Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. 4/3

B. 3/4

C. 3/13

D. 13/3

Back Next

Find the number c such that the function $f(x,y) = cxy^2$ with $0 \le x \le 1$, $1 \le y \le 3$ satisfies conditions to be the joint probability function of some continuous random variables X and Y.

$$\int_0^1 \int_1^3 f(x,y)\,dy\,dx$$

$$\Leftrightarrow \underline{\int_0^1 \int_1^3 cxy^2\,dy\,dx}$$

$$cx.\int_1^3 y^2\,dy = \frac{26}{3}cx \Rightarrow \int_0^1 \frac{26}{3}cx\,dx$$

$$\Leftrightarrow \frac{26}{3}c.\frac{1}{2} = 1 \Rightarrow c = \frac{3}{13}$$

[ ] ...to finish the exam.

Finish

Exit

Answer

A

B

C

D

(Choose 1 answer)

(See picture)

A. (3,7)

B. (3,-7)

C. (-3,-7)

D. (-3,7)

Back Next

Find all values of x such that the matrix

$$A = \begin{pmatrix} 1 & x & 3 \\ -1 & 2 & 1 \\ 2 & 1 & x \end{pmatrix}$$

has determinant equal to 5.

...t to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

Back Next

(Choose 1 answer)

Let A be a matrix of size 4x4 with det(A) = 24. Assume A has four singular values in which three of them are 1, 2 and 3. Find the remaining singular value of A.

A. 4

B. 9

C. 16

D. 8

$$\det = \sigma_1 \cdot \sigma_2 \cdot \sigma_3 \cdots \sigma_n$$

$$\Rightarrow 1 \cdot 2 \cdot 3 \cdot \sigma_4 = 24$$

$$\Rightarrow \sigma_4 = 4$$

I want to finish the exam.

Finish

Exit

Answer

(Choose 1 answer)

(See picture)

- [ ] A
- [ ] B
- [ ] C
- [ ] D

A. (ii)

B. (iv)

C. (iii)

D. (i)

Back Next

Find the dual Lagrangian $\mathfrak{D}(\lambda)$ of the following quadratic programming

$$\min\ x_1^2 + 3x_2^2$$

$$\text{subject to}\quad x_1 + x_2 \le -1$$

$$2x_1 - x_2 \le 2$$

(i) $-\frac{1}{3}\lambda_1^2 - \frac{5}{6}\lambda_1\lambda_2 - \frac{13}{12}\lambda_2^2 - \lambda_1 + 2\lambda_2$

(ii) $-\frac{1}{3}\lambda_1^2 - \frac{5}{6}\lambda_1\lambda_2 - \frac{13}{12}\lambda_2^2 + \lambda_1 - 2\lambda_2$

(iii) $-\frac{1}{3}\lambda_1^2 + \frac{5}{6}\lambda_1\lambda_2 - \frac{13}{12}\lambda_2^2 - \lambda_1 + 2\lambda_2$

(iv) $-\frac{1}{3}\lambda_1^2 - \frac{5}{6}\lambda_1\lambda_2 + \frac{13}{12}\lambda_2^2 + \lambda_1 - 2\lambda_2$

Want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. 0.00694

B. -0.0017

C. -0.00694

D. 0.0017

Back | Next

Let $p(x,y) = (1/2)(y-x)$ with $0 \le x \le 1$, $2 \le y \le 3$ be the joint probability function of some continuous random variables X and Y. Given the following marginal probability functions of X and Y.

$p(x) = (5/4)-(1/2)x, \quad 0 \le x \le 1$

$p(y) = (1/2)y-(1/4), \quad 2 \le y \le 3$

Find the covariance of X and Y.

$\mathrm{cov}[x,y] = E[x,y] - E[x]E[y]$

$E(x) = \int_0^1 x \cdot p(x)\,dx$

$\Leftrightarrow E(x) = \int_0^1 x\left(\frac{5}{4} - \frac{1}{2}x\right)dx = \frac{11}{24}$

$E(y) = \int_2^3 y \cdot p(y)\,dy = \int_2^3 y\left(\frac{1}{2}y - \frac{1}{4}\right)dy = \frac{61}{24}$

$E[x,y] = \int_2^3\int_0^1 xy \cdot p(x,y)\,dy\,dx = \int_2^3\int_0^1 xy \cdot \frac{1}{2}(y-x)\,dy\,dx$

$\int_2^3 \frac{1}{2}\int_0^1 xy^2 - x^2y\,dy\,dx = \int_2^3 \frac{1}{2}\left(x\int_0^1 y^2\,dy - x^2\int_0^1 y\,dy\right)$

$\int_2^3 \frac{1}{2}\left(\frac{1}{3}x - \frac{1}{2}x^2\right)dx = -\frac{7}{6}$

$\mathrm{cov}[x,y] = -\frac{7}{6} - \frac{11}{24}\cdot\frac{61}{24}$

- [ ] I want to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. (i)

B. (ii)

C. (iv)

D. (iii)

Back Next

Let $p(x,y) = (3/13)xy^2$ with $0 \le x \le 1$, $1 \le y \le 3$ be the joint probability function of some continuous random variables X and Y. Find the marginal probability distribution p(x) of X.

