CmpE 548: Monte Carlo Methods
Assignment 2

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Part I: Gibbs Sampler

Let Z be a $I \times J$ binary matrix that describes an image where each Z_{ij} is either $+1$ or -1 (i.e. black or white). We assume that Z is generated from the following simple Ising model with parameter β:

$$E(Z) = -\beta \sum_{(i,j)\sim(k,l)} Z_{ij} Z_{kl} \quad (1)$$

$$p(Z) \propto e^{-E(Z)} \quad (2)$$

Here, the summation is over the all connected pairs of Z_{ij} and Z_{kl}. Two pixels are connected if they share a common edge on the image, i.e. they are neighbours.

Things to do

- Derive an expression for the conditional distribution of Z_{ij} given all the other elements of Z.

$$p(Z_{ij} \mid Z_{-ij}) \quad (3)$$

  Note: Z_{-ij} denotes all elements of Z except Z_{ij}.

- For different values of β, draw samples $Z \sim p(Z)$ using Gibbs sampler by iteratively sampling $Z_{ij} \sim p(Z_{ij} \mid Z_{-ij})$. Plot the resulting samples.

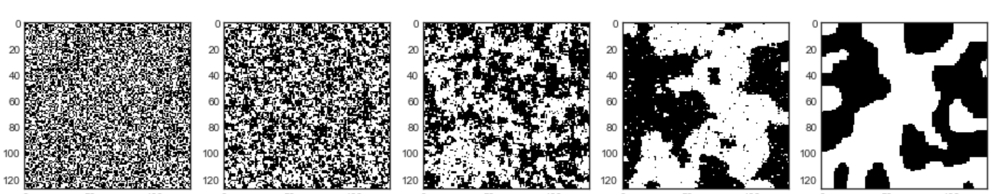

Figure 1: 5 different samples of size 128×128

Part II: Image Denoising

Let Z be a $I \times J$ binary matrix that describes an image where each Z_{ij} is either $+1$ or -1 (i.e. black or white). By randomly flipping some of the pixels of Z, we obtain the noisy image X which we will be observed. Assume the joint distribution of X and Z is the following Boltzmann distribution:

$$E(Z, X) = -\gamma \sum_{ij} Z_{ij} X_{ij} - \beta \sum_{(i,j)\sim(k,l)} Z_{ij} Z_{kl} \tag{4}$$

$$P(Z, X) \propto e^{-E(Z,X)} \tag{5}$$

Here, the second summation is over the all connected pairs of Z_{ij} and Z_{kl}. In this formulation, higher γ implies lower noise on the X. Similarly, we expect more consistency between the neighbouring pixels if β is higher (as you can see from Part).

Things to do

- Read the *lena.pgm* file into a 640×640 matrix Y. Then, construct the binary matrix X as follows: If $Y_{ij} > 127$, set $X_{ij} = 1$. Otherwise, set $X_{ij} = -1$. Plot X.
- Derive an expression for the conditional distribution of Z_{ij} given all the other elements of Z and all the elements of X.

$$p(Z_{ij} \mid Z_{-ij}, X) \tag{6}$$

  Note: Z_{-ij} denotes all elements of Z except Z_{ij}.

- For different values of β and γ, draw samples $Z \sim p(Z)$ using Gibbs sampler by iteratively sampling $Z_{ij} \sim p(Z_{ij} \mid Z_{-ij})$. Plot the resulting samples. Explain the results briefly.

Deadline: 8 November, 2017 10:00