Kizspy | Question: 1

(Choose 1 answer)

(see picture)

A. 5
B. 7
C. 4
D. 25

Let $x = [1\ \ 1\ \ 1]^T$ and $y = [-2\ \ 1\ \ 2]^T$ be vectors in $\mathbb{R}^3$. Find the Euclidean norm of $2x + y$.

$2x + y = 2[1\ 1\ 1]^T + [-2\ 1\ 2]^T$

$\Leftrightarrow 2x + y = [0\ 3\ 4]$

Euclidean norm : $\sqrt{0^2 + 3^2 + 4^2} = 7$

Kizspy | Question: 2

(Choose 1 answer)

(see picture)

A. Only Q
B. Neither P nor Q
C. Only P
D. Both P and Q

Which of the following is a symmetric and positive definite matrix?

$$P = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}, \quad Q = \begin{bmatrix} 16 & 0 \\ 0 & 9 \end{bmatrix}.$$

P đg chéo phụ : 0 , đg chéo chính : -1

=> ko phải symetric

Kizspy | Question: 3

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

Compute the length of the vector $\mathbf{x} = [1\ \ -3]^T$ using the inner product

$$<\mathbf{x}, \mathbf{y}> = \mathbf{x}^T A \mathbf{y},$$

in which $A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$.

(i) 5
(ii) $\sqrt{17}$
(iii) 4
(iv) $\sqrt{15}$

$\langle x, x \rangle = [1, -3] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$\langle x, x \rangle = 17$

lengh $= \sqrt{\langle x, x \rangle} = \sqrt{17}$

Kizspy | Question: 4

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

$$\boxed{\cos\alpha = \frac{\langle x,y\rangle}{\sqrt{\langle x,x\rangle\cdot\langle y,y\rangle}}}$$

Let x $= [1 \quad 1]^T$ and y $= [-1 \quad 2]^T$. Determine the angle between x and y using the following inner product

$$\langle x, y\rangle = x^T A x, \quad A = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}.$$

(i) Π

(ii) Π/4

(iii) Π/2

(iv) Π/6

$\langle x,y\rangle = x^T A y = [1 \ 1]\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\Leftrightarrow x^T A y = [0]$

$\langle x,x\rangle = x^T A x = [6]$

$\langle y,y\rangle = [3]$

$\cos\alpha = \frac{\langle x,y\rangle}{\sqrt{\langle x,x\rangle\cdot\langle y,y\rangle}} \Leftrightarrow \cos\alpha = 0$

$\Rightarrow \cos\alpha = \frac{\pi}{2}$

Kizspy | Question: 5

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

Using Gram-schmidt process to convert the basis

$\mathbf{b_1} = [1 \quad 2 \quad -1]^T, \quad \mathbf{b_2} = [0 \quad 1 \quad 3]^T$

into an orthogonal basis $\{\mathbf{c_1}, \mathbf{c_2}\}$. Find the vector $\mathbf{c_2}$.

(i) $(1/6)[1 \quad 8 \quad 17]^T$

(ii) $(1/6)[1 \quad -8 \quad 17]^T$

(iii) $(1/6)[-1 \quad -8 \quad -17]^T$

(iv) $(1/6)[-1 \quad 8 \quad -17]^T$

$c_1 = b_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$c_2 = b_2 - \frac{\langle b_2, c_1 \rangle}{\langle c_1, c_1 \rangle} c_1 = \begin{bmatrix} 1/6 \\ 4/3 \\ 17/6 \end{bmatrix}$

$\Leftrightarrow \quad c_2 = \frac{1}{6} \cdot \begin{bmatrix} 1 \\ 8 \\ 17 \end{bmatrix}$

Kizspy | Question: 6

(Choose 1 answer)

(See picture)

A. 3/2
B. 1
C. -1/2
D. All of the other choices are incorrect

$PROJ_U(u) = A(A^TA)^{-1}A^T.u$

$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ -1 & 2 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$A^TA = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$(A^TA)^{-1} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 0{,}125 \end{bmatrix}$

$A(A^TA)^{-1}A^T.u = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ - & \end{bmatrix}$

Let $[a \;\; b \;\; c]^T$ be the projection of the vector $u=[1 \;\; 2 \;\; 0]^T$ on the subspace U spanned by the set $\{[1 \;\; 1 \;\; -1]^T, [2 \;\; 0 \;\; 2]^T\}$. Find a.

