

Find max-min of $g(x) = x^3 - 9x^2 + 23x - 15$
on $[0.5] - [5.5]$ or $[\frac{1}{2}, \frac{11}{2}]$

$$\left(\frac{1}{2}\right)^3 - 9\left(\frac{1}{2}\right)^2 + 23\left(\frac{1}{2}\right) - 15 = \frac{1}{8} - \frac{18}{8} + \frac{92}{8} - \frac{120}{8}$$

$$= * \cancel{g}(0.5) = -\frac{45}{8} \text{ or } -5.625 \rightarrow \text{Min}$$

$$\left(\frac{11}{2}\right)^3 - 9\left(\frac{11}{2}\right)^2 + 23\left(\frac{11}{2}\right) - 15 = \frac{1331}{8} - \frac{2178}{8} + \frac{1012}{8} - \frac{120}{8}$$

$$= * \cancel{g}(5.5) = +\frac{45}{8} \text{ or } 5.625 \rightarrow \text{Max}$$

$$g(x) = x^3 - 9x^2 + 23x - 15, \quad \frac{d}{dx} g(x) = 3x^2 - 18x + 23$$

$$* (x-3)^3 = x^3 - 9x^2 + 27x - 27$$

$$g(x) = (x-3)^3 - 4x + 12 = (x-3)^3 - 4(x-3)$$

$$= (x-3)((x-3)^2 - 4)$$

$$* (x-1)(x-3)(x-5) = x-3 \left(\underset{x}{x^2} - \underset{-5}{6x} + 5 \right)$$

$$* \underline{g(x) = (x-1)(x-3)(x-5)}, \quad \underline{\frac{d}{dx} g(x) = 3x^2 - 18x + 23}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{18 \pm \sqrt{48}}{6}$$

$$= \frac{9 \pm 2\sqrt{3}}{3}, \quad * \left(\underbrace{\frac{9+2\sqrt{3}}{3}}_x - 1 \right) = \frac{6+2\sqrt{3}}{3}$$

$$* \left(\underbrace{\frac{9+2\sqrt{3}}{3}}_x - 3 \right) = \frac{2\sqrt{3}}{3} \quad * \left(\frac{9+2\sqrt{3}}{3} - 5 \right) = \frac{-6+2\sqrt{3}}{3}$$

$$\rightarrow \left(\frac{6+2\sqrt{3}}{3} \right) \cdot \left(\frac{2\sqrt{3}}{3} \right) \cdot \left(\frac{-6+2\sqrt{3}}{3} \right) = \boxed{\frac{\pm 16\sqrt{3}}{9} = \pm 3.0792}$$