Assignment03

October 18, 2024

[1]:
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# B.Sc. Software Engineering (1st year)
# Submitted 10/18/2024
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

[3]:
```
clear_vars()
# 1.)
# -a.) Use SageMath to find a general solution to this differential equation.
x = var('x')
y = function('y')(x)
diff_eq = x * diff(y, x) == y
solution = desolve(diff_eq, y)
show(diff_eq)
show("1a->",solution)

# -b.) Use SageMath to find a solution to this differential equation satisfying␣
↪the initial condition y = 1 when x = 1.
solution = desolve(diff_eq,y,[1,1])
show("1b->", solution)

# -c.) Verify - by hand! - that the solution you obtained in part b satisfies␣
↪the given differential equation with the given initial conditions.
dy_dx = diff(solution, x)
left = x * dy_dx
right = solution
show("1c:")
show("dy/dx equal to -> ",dy_dx)
show("Equation becomes -> ", left==right)
show("Is given conditions satisfies the equation? -> ",bool(left == right))
show("Function bool(x * dy_dx == x) ensures the conditions satisfies the␣
↪equation")
```

$$x\frac{\partial}{\partial x}y\left(x\right) = y\left(x\right)$$

1a->Cx

1b->x

1c:

dy/dx equal to ->1

Equation becomes ->$x = x$

Is given conditions satisfies the equation? ->True

Function bool(x * dy_dx == x) ensures the conditions satisfies the equation

[5]:
```
clear_vars()
# 2.)
# -a.) Use SageMath to find a general solution to this differential equation.
x = var('x')
y = function('y')(x)
diff_eq = diff(y, x) == x*y
show(diff_eq)
show("2a=> ",desolve(diff_eq, y))

# -b.) Use SageMath to find a solution to this differential equation satisfying␣
↪the initial condition y = 1 when x = 0.
solution = desolve(diff_eq,y,[0,1])
show("2b=>", solution)

# -c.) Verify - by hand! - that the solution you obtained in part b satisfies␣
↪the given differential equation with the given initial conditions.
dy_dx = diff(solution, x)
left = dy_dx
right = x * solution
show("2c:")
show("dy/dx equal to -> ",dy_dx)
show("Equation becomes -> ", left == right)
show("Is given conditions satisfies the equation? -> ", bool(left == right))
show("Function bool(xe^([1/2]x^2) = xe^([1/2]x^2) ensures the conditions␣
↪satisfies the equation")
```

$$\frac{\partial}{\partial x} y\left(x\right) = x y\left(x\right)$$

2a=>$C e^{\left(\frac{1}{2} \, x^{2}\right)}$

2b=>$e^{\left(\frac{1}{2} \, x^{2}\right)}$

2c:

dy/dx equal to ->$x e^{\left(\frac{1}{2} \, x^{2}\right)}$

Equation becomes ->$x e^{\left(\frac{1}{2} \, x^{2}\right)} = x e^{\left(\frac{1}{2} \, x^{2}\right)}$

Is given conditions satisfies the equation? ->True

Function bool(xe^([1/2]x^2) = xe^([1/2]x^2) ensures the conditions satisfies the equation

[6]:
```
clear_vars()
# 3.)
# -a.) Use SageMath to find a general solution to this differential equation.
x = var('x')
y = function('y')(x)
diff_eq = diff(y, x) == (y^2)+1
show("Equation -> ",diff_eq)
show("3a=> ",desolve(diff_eq, y))

# -b.) Use SageMath to find a solution to this differential equation satisfying␣
 ↪the initial condition y = 0 when x = 0.
solution = desolve(diff_eq,y,[0,0])
show("3b=>", solution)

# -c.) Verify - by hand! - that the solution you obtained in part b satisfies␣
 ↪the given differential equation with the given initial conditions.
show("There fore at y=0 and x=0 must 0")
```

Equation -> $\frac{\partial}{\partial x}y\left(x\right) = y\left(x\right)^{2} + 1$

3a=> $\arctan\left(y\left(x\right)\right) = C + x$

3b=> $\arctan\left(y\left(x\right)\right) = x$

There fore at y=0 and x=0 must 0

[7]:
```
clear_vars()
# 4.)
# Explanation
x = var('x')
y = function('y')(x)
org_diff = diff(y, x)^2 + (x + y)*(diff(y, x)) + x*y == 0
diff_eq1 = (diff(y,x) + x)
diff_eq2 = (diff(y,x) + y)

show("Original differential equation was: (dy/dx)^2 + (x+y)(dy/dx)+xy = 0")
show(org_diff)

show("Given differential equation can be factorized as [(dy/dx)+x] * [(dy/
 ↪dx)+y]")
show(diff_eq1)
show(diff_eq2)
```

Original differential equation was: (dy/dx)^2 + (x+y)(dy/dx)+xy = 0

$x y\left(x\right) + \left(x + y\left(x\right)\right)\frac{\partial}{\partial x}y\left(x\right) + \frac{\partial}{\partial x}y\left(x\right)^{2} = 0$

Given differential equation can be factorized as [(dy/dx)+x] * [(dy/dx)+y]

$$x + \frac{\partial}{\partial x} y(x)$$

$$y(x) + \frac{\partial}{\partial x} y(x)$$

[8]:
```
clear_vars()
# 4.)
# Use SageMath to help find all the general solutions to the differential␣
↪equation
x = var('x')
y = function('y')(x)

diff_eq1 = (diff(y,x) + x)
diff_eq2 = (diff(y,x) + y)
result = ((diff_eq1*diff_eq2)==0)

show("4.) result=> ",desolve(result,y,contrib_ode=True))
```

4.) result=> $\left[y(x) = -\frac{1}{2} x^2 + C, y(x) = Ce^{(-x)}\right]$