

MPC for Robot Manipulators With Integral Sliding Modes Generation

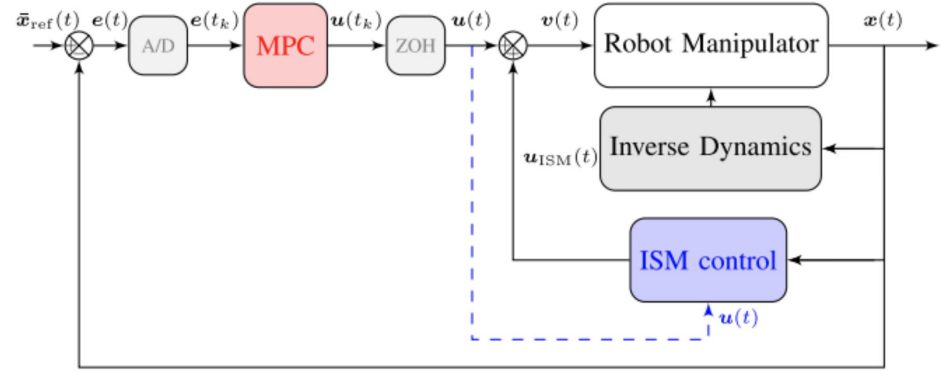


Incremona, Ferrara, Magni; IEEE/ASME Transactions on Mechatronics

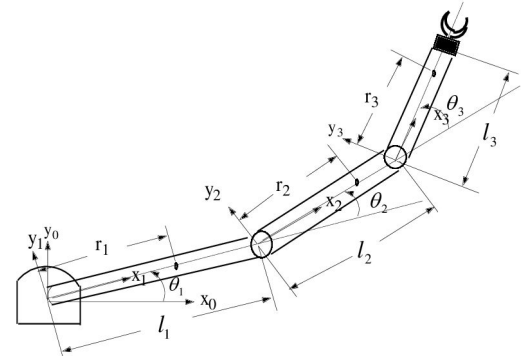
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Team 1

Problem Statement

- MPC struggles with robot systems due to:
 - Nonlinear MIMO dynamics
 - Modelling uncertainties
 - High computational cost
- Proposed Solution:
 - Hierarchical control: Inverse Dynamics + ISM + MPC
 - ISM handles uncertainties first
 - Leads to a simplified MPC optimization
- Key Novelty:
 - ISM enforces sliding modes from initial time, creating uncertainty-free dynamics
 - Multi-rate control (fast ISM, slower MPC)
 - Significantly reduced computational complexity



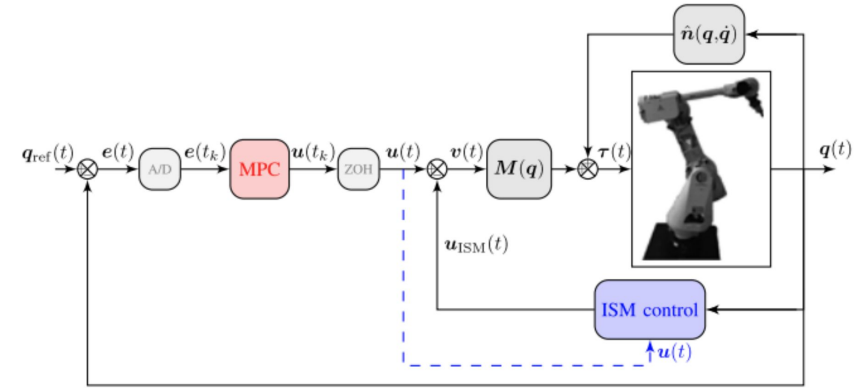
$$M(q) \ddot{q} + h(\dot{q}, q) + g(q) = F(t)$$



Controller Overview

Three Layer Control Architecture

- 1) Inner Loop: Inverse Dynamics Controller
 - a) Linearizes nonlinear MIMO system
 - b) Transforms to n decoupled double integrators
- 2) Middle Loop: Integral Sliding Mode (ISM)
 - a) Rejects matched uncertainties
 - b) Fast sampling rate (high frequency)
- 3) Outer Loop: MPC
 - a) Optimal trajectory tracking
 - b) Handles constraints
- 4) Combine Control Law:
 - a) $v = u_MPC + u_ISM$



The full coupled robot model:

$$\underbrace{M(q)}_{\text{inertia matrix}} \ddot{q} + \underbrace{n(q, \dot{q})}_{\text{Coriolis/centripetal gravity + friction}} = \tau$$