Hình chiếu của u lên subspace U

$PROJ_U(u) = c_1\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

$u = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{cases} c_1 + 2c_2 = 1 \\ c_1 + 0c_2 = 2 \\ -c_1 + 2c_2 = 0 \end{cases} \Leftrightarrow \begin{cases} c_1 = 2 \\ c_2 = -\frac{1}{2} \end{cases}$

$\Rightarrow PROJ_U(u) = 2\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \rightarrow$ $a = 1$, $b = 2$, $c = -3$

$\Rightarrow a = 1$

Kizspy | Question: 7

(Choose 1 answer)

(See picture)

A. 1 and 2
B. 1,2 and 3
C. 3 only
D. 2 only

Select the correct answer(s):

1. $Trace(AB) = Trace(A)\,Trace(B)$ for all square matrices A, B.

2. $\boxed{Trace(CAC^{-1}) = Trace(A) \text{ for all invertible square matrices C,A.}}$

3. $Trace(A^{-1}) = \frac{1}{Trace(A)}$ for all invertible square matrices A.

Kizspy | Question: 8

(Choose 1 answer)

(See picture)

A. 1 and 5
B. -1 and -5
C. 2 and 4
D. -2 and -4

Let $A = \begin{pmatrix} 4 & \sqrt{3} \\ \sqrt{3} & 2 \end{pmatrix}$. What are the two eigenvalues of A ?

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{vmatrix} 4-\lambda & \sqrt{3} \\ \sqrt{3} & 2-\lambda \end{vmatrix}\right) = 0$$

$$\Rightarrow \lambda^2 - 6\lambda + 5 = 0$$

$$\Rightarrow \left[\begin{array}{l} \lambda_1 = 1 \\ \lambda_2 = 5 \end{array}\right.$$

Kizspy | Question: 9

(Choose 1 answer)

(See picture)

$\boxed{\text{A.}}$ [1, 0, 0]
B. [0, 1, 0]
C. [0, 0, 1]
D. [1, 1, 1]

Given that $\lambda = 2$ is a eigenvalue of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 5 & 1 \\ 0 & 6 & 7 \end{bmatrix}$$

Find an eigenvector corresponding to eigenvalue $\lambda = 2$.

$$\det(A - \lambda I) = 0$$

$$\lambda = 2$$

$$\Rightarrow \begin{vmatrix} 0 & 3 & 4 \\ 0 & 3 & 1 \\ 0 & 6 & 5 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 3y + 4z = 0 \\ 3y + 1z = 0 \\ 6y + 5z = 0 \end{cases} \Leftrightarrow \begin{cases} y = -\frac{4}{3}z \\ y = -z \\ z = 0 \end{cases} \Leftrightarrow \begin{cases} y = 0 \\ z = 0 \end{cases}$$

$$\Rightarrow (x, 0, 0)$$

Kizspy | Question: 10

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)
E. (v)

Given the eigendecomposition:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}.$$

Compute the (1, 2)-entry of the matrix A^{2023}.

(i) $2^{2023} - 3^{2023}$
(ii) 2^{2023}
(iii) 3^{2023}
(iv) $2^{2023} + 3^{2023}$
(v) 0

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$$

(1, 2)-entry of matrix, $A = 0^{2023}$

$= 0$

Kizspy | Question: 11

(Choose 1 answer)

(See picture)

A. 0
B. 1
C. 2
D. 3

How many nonzero singular values does

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$$

have?

$$A^TA = \begin{bmatrix} 21 & 9 \\ 9 & 11 \end{bmatrix}$$

$$\det|A^TA - \lambda I| = 0$$

$$\Leftrightarrow \begin{vmatrix} 21-\lambda & 9 \\ 9 & 11-\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - 32\lambda + 150 = 0$$

$$\Rightarrow \left[\begin{array}{l} \lambda > 0 \\ \lambda > 0 \end{array}\right.$$

$\Rightarrow$ 2 Singular value nonzero

Kizspy | Question: 12

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

What is the rank of a matrix A in terms of its singular values?

