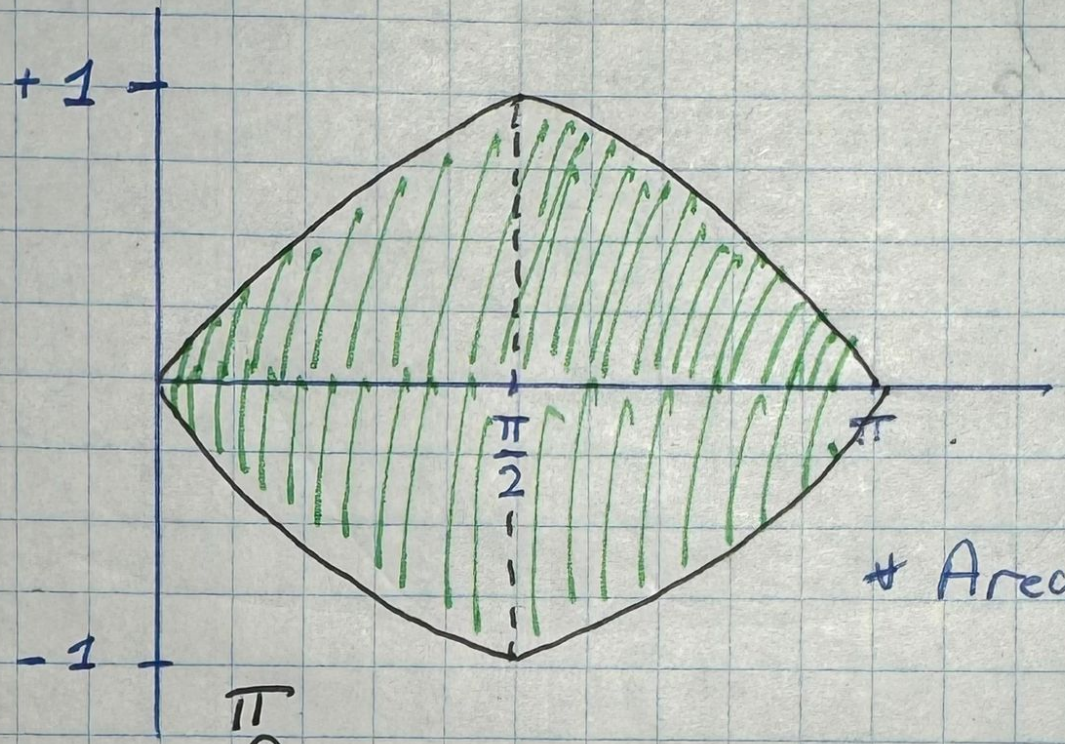
Region between $y = \sin(x)$, $y = -\sin(x)$ $[0, \pi]$



$$* \text{Area} = \int_a^b \underbrace{F(x)}_{\text{top}} - \underbrace{S(x)}_{\text{bottom}}$$