$$e = q_{ref} - q$$

$I_1 = m_1 \cdot m_4 \cdot (r_{12}^2 + r_{13}^2) + (m_1 + m_2 + m_3 + m_4) \cdot a_2^2$ $I_2 = -I_{222} + I_{223} + (m_2 + m_3 + m_4 + m_1) \cdot a_2^2$ $I_3 = m_1 \cdot r_{12}^2 + m_1 \cdot r_{13}^2$ $I_4 = m_2 \cdot r_{23} \cdot (d_2 + r_{12}) + m_3 \cdot a_2 \cdot r_{13}$ $+ (m_1 + m_2 + m_3 + m_4) \cdot a_2 \cdot (d_2 + d_3)$ $I_5 = -m_2 \cdot a_2 \cdot r_{23} + (m_1 + m_2 + m_3) \cdot a_2 \cdot d_4 + m_4 \cdot a_2 \cdot r_{14}$ $I_6 = I_{222} + m_2 \cdot r_{12}^2 + m_2 \cdot a_2^2 + m_3 \cdot (d_4 + r_{14})^2 + I_{223}$ $+ m_3 \cdot a_2^2 + m_3 \cdot d_4^2 + I_{233} + m_4 \cdot a_2^2 + m_4 \cdot d_4^2$ $+ m_4 \cdot r_{14}^2 + I_{234}$ $I_7 = m_3 \cdot r_{23}^2 + I_{222} - I_{223} + m_4 \cdot r_{14}^2 + 2 \cdot m_4 \cdot d_4 \cdot r_{14}$ $+ (m_1 + m_2 + m_3) \cdot (d_4^2 - a_2^2) + I_{224} - I_{225} + I_{226}$ $- I_{227} + m_4 \cdot r_{14}^2 - I_{228} + I_{229}$ $I_8 = -m_4 \cdot (d_4 + d_5) \cdot (d_4 + r_{14}) - (m_1 + m_2) \cdot (d_2 + d_3) \cdot d_4$ $+ m_3 \cdot r_{23} \cdot r_{13} + m_3 \cdot (d_2 + d_3) \cdot r_{13}$ $I_9 = m_2 \cdot r_{12} \cdot (d_2 + r_{12})$ $I_{10} = 2 \cdot m_1 \cdot a_2 \cdot r_{12} + 2 \cdot (m_1 + m_2 + m_3) \cdot a_2 \cdot d_1$ $I_{11} = -2 \cdot m_2 \cdot r_{12} \cdot r_{13}$ $I_{12} = (m_4 + m_3 + m_4) \cdot a_2 \cdot a_3$ $I_{13} = (m_1 + m_2 + m_3) \cdot a_2 \cdot (d_2 + d_3)$ $I_{14} = I_{224} + I_{225} + I_{226}$ $I_{15} = m_4 \cdot d_4 \cdot r_{14}$ $I_{16} = m_4 \cdot a_2 \cdot r_{14}$ $I_{17} = I_{225} + I_{226} + m_4 \cdot r_{14}^2$ $I_{18} = m_4 \cdot (d_2 + d_3) \cdot r_{14}$ $I_{19} = I_{224} - I_{225} + I_{226} - I_{227} + m_4 \cdot r_{14}^2 + I_{228} - I_{229}$ $I_{20} = I_{225} - I_{226} - m_4 \cdot r_{14}^2 + I_{226} - I_{227}$ $I_{21} = I_{226} - I_{227} + I_{228} - I_{229}$ $I_{22} = I_{224}$	<p>Table A4. The expressions giving the elements of the kinetic energy matrix. (The Abbreviated Expressions have units of kg-m².)</p> $a_{11} = I_{11} + I_1 \cdot C C 2 + I_2 \cdot S S 2 + I_{10} \cdot C C 2 + I_{11} \cdot S C 2$ $+ I_{12} \cdot S S 5 \cdot (S 2 3 + (1 - C C 4) - 1) - 2 \cdot S C 2 3 \cdot C 4 \cdot S C 5$ $+ I_{13} \cdot S S 2 3 \cdot C C 4 + 2 \cdot (I_1 \cdot C 2 \cdot S 2 3 + I_{12} \cdot C 2 \cdot C 2 3$ $+ I_{13} \cdot (S S 2 3 \cdot C 5 + S C 2 3 \cdot C 4 \cdot S 5)$ $+ I_{14} \cdot C 2 \cdot (S 2 3 \cdot C 5 + C 2 3 \cdot C 4 \cdot S 5)$ $+ I_{15} \cdot S 4 \cdot S 5 + I_{13} \cdot (S C 2 3 \cdot C 5 + C C 2 3 \cdot C 4 \cdot S 5) \}$ $\approx 2.57 + 1.38 \cdot C C 2 + 0.30 \cdot S S 2 + 7.44 \times 10^{-3} \cdot C 2 \cdot S 2 3$ $a_{12} = I_1 \cdot S 2 + I_2 \cdot C 2 + I_3 \cdot S 2 + I_{10} \cdot C 2 \cdot S 4 + I_{11} \cdot C 2 \cdot S 5$ $+ I_{12} \cdot S 2 \cdot S 4 \cdot S 5 + I_{13} \cdot (S 2 3 \cdot C 4 \cdot S 5 - C 2 3 \cdot C 5)$ $+ I_{14} \cdot S 2 \cdot S C 4 + I_{15} \cdot S 4 \cdot S 5 \cdot (S 2 3 \cdot C 4 \cdot C C 5 + C 2 3 \cdot S C 5)$ $+ I_{16} \cdot S 2 3 \cdot S 4 \cdot S 5$ $\approx 6.90 \times 10^{-3} \cdot S 2 - 1.34 \times 10^{-3} \cdot C 2 3 + 2.38 \times 10^{-3} \cdot C 2$ $a_{13} = I_1 \cdot C 2 3 + I_2 \cdot S 2 3 - I_{12} \cdot C 2 3 \cdot S 4 \cdot S 5 + I_{13} \cdot S 2 3 \cdot S C 4$ $+ I_{14} \cdot (S 2 3 \cdot C 4 \cdot S 5 - C 2 3 \cdot C 5) + I_{15} \cdot S 2 3 \cdot S 4 \cdot S 5$ $+ I_{16} \cdot S 4 \cdot (S 2 3 \cdot C 4 \cdot C C 5 + C 2 3 \cdot S C 5) \}$ $\approx -1.34 \times 10^{-3} \cdot C 2 3 + -3.97 \times 10^{-3} \cdot S 2 3$ $a_{14} = I_1 \cdot C 2 3 + I_{10} \cdot S 2 3 + S 4 \cdot S 5 + I_{12} \cdot C 2 \cdot C 4 \cdot S 5$ $+ I_{13} \cdot C 2 3 \cdot S 4 \cdot S 5 - I_{16} \cdot (S 2 3 \cdot C 4 \cdot