

AB distance  $d$  and rate  $r_1, r_2$

$$6 \text{ hour} = t_1 + t_2$$

$$\text{total distance} = 2d$$

Since  $d = r_1 t_1$ ,  $d = r_2 t_2$  therefore;

$$t_1 = \frac{d}{r_1}, \quad t_2 = \frac{d}{r_2}$$

1.)

$$2d = r_1 (t_1 + t_2) \Leftrightarrow 2d = r_1 \left( \frac{d}{r_1} + \frac{d}{r_2} \right)$$

$$2d \cdot \frac{1}{d} = r_1 \left( \frac{d}{r_1} + \frac{d}{r_2} \right) \cdot \frac{1}{d}$$

$$2 = r_1 \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \cdot \frac{1}{2} \Rightarrow 1 = \frac{r_1}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

$$1 = \frac{r_1}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \cdot \frac{1}{r_1} = \frac{1}{r_1} = \frac{1}{2} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) = \left( \frac{1}{r_1} = \frac{r_1 + r_2}{2r_1 r_2} \right)^{-1}$$

$$\Rightarrow r = \frac{2r_1 r_2}{r_1 + r_2}$$

North / South 4 km/h  
East 3 km/h  
West 6 km/h

$$r_1 = \frac{2 \cdot 4 \cdot 4}{8} = \underline{4}$$

$$r_2 = \frac{2 \cdot 6 \cdot 3}{9} = \underline{4}$$

$$2d = 4 (\underbrace{t_1 + t_2}_{6 \text{ hour}})$$

$$2d = 24 \quad \boxed{d = 12}$$

2.) As seen from the figure A to B distance must involve N/S direction

Fastest possible path:

$$r = \frac{2r_1 r_2}{r_1 + r_2}$$

$$\frac{12 \text{ km}}{4.8} = \underline{2.5 \text{ hours}}$$

$$\frac{2 \cdot 4 \cdot 6}{4 + 6} = \frac{48}{10} \Leftrightarrow 4.8 \text{ km/h}$$

$$\boxed{2 \text{ hour} - 2.5 \text{ hour}}$$

Slowest possible path:

$$\frac{2 \cdot 4 \cdot 3}{4 + 3} = \frac{24}{7}$$

$$= \frac{24}{7}$$

$$4 \text{ hour}$$

$$= 3.4 \text{ km/h}$$

$$\frac{12}{3.4} =$$

$$\boxed{3.5 \text{ hour}}$$