Mathematics 1110H (Section A) – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2024

Solutions to Quiz #4
Derivatives

Please do these problems by hand and show all your work.

1. Find the slope of the tangent line to $y = \tan(x)$ at $x = \frac{\pi}{4}$. *[1.5]*

SOLUTION. The slope of the tangent line is given by the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{4}$. We know from class and textbook that $\frac{dy}{dx} = \frac{d}{dx}\tan(x) = \sec^2(x)$. It follows that

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \sec^2\left(\frac{\pi}{4}\right) = \left(\frac{1}{\cos(\pi/4)}\right)^2 = \left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2 = \left(\sqrt{2}\right)^2 = 2,$$

so the slope of the tangent line to $y = \tan(x)$ at $x = \frac{\pi}{4}$ is 2. ■

2. Compute $f'(x)$ if $f(x) = \ln(\tan(x) + \sec(x))$. Simplify your answer as much as you reasonably can. *[1.5]*

SOLUTION. Here we go, using the Chain Rule and the facts that $\frac{d}{dx}\tan(x) = \sec^2(x)$ and $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$, plus a bit of algebra when simplifying.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\ln(\tan(x) + \sec(x)) = \frac{1}{\tan(x)+\sec(x)} \cdot \frac{d}{dx}(\tan(x)+\sec(x)) \\ &= \frac{1}{\tan(x)+\sec(x)} \cdot \left(\sec^2(x) + \sec(x)\tan(x)\right) \\ &= \frac{\sec(x)(\sec(x)+\tan(x))}{\tan(x)+\sec(x)} = \frac{\sec(x)(\tan(x)+\sec(x))}{\tan(x)+\sec(x)} \\ &= \sec(x) \quad \blacksquare \end{aligned}$$

3. Find the slope of the tangent line to $y = \frac{x^2-5x+4}{x^2-1}$ at $x = 2$. *[2]*

SOLUTION. We will try to simplify the function first.

$$y = \frac{x^2-5x+4}{x^2-1} = \frac{(x-1)(x-4)}{(x-1)(x+1)} = \frac{x-4}{x+1}$$

The slope of the tangent line is given by the value of $\frac{dy}{dx}$ when $x = 2$. We will use the Quotient Rule to compute $\frac{dy}{dx}$.

$$\begin{aligned}\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x-4}{x+1}\right) &= \frac{\left[\frac{d}{dx}(x-4)\right](x+1) - (x-4)\left[\frac{d}{dx}(x+1)\right]}{(x+1)^2} \\ &= \frac{1\cdot(x+1) - (x-4)\cdot 1}{(x+1)^2} = \frac{x+1-x+4}{(x+1)^2} = \frac{5}{(x+1)^2}\end{aligned}$$

It follows that the slope of the tangent line to $y = \frac{x^2-5x+4}{x^2-1}$ at $x = 2$ is

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{5}{(2+1)^2} = \frac{5}{3^2} = \frac{5}{9}. \quad \blacksquare$$