(i) The rank is the number of nonzero singular values. ✓
(ii) The rank is the number of singular values. ✗
(iii) The rank is the sum of all singular values. ✗
(iv) The rank is the largest singular value. ✗

Kizspy | Question: 13

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

The spectral norm of $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^T$ is:

(i) $\sqrt{2}$ (ii) $\sqrt{3}$

(iii) 2 (iv) $\sqrt{5}$

The spectral norm $= \sqrt{\max(\text{eigenvalues})}$

$\det|A^TA - \lambda I| = 0$

$A^TA = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow \det|A^TA - \lambda I| = 0$

$\lambda^2 - 4\lambda + 4 = 0 \Rightarrow \lambda = 2 \Rightarrow$ spectral norm $: \sqrt{2}$

Kizspy | Question: 14

(Choose 1 answer)

(See picture)

A. 3.93
B. 3.78
C. 3.51
D. 3.73
E. 3.87

$2^{x+1} = 2^1 . 2^x$

The Taylor expansion of function $f(x) = 2^{x+1}$ centered at 0 is a polynomial of the form

$$a_0 + a_1 x + \ldots + a_n x^n + \ldots$$

What is the value of $a_0 + a_1 + a_2$?

$(a^u)' = u' a^u . \ln a$

$(\ln u)' = \frac{u'}{u}$

$f(x) = 2^{x+1} \Rightarrow f(0) = 2$

$f'(x) = (x+1)' . 2^{x+1} . \ln 2 \Rightarrow f'(0) = 2 . \ln 2$

$f''(x) = (x+1)' . 2^{x+1} (\ln)^2 \Rightarrow f''(0) = 2 . (\ln 2)^2$

$P_n(x) = f(\alpha) + \frac{f'(\alpha)(x-\alpha)^1}{1!} + \frac{f''(\alpha)(x-\alpha)^2}{2!} + \ldots + \frac{f^n(\alpha)(x-\alpha)^n}{n!}$

$\Rightarrow P_2(x) = 2 + \frac{2 \ln 2}{1}(x-0)^1 + \frac{2(\ln 2)^2}{2}(x-0)^2$

$\Rightarrow a_0 = 2$

$a_1 = 2 \ln 2$

$a_2 = (\ln)^2$

$\Rightarrow a_0 + a_1 + a_3 = 3{,}87$

Kizspy | Question: 15

(Choose 1 answer)

(See picture)

A. (iv)
B. (iii)
C. (i)
D. (ii)

$(u^n) = n \cdot u^{n-1} \cdot u'$

Find $\frac{\partial z}{\partial y}(2,-3)$ for $z = \sqrt{x^2+y^2}$

(i) $\sqrt{13}/13$

(ii) $-3\sqrt{13}/13$

(iii) $-\sqrt{13}/13$

(iv) $3\sqrt{13}/13$

$z = (x^2+y^2)^{\frac{1}{2}}$

Đạo hàm theo biến y

$\frac{\partial z}{\partial y} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot (x^2+y^2)'$

$\Leftrightarrow \frac{\partial z}{\partial y} = \frac{1}{2}(x^2+y)^{-\frac{1}{2}} \cdot 2y$

$\Leftrightarrow \frac{\partial z}{\partial y} = -\frac{3\sqrt{13}}{13}$

Kizspy | Question: 16

(Choose 1 answer)

(See picture)

A. 0
B. 1
C. -1
D. 2
E. -2

Let

$$z = x^2 + xy^2; \quad x = r\cos\theta; \quad y = r\sin\theta.$$

Use the Chain Rule to find $\frac{\partial z}{\partial r}$ at $(r,\theta) = (1, \frac{\pi}{2})$.

