MATH 1350,
Exercise Set 8 - Solutions

---

1. (a) How are linear transformations and matrices related?
   (b) How can we associate a matrix to a linear map $T : \mathbb{R}^n \to \mathbb{R}^m$?
   (c) How can computations with T on vectors in $\mathbb{R}^n$ be accomplished with the matrix representation of T?

solution:

(a) Every linear transformation between finite dimensional vector spaces can be represented with a matrix. Conversely, every matrix defines a linear transformation between finite dimensional vector spaces.

(b) To construct a matrix representation of a linear map $T : \mathbb{R}^n \to \mathbb{R}^m$ we begin by choosing a basis $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ for the domain (here $\mathbb{R}^n$) and a basis $\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m\}$ for the codomain (here $\mathbb{R}^m$). We apply T to each basis $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ and write the result as a linear combination of $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m$: e.g.

$$\begin{aligned}
T(\mathbf{v}_1) &= C_{1,1}\mathbf{w}_1 + C_{2,1}\mathbf{w}_2 + \cdots + C_{m,1}\mathbf{w}_m \\
T(\mathbf{v}_2) &= C_{1,2}\mathbf{w}_1 + C_{2,2}\mathbf{w}_2 + \cdots + C_{m,2}\mathbf{w}_m \\
&\vdots \\
T(\mathbf{v}_n) &= C_{1,n}\mathbf{w}_1 + C_{2,n}\mathbf{w}_2 + \cdots + C_{m,n}\mathbf{w}_m
\end{aligned}$$

The matrix representation for T with respect to these two bases is matrix whose entries in column i are the coefficients used in the linear combination of $T(\mathbf{v}_i)$ in terms of $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m$: e.g. the matrix in this case is

$$\begin{pmatrix} C_{1,1} & C_{1,2} & \ldots & C_{1,n} \\ C_{2,1} & C_{2,2} & \ldots & C_{2,n} \\ \vdots & \vdots & & \vdots \\ C_{m,1} & C_{m,2} & \ldots & C_{m,n} \end{pmatrix}$$

The *standard matrix* for T is when we choose $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ and $\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m\}$ to be the standard bases for $\mathbb{R}^n$ and $\mathbb{R}^n$ respectively.

(c) If A is the standard matrix for the linear map $T : \mathbb{R}^n \to \mathbb{R}^m$ then $T(\mathbf{v}) = A\mathbf{v}$ for any $\mathbf{v} \in \mathbb{R}^n$, where $A\mathbf{v}$ is matrix multiplication. *(Note that A is an $m \times n$ matrix in this case, and $\mathbb{R}^n$ is the set of $n \times 1$ matrices.)*

$\square$

2. Find the standard matrix for each of the following linear transformations.
   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a + 3b \\ a - b \end{pmatrix}$.
   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix}$.

(c) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$.

(d) $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2a+3b \\ 4b+5c \end{pmatrix}$.

(e) $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = a+b+c$.

(f) $T : \mathbb{R} \to \mathbb{R}^2$ given by $T\left(a\right) = \begin{pmatrix} a \\ 3a \end{pmatrix}$.

(g) $T : \mathbb{R}^4 \to \mathbb{R}^2$ given by $T\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a+b \\ c+d \end{pmatrix}$.

solution:

(a) Apply T to the standard basis vectors for $\mathbb{R}^2$ (domain) and express these in terms of the standard basis for $\mathbb{R}^2$ (codomain):

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} = 3 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (-1) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The coefficients of each linear combination form the columns of the standard matrix:

$$\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}.$$

Note: This process may seem a little overly complicated when finding the standard matrix; with practice you can often "see" what the standard matrix is without going through the motions of applying T to the standard basis vectors, then expressing the result as a linear combination. However, for non-standard matrix representations (something you will see in a later linear algebra course) it will not be easy to see by inspection. So it will be important to know precisely how a matrix representation is defined.

For some of the questions below we will simply give the standard matrix, without going through the steps.

(b)

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(c) Apply T to the standard basis vectors for $\mathbb{R}^3$ (domain) and express these in terms of the standard basis for $\mathbb{R}^3$ (codomain):

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} = 1\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}$$
$$T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} = 0\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}$$
$$T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} = 0\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}$$

The standard matrix for T is,

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}.$$

(d) Apply T to the standard basis vectors for $\mathbb{R}^3$ (domain) and express these in terms of the standard basis for $\mathbb{R}^2$ (codomain):

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix} = 2\cdot\begin{pmatrix}1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\1\end{pmatrix}$$
$$T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} = 3\cdot\begin{pmatrix}1\\0\end{pmatrix} + 4\cdot\begin{pmatrix}0\\1\end{pmatrix}$$
$$T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\5\end{pmatrix} = 0\cdot\begin{pmatrix}1\\0\end{pmatrix} + 5\cdot\begin{pmatrix}0\\1\end{pmatrix}$$

The standard matrix for T is,

$$\begin{pmatrix}2&3&0\\0&4&5\end{pmatrix}.$$

(e)

$$\begin{pmatrix}1&1&1\end{pmatrix}$$

(f)

$$\begin{pmatrix}1\\3\end{pmatrix}$$

(g)

$$\begin{pmatrix}1&1&0&0\\0&0&1&1\end{pmatrix}$$

$\square$

3. A linear map T has standard matrix
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 1 \end{pmatrix}.$$

   (a) What are the domain and codomain of T (assuming these are vectors spaces of type $\mathbb{R}^n$)?

