See picture)
- B A. -1.852
- C B. 1.54
- D C. 0.795
- D D. 0.823

[Back](#)[Next](#)

Let $f(x,y) = x^2y + 3y$. Applying the gradient descent algorithm with the step-size $\gamma = 0.1$, the **momentum** $\alpha = 0.02$ and the initial point $(x_0, y_0) = (1, -1)$, find y_2 of the point after 2nd iteration.

Answer

(Choose 1 answer)

- A (See picture)
- B
- A. (iii)
- B. (iv)
- C. (ii)
- D. (i)

[Back](#)[Next](#)

Consider the function $f(x, y) = e^{-x^2+y^2}$. What is the Hessian matrix $H(0, 0)$ of f at the point (0, 0)?

- (i) $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$
- (ii) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$
- (iii) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
- (iv) $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

Answer

(Choose 1 answer)

- A (See picture)
- B
- C (i)
- D (ii)
- E (iv)
- D. (iii)
- E. None of the others

Back

Next

Given that $\lambda=1$ is an eigenvalue of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Find a set of basic eigenvectors corresponding to this eigenvalue $\lambda=1$.

(i) $\{[0,0,1]^T\}$

(ii) $\{[1,0,0]^T, [0,0,1]^T\}$

(iii) $\{[1,0,0]^T\}$

Answer

(Choose 1 answer)

- A (See picture)
- B
- C. (ii)
- D. (iii)
- C. (i)
- D. (iv)

[Back](#)[Next](#)

The (2, 3)-entry of the Hessian matrix of

$$f(x, y, z) = 3xyz - x^2 + 3y^2 - 5yz$$

at (1, -1, 2) is

- (i) -2
- (ii) -8
- (iii) -5
- (iv) -3

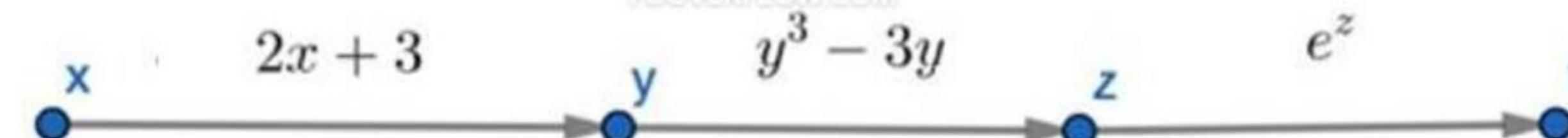
Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. 2 and -3
- D B. 2 and 3
- E C. -2 and 3
- D. -2 and -3
- E. -1 and -2

[Back](#)[Next](#)

A computational graph is given as bellow.



Find all values of x such that the derivative of t with re

Answer

(Choose 1 answer)

- A (See picture)
- B
- A. (iii)
- B. (iv)
- C. (i)
- D. (ii)

[Back](#)[Next](#)

Compute the length of the vector $\mathbf{x} = [2 \ -1]^T$ using the inner product

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{A} \mathbf{y},$$

in which $A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$.

- (i) 4
- (ii) $\sqrt{17}$
- (iii) $\sqrt{13}$
- (iv) $\sqrt{15}$

Answer

(Choose 1 answer)

- A (See picture)
- B
- A. (ii)
- B. (iv)
- C. (iii)
- D. (i)

Back

Next

Consider the following bivariate distribution $f(x, y)$ of two random variables X and Y

$$f(x, y) = \begin{cases} 1.5x + 0.5y & \text{if } 0 \leq x, y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability density function $f(x)$ of X .

6, 575

(i) $f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

(ii) $f(x) = \begin{cases} 0.5x + 0.75 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

(iii) $f(x) = \begin{cases} x + 0.5 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. 0.25
- D B. None of the other choices is correct
- E C. 0.5
- D. 0.75
- E. 0.125

[Back](#)[Next](#)

Let X, Y be independent continuous random variables having uniform distributions over $[0, 2]$.

The probability $P(0 \leq X \leq 1, 0 \leq Y \leq 1)$ is

Answer

(Choose 1 answer)

- A (See picture)
- B
- C
- D

[Back](#)[Next](#)

The second order Taylor polynomial of

$$f(x, y) = \sin(2xy) + \cos y$$

about $(0, 0)$ is:

- (i) $1 - xy + y^2$
- (ii) $1 + xy - y^2$
- (iii) $1 - 2xy + 0.5y^2$
- (iv) $1 + 2xy - 0.5y^2$

Answer

(Choose 1 answer)

- A (See picture)
- B A. (i)
- C B. (ii)
- D C. (iv)
- D. (iii)

[Back](#)[Next](#)

Let A be an invertible square matrix of size 2×2 . Let 1 and 2 be the two singular values of A . Choose the correct answer :

