OnurOnel_Assignment05

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[1]:
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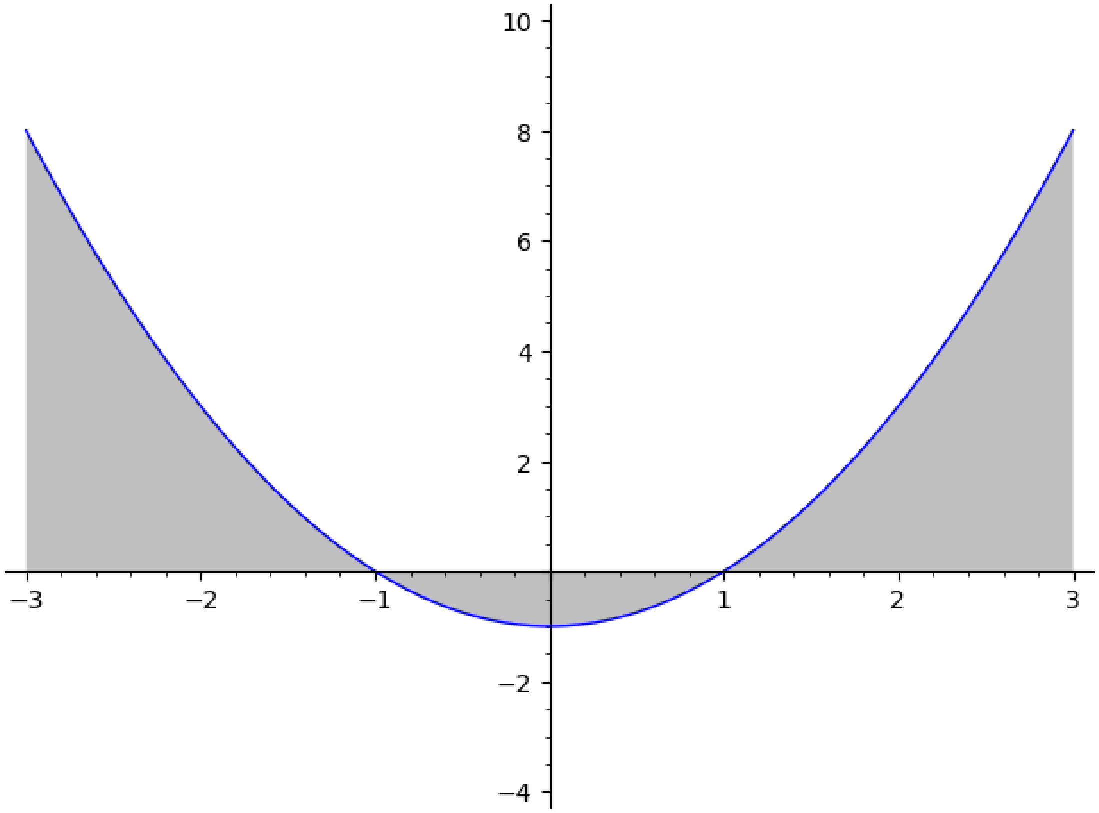
# B.Sc. Software Engineering (1st year)
# Submitted 11/12/2024

# This import statement will allow to receive more than one output from a␣
↪single code cell.
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

[2]:
```
# 1.) Sketch the region whose weighted area is computed by this definite␣
↪integral.
f(x) = x^2 -1
p = plot(f(x), (x, -3, 3), ymin=-4, ymax=10, color='blue',fill = true,␣
↪fillcolor='grey')
show(p)
```



[3]:
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# 2.) Set up the Right-Hand Rule formula for computing this definite integral
f = function('f')(x)

f(x) = x^2 -1 # integrand
a = var('a')  # -1
b = var('b')  # 3
n = var('n')  # upper limit
i = var('i')  # index

s(n) = sum( (b-a)/n * f(a+i*(b-a)/n),i, 1, n ) # Right-Hand Formula
```

[4]:
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# 3.) Evaluate the formula you obtained in solving question 2 using SageMath
s(n) = sum( (4)/n * f(-1+i*(4)/n),i, 1, n )
show(s(n))
show("Lim n → ∞ = ",lim(s(n), n=oo))
```

$$\frac{16\,(n^2 + 3\,n + 2)}{3\,n^2}$$

Lim n → ∞ = $\frac{16}{3}$

[5]:
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# 4.) Evaluate the formula you obtained in part b by hand
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[6]:
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# 5.) Evaluate the given definite integral using SageMath's integral command
show(integral(f(x),x,-1,3))
```

$$\frac{16}{3}$$

4.) Assignment 5 Onur Onel

Right Hand Formula:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left[\frac{b-a}{n}\cdot\sum_{i=1}^{n} f\left(a+i\cdot\frac{b-a}{n}\right)\right]$$

$$\int_{-1}^{3}(x^2-1)\,dx = \lim_{n\to\infty}\left[\frac{3-(-1)}{n}\sum_{i=1}^{n} f\left(-1+i\,\frac{3-(-1)}{n}\right)\right]$$

$$\lim_{n\to\infty}\left[\frac{4}{n}\sum_{i=1}^{n} f\left(-1+\frac{i4}{n}\right)\right] = \lim_{n\to\infty}\left[\frac{4}{n}\sum_{i=1}^{n}\left(\left(-1+\frac{i4}{n}\right)^2-1\right)\right]$$

$$\lim_{n\to\infty}\left[\frac{4}{n}\sum_{i=1}^{n}\left[1-2i\frac{4}{n}+i^2\frac{16}{n^2}-1\right]\right] = \lim_{n\to\infty}\left[\frac{4}{n}\sum_{i=1}^{n}\frac{8}{n}\left(\frac{2}{n}i^2-i\right)\right]$$

$$\lim_{n\to\infty}\left[\frac{32}{n^2}\sum_{i=1}^{n}\left(\frac{2}{n}i^2-i\right)\right] = \lim_{n\to\infty}\left[\frac{32}{n^2}\left[\left(\sum_{i=1}^{n}\frac{2}{n}i^2\right)-\left(\sum_{i=1}^{n} i\right)\right]\right]$$

$$\lim_{n\to\infty}\left[\frac{32}{n^2}\left[\left(\frac{2}{n}\sum_{i=1}^{n} i^2\right)-\frac{n(n+1)}{2}\right]\right]$$

$$\lim_{n\to\infty}\left[\frac{32}{n^2}\left[\frac{2}{n}\,\frac{n(n+1)(2n+1)}{6}-\frac{n^2+n}{2}\right]\right]$$

$$\lim_{n\to\infty}\left[\frac{32}{n^2}\left[\frac{2n^2+3n+1}{3}-\frac{n^2+n}{2}\right]\right]$$

$$\lim_{n\to\infty}\left[\frac{64}{3}+\frac{32}{n}+\frac{32}{3n^2}-16-\frac{16}{n}\right]$$

$$=\frac{64}{3}+0+0-\frac{48}{3}-0$$

$$=\frac{16}{3}$$