S C 5 + C 2 3 \cdot S S 5)$ $+ I_{17} \cdot C 2 3 \cdot C 4 \cdot S 5 \}$ ≈ 0 $a_{15} = I_{11} \cdot S 2 3 \cdot S 4 \cdot C 5 + I_{14} \cdot C 2 \cdot S 4 \cdot C 5 + I_{15} \cdot S 2 3 \cdot S 4$ $+ I_{16} \cdot (S 2 3 \cdot S 5 - C 2 3 \cdot C 4 \cdot C 5) + I_{17} \cdot C 2 3 \cdot S 4 \cdot C 5$ ≈ 0 $a_{16} = I_{12} \cdot (C 2 3 \cdot C 5 - S 2 3 \cdot C 4 \cdot S 5) \}$ ≈ 0 $a_{22} = I_{10} \cdot S 2 + I_2 \cdot S 2 + I_{10} \cdot S S 4 \cdot S S 5 + I_{11} \cdot S S 4$ $+ 2 \cdot (I_1 \cdot S 3 + I_{12} \cdot C 5 + I_{13} \cdot C 5$ $+ I_{14} \cdot (S 3 \cdot C 5 + C 2 3 \cdot C 4 \cdot S 5) + I_{15} \cdot C 4 \cdot S 5) \}$ $\approx 7.44 \times 10^{-3} \cdot S 2 + 1.11 \times 10^{-3} \cdot S S 4 + 1.11 \times 10^{-3} \cdot S S 5$ $+ I_{10} \cdot S S 4 \cdot S S 5 + I_{11} \cdot S S 4 + 2 \cdot (I_{13} \cdot C 5 + I_{12} \cdot C 4 \cdot S 5) \}$ $\approx 333 + 3.72 \times 10^{-3} \cdot S 3 - 1.10 \times 10^{-3} \cdot C 3$ $a_{24} = -I_{13} \cdot S 4 \cdot S 5 - I_{16} \cdot S 3 \cdot S 4 \cdot S 5 + I_{10} \cdot S 4 \cdot S C 5$ ≈ 0 $a_{25} = I_{13} \cdot C 4 \cdot C 5 + I_{16} \cdot (C 3 \cdot S 5 + S 3 \cdot C 4 \cdot C 5)$ $+ I_{17} \cdot C 4 \cdot I_{18} \cdot S 5 \}$ ≈ 0 $a_{26} = I_{12} \cdot S 4 \cdot S 5 \}$ ≈ 0 $a_{33} = I_{10} \cdot S 4 + I_{10} \cdot S S 4 \cdot S S 5 + I_{11} \cdot S S 4$ $+ 2 \cdot (I_{13} \cdot C 5 + I_{12} \cdot C 4 \cdot S 5) \}$ ≈ 1.16 $a_{34} = -I_{13} \cdot S 4 \cdot S 5 + I_{10} \cdot S 4 \cdot S C 5$ $\approx -1.25 \times 10^{-3} \cdot S 4 \cdot S 5$ $a_{35} = I_{13} \cdot C 4 \cdot C 5 + I_{16} \cdot (C 3 \cdot S 5 + S 3 \cdot C 4 \cdot C 5)$ $+ I_{17} \cdot C 4 \cdot I_{18} \cdot S 5 \}$ $\approx 1.25 \times 10^{-3} \cdot C 4 \cdot C 5$ $a_{36} = I_{12} \cdot S 4 \cdot S 5 \}$ ≈ 0 $a_{44} = I_{10} \cdot S 4 + I_{10} \cdot S S 4 \cdot S S 5 + I_{11} \cdot S S 4$ $+ 2 \cdot (I_{13} \cdot C 5 + I_{12} \cdot C 4 \cdot S 5) \}$ ≈ 0.20 $a_{45} = 0$ $a_{46} = I_{12} \cdot C 5 \}$ ≈ 0 $a_{55} = I_{10} \cdot S 4 + I_{17} \}$ ≈ 0.18 $a_{56} = I_{10} \cdot S 4 + I_{17} \}$ ≈ 0.19	$b_{224} = 2 \cdot (-I_{16} \cdot C 3 \cdot S 4 \cdot S 5 + I_{10} \cdot S C 4 \cdot S S 5$ $+ I_{11} \cdot S C 4 - I_{12} \cdot S 4 \cdot S 5) \}$ $\approx -2.48 \times 10^{-3} \cdot C 3 \cdot S 4 \cdot S 5$ $b_{225} = 2 \cdot (-I_{13} \cdot S 5 + I_{16} \cdot (C 3 \cdot C 4 \cdot C 5 - S 3 \cdot S 5)$ $+ I_{10} \cdot S S 4 \cdot S C 5 + I_{12} \cdot C 4 \cdot C 5) \}$ $\approx -2.50 \times 10^{-3} \cdot S 5 + 2.48 \times 10^{-3} \cdot (C 3 \cdot C 4 \cdot C 5 - S 3 \cdot S 5)$ $b_{226} = 0$ $b_{227} = -b_{224}$ $b_{228} = 0$ $b_{229} = 2 \cdot (-I_{13} \cdot S 4 \cdot C 5 - I_{16} \cdot S 3 \cdot S 4 \cdot C 5)$ $- I_{17} \cdot S 4 + I_{20} \cdot S 4 \cdot (1 - 2 \cdot S S 5) \}$ ≈ 0 $b_{244} = I_{12} \cdot C 4 \cdot S 5 \}$ ≈ 0 $b_{245} = I_{13} \cdot S 4 \cdot C 5 \}$ ≈ 0 $b_{246} = 0$ $b_{247} = 2 \cdot (-I_{13} \cdot C 2 3 \cdot S 4 \cdot S 5 + I_{12} \cdot S 2 3 \cdot C 4 \cdot S 5$ $+ I_{10} \cdot (S 2 3 \cdot (C C 5 \cdot C C 4 - 0.5) + C 2 3 \cdot C 4 \cdot S C 5) \}$ $+ I_{14} \cdot S 2 3 + I_{15} \cdot S 2 3 \cdot (1 - (2 \cdot S S 4)) \}$ $\approx -2.50 \times 10^{-3} \cdot C 2 3 \cdot S 4 \cdot S 5 + 1.64 \times 10^{-3} \cdot S 2 3$ $+ 0.30 \times 10^{-3} \cdot S 2 3 \cdot (1 - 2 \cdot S S 4)$ $b_{248} = 2 \cdot (-I_{13} \cdot C 2 3 \cdot S 4 \cdot C 5 + I_{12} \cdot S 2 3 \cdot S 4 \cdot C 5)$ $- I_{17} \cdot C 2 3 \cdot S 4$ $+ I_{20} \cdot S 4 \cdot (C 2 3 \cdot (1 - 2 \cdot S S 5) - 2 \cdot S 2 3 \cdot C 4 \cdot S C 5) \}$ $\approx -2.50 \times 10^{-3} \cdot C 2 3 \cdot S 4 \cdot C 5 - 6.42 \times 10^{-4} \cdot C 2 3 \cdot