MATH 1350,
Exercise Set 4

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1. Sketch the following vectors on a set of axes with the initial points located at the origin (i.e. in standard position).

$$\mathbf{v}_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -4 \\ -3 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 5 \\ -4 \end{pmatrix},$$

$$\mathbf{v}_5 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \quad \mathbf{v}_6 = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}, \quad \mathbf{v}_7 = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix}, \quad \mathbf{v}_8 = \begin{pmatrix} 0 \\ 0 \\ -5 \end{pmatrix}.$$

2. Write the vector $\overrightarrow{PQ}$, in column matrix form, for the following points P and Q.
   (a) $P(4, 8)$, $Q(3, 7)$
   (b) $P(-5, 0)$, $Q(-3, 1)$
   (c) $P(3, -7, 2)$, $Q(-2, 5, -4)$
   (d) $P(a, b, c)$, $Q(0, 0, 0)$
   (e) $P(0, 0, 0)$, $Q(a, b, c)$

3. Find a point Q that creates a nonzero vector $\overrightarrow{PQ}$, with initial point $P(-1, 3, -5)$, which points in the same direction as the vector $\begin{pmatrix} 6 \\ 7 \\ -3 \end{pmatrix}$.

4. Let

$$\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 4 \\ 0 \\ -8 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 6 \\ -1 \\ -4 \end{pmatrix}.$$

   Write the vector (i.e. the column matrix) representing the following:
   (a) $\mathbf{u} + \mathbf{v}$
   (b) $\mathbf{v} - \mathbf{w}$
   (c) $3\mathbf{u} + 4\mathbf{w}$
   (d) $5(\mathbf{v} - 4\mathbf{u})$
   (e) $(2\mathbf{u} - 7\mathbf{w}) - (8\mathbf{v} + \mathbf{u})$
   (f) The vector $\mathbf{x}$ such that $2\mathbf{u} - \mathbf{v} + \mathbf{x} = 7\mathbf{x} + \mathbf{w}$.

5. Draw a picture that shows four nonzero vectors whose sum is the vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

6. Find the lengths of the following vectors

$$\mathbf{v}_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -4 \\ -3 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 5 \\ -4 \end{pmatrix},$$

$$\mathbf{v}_5 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \quad \mathbf{v}_6 = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}, \quad \mathbf{v}_7 = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix}, \quad \mathbf{v}_8 = \begin{pmatrix} 0 \\ 0 \\ -5 \end{pmatrix}.$$

7. Show that if $\mathbf{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is a nonzero vector, then $\frac{1}{||\mathbf{v}||}\mathbf{v}$ is a unit vector.

8. Show that the vectors
$$\mathbf{u} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 5 \\ -2 \end{pmatrix},$$
satisfy the Cauchy-Schwarz inequality, $|\mathbf{u} \cdot \mathbf{v}| \leq ||\mathbf{u}||\ ||\mathbf{v}||$.

9. Show that the vectors
$$\mathbf{u} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix},$$
satisfy the triangle inequality, $||\mathbf{u} + \mathbf{v}|| \leq ||\mathbf{u}|| + ||\mathbf{v}||$.

10. Find the angle between vectors $\mathbf{u}$ and $\mathbf{v}$ for the following.
    (a) $\mathbf{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 5 \\ -7 \end{pmatrix}$
    (b) $\mathbf{u} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$
    (c) $\mathbf{u} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$