(i) $p(x) = 2x, \quad 0 \le x \le 1$

(ii) $p(x) = x + 0.5, \quad 0 \le x \le 1$

(iii) $p(x) = 1.5x - 0.5, \quad 0 \le x \le 1$

(iv) $p(x) = 4x - 1, \quad 0 \le x \le 1$

$$\int_1^3 \frac{3}{13} x y^2 \, dy = \frac{3}{13} x \cdot \int_1^3 y^2 dy$$

$$p(x) = 2x \quad 0 \le x \le 1$$

... to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. (iv)

B. (iii)

C. (ii)

D. (i)

Back Next

Let X and Y be independent random variables, each taking the values -1, 0 or 1. Let $Z = X + 2Y$. How many possible values does Z have?

(i) 9 (ii) 8

$\boxed{\text{(iii) } 7}$ (iv) 6

$2^3 = 8$

$Y \backslash X$	-1	0	1
-1	-3	-2	-1
0	-1	0	1
1	1	2	3

possible value : $-3, -2, -1, 0, 1, 2, 3$

$\Rightarrow 7$

I want to finish the exam.

Exit

Answer

(Choose 1 answer)

(See picture)

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

A. -15

B. 8

C. 0

D. 2

E. -1

Back Next

Suppose that the characteristic polynomial of a matrix A is

$$p_A(\lambda) = -\lambda^3 + 8\lambda^2 + 2\lambda - 15.$$

What is the value of the trace of A?

$$P_A(\lambda) = (-1)^n \left(\lambda^n - (\text{trace of } A)\lambda^{n-1} + \dots + \det(A)\right)$$

$$\Rightarrow P_A(\lambda) = -\lambda^3 + \underbrace{8}_{\text{trace}}\lambda^2 + 2\lambda \underbrace{-15}_{\det(A)}$$

$$\Rightarrow \text{trace of } A = 8$$

[ ] Want to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

Back | Next

(Choose 1 answer)

(See picture)

A. 2/3

B. 8/3

C. 2

D. -4/3

E. -1

Given a dataset consisting of the following vectors

$x_1 = [0\ \ 2\ \ 2]^T,\quad x_2 = [1\ \ 0\ \ 2]^T,\quad x_3 = [2\ \ -2\ \ -1]^T.$

Let $S = [s_{ij}]$ be the covariance matrix of this dataset. Find s_{22}.

$$\bar{x} = \frac{1}{3}(x_1 + x_2 + x_3) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 - \bar{x} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$x_2 - \bar{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x_3 - \bar{x} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$$

$$s_{22} = \frac{1}{n} \sum_{i=n}^{n} (x_{i2} - \bar{x}_2)^2$$

$$s_{22} = \frac{1}{3}\left[2^2 + 0^2 + (-2)^2\right]$$

$$s_{22} = \frac{1}{3}(4+4) = 4$$

I want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

(Choose 1 answer)

(See picture)

A. -48

$\boxed{\text{B. -42}}$

C. -50

D. -52

E. -45

Back Next

Find the determinant of the Hessian of

$$f(x,y,z) = x^2 + 2y^2 - z^2 + 2xy + xz - 3yz + x + 2y + 3z + 5$$

$$\text{Hessian} \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial z \partial x} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial z \partial y} \\ \frac{\partial^2 f}{\partial x \partial z} & \frac{\partial^2 f}{\partial y \partial z} & \frac{\partial^2 f}{\partial z^2} \end{vmatrix}$$

$2x + 2y + z + 1 \rightarrow \frac{\partial^2 f}{\partial x^2} = 2, \frac{\partial^2 f}{\partial x \partial y} = 2, \frac{\partial^2 f}{\partial x \partial z} = 1$

$4y + 2x - 3z + 2 \rightarrow \frac{\partial^2 f}{\partial y \partial x} = 2, \frac{\partial^2 f}{\partial y^2} = 4, \frac{\partial^2 f}{\partial y \partial z} = -3$

$-2z + x - 3y + 3 \rightarrow \frac{\partial^2 f}{\partial z \partial x} = 1, \frac{\partial^2 f}{\partial z \partial y} = -3, \frac{\partial^2 f}{\partial z^2} = -2$

$$\Rightarrow H = \begin{vmatrix} 2 & 2 & 1 \\ 2 & 4 & -3 \\ 1 & -3 & -2 \end{vmatrix} \Rightarrow \det(H) = -42$$

- [ ] I want to finish the exam.

