Middle East Technical University Department of Computer Engineering

CENG 382

Analysis of Dynamic Systems

Fall 2019-2020

Homework Assignment 1

Due date: October 23 2019, Wednesday

Juliet is in love with Romeo. However, in this version of the story, Romeo is an unreliable lover. The more Juliet loves him, the more he begins to dislike her. But, when she loses interest, his feelings for her begin to warm up. She, on the other hand, tends to have emotions that are in parallel to his: her love grows when he loves her more, and turns to hate when he hates her.
A simple model for this relationship (in continuous-time) is given by the following differential equation:

$$\begin{aligned} \frac{dR(t)}{dt} &= -aJ(t) \\ \frac{dJ(t)}{dt} &= \ bR(t) \\ R(0) &= 1 \\ R'(0) &= 0 \end{aligned} \tag{1}$$

where $R(t)$ and $J(t)$ are functions of time quantifying the feelings of Romeo and Juliet towards each other, respectively, with positive value signifying love. The constants a and b are positive and determine reaction times.

This problem is adapted from [1].

Question 1

Write a second order differential equation only in terms of $R(t)$ based on the differential equation given above (eqn. 1). Once that is done, find the solution for $R(t)$. Then, find the corresponding expression for $J(t)$.

Question 2

First, plot $R(t)$ and $J(t)$ in MATLAB as a function of time $t \in [0, 10]$ on the same graph for $a = 1$ and $b = 1$. On a separate graph, plot the $R(t)$ versus $J(t)$ graph to illustrate the behavior of this dynamical system. Based on these solutions, describe qualitatively what happens to Romeo and Juliet's relationship in time.

Question 3

Sometimes, a good method for understanding the behavior of a continuous dynamical system is to convert it to a discrete difference equation. This will inevitably be an approximation, for which a simple alternative is given by

$$\begin{aligned} R[k+1] - R[k] &= -AJ[k] \\ J[k+1] - J[k] &= BR[k] \end{aligned} \tag{2}$$

with the indices k and $k+1$ corresponding to discrete time instance t_k and t_{k+1} such that the time difference $\Delta t = t_{k+1} - t_k$ remains constant.

Find expressions for the constants A and B in the difference equation above (eqn. 2) corresponding to the constants a and b from the differential equation. You can assume that the differential equation is approximated such that the values $R(t)$ and $J(t)$ remain constant from t_k until t_{k+1} and then undergo a step change.

Question 4

Convert these two difference equations into a second order difference equation written in terms of Romeo's love $R[k]$. Subsequently, find the solution $R[k]$ to this difference equation.

Question 5

Plot the $R[k]$ versus $J[k]$ graph in MATLAB using the same parameters a and b from **Question 2**, but using three different values $\Delta t = 0.5$ (plot for 20 samples), $\Delta t = 0.1$ (plot for 100 samples), and $\Delta t = 0.01$ (plot for 1000 samples). Are these graphs similar to the graph you obtained in **Question 2**? What is the effect of Δt on the solutions? Based on these results, is it a good idea to use the discrete approximation to the continuous love dynamics? Briefly explain your answer.

Submission

Write **a report** with your answers to above questions, including graphs that you generated using MATLAB. Make sure your graphs have proper axis labels and descriptive captions with explanations of what is being shown.
You should also submit your single MATLAB script named as **hw1.m** that generates all the plots you included in your report when executed from within MATLAB.

Regulations

1. It is the student's responsibility to check the validity of their submitted files.
2. **Late Submission:** Not allowed.
3. **Cheating:** Do not cheat.
4. **Evaluation:** Your submitted scripts will be run on **MATLAB R2016b** that are installed on ineks. If you have used another version of MATLAB and your scripts do not properly run on this version, please specify the version of MATLAB you used on your reports.

References

[1] Strogatz, S. H. (2018). Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. CRC Press.