Chain Rule:

$$\frac{\partial z}{\partial R} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial R} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial R}$$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^2 + xy^2) = 2x + y^2 = 2(R\cos\alpha) + (R.\sin\alpha)^2 =$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(x^2 + xy^2) = 2y = 2(R.\sin\alpha)$$

$$\frac{\partial x}{\partial R} = \frac{\partial}{\partial R}(R\cos\alpha) = \frac{\partial}{\partial z}(R).\cos\alpha + \frac{\partial}{\partial R}(\cos\alpha).R = \cos\alpha + R.0 =$$

$$\frac{\partial y}{\partial R} = \frac{\partial}{\partial R}(R.\sin\alpha) = \frac{\partial}{\partial R}(R).\sin\alpha + \frac{\partial}{\partial R}(\sin\alpha).R = \sin\alpha + R.0$$

$$\Rightarrow \frac{\partial z}{\partial R} = (2x + y^2)\cos\alpha + 2y.\sin\alpha$$

at $(R,\alpha) = (1, \frac{\pi}{2}) \Rightarrow \frac{\partial z}{\partial R} \approx 2$

Kizspy | Question: 17

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)
E. (v)

$\Rightarrow J_f(0,-2) = \begin{bmatrix} 0 & -1 \\ -6 & 0 \end{bmatrix}$

$\Rightarrow \det(J_f(0,-2)) = -6$

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$:

$$f(x, y) = \begin{bmatrix} \sqrt{x^4 + y^2} \\ x^3 + 3xy \end{bmatrix}.$$

The Jacobian determinant of f at $(x, y) = (0, -2)$ is:

(i)	-6	(ii)	6
(iii)	-3	(iv)	3
(v)	0		

$f(x,y) = \begin{bmatrix} \sqrt{x^4+y^2} \\ x^2+3xy \end{bmatrix} \rightarrow$

$f_1(x,y) = \sqrt{x^4+y^2}$

$f_2(x,y) = x^2 + 3xy$

$(x, y) = (0, -2)$

$\text{Jacobian}_f(x,y) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}$

$\frac{\partial f_1}{\partial x} = \left(\sqrt{x^4+y^2}\right)' = \frac{1}{2}(x^4+y^2)^{-\frac{1}{2}} \cdot 4x^3 = 0$, $\frac{\partial f_1}{\partial y} = \frac{1}{2}(x^4+y^2)^{-\frac{1}{2}} \cdot 2y = -1$

$\frac{\partial f_2}{\partial x} = x^2 + 3xy = 2x + 3y = -6$, $\frac{\partial f_2}{\partial y} = 3x = 0$

Kizspy | Question: 18

(Choose 1 answer)

(See picture)

A. 2.207
B. 3.524
C. 2.521
D. 3.107
E. 3.231

$t = h(g(f(x)))$

$y = f(x) = 2x + 3$

$\frac{\partial y}{\partial x} = f'(x) = 2$

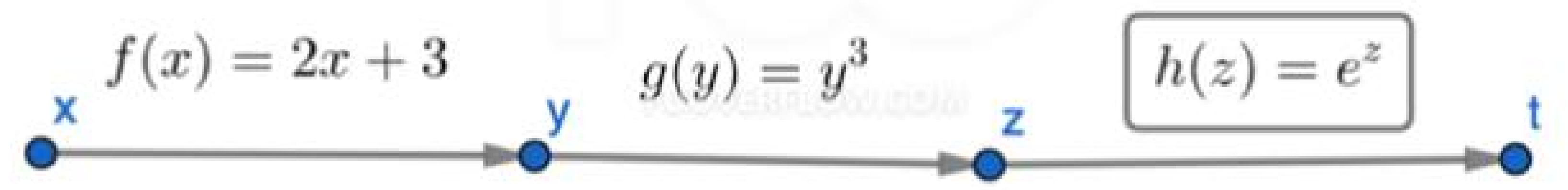
$z = g(y) = y^3$

$\frac{\partial z}{\partial y} = g'(y) = 3y^2$

$t = h(z) = e^z$

$\frac{\partial t}{\partial z} = h'(z) = e^z$

A computational graph is given as bellow.
Compute the derivative of t with respect to x at $x = -2$.