- (i) 1 and $1/2$ are two singular values of matrix A^{-1}
- (ii) 1 and $1/2$ are two singular values of matrix A^T
- (iii) -1 and -2 are two singular values of matrix A^{-1}
- (iv) 1 and $\sqrt{2}$ are two singular values of matrix A^T

Answer

(Choose 1 answer)

- A (See picture)
- B
- C
- D

[Back](#)[Next](#)

Find the dual optimization problem to the following linear programming

$$\begin{aligned} \min \quad & 2x_1 - 3x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 1 \\ & 3x_1 - 4x_2 \leq -2 \\ & x_2 \leq -1 \end{aligned}$$

(i) $\max -\lambda_1 + 2\lambda_2 + \lambda_3$
subject to $\lambda_1 + 3\lambda_2 = -2$
 $\lambda_1 - 4\lambda_2 + \lambda_3 = 3$
 $\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0$

(iii) $\max \lambda_1 + 2\lambda_2 - \lambda_3$
subject to $\lambda_1 + 3\lambda_2$
 $\lambda_1 - 4\lambda_2 + \lambda_3$
 $\lambda_1 \geq 0, \lambda_2 \geq 0, \dots$

(ii) $\max -\lambda_1 + 2\lambda_2 + \lambda_3$
subject to $\lambda_1 + 3\lambda_2 = 2$
 $\lambda_1 - 4\lambda_2 + \lambda_3 = -3$
 $\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0$

(iv) $\max \lambda_1 + 2\lambda_2 - \lambda_3$
subject to $\lambda_1 + 3\lambda_2$
 $\lambda_1 - 4\lambda_2 + \lambda_3$
 $\lambda_1 \geq 0, \lambda_2 \geq 0, \dots$

Answer

(Choose 1 answer)

- A (See picture)
- B A. (i)
- C B. (iii)
- D C. (iv)
- D. (i)

Back

Next

The first order Taylor polynomial of $f(x, y) = (x^2 + 2xy)^{-2}$ at $(1, 1)$ is of the form $a + b(x - 1) + c(y - 1)$.

The value of $b + c$ is:

- (i) $\frac{4}{9}$
- (ii) $-\frac{4}{9}$
- (iii) $\frac{2}{9}$
- (iv) $-\frac{2}{9}$



Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. (iv)
- D B. (iii)
C. (i) D. (ii)

Back

Next

All nonzero singular values of $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix}$ are:

- (i) 3 and 1
- (ii) $\sqrt{3}$ and 1
- (iii) 2 and 1
- (iv) $\sqrt{2}$ and 1



Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. (iv)
- D B. (iii)
- C. (i) C
- D. (ii)

Back

Next

All nonzero singular values of $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix}$ are:

- (i) 3 and 1
- (ii) $\sqrt{3}$ and 1
- (iii) 2 and 1
- (iv) $\sqrt{2}$ and 1

Answer

(Choose 1 answer)

- A (See picture)
- B A. 20
- C B. -64
- D C. -32
- D D. 74

[Back](#)[Next](#)

Find $\frac{\partial z}{\partial x}(4, -4)$ for $z = (4x + 6y)^2$

Answer

(Choose 1 answer)

- A (See picture)
- B
- C
- D. 0.8
- E. 0.7
- F. None of the other choices is correct
- G. 0.5

Back

Next

Consider the following bivariate distribution $p(x, y)$ of two discrete random variables X and Y .

		Y		
		1	2	3
X	-1	0.2	0.3	0.1
	1	0.15	0.15	0.1

Compute $P(X \leq 1, Y \leq 2)$.

$$\text{G, Z} + \text{Y} \geq + 0_1^{15} \text{ to } 1^5$$

Answer

(Choose 1 answer)

- A (See picture)
- B
- A. (ii)
- B. (iii)
- C. (iv)
- D. (i)

Back

Next

Let $W = [X \ Y]^T$ be a 2-dimensional random variable with normal distribution $W \sim N(\mu, \Sigma)$, in which

$$\mu = [0 \ 1]^T, \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.$$

Find the formula for the probability density function $p(x,y)$ of W .