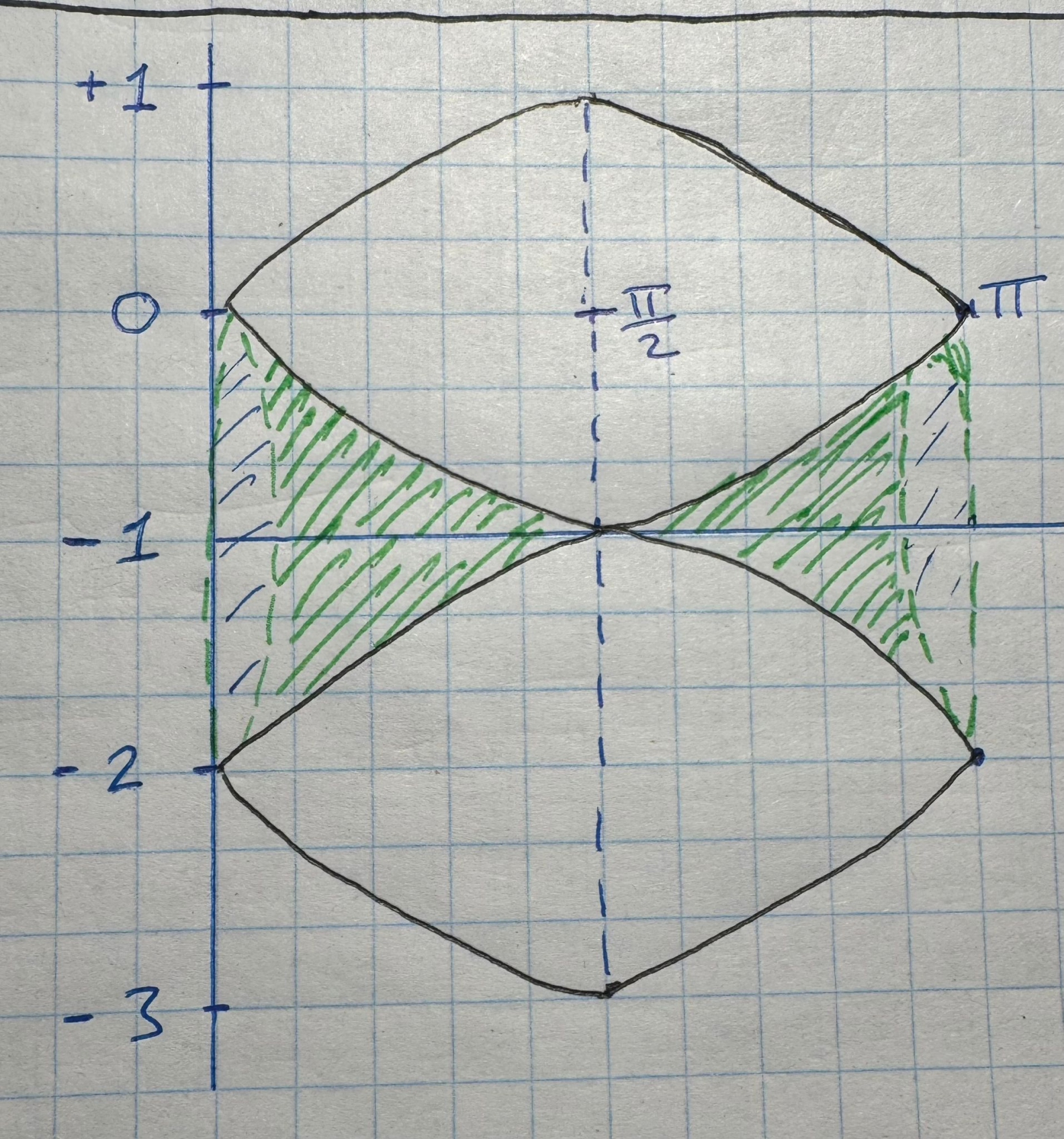
$$\text{Area} = \int_0^{\pi} \sin x - (-\sin x)\, dx = \int_0^{\pi} 2\sin(x)\, dx$$

$$= -2\cos(x)\Big|_0^{\pi}$$

$$-2(\underbrace{\cos\pi}_{-1}) - -2\underbrace{\cos(0)}_{1}$$

$$+2 + 2 = 4$$

$$\text{Area} = 4$$


+1
0
-1
-2
-3
π/2
π

Disk Method:

$$V = \pi \int_a^b (f(x))^2 dx$$

$$V = \pi \int_0^{\pi} (\sin x + 1)^2 dx$$

* $f(x)$ is radius of disc
* dx height of disc

$$(\sin x + 1)^2 = \sin^2(x) + 2\sin(x) + 1$$

$$V = \pi \int_0^{\pi} \sin^2 x + 2\sin x + 1$$

$$\int \sin^2(x) = \frac{x}{2} - \frac{\sin 2x}{4}$$

$$\Longrightarrow \quad \pi \left[\frac{x}{2} - \frac{\sin 2x}{4} + -2\cos x + x \right]_0^{\pi}$$

$x = \pi \Rightarrow \frac{\pi}{2} - 0 - (-2) + \pi = \frac{\pi}{2} + 2 + \pi$

$x = 0 \Rightarrow 0 - 0 - 2 + 0 = -2$

$$\pi \left[\left(\frac{\pi}{2} + 2 + \pi \right) - (-2) \right]$$

$$\Longrightarrow \quad V = \pi \left(\frac{3\pi}{2} + 4 \right) \qquad V = \frac{3\pi^2}{2} + 4\pi$$