Finish

Exit

Answer

- [x] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

Back Next

(Choose 1 answer)

(See picture)

A. 2.5

B. 3.5

C. None of the other choices is correct

D. 5.5

E. 4.5

Consider the mixture of two independent Gaussians X and Y

$$p\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \overset{w_1 = 5}{0.5}\mathcal{N}\left(\begin{bmatrix} 8 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right) \underset{w_2 = 1}{+} \mathcal{N}\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}\right).$$

Find the variance of Y.

$$\Sigma = \begin{vmatrix} Var(x_1) & Cov(x_1, x_2) & \cdots & Cov(x_1, x_n) \\ Cov(x_2, x_1) & Var(x_2) & \cdots & Cov(x_2, x_n) \\ \vdots & \vdots & \ddots & \\ Cov(x_n, x_1) & Cov(x_n, x_2) & \cdots & Var(x_n) \end{vmatrix}$$

$$\Sigma \begin{vmatrix} Var(x) & Cov(x, y) \\ Cov(y, x) & Var(y) \end{vmatrix}$$

$$\mu = 0{,}5 \begin{bmatrix} 8 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7/2 \end{bmatrix}$$

$$Var(Y) = \sum_i w_i \left[Var(Y) + (\mu_i(y) - \mu(y))^2 \right]$$

$$\Leftrightarrow Var(Y) = 5 \cdot \left[2 + \left(3 - \tfrac{7}{2}\right)^2 \right] + 1 \left[4 + \left(2 - \tfrac{7}{2}\right)^2 \right]$$

$$\Leftrightarrow Var(Y) = 7{,}375$$

I want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. (ii)

B. (iii)

C. (i)

D. (iv)

Back Next

Let c be the number such that $f(x,y) = c(x+y)$ is the joint probability distribution of two discrete random variables X and Y, where $x = 2, 3, 4$ and $y = 0, 1, 2$. Describe the marginal probability distribution of Y.

(i)

Y	0	1	2
P(Y=k)	10c	14c	18c

(ii)

Y	1	2	3
P(Y=k)	10c	13c	16c

(iii)

Y	1	1	3
P(Y=k)	9c	13c	17c

Want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. (ii)

B. (iii)

C. (i)

D. (iv)

Back Next

Let $\mathbf{x} = \begin{bmatrix} 1 & -1 & 7 \end{bmatrix}^T$, $\mathbf{y} = \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}^T$ in $\mathbb{R}^3$.

The Manhattan norm (or the ℓ_1 norm) of $2\mathbf{x} - \mathbf{y}$ is:

(i) 12

(ii) 18

(iii) $\sqrt{146}$

(iv) $\sqrt{118}$

$$2x - y = \begin{bmatrix} 4 \\ -3 \\ \underline{11} \end{bmatrix}$$

$$\text{Manhattan norm} = \sum_{1} |x_1|$$

$$\Rightarrow \ell_1 \text{ norm} = |4| + |-3| + |\underline{11}| = \underline{18}$$

☐ I want to finish the exam.

Finish

Exit

Answer

(Choose 1 answer)

(See picture)

- [ ] A
- [ ] B
- [ ] C
- [ ] D

A. (iv)

B. (iii)

C. (i)

D. (ii)

Back Next

The second order Taylor polynomial of

$$f(x,y) = (x^2 + 2y^2)^5 \text{ about } (1,0) \text{ is}$$

$$a_1x^2 + a_2xy + a_3y^2 + a_4x + a_5y + a_6.$$

The value of a_4 is

(i) 10 (ii) -80

(iii) 45 (iv) -90

Want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

Back Next

(Choose 1 answer)

(See picture)

A. Both x and y

B. x only

C. None of x or y

D. y only

Let $U = \text{span}\{[1 \;\; 1 \;\; -2]^T, \; [1 \;\; 0 \;\; 1]^T\}$.
Which of the following vectors are in $U^{\perp}$?