S 4$ $b_{249} = -b_{246}$ $b_{250} = 0$ $b_{251} = 2 \cdot (-I_{10} \cdot S C 4 \cdot S S 5 + I_{11} \cdot S C 4 - I_{12} \cdot S 4 \cdot S 5) \}$ ≈ 0 $b_{252} = 2 \cdot (-I_{10} \cdot S S 5 + I_{11} \cdot S S 5 + I_{10} \cdot S 4 \cdot C 5 + I_{12} \cdot C 4 \cdot C 5)$ $- 2 \cdot (I_{13} \cdot C 2 3 \cdot S 4 \cdot S 5 + I_{16} \cdot S 2 3 \cdot S 4 \cdot S 5)$ $- I_{17} \cdot S 2 3 \cdot C 4 \}$ ≈ 0 $b_{253} = -I_{10} \cdot S 2 3 \cdot S 4 \cdot S 5 + I_{13} \cdot S 2 3 \cdot (1 - (2 \cdot S S 4))$ $+ I_{16} \cdot S 2 3 \cdot (1 - 2 \cdot S S 4 \cdot C C 5) - I_{14} \cdot S 2 3 \}$ ≈ 0 $b_{255} = I_{17} \cdot C 2 3 \cdot S 4 + I_{18} \cdot 2 \cdot (S 2 3 \cdot C 4 \cdot C 5 + C 2 3 \cdot S 5)$ $+ I_{19} \cdot S 4 \cdot (C 2 3 \cdot (1 - 2 \cdot S S 5) - S 2 3 \cdot C 4 \cdot 2 \cdot S C 5) \}$ ≈ 0 $b_{256} = -I_{16} \cdot (S 2 3 \cdot C 5 + C 2 3 \cdot C 4 \cdot S 5) \}$ ≈ 0 $b_{257} = -I_{16} \cdot (C 2 3 \cdot C 4 \cdot S 5 + S 2 3 \cdot C 5) + I_{19} \cdot C 2 3 \cdot S C 4$ $+ I_{20} \cdot S 4 \cdot (C 2 3 \cdot C 4 \cdot C C 5 - S 2 3 \cdot S C 5)$ $- I_{17} \cdot C 2 3 \cdot S 4 \cdot S 5 \}$ $\approx -0.42 \times 10^{-4} \cdot S 2 3 \cdot C 4$ $b_{258} = 0$ $b_{259} = -b_{256}$ $b_{260} = 0$ $b_{261} = I_{17} \cdot S 4 + I_{20} \cdot S 4 \cdot (1 - 2 \cdot S S 5) \}$ $\approx 6.42 \times 10^{-4} \cdot S 4$ $b_{262} = -b_{260}$ $b_{263} = -b_{264}$ $b_{264} = -b_{264}$ $b_{265} = -I_{10} \cdot (S 2 3 \cdot C 4 \cdot (1 - 2 \cdot S S 5) + 2 \cdot C 2 3 \cdot S C 5)$ $- I_{17} \cdot S 2 3 \cdot C 4 \}$ $\approx -0.42 \times 10^{-4} \cdot S 2 3 \cdot C 4$ $b_{266} = 0$ $b_{267} = -b_{264}$ $b_{268} = 0$ $b_{269} = -I_{13} \cdot S 4 \cdot C 5 - I_{17} \cdot S 4 + I_{20} \cdot S 4 \cdot (1 - 2 \cdot S S 5) \}$ $\approx -2.50 \times 10^{-3} \cdot S 4 \cdot C 5$ $b_{269} = -b_{264}$ $b_{270} = -b_{264}$ $b_{271} = -b_{264}$ $b_{272} = -b_{264}$ $b_{273} = -b_{264}$ $b_{274} = -b_{264}$ $b_{275} = -b_{264}$ $b_{276} = -b_{264}$ $b_{277} = -b_{264}$ $b_{278} = -b_{264}$ $b_{279} = -b_{264}$ $b_{280} = -b_{264}$ $b_{281} = -b_{264}$ $b_{282} = -b_{264}$ $b_{283} = -b_{264}$ $b_{284} = -b_{264}$ $b_{285} = -b_{264}$ $b_{286} = -b_{264}$ $b_{287} = -b_{264}$ $b_{288} = -b_{264}$ $b_{289} = -b_{264}$ $b_{290} = -b_{264}$ $b_{291} = -b_{264}$ $b_{292} = -b_{264}$ $b_{293} = -b_{264}$ $b_{294} = -b_{264}$ $b_{295} = -b_{264}$ $b_{296} = -b_{264}$ $b_{297} = -b_{264}$ $b_{298} = -b_{264}$ $b_{299} = -b_{264}$ $b_{300} = -b_{264}$	<p>Table A6. The expressions for the terms of the centrifugal matrix. (The Abbreviated Expressions have units of kg-m².)</p> $c_{11} = 0$ $c_{12} = +I_4 \cdot C 2 - I_8 \cdot S 2 3 - I_9 \cdot S 2 + I_{13} \cdot C 2 3$ $+ I_{14} \cdot S 2 3 \cdot S 4 \cdot S 5 + I_{15} \cdot C 2 \cdot S 4 \cdot S 5$ $+ I_{16} \cdot (C 2 3 \cdot C 4 \cdot S 5 + S 2 3 \cdot C 5) + I_{19} \cdot C 2 3 \cdot S C 4$ $+ I_{20} \cdot S 4 \cdot (C 2 3 \cdot C 4 \cdot C C 5 - S 2 3 \cdot S C 5)$ $\approx 6.90 \times 10^{-3} \cdot C 2 + 1.34 \times 10^{-3} \cdot S 2 3 - 2.38 \times 10^{-3} \cdot S 2$ $c_{13} = 0.5 \cdot b_{123}$ $c_{14} = -I_{15} \cdot S 2 3 \cdot S 4 \cdot S 5 - I_{16} \cdot C 2 \cdot S 4 \cdot S 5$ $+ I_{17} \cdot C 2 3 \cdot C 4 \cdot S 5 + I_{18} \cdot S 2 3 \cdot S 4 \cdot S 5$ $- I_{19} \cdot C 2 3 \cdot S 4 \cdot S 5 \}$ ≈ 0 $c_{15} = -I_{14} \cdot S 2 3 \cdot S 4 \cdot S 5 - I_{16} \cdot C 2 \cdot S 4 \cdot S 5$ $+ I_{18} \cdot (S 2 3 \cdot C 5 + C 2 3 \cdot C 4 \cdot S 5) - I_{19} \cdot C 2 3 \cdot S 4 \cdot S 5$ ≈ 0 $c_{16} = 0$ $c_{17} = 0$ $c_{18} = 0.5 \cdot b_{123}$ $c_{19} = -I_{15} \cdot S 2 3 \cdot S 4 \cdot S 5 - I_{16} \cdot