

$$\bullet \lim_{x \rightarrow 1} \left(-\frac{2x}{3} + \frac{1}{3} \right) = -\frac{1}{3}$$

$$|x-1| < \delta, \quad \left| -\frac{2x}{3} + \frac{1}{3} + \frac{1}{3} \right| < \varepsilon$$

$$-\frac{2}{3}|x-1| < \varepsilon \Leftrightarrow \left| -\frac{2x}{3} + 2 \right| < \varepsilon$$

$$|x-1| < \frac{3\varepsilon}{2} \quad * \delta = \frac{3\varepsilon}{2} \quad \lim_{x \rightarrow 1} \left(-\frac{2x}{3} + \frac{1}{3} \right) = -\frac{1}{3}$$

$$\bullet \lim_{x \rightarrow 0} x^2 = 0 \quad |x| < \delta, \quad |x^2| < \varepsilon$$

$$\lim_{x \rightarrow 0} x^2 = 0 \quad * \underline{\underline{\delta = \sqrt{\varepsilon}}} \quad |x| < \sqrt{\varepsilon}$$

$$\bullet \lim_{x \rightarrow 2} x^2 = 4 \quad |x-2| < \delta, \quad |x^2-4| < \varepsilon$$

$$|x-2| < \delta < 1 \quad x+2| |x-2| < \varepsilon$$

$$|x-2| < 1$$

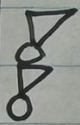
$$-1 < x-2 < 1$$

$$1 < x < 3$$

$$3 < x+2 < 5$$

$$* \min \left\{ 1, \frac{\varepsilon}{5} \right\}$$

$$x-2 < \frac{\varepsilon}{5}$$



$$x \neq -2$$

ε can't depend x

$$\Rightarrow \frac{1}{3} > \frac{1}{x+2} > \frac{1}{5} \Leftrightarrow \frac{\varepsilon}{3} > \frac{\varepsilon}{x+2} > \frac{\varepsilon}{5}$$

$$\delta = \frac{\varepsilon}{5}$$

$$\lim_{x \rightarrow 2} x^2 = 4$$

$$\bullet \lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \frac{0}{0} \quad \frac{(x-5)(x+5)}{x-5}$$

$$x+5 = \underline{\underline{10}}$$

$$\bullet |x-5| < \delta$$

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = 10$$

$$\bullet \left| \frac{\cancel{x-5}(x+5)}{\cancel{x-5}} - 10 \right| < \varepsilon \iff |x+5-10| < \varepsilon$$

$$|x-5| < \varepsilon$$

$$\underline{\underline{\delta = \varepsilon}}$$

$$\bullet \lim_{x \rightarrow 3} \frac{1}{x} = \frac{1}{3}$$

$$|x-3| < \delta, \quad \left| \frac{1}{x} - \frac{1}{3} \right| < \varepsilon$$

$$|x-3| < \varepsilon |3x| \iff \frac{|x-3|}{3|x|} < \varepsilon \iff \frac{|3-x|}{|3x|} < \varepsilon$$

$$|x-3| < \delta < \frac{1}{2} \quad |x-3| < \frac{1}{2}$$

$$\frac{5}{2} < x < \frac{7}{2} \iff -\frac{1}{2} < x-3 < \frac{1}{2}$$

$$\frac{15}{2} < 3x < \frac{21}{2}$$

$$\min \left\{ \frac{1}{2}, \frac{15}{2} \right\} * \delta = \underline{\underline{\frac{\varepsilon 15}{2}}}$$

For any $\varepsilon > 0$ choosing $\delta = \frac{\varepsilon 15}{2}$ will ensure $\left| \frac{1}{x} - \frac{1}{3} \right| < \varepsilon$ when $0 < |x-3| < \delta$ proving $\lim_{x \rightarrow 3} \frac{1}{x} = \frac{1}{3} \checkmark$