$\mathbf{x} = [1 \;\; -3 \;\; -1]^T \quad \mathbf{y} = [0 \;\; 0 \;\; -1]^T$

$$U = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$v^T \cdot U = 0 \Rightarrow \perp$$

I want to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. (iii)

B. (ii)

C. (i)

D. (iv)

Back Next

Let $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T, \mathbf{y} = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T$ in $\mathbb{R}^2$. The length of the vector $\begin{bmatrix} -2 & 1 \end{bmatrix}^T$ using the inner product

$$\langle \mathbf{x}, \mathbf{y} \rangle = x_1 y_1 + x_1 y_2 + x_2 y_1 + 3 x_2 y_2$$

is:

$x = [-2 \quad 1], \; y = [-2 \quad 1]$

(i) 3

$\langle x, y \rangle = -2 \cdot -2 + (-2) \cdot 1 + 1 \cdot (-2) + 3(1)(1)$

(ii) $\sqrt{3}$

(iii) $\sqrt{11}$

$\Leftrightarrow \quad \langle x, y \rangle = 3$

length $= \sqrt{3}$

[ ] ... to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F

Back Next

(Choose 1 answer)

(See picture)

A. 4.5

B. 3.5

C. 4.0

D. None of the other choices is correct

E. 3.0

F. 2.0

Assume $f = f(x,y)$, $x = x(t)$, $y = y(t)$.

We know that at some point $t = t_0$, the partial derivaties are as follows:

$dx/dt = 14$, $\partial f/\partial x = 2$, $\partial f/\partial y = 2$, $df/dt = 34$.

Compute dy/dt.

$$\frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$$

$$\Leftrightarrow 34 = 2 \cdot 14 + 2 \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = 3$$

I want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. Both (i) and (ii)

$\boxed{\text{B. Neither (i) nor (ii)}}$

C. Only (i)

D. Only (ii)

Back Next

Which of the following is a symmetric, positive definite matrix?

(i) $\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$

$\det \begin{vmatrix} 1 & 3 \\ 3 & 5 \end{vmatrix} = -4 \Rightarrow$ non symmetric

$\det \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} = -1 \Rightarrow$ nonsymmetric

- [ ] ... to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

Back Next

(Choose 1 answer)

(See picture)

A. -1

B. None of the other choices is correct

C. 2

D. -2

E. 1

If we apply the Gram-Schmidt algorithm to find an orthogonal basis $\{E_1, E_2\}$ from the basis

$$\{[0 \quad 1 \quad 1]^T, [-2 \quad -4 \quad -2]^T\}$$

then what is the third coordinate of E_2 ?

Gram-Schidt of E_1

$$E_2 = v_2 - \frac{\langle v_2 \cdot v_1 \rangle}{\langle v_1 \cdot v_1 \rangle} v_1 = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

$\Rightarrow$ third coordinate : 1

- [ ] I want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

Back Next

(Choose 1 answer)

(See picture)

A. (iv)

B. (iii)

C. None of the other choices is correct

D. (i)

E. (ii)

Find the Taylor series of the fuction

$$f(x) = 1 + x + x^2$$

at $a = 1$.

(i) $3 - 3(x-1) - (x-1)^2$

(ii) $3 - 3(x-1) + (x-1)^2$

(iii) $3 + 3(x-1) - (x-1)^2$

(iv) $3 + 3(x-1) + (x-1)^2$

I want to finish the exam.

Finish

Exit

Answer

(Choose 1 answer)

(See picture)

- [ ] A
- [ ] B
- [ ] C
- [ ] D

A. (i)

B. (iv)

C. (ii)

D. (iii)

Back Next

Find the dual optimization problem to the following linear programming

$$\begin{aligned} \min\ & 3x_1 - 2x_2 \\ \text{subject to}\ & x_1 + x_2 \le 1 \\ & 3x_1 - 4x_2 \le -2 \\ & x_2 \le -1 \end{aligned}$$

(i)
$$\begin{aligned} \max\ & -\lambda_1 + 2\lambda_2 + \lambda_3 \\ \text{subject to}\ & \lambda_1 + 3\lambda_2 = -2 \\ & \lambda_1 - 4\lambda_2 + \lambda_3 = 3 \\ & \lambda_1 \ge 0, \lambda_2 \ge 0, \lambda_3 \ge 0 \end{aligned}$$

(ii)
$$\begin{aligned} \max\ & -\lambda_1 + 2\lambda_2 + \lambda_3 \\ \text{subject to}\ & \lambda_1 + 3\lambda_2 = -3 \\ & \lambda_1 - 4\lambda_2 + \lambda_3 = 2 \\ & \lambda_1 \ge 0, \lambda_2 \ge 0, \lambda_3 \ge 0 \end{aligned}$$