C 2 \cdot S 4 \cdot S 5$ $+ I_{17} \cdot C 2 3 \cdot C 4 \cdot S 5 + I_{18} \cdot S 2 3 \cdot S 4 \cdot S 5$ $- I_{19} \cdot C 2 3 \cdot S 4 \cdot S 5 \}$ ≈ 0 $c_{20} = 0$ $c_{21} = -0.5 \cdot b_{123}$ $c_{22} = 0$ $c_{23} = 0.5 \cdot b_{123}$ $c_{24} = -I_{15} \cdot C 4 \cdot S 5 - I_{16} \cdot S 3 \cdot C 4 \cdot S 5 + I_{20} \cdot C 4 \cdot S C 5$ ≈ 0 $c_{25} = -I_{13} \cdot C 4 \cdot S 5 + I_{16} \cdot (C 3 \cdot C 5 - S 3 \cdot C 4 \cdot S 5)$ $+ I_{12} \cdot C 5 \}$ ≈ 0 $c_{26} = 0$ $c_{27} = -0.5 \cdot b_{123}$ $c_{28} = 0$ $c_{29} = -I_{13} \cdot C 4 \cdot S 5 + I_{20} \cdot C 4 \cdot S C 5$ $\approx -1.25 \times 10^{-3} \cdot C 4 \cdot S 5$ $c_{30} = -I_{13} \cdot C 4 \cdot S 5 + I_{20} \cdot C 4 \cdot S C 5$ ≈ 0 $c_{31} = -0.5 \cdot b_{123}$ $c_{32} = 0$ $c_{33} = 0.5 \cdot b_{123}$ $c_{34} = 0$ $c_{35} = 0$ $c_{36} = 0$ $c_{37} = -0.5 \cdot b_{123}$ $c_{38} = 0$ $c_{39} = 0$ $c_{40} = 0$ $c_{41} = -0.5 \cdot b_{123}$ $c_{42} = -0.5 \cdot b_{123}$ $c_{43} = 0.5 \cdot b_{123}$ $c_{44} = 0$ $c_{45} = 0$ $c_{46} = 0$ $c_{47} = -0.5 \cdot b_{123}$ $c_{48} = 0$ $c_{49} = 0$ $c_{50} = 0$ $c_{51} = 0$ $c_{52} = 0$ $c_{53} = 0$ $c_{54} = 0$ $c_{55} = 0$ $c_{56} = 0$ $c_{57} = 0$ $c_{58} = 0$ $c_{59} = 0$ $c_{60} = 0$	<p>Table A7. Gravity Terms. (The Abbreviated Expressions have units of newton-meters.)</p> $g_1 = 0$ $g_2 = g_1 \cdot C 2 + g_2 \cdot S 2 3 + g_3 \cdot S 2 + g_4 \cdot C 2 3$ $+ g_5 \cdot (S 2 3 \cdot C 5 + C 2 3 \cdot C 4 \cdot S 5)$ $\approx -37.2 \cdot C 2 - 8.4 \cdot S 2 3 + 1.02 \cdot S 2$ $g_3 = g_2 \cdot S 2 3 + g_4 \cdot C 2 3 + g_5 \cdot (S 2 3 \cdot C 5 + C 2 3 \cdot C 4 \cdot S 5) \}$ $\approx -8.4 \cdot S 2 3 + 0.25 \cdot C 2 3$ $g_4 = -g_5 \cdot S 2 3 \cdot S 4 \cdot S 5$ $\approx 2.8 \times 10^{-3} \cdot S 2 3 \cdot S 4 \cdot S 5$ $g_5 = g_6 \cdot (C 2 3 \cdot S 5 + S 2 3 \cdot C 4 \cdot C 5) \}$ $\approx -2.8 \times 10^{-3} \cdot (C 2 3 \cdot S 5 + S 2 3 \cdot C 4 \cdot C 5)$ $g_6 = 0$	$h_{281} = g_{281}$ $h_{282} = g_{282}$ $h_{283} = g_{283}$ $h_{284} = 0$ $h_{285} = 0$ $h_{286} = 0$ $h_{287} = 0$ $h_{288} = 0$ $h_{289} = 0$ $h_{290} = 0$ $h_{291} = 0$ $h_{292} = 0$ $h_{293} = 0$ $h_{294} = 0$ $h_{295} = 0$ $h_{296} = 0$ $h_{297} = 0$ $h_{298} = 0$ $h_{299} = 0$ $h_{300} = 0$ $h_{301} = -b_{215}$ $h_{302} = -b_{213}$ $h_{303} = -b_{213}$ $h_{304} = -b_{213}$ $h_{305} = -b_{213}$ $h_{306} = -b_{213}$ $h_{307} = -b_{213}$ $h_{308} = -b_{213}$ $h_{309} = -b_{213}$ $h_{310} = -b_{213}$ $h_{311} = -b_{213}$ $h_{312} = -b_{213}$ $h_{313} = -b_{213}$ $h_{314} = -b_{213}$ $h_{315} = -b_{213}$ $h_{316} = -b_{213}$ $h_{317} = -b_{213}$ $h_{318} = -b_{213}$ $h_{319} = -b_{213}$ $h_{320} = -b_{213}$ $h_{321} = -b_{213}$ $h_{322} = -b_{213}$ $h_{323} = -b_{213}$ $h_{324} = -b_{213}$ $h_{325} = -b_{213}$ $h_{326} = -b_{213}$ $h_{327} = -b_{213}$ $h_{328} = -b_{213}$ $h_{329} = -b_{213}$ $h_{330} = -b_{213}$ $h_{331} = -b_{213}$ $h_{332} = -b_{213}$ $h_{333} = -b_{213}$ $h_{334} = -b_{213}$ $h_{335} = -b_{213}$ $h_{336} = -b_{213}$ $h_{337} = -b_{213}$ $h_{338} = -b_{213}$ $h_{339} = -b_{213}$ $h_{340} = -b_{213}$ $h_{341} = -b_{213}$ $h_{342} = -b_{213}$ $h_{343} = -b_{213}$ $h_{344} = -b_{213}$ $h_{345} = -b_{213}$ $h_{346} = -b_{213}$ $h_{347} = -b_{213}$ $h_{348} = -b_{213}$ $h_{349} = -b_{213}$ $h_{350} = -b_{213}$ $h_{351} = -b_{213}$ $h_{352} = -b_{213}$ $h_{353} = -b_{213}$ $h_{354} = -b_{213}$ $h_{355} = -b_{213}$ $h_{356} = -b_{213}$ $h_{357} = -b_{213}$ $h_{358} = -b_{213}$ $h_{359} = -b_{213}$ $h_{360} = -b_{213}$
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Example Lagrangian expansion for a 4 DOF robot arm