(i) $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{(x-1)^2}{2}-\frac{y^2}{4}}$

(ii) $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{(x-1)^2}{4}-\frac{y^2}{2}}$

(iii) $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{x^2}{4}-\frac{(y-1)^2}{2}}$

(iv) $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{x^2}{2}-\frac{(y-1)^2}{4}}$

Answer

(Choose 1 answer)

- A (See picture)
- B A. Only g
- C B. Neither f nor g
- D C. Both f and g
- D. Only f

[Back](#)[Next](#)

Which the following functions are convex on \mathbb{R}^2 ?

$$f(x, y) = x^2 + 2y^2$$

$$g(x, y) = x^2 + xy + y^2.$$

Answer

(Choose 1 answer)

- A (See picture)
- B A. -0.00694
- C B. 0.0017
- D C. -0.0017
- D D. 0.00694

[Back](#)[Next](#)

Let $p(x,y) = y-x$ with $0 \leq x \leq 1$, $1 \leq y \leq 2$ be the joint probability function of some continuous random variables X and Y . Given the following marginal probability functions of X and Y .

$$p(x) = 1.5-x, \quad 0 \leq x \leq 1$$

$$p(y) = y-0.5, \quad 1 \leq y \leq 2$$

Find the covariance of X and Y .

Answer

(Choose 1 answer)

- A (See picture)
- B
- C
- D

[Back](#)[Next](#)

Consider the following bivariate distribution $p(x, y)$ of two random variables X and Y

$$p(x, y) = \begin{cases} xy, & \text{if } 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

The probability $P(0 < X < 1, 0 \leq Y \leq 0.5)$ is

- (i) 0.05
- (ii) 0.0625
- (iii) 0.075
- (iv) 0.0825

Answer

(Choose 1 answer)

- A (See picture)
- B A. -2
- C B. $\frac{2}{3}$
- D C. 2
- E D. $-\frac{4}{3}$
- F E. 1

[Back](#)[Next](#)

Given a dataset consisting of the following vectors

$$\mathbf{x}_1 = [1 \ 0 \ 2]^T, \quad \mathbf{x}_2 = [0 \ 2 \ -1]^T, \quad \mathbf{x}_3 = [2 \ -2 \ 2]^T.$$

Let $S = [s_{ij}]$ be the covariance matrix of this dataset. Find s_{33} .

Answer

(Choose 1 answer)

- A (See picture)
- B A. -2
- C B. $\frac{2}{3}$
- D C. 2
- E D. $-\frac{4}{3}$
- F E. 1

[Back](#)[Next](#)

Given a dataset consisting of the following vectors

$$\mathbf{x}_1 = [1 \ 0 \ 2]^T, \quad \mathbf{x}_2 = [0 \ 2 \ -1]^T, \quad \mathbf{x}_3 = [2 \ -2 \ 2]^T.$$

Let $S = [s_{ij}]$ be the covariance matrix of this dataset. Find s_{33} .

Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. -1 and 30
- D B. -5 and -11
- E C. 5 and 1
- D. 11 and -30
- E. 1 and 11

[Back](#)[Next](#)

Suppose that the characteristic polynomial of a matrix A is

$$p_A(\lambda) = \lambda^4 + \lambda^3 - 11\lambda^2 - 5\lambda + 30.$$

The trace and the determinant of A respectively are:

Answer

(Choose 1 answer)

- A (See picture)
- B A. 3.231
- C B. 3.196
- D C. 3.017
- D D. 3.162

[Back](#)[Next](#)

Let \hat{A}_1 be the rank-1 approximation of the matrix A given by:

$$A = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix}$$

What is the spectral norm of $A - \hat{A}_1$?

Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. (iv)
- D B. (ii)
- C. (iii)
- D. (i)

[Back](#)[Next](#)

For the following functions, calculate $\frac{df}{dt}$ in matrix form:

$$f(x) = f(x_1, x_2) = x_1^4 \sin(x_2)$$

$$x_1 = 2t$$

$$x_2 = 1 + t^2$$

(i)
$$\begin{bmatrix} 4x_1^3 \sin(x_2) \\ x_1^4 \cos(x_2) \end{bmatrix} [2 \quad 2t]$$

(ii)
$$\begin{bmatrix} 4x_1^3 \sin(x_2) & x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2 \\ 2t \end{bmatrix}$$

(iii)
$$\begin{bmatrix} 4x_1^3 \sin(x_2) \\ x_1^4 \cos(x_2) \end{bmatrix} [2t \quad 1 + t^2]$$

(iv)
$$\begin{bmatrix} 4x_1^3 \sin(x_2) & x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2t \\ 1 + t^2 \end{bmatrix}$$

Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. (iv)
- D B. (ii)
- C. (iii)
- D. (i)

[Back](#)[Next](#)

For the following functions, calculate $\frac{df}{dt}$ in matrix form:

$$f(x) = f(x_1, x_2) = x_1^4 \sin(x_2)$$

$$x_1 = 2t$$

$$x_2 = 1 + t^2$$

(i)
$$\begin{bmatrix} 4x_1^3 \sin(x_2) \\ x_1^4 \cos(x_2) \end{bmatrix} [2 \quad 2t]$$

(ii)
$$\begin{bmatrix} 4x_1^3 \sin(x_2) & x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2 \\ 2t \end{bmatrix}$$