(iii)
$$\begin{aligned} \max\ & \lambda_1 + 2\lambda_2 - \lambda_3 \\ \text{subject to}\ & \lambda_1 + 3\lambda_2 = -3 \\ & \lambda_1 - 4\lambda_2 + \lambda_3 = 2 \\ & \lambda_1 \ge 0, \lambda_2 \ge 0, \lambda_3 \ge 0 \end{aligned}$$

(iv)
$$\begin{aligned} \max\ & \lambda_1 + 2\lambda_2 - \lambda_3 \\ \text{subject to}\ & \lambda_1 + 3\lambda_2 = 2 \\ & \lambda_1 - 4\lambda_2 + \lambda_3 = -3 \\ & \lambda_1 \ge 0, \lambda_2 \ge 0, \lambda_3 \ge 0 \end{aligned}$$

- [ ] Want to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. (iv)

B. (i)

C. (iii)

D. (ii)

Back Next

Consider the function $f(x,y) = x^3 + y^3 - 3x^2y$. Find the Hessian matrix of f at $(x,y) = (-1,1)$.

(i) $\begin{bmatrix} -12 & 6 \\ 6 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} -12 & 6 \\ 6 & -6 \end{bmatrix}$

(iii) $\begin{bmatrix} -12 & -6 \\ -6 & -6 \end{bmatrix}$ (iv) $\begin{bmatrix} -12 & -6 \\ -6 & 6 \end{bmatrix}$

Want to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. (i)

B. (iv)

C. (ii)

D. (iii)

Back | Next

Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$. Compute A^5 :

(i) $\begin{pmatrix} 34 & -66 \\ 33 & -65 \end{pmatrix}$

(ii) $\begin{pmatrix} 34 & 66 \\ -33 & -65 \end{pmatrix}$

(iii) $\begin{pmatrix} -34 & 66 \\ -33 & 65 \end{pmatrix}$

(iv) $\begin{pmatrix} -34 & -66 \\ 33 & 65 \end{pmatrix}$

- [ ] Want to finish the exam.

Finish

Exit

Answer

☐ A

☐ B

☐ C

☐ D

(Choose 1 answer)

(See picture)

A. -1/2

B. 1

C. 3/2

D. All of the other choices are incorrect

Back Next

Let $[a\ \ b\ \ c]^T$ be the projection of the vector $u=[1\ \ 2\ \ 0]^T$ on the subspace U spanned by the set $\{[1\ \ 1\ \ -1]^T, [2\ \ 0\ \ 2]^T\}$. Find b.

$$U = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ -1 & 2 \end{bmatrix} \qquad u = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\mathrm{Proj}_U(u) = U\left(U^T U\right)^{-1} U^T \cdot u$$

$$\Rightarrow \mathrm{Proj}_U(v) = \begin{bmatrix} 1,5 \\ 1 \\ -0,5 \end{bmatrix}$$

☐ I want to finish the exam.

Exit

Answer

- A
- B
- C
- D
- E

(Choose 1 answer)

(See picture)

A. -1

B. 1

C. -2

D. 0

E. 2

Back Next

Let

$$z=\ln\left(x^2+y^2\right);\quad x=2s+t;\qquad y=s-2t.$$

$\rightarrow x = 3$

$\rightarrow y = -1$

Use the Chain Rule to find $\frac{\partial z}{\partial s}$ at $(s,t)=(1,1)$.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial x} = \frac{(x^2+y^2)'}{x^2+y^2} = \frac{2x}{x^2+y^2} \quad \Big| \quad \frac{\partial z}{\partial y} = \frac{2y}{x^2+y^2}$$

$$\frac{\partial x}{\partial s} = 2 \qquad \frac{\partial y}{\partial s} = 1$$

$$\Rightarrow \frac{\partial z}{\partial x}(1,1) = 2\cdot\frac{2x}{x^2+y^2} + \frac{2y}{x^2+y^2} = 1$$

Mark to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(see picture)

A. -1

B. -1/2

C. -3/4

D. -1/4

Back Next

Let $x = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & c \end{bmatrix}^T$ be a vector in $\mathbb{R}^4$, where c is a **negative** real number. Find c such that x is a unit vector with respect to the Euclidean norm.

Want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

Back Next

(Choose 1 answer)

(See picture)

A. (i)

B. (ii)

C. (iii)

D. (iv)

Let $W = [X \;\; Y]^T$ be a 2-dimensional random variable with normal distribution $W = N(\mu, \Sigma)$, in which

$$\mu = [0 \;\; 1]^T, \Sigma = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

Find the formula for the probability density function p(x,y) of W.

