- $\lim_{x \to 1} \left(-\frac{2x}{3} + \frac{1}{3} \right) = -\frac{1}{3}$

$|x-1| < \delta$, $\left| -\frac{2x}{3} + \frac{1}{3} + \frac{1}{3} \right| < \varepsilon$

$-\frac{2}{3}|x-1| < \varepsilon \iff \left| -\frac{2x}{3} + 2 \right| < \varepsilon$

$|x-1| < \frac{3\varepsilon}{2}$ $\quad * \delta = \frac{3\varepsilon}{2}$ $\quad \lim_{x \to 1} \left(-\frac{2x}{3} + \frac{1}{3} \right) = -\frac{1}{3}$

- $\lim_{x \to 0} x^2 = 0$ $\quad |x| < \delta$, $|x^2| < \varepsilon$

$\lim_{x \to 0} x^2 = 0$ ✓ $\quad * \underline{\underline{\delta = \sqrt{\varepsilon}}}$ $\quad |x| < \sqrt{\varepsilon}$

- $\lim_{x \to 2} x^2 = 4$ $\quad |x-2| < \delta$, $|x^2 - 4| < \varepsilon$

$|x-2| < \delta < 1$ $\quad x+2|x-2| < \varepsilon$

$|x-2| < \frac{\varepsilon}{x+2}$

$|x-2| < 1$

$-1 < x-2 < 1$

$1 < x < 3$

$\boxed{* \min \left\{ 1, \frac{\varepsilon}{5} \right. \quad x-2 < \frac{\varepsilon}{5}}$

$\forall$ $x \neq -2$

ε can't depend x

$3 < x+2 < 5 \implies \frac{1}{3} > \frac{1}{x+2} > \frac{1}{5} \iff \frac{\varepsilon}{3} > \frac{\varepsilon}{x+2} > \frac{\varepsilon}{5}$

$\delta = \frac{\varepsilon}{5}$ $\quad \lim_{x \to 2} x^2 = 4$

- $\lim_{x\to 5} \frac{x^2-25}{x-5} = \frac{0}{0}$ $\qquad \frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}}$

$x+5 = \underline{\underline{10}}$

$$\lim_{x\to 5} \frac{x^2-25}{x-5} = 10$$

- $|x-5| < \delta$

- $\left| \frac{\cancel{x-5}(x+5)}{\cancel{x-5}} - 10 \right| < \varepsilon \iff |x+5-10| < \varepsilon$

$|x-5| < \varepsilon$

$\underline{\underline{\delta = \varepsilon}}$

- $\lim_{x\to 3} \frac{1}{x} = \frac{1}{3}$ $\qquad |x-3| < \delta, \ \left|\frac{1}{x} - \frac{1}{3}\right| < \varepsilon$

$|x-3| < \varepsilon|3x| \iff \frac{|x-3|}{3|x|} \iff \frac{|3-x|}{|3x|} < \varepsilon$

$|x-3| < \delta < \frac{1}{2}$ $\qquad |x-3| < \frac{1}{2}$

$\frac{5}{2} < x < \frac{7}{2} \iff -\frac{1}{2} < x-3 < \frac{1}{2}$

$\frac{15}{2} < 3x < \frac{21}{2}$ $\qquad \min\left\{\frac{1}{2}, \frac{15}{2}\right.$ $\ * \underline{\underline{\delta = \frac{\varepsilon 15}{2}}}$

For any $\varepsilon > 0$ chosing $\delta = \frac{\varepsilon 15}{2}$ will ensure $\left|\frac{1}{x} - \frac{1}{3}\right| < \varepsilon$ when $0 < |x-3| < \delta$ proving $\lim_{x\to 3} \frac{1}{x} = \frac{1}{3}$ ✓