

$$\lim_{x \rightarrow 0} 2 \sin(2x) = 0 \quad |x| < \delta, \quad \begin{aligned} |2 \sin(2x)| &< \varepsilon \\ |\sin(2x)| &< \frac{\varepsilon}{2} \end{aligned}$$

* for any $\varepsilon > 0$, there are $\delta > 0$ such that $|x| < \delta$

$$|\sin(2x)| < 2|x| < \frac{\varepsilon}{2}$$

$$|\sin(2x)| < |x| < \frac{\varepsilon}{4}$$

* For every $\varepsilon > 0$, $\delta = \frac{\varepsilon}{4}$ then $|x| < \delta$ proves

$$|2 \sin(2x)| < \varepsilon$$