$x = -2 \rightarrow f(x) = -1 \Rightarrow y = -1$

$y = -1 \rightarrow g(y) = -1 \Rightarrow z = -1$

$\Rightarrow \left[\begin{array}{l} x = -2 \\ y = -1 \\ z = -1 \end{array}\right.$

$\frac{\partial t}{\partial x} = f'(x) \cdot g'(x) \cdot h'(z) = 2 \cdot 3y^2 \cdot e^z = 2,2072$

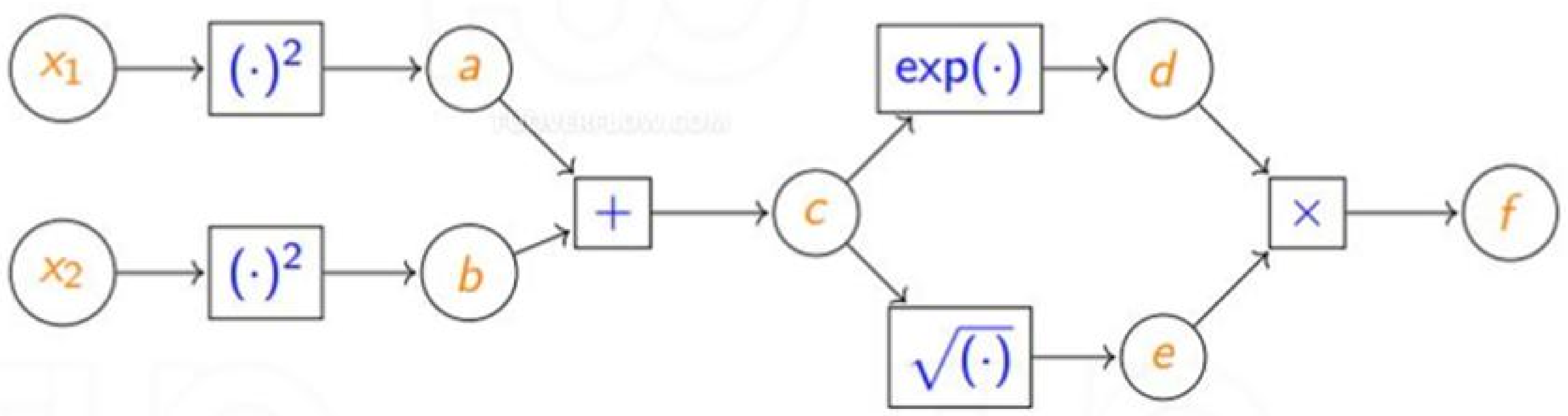
Kizspy | Question: 19

(Choose 1 answer)

(See picture)

A. 1460.19
B. -730.10
C. 835.40
D. -417.70

Given the computation graph. Find the partial derivative $\frac{\partial f}{\partial x_1}$ at $x_1 = 2$, $x_2 = -1$.



Kizspy | Question: 20

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

Suppose that the Hessian matrix of $f(x, y, z)$ at $(0, 1, 1)$ is

$$\begin{bmatrix} 2 & -1 & 4 \\ -1 & 3 & 5 \\ 4 & 5 & -1 \end{bmatrix}.$$

The value of $2\frac{\partial^2 f}{\partial x \partial z} + \frac{\partial^2 f}{\partial y^2}$ at $(0, 1, 1)$ is

(i) 1 (ii) 13

(iii) 0 (iv) 11

$$\frac{\partial^2 f}{\partial x \partial z} = 4, \quad \frac{\partial^2 f}{\partial y^2} = 3$$

$$\Rightarrow 2\frac{\partial^2 f}{\partial x \partial z} + \frac{\partial^2 f}{\partial y^2} = \underline{11}$$

Kizspy | Question: 21

(Choose 1 answer)

(See picture)