Controller Formulation - ISM

Three Layer Control Architecture

1) Inner Loop: Inverse Dynamics Controller

$$\ddot{q} = v - \eta(q, \dot{q}), \text{ where } \eta(q, \dot{q}) = -M^{-1}(q)(\hat{n}(q, \dot{q}) - n(q, \dot{q})).$$

N-decoupled

$$\begin{cases} \dot{x}_{1_i}(t) = x_{2_i}(t) \\ \dot{x}_{2_i}(t) = v_i(t) - \eta_i(t) \end{cases}$$

Double Integrators

2) Middle Loop: Integral Sliding Mode (ISM)

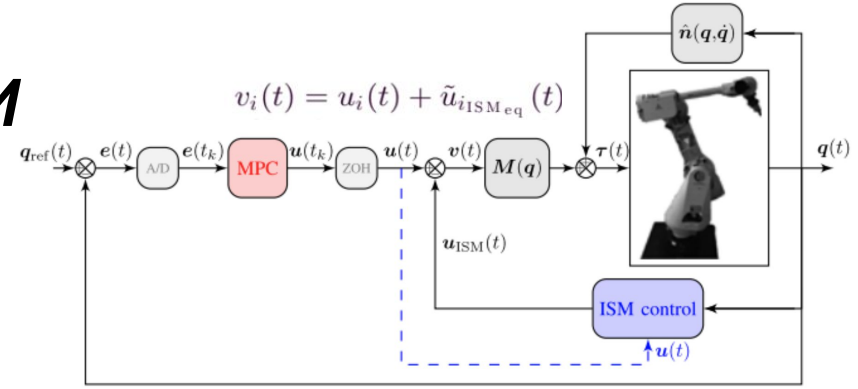
$$\tilde{u}_{i\text{ISM}_{eq}}(t) = \frac{1}{\mu_i} \int_{t_0}^t e^{-\frac{1}{\mu_i}(t-\zeta)} u_{i\text{ISM}}(\zeta) d\zeta$$

$$\begin{cases} \dot{x}_{1_i}(t) = x_{2_i}(t) \\ \dot{x}_{2_i}(t) = u_i(t) \end{cases}$$

$$\sigma_i(x_i(t)) =$$

$$S_i \left(x_i(t) - x_i(t_0) - \int_{t_0}^t [x_{2_i}(\zeta), v_i(\zeta) - u_{i\text{ISM}}(\zeta)]^T d\zeta \right)$$

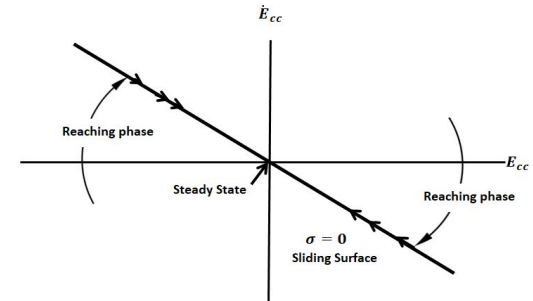
$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t)$$



Sliding Mode Control

$$\sigma_i = S_i \left[x_i(t) - x_i(t_0) - \int_{t_0}^t \begin{bmatrix} x_{2_i}(\zeta) \\ u_i(\zeta) \end{bmatrix} d\zeta \right], \quad S_i = [c_i \ 1]$$

$$u_i^{\text{ISM}} = -U_i^{\max} \text{sgn}(\sigma_i)$$



Sliding surface and sliding variable. [2]

Controller Formulation - MPC

Outer Loop: MPC

Recap: $\ddot{q} = v + M^{-1}(q)(\hat{n}(q, \dot{q}) - n(q, \dot{q})) = v - \eta(q, \dot{q})$

$$v_i(t) = u_i(t) + \tilde{u}_{i_{\text{ISM}_{\text{eq}}}}(t)$$

$$x_{k+1} = \tilde{A} x_k + \tilde{B} u_k, \quad \tilde{A} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$$

Cost Function:

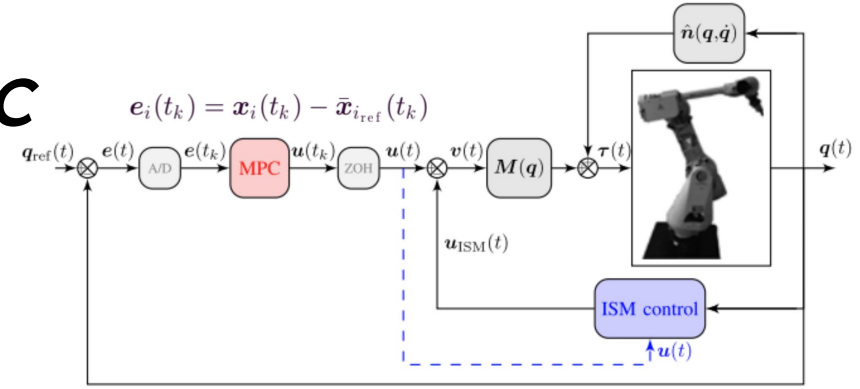
$$J(e_i(t_k), \mathbf{u}_{i|t_k, t_k+N-1|t_k}, N) =$$

$$\sum_{j=0}^{N-1} \|e_i(t_{k+j})\|_{Q_i}^2 + \|u_i(t_{k+j})\|_{R_i}^2 + \|e_i(t_{k+N})\|_{\Pi_i}^2$$

Subject to: $x_i(t_{k+j}) \in \mathcal{X}$

$$x_i(t_{k+N}) \in \mathcal{X}_f$$

$$\|u_i(t_{k+j})\| \leq v_{i_{\max}} - U_{i_{\max}}$$



Terminal Set and Terminal Control:

$$\mathcal{X}_f := \{x_i \mid \|x - \bar{x}_{i_{\text{ref}}}\|_{\Pi}^2 \leq \rho\}, \quad \mathcal{X}_f \subseteq \mathcal{X}$$

K_{LQ} is the control gain of an infinite horizon Linear Quadratic (LQ) controller with the same cost function

$$\kappa_{i_f}(e_i(t_k)) = K_{\text{LQ}} e_i(t_k)$$

$$x_i(t_k) \in \mathcal{X}_f$$

$$\|\kappa_{i_f}(e_i(t_k))\| \leq v_{i_{\max}} - U_{i_{\max}}$$

Solution to the Riccati Equation

$$(\tilde{A}_i - \tilde{B}_i K_{\text{LQ}})^T \Pi_i (\tilde{A}_i - \tilde{B}_i K_{\text{LQ}}) - \Pi_i = -Q_i - K_{\text{LQ}}^T R_i K_{\text{LQ}}$$

Paper Results

- 3-joint COMAU Smart3-S2 industrial robot manipulator
- Realistic uncertainty injection (identified from real robot data)
- Constraints: See Table I (position, velocity, acceleration limits)
- Target position: $q_ref = [\pi/4, \pi/3, 2\pi/4]^T$, from $q_0 = [0, 0, 0]$
- Controller parameters:
 - $C_i = [10, 10, 10]$
 - ISM gains: $[20, 35, 85]$
 - MPC: $Q = \text{diag}(100, 100)$, $R = 0.1$,
 - Terminal Weight $\Pi_i = \begin{bmatrix} 5213.4 & 165.8 \\ 165.8 & 221.3 \end{bmatrix}$
 - Horizon $N=10$
 - Sampling: MPC = 20ms, ISM = 1ms

Key Results:

- MPC/ISM reduces RMS error dramatically:
 - Joint 1: $0.2105 \rightarrow 0.0070$ rad (30× decrease)
 - Joint 3: $1.0458 \rightarrow 0.0152$ rad (69× decrease)
- Similar control effort but better performance
- Computational efficiency:
 - MPC: 18ms average execution time
 - ISM: only 29μs average execution time
 - Suitable for real-time implementation
- MPC alone violates constraints,
- MPC/ISM ensures constraint satisfaction

TABLE I
STATE AND INPUT CONSTRAINTS FOR EACH JOINT

Joint i	q_{i_max} (rad)	\dot{q}_{i_max} (rad s ⁻¹)	v_{i_max} (rad/s ²)
1	1.83	2	145
2	2.71	3.5	250
3	3.49	6.3	350

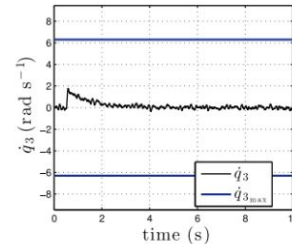
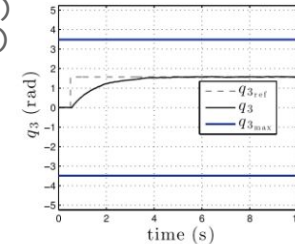
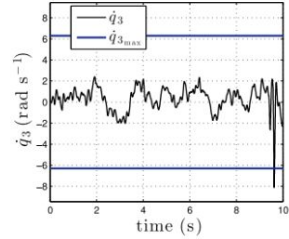
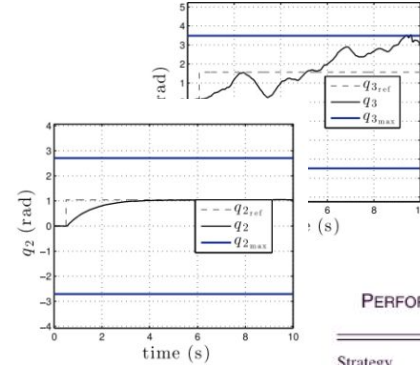
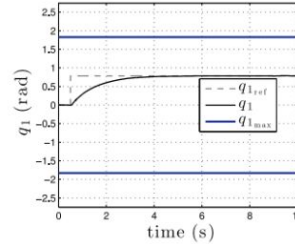
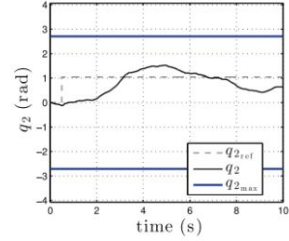
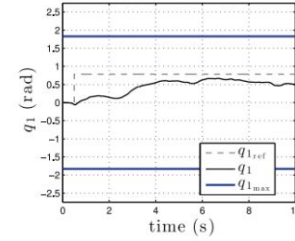


TABLE II
PERFORMANCE INDEXES FOR EACH JOINT

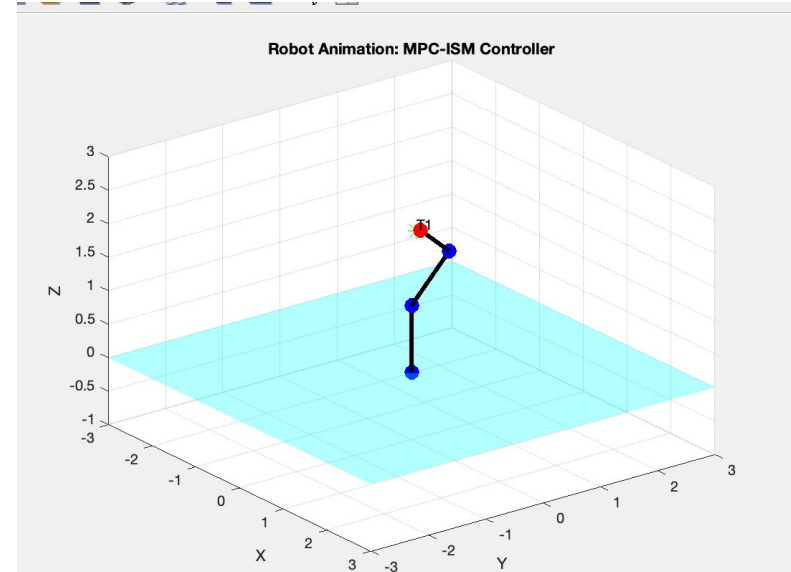
Strategy	Joint i	e_{RMS_i} (rad)	E_{c_i} (rad/s ²)
MPC	1	0.2105	8.0303
	2	0.3749	13.4839
	3	1.0458	41.2172
MPC/ISM	1	0.0070	8.0600
	2	0.0102	13.4217
	3	0.0152	34.8530

TABLE III
TIME CONSUMPTION OF THE PROPOSED CONTROL STRATEGY IN SECONDS

Algo.	Mean	Min.	Max.	Std. dev.
MPC	0.018	0.017	0.71	0.011
ISM	2.9×10^{-5}	2.7×10^{-5}	0.0086	1.4×10^{-4}

Implementation on Matlab

- 1) Robot Joint Angle target : $\pi/2$, $\pi/3$, $\pi/4$
- 2) Joint trajectory from MPC and MPC-ISM controllers compared to reference cubic polynomial trajectory
- 3) Sinusoidal noise injected to see the benefits of the MPC-ISM Combined Controller Algorithm



Uncertainty Formulation and Injection

1.) Uncertainty formulation:

```
% Deterministic component based on position
pos_comp = 0.5 * max_eta * sin(q(i));

% Deterministic component based on velocity
vel_comp = 0.2 * max_eta * sign(q_dot(i)) * min(abs(q_dot(i)), 1);
|
% Random component (bounded)
rand_comp = 0.3 * max_eta * (2*rand() - 1);

% Combined uncertainty
eta(i) = pos_comp + vel_comp + rand_comp;
```

2.) Uncertainty injection:

Where M is the inertia matrix,
 n contains the nonlinear terms
(friction, coriolis effect), and η is
the uncertainty

$$\ddot{q} = M^{-1}(u - n + \eta)$$

```
% Compute acceleration: q_ddot = M^{-1}(\tau - n + \eta)|
q_ddot = M \ (u - n + eta);
```

3.) This is done for all 3 joints.