(iii)
$$\begin{bmatrix} 4x_1^3 \sin(x_2) \\ x_1^4 \cos(x_2) \end{bmatrix} [2t \quad 1 + t^2]$$

(iv)
$$\begin{bmatrix} 4x_1^3 \sin(x_2) & x_1^4 \cos(x_2) \end{bmatrix} \begin{bmatrix} 2t \\ 1 + t^2 \end{bmatrix}$$

Answer

(Choose 1 answer)

- A (See picture)
- B A. 0.23
- C B. 0.07
- D C. 0.16
- D D. 0.45

[Back](#)[Next](#)

Consider the problem of minimizing $f(x, y) = x^2 + xy + 2y^2$ using the gradient descent method. Starting from the point $(x_0, y_0) = (1, 1)$ and step-size $\gamma = 0.1$. Find y_2 .

FUOVERFLOW.COM

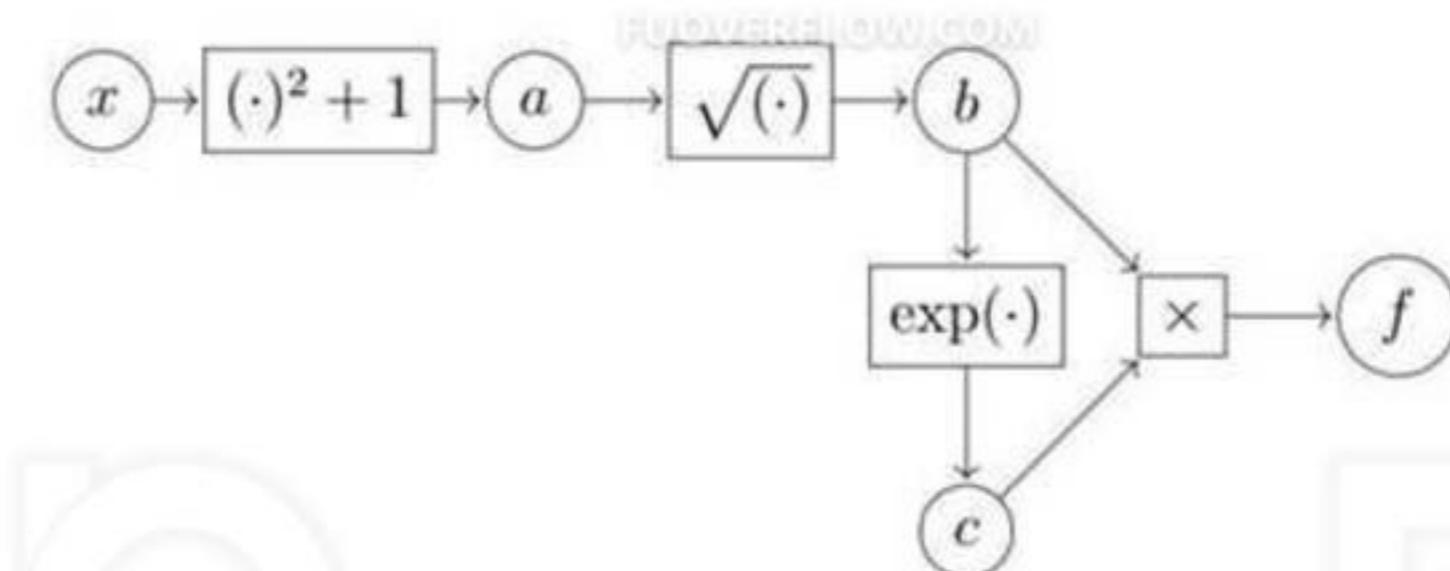
Answer

(Choose 1 answer)

- A (See picture)
- B
- C A. (ii)
- D B. (iii)
- C. (iv)
- D. (i)

[Back](#)[Next](#)

Consider a computation graph with inputs x , function value f and intermediate variables a, b, c . Find the formula of f .



- (i) $(x^2 + 1) \exp(x^2 + 1)$
- (ii) $(x^2 + 1) \exp(\sqrt{x^2 + 1})$
- (iii) $\sqrt{x^2 + 1} \exp(x^2 + 1)$
- (iv) $\sqrt{x^2 + 1} \exp(\sqrt{x^2 + 1})$

Answer

(Choose 1 answer)

- A (See picture)
- B A. Both x and y
- C B. x only
- D C. y only
- D. None of x or y

[Back](#)[Next](#)

Let $U = \text{span}\{[1 \ 1 \ -2]^T, [1 \ 0 \ 1]^T\}$.
Which of the following vectors are in U^\perp ?

$$\mathbf{x} = [1 \ -3 \ -1]^T \quad \mathbf{y} = [0 \ 0 \ -1]^T$$