A. 2
B. -2
C. 4
D. -4

$$\boxed{\frac{\partial^2 f}{\partial y\, \partial x}}$$

The $(2,1)$-entry of the Hessian matrix of

$$f(x,y) = (x^2 - xy)\sin y$$

at $(x,y) = (1,0)$ is ______

$$\frac{\partial f}{\partial y\, \partial x} = \frac{\partial}{\partial y}(x^2 - xy)\sin y + \frac{\partial}{\partial x}(x^2 - xy)\sin y$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 - xy).\sin y + (x^2 - xy)\frac{\partial}{\partial y}\sin y$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 - xy).\sin y + (x^2 - xy).\frac{\partial}{\partial x}.\sin y$$

$$\frac{\partial f}{\partial y} = -1x.\sin y + (x^2 - xy).\cos y$$

$$\frac{\partial f}{\partial x} = (2x - y)\sin y + (x^2 - xy).0$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\right) = \frac{\partial}{\partial y}(2x - y)\sin y. = \frac{\partial}{\partial y}(2x - y).\sin y + \frac{\partial}{\partial y}.\sin y\,(2x - y)$$

$$= 2x.\sin y + \cos y.(2x) = 2$$

Kizspy | Question: 22

(Choose 1 answer)

(See picture)

A. 37
B. 74
C. 160
D. 80

Consider the bivariate function

$$f(x, y) = (x^2y - xy^2)^2.$$

The second-order Taylor polynomial of $f(x, y)$ about $(x, y) = (-1, 2)$ is:

$$T_2(x, y) = c_0 + c_1(x+1) + c_2(y-2) + c_3(x+1)^2 + c_4(x+1)(y-2) + c_5(y-2)^2.$$

The value of c_5 is ______

Kizspy | Question: 23

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)
E. None of the other choices is correct

Suppose that X has a discrete uniform distribution on the integers 0 through 99.
The probability such that $2^X \leq 50$ is

(i) 0.05 (ii) 0.06

(iii) 0.07 (iv) 0.08

discrete uniform distribution

$n = b - a + 1 = 100$

$\Rightarrow$ Probability $= \frac{1}{100}$

with $X = [0, 1, 2, 3, 4, 5]$, $2^x \leqslant 50$

$\Rightarrow P(2^x \leqslant 50) = \frac{6}{100} = 0{,}06$

Kizspy | Question: 24

(Choose 1 answer)

(See picture)

A. 3/4
B. 1/4
C. 1/2
D. 2/3
E. 5/8

Given the joint probability distribution of two discrete random variables X and Y.

X \ Y	**0**	**1**	**2**	**3**
1	0.05	0.15	0.2	0.05
2	0	0.05	0.1	0
3	0.15	0	0.1	0.15

Find $P(X > 1 | Y = 2)$.

$$P(x>1, y=2) = 0{,}1 + 0{,}1 = 0{,}2$$

$$P(Y=2) = 0{,}2 + 0{,}1 + 0{,}1 = 0{,}4$$

$$P(X>1 | Y=2) = \frac{P(x>1, y=2)}{P(Y=2)} = \frac{0{,}2}{0{,}4}$$

$$= 1/2$$

Kizspy | Question: 25

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

$$P(y) = \int_0^1 P(x,y)\,dx$$

$$\Leftrightarrow P(y) = \int_0^1 \frac{3}{13} x y^2\,dx$$

$$\Leftrightarrow P(y) = \frac{3}{13} \cdot y^2 \cdot \int_0^1 x\,dx$$

$$\Leftrightarrow P(y) = \frac{3}{13} \cdot y^2 \cdot \frac{1}{2} = \frac{3}{26} \cdot y^2$$

Let $p(x,y) = (3/13)xy^2$ with $0 \le x \le 1$, $1 \le y \le 3$ be the joint probability function of some continuous random variables X and Y. Find the marginal probability distribution p(y) of Y.

(i) $p(y) = (1/4)y, \quad 1 \le y \le 3$

(ii) $p(y) = (3/13)y^2 - (1/2), \quad 1 \le y \le 3$

(iii) $p(y) = (3/26)y^2, \quad 1 \le y \le 3$

(iv) $p(y) = (1/2)y - (1/2), \quad 1 \le y \le 3$

$$1 \le y \le 3$$

Kizspy | Question: 26

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

Let c be the number such that f(x,y) = c(x+y) is the joint probability distribution of two discrete random variables X and Y, where x = 1, 2, 3, 4 and y = 0, 1, 2. Describe the marginal probability distribution of X.