Results

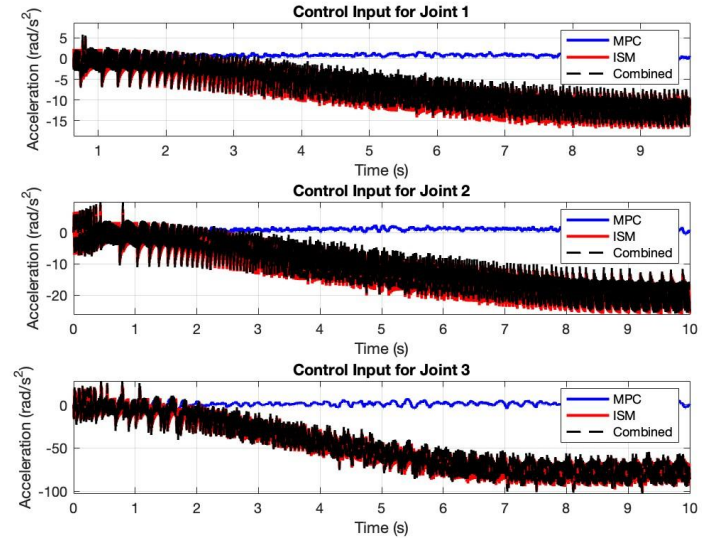
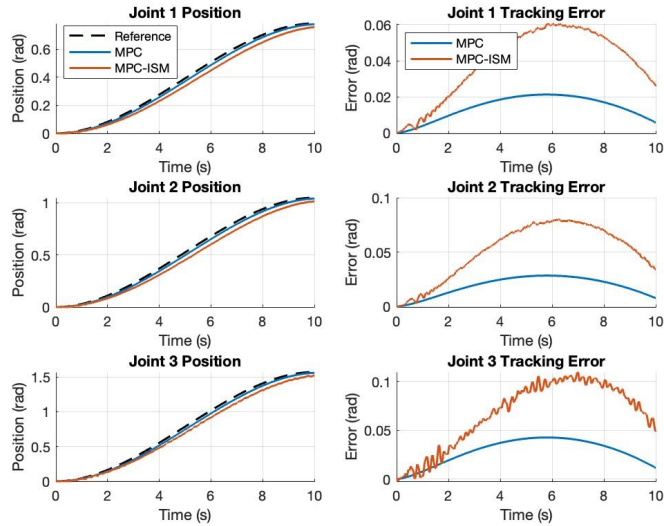
- 1) The results were not what was expected based on the paper.
- 2) MPC Controller was very well tuned to account for the noise and gave highly accurate results.
- 3) MPC-ISM performed slightly worse than MPC.
- 4) However, the difference seems negligible - both controllers have an error within hundredths of a radian.

```
Controller: MPC
RMS Tracking Error (rad):
  Joint 1: 0.0156
  Joint 2: 0.0207
  Joint 3: 0.0311
Control Effort (rad/s^2):
  Joint 1: 0.0259
  Joint 2: 0.0345
  Joint 3: 0.0518
Steady-state Error (rad):
  Joint 1: 0.005942
  Joint 2: 0.007923
  Joint 3: 0.011884
```

```
Controller: MPC-ISM
RMS Tracking Error (rad):
  Joint 1: 0.0434
  Joint 2: 0.0571
  Joint 3: 0.0748
Control Effort (rad/s^2):
  Joint 1: 8.4462
  Joint 2: 13.5679
  Joint 3: 57.2287
Steady-state Error (rad):
  Joint 1: 0.026736
  Joint 2: 0.034350
  Joint 3: 0.050732
```

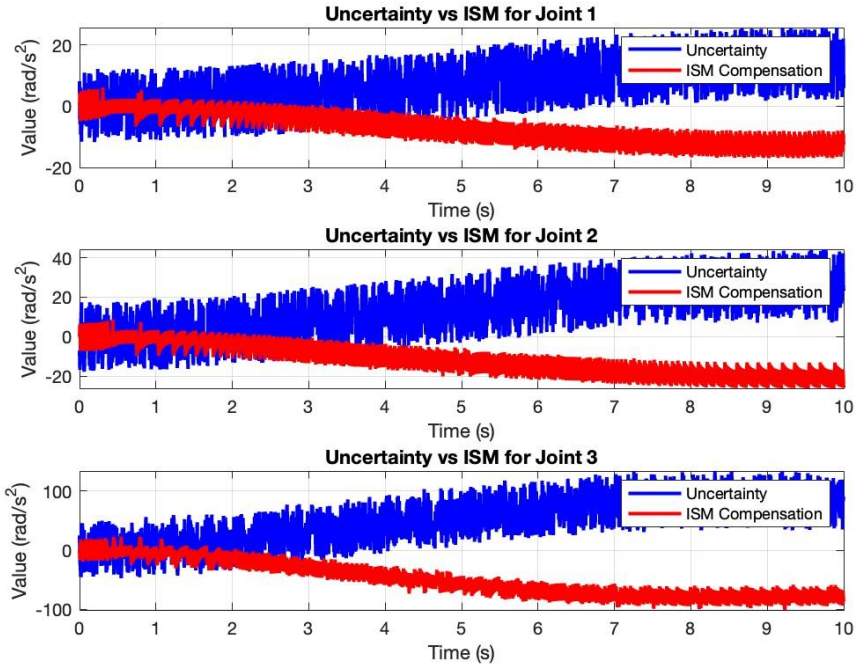
Results

- 1) We can see however, that the ISM Controller adjusts the control input for the joints to account for the uncertainty



Results

- 1) As the uncertainty goes higher in magnitude, the ISM controllers counter action increases as well



Possible considerations

- 1) Uncertainty modeling - The simulated uncertainty might have not been aggressive enough to impact MPC performance as the uncertainty injected by the authors
- 2) Hardware Differences: The COMAU Smart3-S2 robot in the paper may have different characteristics than our simulated model
 - a) The paper notes that their model of the COMAU Smart3-S2 on Matlab Simulink was based on data from real experiments.
 - b) However, we are simulating a perfect model on which the MPC acts upon. Due to this, the performance of MPC itself is highly accurate.
 - c) The performance difference between MPC and ISM is marginally different, and seems to be negligible due to this .

References

- [1] Incremona, Gian Paolo, et al. "MPC for robot manipulators with integral sliding modes generation." IEEE/ASME Transactions on Mechatronics, vol. 22, no. 3, June 2017, pp. 1299–1307, <https://doi.org/10.1109/tmech.2017.2674701>.
- [2] Adaptive chaos control of a humanoid robot arm: a fault-tolerant scheme - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Sliding-surface-and-sliding-variable_fig1_370320084
- [3] ECE 5463 taught at Ohio State University by Prof. Wei Zhang https://storage1.ucsd.edu/slides/CSE291Robo/L9_lagrangian.html#/title
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