Mathematics 1110H (Section A) – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2024

Solution to Quiz #2
Sin$\varepsilon\delta$†

In doing this quiz, you may freely use the fact it true that for all real numbers t, $|\sin(t)| \leq |t|$.

1. Use the ε–δ definition of limits to verify that $\lim_{x\to 0} 2\sin(2x) = 0$. *[5]*

SOLUTION. To verify that $\lim_{x\to 0} 2\sin(2x) = 0$ using he ε–δ definition of limits, we need to check that for any $\varepsilon > 0$ we can find a $\delta > 0$ such that if $|x - 0| < \delta$, then $|2\sin(2x)| < \varepsilon$. As usual, we attempt to reverse-engineer the δ we need.

Suppose we are given an $\varepsilon > 0$; then

$$\begin{aligned} |2\sin(2x)| < \varepsilon &\iff 2\,|\sin(2x)| < \varepsilon \\ &\iff |\sin(2x)| < \frac{\varepsilon}{2}. \end{aligned}$$

At this point we xan exploit the fact that $|\sin(t)| \leq |t|$. for all t. Applying this fact with $t = 2x$ tells us that $|\sin(2x)| \leq |2x|$, so if we can get $|2x| < \frac{\varepsilon}{2}$, then we will have $|\sin(2x)| \leq |2x| < \frac{\varepsilon}{2}$. Note that

$$\begin{aligned} |2x| < \frac{\varepsilon}{2} &\iff 2|x| < \frac{\varepsilon}{2} \\ &\iff |x| < \frac{\varepsilon}{4} \\ &\iff |x - 0| < \frac{\varepsilon}{4}. \end{aligned}$$

Since each of the steps immediately above is reversible, if we let $\delta = \frac{\varepsilon}{4}$, it will follow that $|2x| < \frac{\varepsilon}{2}$, and since $|\sin(t)| \leq |t|$. for all t, it then follows that $|\sin(2x)| \leq |2x| < \frac{\varepsilon}{2}$. As the steps in our first calculation above are reversible, it then follows that $|2\sin(2x)| < \varepsilon$, as required. ∎

† With apologies to the creators of *Buckaroo Banzai*. "Remember: no matter where you go, there you are."