(i)

X	1	2	3	4
P(X=k)	6c	9c	12c	15c

(ii)

X	1	2	3	4
P(X=k)	10c	14c	18c	22c

(iii)

X	1	1	3	4
P(X=k)	9c	12c	15c	18c

(iv)

X	1	2	3	4
P(X=k)	3c	7c	11c	15c

$$\sum_i \sum_j c(x_i + y_j)$$

$x = 1$

$P(X=1, Y=0) = c(1+0)$

$P(X=1, Y=1) = c(1+1)$

$P(x=1, y=2) = c(1+2)$

$x = 1 \Rightarrow P(X=1) = 6c$

. . . . .

Kizspy | Question: 27

(Choose 1 answer)

(See picture)

A. 0.1
B. 0.1125
C. 0.2
$\boxed{\text{D. 0.2475}}$
E. None of the other choices is correct

Consider the following bivariate distribution $p(x, y)$ of two discrete random variables X and Y

		Y		
		1	2	3
X	1	0.17	0.13	0.25
	2	0.1	0.3	0.05

Find the variance of X.

$$VAR(X) = E(X^2) - E(X)^2$$

$$E(X^2) = \sum_i x_i^2 \cdot P(X=i)$$

$$\Rightarrow E(X^2) = 1^2 \cdot P(X=1) + 2^2 \cdot P(X=2)$$

$$\Leftrightarrow E(X^2) = 1^2 \cdot (0{,}17 + 0{,}13 + 0{,}25) + 2^2 (0{,}1 + 0{,}2 + 0{,}05) = 2{,}35$$

$$E(X) = \sum_i x_i P(X=i)$$

$$\Rightarrow E(X) = 1 \cdot (0{,}17 + 0{,}13 + 0{,}25) + 2 (0{,}1 + 0{,}2 + 0{,}05) = 1{,}45$$

$$\Rightarrow Var(X) = 2{,}35 - 1{,}45^2$$

$$= 0{,}2475$$

Kizspy | Question: 28

(Choose 1 answer)

(See picture)

A. 2/3
B. -4/3
C. 8/3
D. 2
E. 1

Given a dataset consisting of the following vectors

$x_1 = [1 \ \ 0 \ \ 2]^T, \quad x_2 = [2 \ \ -2 \ \ 2]^T, \quad x_3 = [0 \ \ 2 \ \ -1]^T.$

Let $S = [s_{ij}]$ be the covariance matrix of this dataset. Find s_{13}.

Kizspy | Question: 29

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

Let $W = [X \;\; Y]^T$ be a 2-dimensional random variable with normal distribution $W = N(\mu,\Sigma)$, in which

$$\mu = [1 \;\; 0]^T, \Sigma = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

Find the formula for the probability density function p(x,y) of W.

(i) $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{(x-1)^2}{2}-\frac{y^2}{4}}$

(ii) $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{(x-1)^2}{4}-\frac{y^2}{2}}$

(iii) $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{x^2}{4}-\frac{(y-1)^2}{2}}$

(iv) $p(x,y) = \frac{1}{2\pi\sqrt{2}} e^{-\frac{x^2}{2}-\frac{(y-1)^2}{4}}$

Kizspy | Question: 30

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

Consider the mixture of two Gaussians X and Y

$$p\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = 0.2\mathcal{N}\left(\begin{bmatrix} 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) + 0.6\mathcal{N}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\right).$$

The mean of $X - 2Y$ is

(i) 1.6 (ii) 2

(iii) 2.2 $\boxed{\text{(iv) 1.8}}$

trung bình của x : 0,2.8 + 0,6.1 = 2,2

trung bình của Y: 0,2.1 + 0,6.0 = 0,2

$X - 2Y = 2,2 - 2.0,2 = 1,8$

Kizspy | Question: 31

(Choose 1 answer)

(See picture)

A. (0.779, -1.202)
B. (-1.181, 0.798)
C. (1.536, -1.844)
D. (-0.704, 0.236)

Let $f(x,y) = xy^2+3y$. Applying the gradient descent algorithm with the step-size $\gamma = 0.1$ and the initial point $(x_0,y_0) = (1, -1)$, what is the point (x_2,y_2) after the 2^{nd} iteration?