(i) $$p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{(x-1)^2}{2} - \frac{y^2}{4}}$$

(ii) $$p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{(x-1)^2}{4} - \frac{y^2}{2}}$$

(iii) $$p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{x^2}{4} - \frac{(y-1)^2}{2}}$$

(iv) $$p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{x^2}{2} - \frac{(y-1)^2}{4}}$$

$\det(\Sigma) = 2$

$\Sigma^{-1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}$

$$\left(-\frac{1}{2}\begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}^T \cdot \Sigma^{-1} \cdot \begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}\right)$$

$$= \left(-\frac{1}{2}\begin{bmatrix} x-0 \\ y-1 \end{bmatrix}^T \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x-0 \\ y-1 \end{bmatrix}\right)$$

$$= \left(-\frac{1}{2}\begin{bmatrix} \frac{1}{2}x & y-1 \end{bmatrix}\begin{bmatrix} x \\ y-1 \end{bmatrix}\right)$$

$$= \left[-\frac{1}{2}\left(\frac{1}{2}x^2 + (y-1)^2\right)\right]$$

$$\left[-\frac{1}{4}x^2 - \frac{(y-1)^2}{2}\right]$$

$$p(x,y) = \frac{1}{2\pi\sqrt{\det\Sigma}} \cdot e^{\left(-\frac{1}{2}\begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}^T \Sigma^{-1} \cdot \begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}\right)}$$

- [ ] I want to finish the exam.

Finish

Exit

Answer

(Choose 1 answer)

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(See picture)

A. (iv)

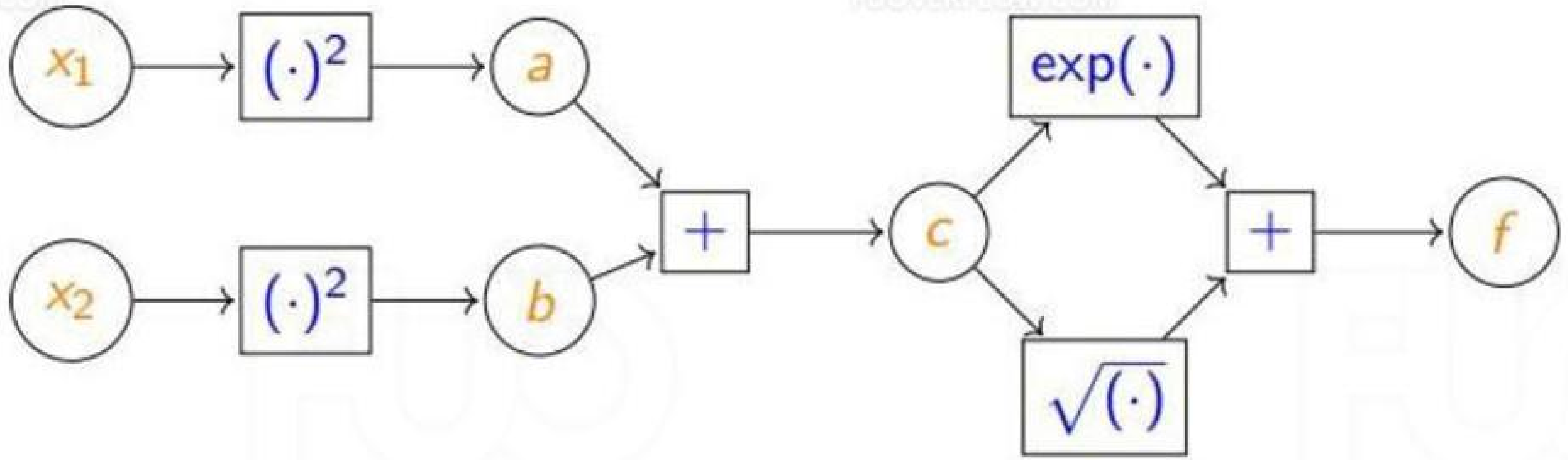
B. (i)

C. (iii)

D. (ii)

Back Next

Consider a computation graph with inputs x_1, x_2, function value f and intermediate variables a, b, c. Find the formula of f.



(i) $e^{x_1+x_2} + \sqrt{x_1^2 + x_2^2}$ (ii) $e^{x_1^2+x_2^2} + \sqrt{x_1 + x_2}$

- [ ] Want to finish the exam.

Exit

Answer

☐ A

☐ B

☐ C

☐ D

(Choose 1 answer)

(See picture)

A. Both 7 and 3 are eigenvalues of A

B. 3 is an eigenvalue of A

C. 7 is an eigenvalue of A

D. None of the other choices is correct

Back Next

Given $A = \begin{bmatrix} 5 & -3 & 1 \\ 3 & 0 & 4 \\ 2 & 1 & 5 \end{bmatrix}$.

