OnurOnel_Assignment04

November 9, 2024

[1]:
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# B.Sc. Software Engineering (1st year)
# Submitted 11/08/2024
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

[2]:
```
# 1a.)
k = var('k')
n = var('n')
show(sum(k,k,1,n))
```

$$\frac{1}{2}\,n^2+\frac{1}{2}\,n$$

[4]:
```
# 1b.)
k = var('k')
n = var('n')
show(sum(k^2,k,1,n))
```

$$\frac{1}{3}\,n^3+\frac{1}{2}\,n^2+\frac{1}{6}\,n$$

[5]:
```
# 1c.)
k = var('k')
n = var('n')
show(sum(k^3,k,1,n))
```

$$\frac{1}{4}\,n^4+\frac{1}{2}\,n^3+\frac{1}{4}\,n^2$$

[6]:
```
# 1d.)
k = var('k')
n = var('n')
show(sum(k^4,k,1,n))
```

$$\frac{1}{5}\,n^5+\frac{1}{2}\,n^4+\frac{1}{3}\,n^3-\frac{1}{30}\,n$$

[17]:
```
# 2.) Give an argument that verifies that the summation formula is true for all␣
  ↪n   1
k = var('k')
n = var('n')
assume(n >= 1)

show(sum(k, k, 1, n))
formula = (n * (n + 1)) / 2
show("Does the formula represent the series for n >= 1? ->", bool(sum(k, k, 1,␣
  ↪n) == formula))
show("Does the formula represent the series when n is negative? ->",␣
  ↪bool(sum(k, k, 1, -n) == formula))
show("Does the formula represent the series when n = 0? ->", bool(sum(k, k, 1,␣
  ↪0) == formula))
show("The boolean comparisons verifies that the summation formula is true for␣
  ↪all n   1")
```

$$\frac{1}{2}n^2 + \frac{1}{2}n$$

Does the formula represent the series for n >= 1? ->True

Does the formula represent the series when n is negative? ->False

Does the formula represent the series when n = 0? ->False

The boolean comparisons verifies that the summation formula is true for all n   1

[9]:
```
# 3a.) Explain why this infinite sum adds up to 2.
k = var('k')
sum_expression = sum(1/2^k, k, 0, oo)
first_term = 1/2^0

show("First term is ->",first_term)
show("Each iteration adds up half of the previous term")
show("Requires infinite iterations to reach exactly 2")
show("However, with each term added, the sum gets closer and closer to 2")
show("This series approaches but never exceeds twice the amount of the first␣
  ↪term")
```

First term is ->1

Each iteration adds up half of the previous term

Requires infinite iterations to reach exactly 2

However, with each term added, the sum gets closer and closer to 2

This series approaches but never exceeds twice the amount of the first term

[10]:
```
# 3b.) Use your algebra or SageMath skills to find a summation formula in terms␣
  ↪of n for the finite sum
```

```
k = var('k')
n = var('n')
show(sum(1/2^k, k, 0, n))
```

$$\frac{2^{n+1} - 1}{2^n}$$

[5]:
```
# 3c.) Whether by hand or SageMath, compute the infinite sum by taking the
↪limit as n → ∞ of the formula you obtained in part b
n = var('n')
formula = (2^(n + 1) - 1) / 2^n
limit = lim(formula, n = oo)
show("Limit of formula is -> ", limit)
```

Limit of formula is ->2

[12]:
```
# 3d.) Use SageMath to compute the infinite sum directly.
k = var('k')
sum_expression = sum(1/2^k, k, 0, oo)
show(sum_expression)
```

2

[14]:
```
# 4a.) Explain why this infinite sum adds up to 1
k = var('k')
formula = 1 / (k^2 + k)
newFormula = 1/k - 1/(k + 1)
show("Formula of the series ->", formula)
show("Using partial terms, the formula becomes ->", newFormula)
show("When the series is expanded, each term cancels with the following term,
↪leaving only 1 from the first term.")
show("This domino effect requires infinitely many terms to fully cancel out
↪each part, resulting in exactly 1.")
```

Formula of the series ->$\frac{1}{k^2 + k}$

Using partial terms, the formula becomes ->$-\frac{1}{k+1} + \frac{1}{k}$

When the series is expanded, each term cancels with the following term, leaving only 1 from the

This domino effect requires infinitely many terms to fully cancel out each part, resulting in e

[17]:
```
# 4b.) Use your algebra or SageMath skills to find a summation formula in terms
↪of n for the finite sum
k = var('k')
n = var('n')
show("In Assignment #4, Question 4.b), the series is defined with k=0. However,
↪this results in an undefined value at the first term due to division by zero.
↪")
```

```
show("Summation formula for k=1 -> ",sum(1 / (k^2 + k), k, 1, n))
```

In Assignment #4, Question 4.b), the series is defined with k=0. However, this results in an ur

Summation formula for k=1 -> $\frac{n}{n+1}$

[16]:
```
# 4c.) Whether by hand or SageMath, compute the infinite sum by taking the␣
↪limit as n → ∞ of the formula you obtained in part b
n = var('n')
formula = n / (n + 1)
limit = lim(formula, n = oo)
show("Limit of formula is -> ",limit)
```

Limit of formula is -> 1

[17]:
```
# 4d.) Use SageMath to compute the infinite sum directly
k = var('k')
show(sum(1 / (k^2 + k), k, 1, oo))
```

1

[18]:
```
# 5.) What does the infinite sum add up to?
```

[20]:
```
k = var('k')
serie = sum(1/k, k, 1, n)
show(serie)
```

H_n

[21]:
```
show("This series diverges, meaning that it does not add up to a finite value.")
```

This series diverges, meaning that it does not add up to a finite value.

[ ]:
```

```