Kizspy | Question: 32

(Choose 1 answer)

(See picture)

A. 1.54
B. -1.852
C. 0.795
D. 0.823

Let $f(x,y) = x^2y+3y$. Applying the gradient descent algorithm with the step-size $\gamma = 0.1$, the **momentum** $\alpha = 0.02$ and the initial point $(x_0,y_0) = (1, -1)$, find x_2 of the point after 2^{nd} iteration.

Kizspy | Question: 33

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

Consider the linear program

$$\min_{x \in \mathbb{R}^3} (2x_1 - x_2 - x_3)$$

$$\text{subject to } \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

Find the dual Lagrangian $\mathcal{D}(\lambda)$.

(i) $2\lambda_1 - \lambda_2$

(ii) $-2\lambda_1 + \lambda_2$

(iii) $\lambda_1 + 2\lambda_2 + \lambda_3$

(iv) $\lambda_1 - 3\lambda_2 - 5\lambda_3$

Kizspy | Question: 34

(Choose 1 answer)

(See picture)

A. Only f
B. Only g
C. Both f and g
D. Neither f nor g

Which of the following functions are convex on $\mathbb{R}^2$?

$$f(x, y) = x^2 - 6xy + y^2$$
$$g(x, y) = x^3 + xy + y^2$$

Kizspy | Question: 35

(Choose 1 answer)

(See picture)

A. Only U
B. Only V
C. Both U and V
D. Neither U nor V

Which of the following sets are convex?

$$U = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$$
$$V = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \geq 1\}.$$

Kizspy | Question: 36

(Choose 1 answer)

(See picture)

A. -4/5
B. 5/4
C. -5/4
D. 4/5

Find the number a such that the vectors $\mathbf{x} = [1\ \ -3]^T$ and $\mathbf{y} = [a\ \ -1]^T$ are orthogonal using the inner product

$$<\mathbf{x}, \mathbf{y}> = \mathbf{x}^T A \mathbf{y},$$

in which $A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$.

Kizspy | Question: 37

(Choose 1 answer)

(See picture)

A. 0
B. abc - a - 2b - 3c
C. a+b+c
D. a(b+1)(c+2)
E. None of the other choices is correct

Find the determinant of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ a & b+1 & c+2 \\ a+1 & b+2 & c+3 \end{bmatrix}$$

Kizspy | Question: 38

(Choose 1 answer)

(See picture)

A. 2x
B. 1
C. y
D. x

Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be given by

$$f(x,y) = \begin{bmatrix} x^2 + y \\ xy \\ 2x + 3y \end{bmatrix}$$

Find the $(2,2)$-entry of the Jacobian of f.

Kizspy | Question: 39

(Choose 1 answer)

(See picture)

A. 2+(x-1)-2(y-2)+6z
B. 2+(x-1)+6(y-2)-2z
C. 2+6(x-1)-2(y-2)+z
D. 2+6(x-1)+(y-2)-2z

Find the first order Taylor expansion of the function

$$f(x, y, z) = x^3y + 2xz^2 - yz + z^2$$

at the point $(x, y, z) = (1, 2, 0)$.

Kizspy | Question: 40

(Choose 1 answer)

(See picture)

A. (i)
B. (ii)
C. (iii)
D. (iv)

Consider the following bivariate distribution $p(x, y)$ of two random variables X and Y

$$p(x, y) = \begin{cases} xy, & \text{if } 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

The probability $P(0 < X < 1, 0 \leq Y \leq 0.5)$ is

(i) 0.05 (ii) 0.0625

(iii) 0.075 (iv) 0.0825