Choose the correct statement.

$\lambda = 7$

$\Leftrightarrow \det(A - \lambda I) = 116$

$\Rightarrow \lambda = 7$ k° phai

$\lambda = 3$

$\Rightarrow \det(A - \lambda I) = -17$

$\Rightarrow \lambda = 3$ k° phai

I want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

(Choose 1 answer)

(See picture)

$\boxed{\text{A. -0.92}}$

B. 2.12

C. -2.48

D. 3.64

E. 1.02

Back Next

We are using the Gradient Descent algorithm to solve the following problem:

$$\text{Min } f(x,y) = 3x^2 + y^2 + 2xy + x - 3y + 1$$

The current iterate is $(x_i, y_i) = (1, 1)$. Using the learning rate (step-size) $\gamma = 0.2$ for the gradient descent algorithm, what is the value of the objective function at the next iterate?

$$\frac{\partial f}{\partial x} = 6x + 2y + 1 = 9$$

$$\frac{\partial f}{\partial y} = 2y + 2x - 3 = 1$$

$$x_{new} = x_{old} - \gamma \cdot \frac{\partial f}{\partial x} = 1 - 0{,}2 \cdot 9 = -0{,}8$$

$$y_{new} = y_{old} - \gamma \cdot \frac{\partial f}{\partial y} = 1 - 0{,}2 \cdot 2 = 0{,}8$$

$$\text{Min } f(-0{,}8, 0{,}8) = -0{,}92$$

I want to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. (iv)

B. (i)

C. (iii)

D. (ii)

Back Next

Let A be an invertible square matrix of size 2x2. Let 1 and 2 be the two singular values of A. Choose the correct answer :

(i) 1 and 1/2 are two singular values of matrix A^{-1}

(ii) 1 and 1/2 are two singular values of matrix A^{T}

(iii) -1 and -2 are two singular values of matrix A^{-1}

(iv) 1 and $\sqrt{2}$ are two singular values of matrix A^{T}

A^{-1}

Singular value A

1, 2

$\Rightarrow \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \Sigma^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & +\frac{1}{2} \end{bmatrix}$

$\Rightarrow$ 1 and $\frac{1}{2}$ sv of A^{-1}

I want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

Back Next

(Choose 1 answer)

(See picture)

A. 0.5

B. 0

C. 0.25

D. 1

E. None of the other choices is correct

Suppose that the random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{2} & \text{if } 0 \leq x \leq 2 \\ 1 & \text{if } x > 2 \end{cases}$$

Let $Y = X^2$. Find the probability $P(Y \leq X)$.

$F(x) = \frac{x}{2}$

$Y \leq X$

$\Rightarrow x^2 \leq x$

$0 \leq x \leq 2 \Rightarrow X \in [0, 1]$

$Y = x^2 \Rightarrow Y \in [0, 4]$

$0 = 0$

$1 = 1$

$P(x \leq 1) = F(1) - F(0) = \frac{1}{2} - \frac{0}{2}$

... to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

Back Next

(Choose 1 answer)

(See picture)

A. 0

B. -6

C. -3

D. 3

E. 6

Compute

$$\begin{vmatrix} 1 & -1 & 1 & -1 \\ 1 & 2 & 1 & 2 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}.$$

I want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

Back Next

(Choose 1 answer)

(See picture)

A. (iii)

B. (iv)

C. (ii)

D. (i)

What is the **second** derivative of

$f(x) = \sin(2x+1) - 3x\cos(2x+1)$?

(i) $8\sin(2x+1) + 12x\cos(2x+1)$

(ii) $4\sin(2x+1) + 6x\cos(2x+1)$

(iii) $4\sin(2x+1)\cos(2x+1) - 6x\cos(2x+1)\sin(2x+1)$

(iv) $4\sin(2x+1) - 6x\cos(2x+1)$

Want to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

Back Next

(Choose 1 answer)

(See picture)

A. -6

B. -1

C. -2

D. -3

E. 0

Given the SVD of A as followings

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{6} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

Find the $(2,2)$-entry of the rank-1 approximation of A.

$$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \sqrt{6} \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -2 & 1 \end{bmatrix}$$

[ ] I want to finish the exam.

Finish

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

Back Next

(Choose 1 answer)

(See picture)

A. -85

B. -87

C. -88

D. -86

E. -84

Given a function $f : \mathbb{R}^2 \to \mathbb{R}^2$

$$f(\mathbf{x}) = \begin{pmatrix} x_1^2 - 3x_1x_2 + 2x_2^2 \\ x_1^3 - 2x_1^2x_2 + 3x_2^3 \end{pmatrix}$$

Let J be the Jacobian of $f(\mathbf{x})$ at (-1,1). Compute $\det(J)$.

