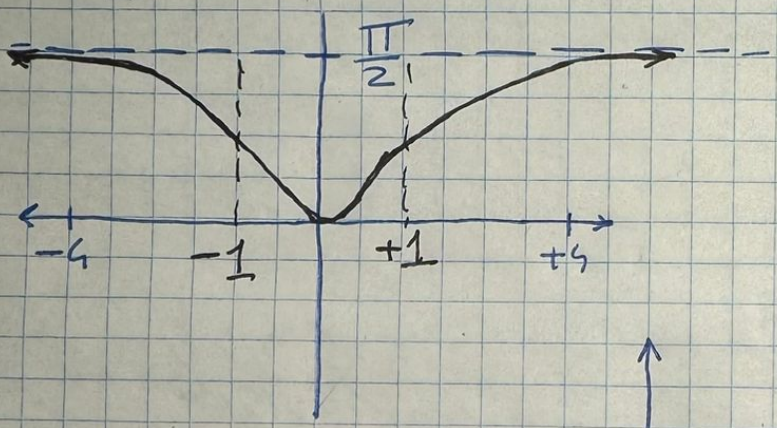
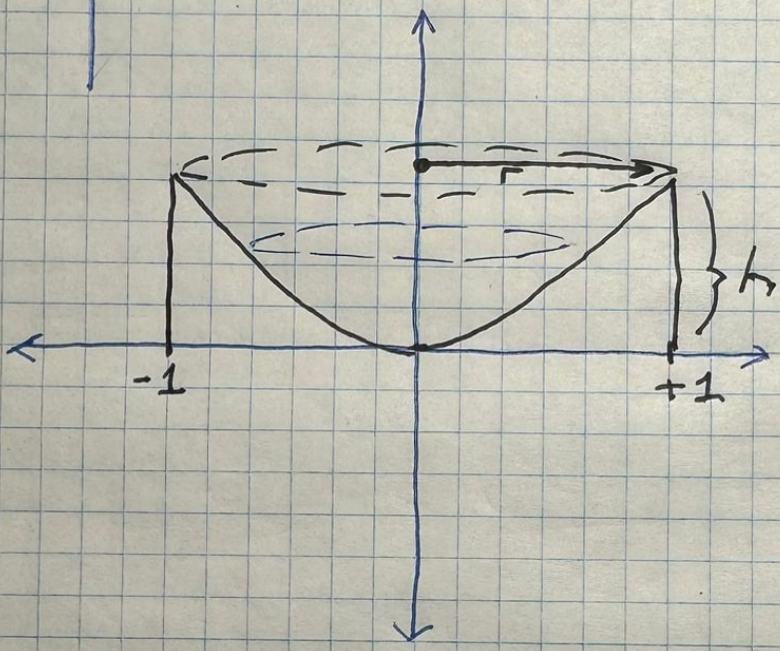
$y = \arctan(x^2)$, $0 \le x \le 1$

revolve about y-axis



Horizontal Asymptote at $y = \frac{\pi}{2}$ as $x \to \pm\infty$



$r = 1$

$h = \frac{\pi}{4}$

$\tan(\arctan(x^2)) = \tan(y)$

$x^2 = \tan(y)$

$x = \sqrt{\tan(y)}$

$0 \le x \le 1$

$x = 0 \quad y = 0$

$x = 1 \quad y = \frac{\pi}{4}$

$$\pi \int_0^{\frac{\pi}{4}} \sqrt{\tan}^{x} \, dy$$

$$\Rightarrow \pi \left(-\ln(\cos y) \Big|_0^{\frac{\pi}{4}} \right)$$

$$\Rightarrow \pi \left(-\ln\cos\tfrac{\pi}{4} - (-\ln\cos 0) \right)$$

$$\pi \left(-\ln\cos\tfrac{\pi}{4} + \overbrace{\ln(1)}^{0} \right)$$

$$\pi \cdot -\ln\cos\left(\tfrac{\pi}{4}\right) = \pi(\ln\sqrt{2})$$

$\ln\sqrt{2}$

* $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

* $-\ln\frac{1}{\sqrt{2}} = \ln\left(\frac{1}{\sqrt{2}}\right)^{-1}$