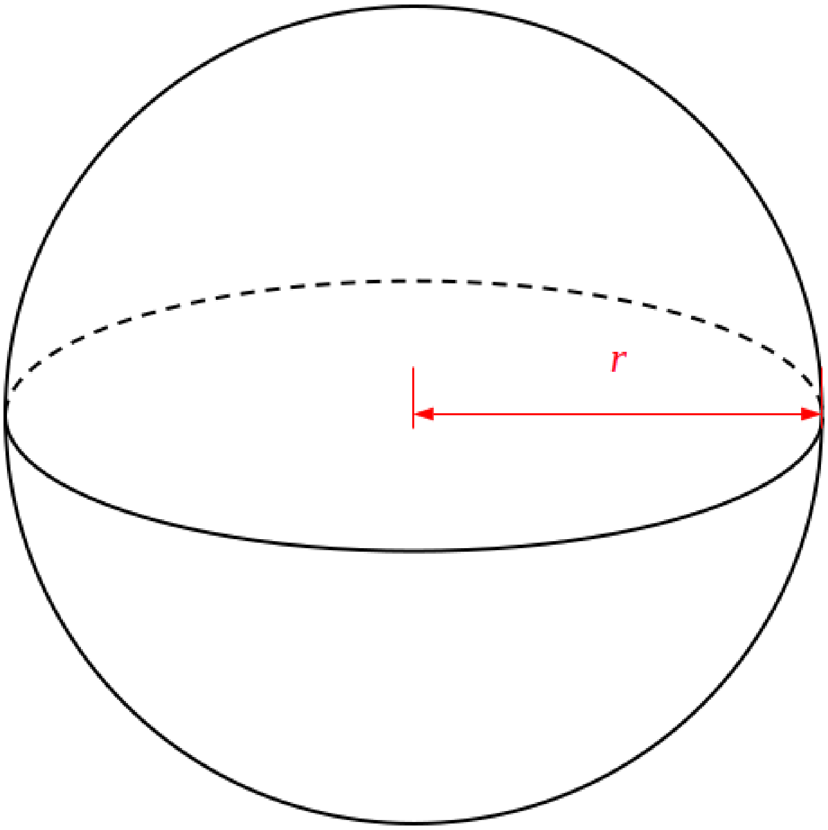
Mathematics 1110H (Section A) – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2024

Solution to Quiz #8
Inflation



Note that a sphere of radius r has volume $V = \dfrac{4\pi r^3}{3}$ and surface area $A = 4\pi r^2$.

1. A spherical balloon is being inflated at a constant rate of $36\pi\ cm^3/s$. How is the surface area of the balloon changing at the instant that the radius of the balloon is $3\ cm$? *[5]*

SOLUTION. We are told that $\left.\dfrac{dV}{dt}\right|_{r=3} = 36\pi$ and we wish to know $\left.\dfrac{dA}{dt}\right|_{r=3}$. The link between these is the fact that the formulas for both the volume and surface area of a sphere are given in terms of the radius r of the sphere.

In particular,

$$36\pi = \left.\frac{dV}{dt}\right|_{r=3} = \left[\frac{d}{dt}\left(\frac{4\pi r^3}{3}\right)\right]_{r=3} = \left[\left(\frac{d}{dr}\frac{4\pi r^3}{3}\right)\frac{dr}{dt}\right]_{r=3} = \left[\left(4\pi\frac{3r^2}{3}\right)\frac{dr}{dt}\right]_{r=3}$$
$$= \left[4\pi r^2\frac{dr}{dt}\right]_{r=3} = 4\pi 3^2\left.\frac{dr}{dt}\right|_{r=3} = 36\pi\left.\frac{dr}{dt}\right|_{r=3}$$

It follows that $\left.\frac{dr}{dt}\right|_{r=3} = \frac{36\pi}{36\pi} = 1$. This information, in turn, lets us compute $\left.\frac{dA}{dt}\right|_{r=3}$.

$$\begin{aligned}\left.\frac{dA}{dt}\right|_{r=3} &= \left[\frac{d}{dt}4\pi r^2\right]_{r=3} = \left[\left(\frac{d}{dr}4\pi r^2\right)\frac{dr}{dt}\right]_{r=3} = \left[8\pi r\frac{dr}{dt}\right]_{r=3} = 8\pi 3\left.\frac{dr}{dt}\right|_{r=3} \\ &= 24\pi \cdot 1 = 24\pi\end{aligned}$$

Thus the surface area of the balloon is increasing at a rate of $24\pi\ cm^2/s$ at the instant that the radius of the balloon is 3 cm. ■