Want to finish the exam.

Finish

Exit

Answer

(Choose 1 answer)

(See picture)

- [ ] A
- [ ] B
- [ ] C
- [ ] D

A. -4/5

B. 5/4

C. 4/5

D. -5/4

Back Next

Find the number a such that the vectors $\mathbf{x} = [1\ \ -3]^T$ and $\mathbf{y} = [a\ \ -1]^T$ are orthogonal using the inner product

$$<\mathbf{x}, \mathbf{y}> = \mathbf{x}^T A \mathbf{y},$$

in which $A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a & -1 \end{bmatrix}^T = 0$$

$$\begin{bmatrix} 5 & -4 \end{bmatrix} \begin{bmatrix} a \\ -1 \end{bmatrix} = 0$$

$$5a + 4 = 0 \Rightarrow a = -\frac{4}{5}$$

[ ] ... to finish the exam.

Exit

Answer

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(Choose 1 answer)

(See picture)

A. Only V

B. Both U and V

C. None of U or V

D. Only U

Back Next

Which of the following sets in R^2 are convex?

$$U = \{[x \;\; y]^T \mid x^2 + 2y^2 = 1\}$$
$$V = \{[x \;\; y]^T \mid x^2 + 2y^2 \geq 1\}$$

Want to finish the exam.

Finish

Exit

Answer

(Choose 1 answer)

- [ ] A
- [ ] B
- [ ] C
- [ ] D

(See picture)

A. (iv)

B. (iii)

C. (ii)

D. (i)

Back Next

The first-order Taylor polynomial of

$$f(x, y) = (x^2 + 2xy^2 + 1)^2$$

about $(1, -1)$ is

$$ax + by + c.$$

The value of $a - b$ is

(i) 0 (ii) 32

(iii) 64 (iv) 128

Want to finish the exam.

Finish

Exit

Answer

(Choose 1 answer)

(See picture)

- [ ] A
- [ ] B
- [ ] C
- [ ] D

A. 1460.19

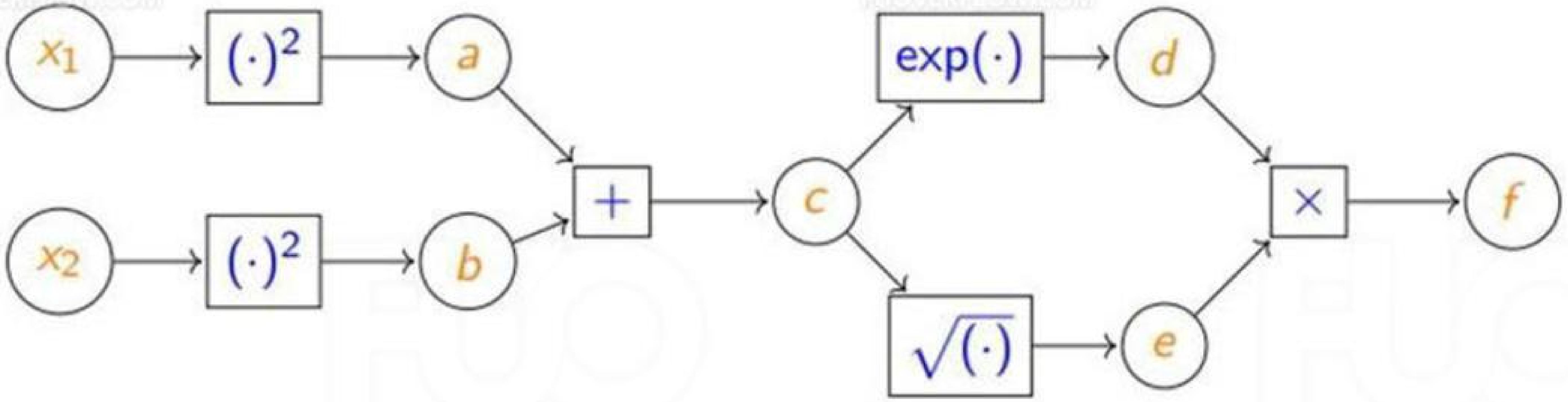
B. -730.10

C. -417.70

D. 835.40

Back Next

Given the computation graph. Find the partial derivative $\frac{\partial f}{\partial x_2}$ at $x_1 = 2$, $x_2 = -1$.



Want to finish the exam.

Exit