   (b) Find a rule for T which describes its output (or image) vector in terms of the entries of an arbitrary input vector from its domain.

solution:

(a) The domain is $\mathbb{R}^3$ and the codomain is $\mathbb{R}^2$.

(b) By the definition of the standard matrix, from the columns we see that
$$T\begin{pmatrix}1\\0\\0\end{pmatrix} = 1\cdot\begin{pmatrix}1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$
$$T\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\cdot\begin{pmatrix}1\\0\end{pmatrix} + 3\cdot\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$
$$T\begin{pmatrix}0\\0\\1\end{pmatrix} = 0\cdot\begin{pmatrix}1\\0\end{pmatrix} + 1\cdot\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

Thus
$$T\begin{pmatrix}a\\b\\c\end{pmatrix} = a\cdot T\begin{pmatrix}1\\0\\0\end{pmatrix} + b\cdot T\begin{pmatrix}0\\1\\0\end{pmatrix} + c\cdot T\begin{pmatrix}0\\0\\1\end{pmatrix} = a\cdot\begin{pmatrix}1\\0\end{pmatrix} + b\cdot\begin{pmatrix}2\\3\end{pmatrix} + c\cdot\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}a+2b\\3b+c\end{pmatrix}.$$

□

4. A linear map T has standard matrix
$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

   (a) What are the domain and codomain of T (assuming these are vectors spaces of type $\mathbb{R}^n$)?

   (b) Find the image under T of each of the standard basis vectors (i.e. find T applied to each of the standard basis vectors).

   (c) Find $T\begin{pmatrix}5\\-3\\4\\2\end{pmatrix}$.

   (d) Find a rule for T which describes its output (or image) vector in terms of the entries of an arbitrary input vector from its domain.

solution:

(a) The domain is $\mathbb{R}^4$ and the codomain is $\mathbb{R}^3$.

(b) By the definition of the standard matrix, from the columns we see that

$$T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = 1\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$T\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = 1\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$T\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = 0\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} + 1\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$T\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = 0\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} + 1\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

(c)

$$\begin{aligned} T\begin{pmatrix}5\\-3\\4\\2\end{pmatrix} &= 5\cdot T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + (-3)\cdot T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + 4\cdot T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + 2\cdot T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \\ &= 5\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + (-3)\cdot\begin{pmatrix}1\\1\\0\end{pmatrix} + 4\cdot\begin{pmatrix}0\\1\\1\end{pmatrix} + 2\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}2\\1\\6\end{pmatrix}. \end{aligned}$$

(d)

$$\begin{aligned} T\begin{pmatrix}a\\b\\c\\d\end{pmatrix} &= a\cdot T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + b\cdot T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + c\cdot T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + d\cdot T\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \\ &= a\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + b\cdot\begin{pmatrix}1\\1\\0\end{pmatrix} + c\cdot\begin{pmatrix}0\\1\\1\end{pmatrix} + d\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}a+b\\b+c\\c+d\end{pmatrix}. \end{aligned}$$

$\square$

5. Find the standard matrix for each linear map $T : \mathbb{R}^n \to \mathbb{R}^m$ and use it to compute $T(\mathbf{v})$.

   (a) Let $T\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}a-b\\c-2a\end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix}2\\3\\4\end{pmatrix}$.

(b) Let $T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ b+c \\ a-2c \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

(c) Let $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3b \\ 2a-5b \\ a \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$.

solution:

(a) Standard matrix: $\begin{pmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$.

$$T(\mathbf{v}) = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

(b) Standard matrix: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -2 \end{pmatrix}$.

$$T(\mathbf{v}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

(c) Standard matrix: $\begin{pmatrix} 0 & 3 \\ 2 & -5 \\ 1 & 0 \end{pmatrix}$.

$$T(\mathbf{v}) = \begin{pmatrix} 0 & 3 \\ 2 & -5 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 21 \\ -31 \\ 2 \end{pmatrix}$$

$\square$

6. For each pair of linear maps, S and T, find the standard matrices for S, T, and where possible, $S \circ T$ and $T \circ S$.

(a) $S : \mathbb{R}^2 \to \mathbb{R}^3$ with $S\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \\ a+3b \end{pmatrix}$ and $T : \mathbb{R}^3 \to \mathbb{R}^2$ with $T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a+b \\ a-c \end{pmatrix}$.

(b) $S : \mathbb{R}^4 \to \mathbb{R}^2$ with $S\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a+3b+2c \\ c-2d \end{pmatrix}$ and $T : \mathbb{R}^2 \to \mathbb{R}^3$ with $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ -a \\ 2b \end{pmatrix}$.

(c) $S : \mathbb{R}^2 \to \mathbb{R}^2$ with $S\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a+b \\ a-3b \end{pmatrix}$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$ with $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a-b \\ a+b \end{pmatrix}$.

solution:

(a) Standard matrix for S: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 3 \end{pmatrix}$.

Standard matrix for T: $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$.

Standard matrix for $S \circ T$: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 4 & 1 & -3 \end{pmatrix}$.

Standard matrix for $T \circ S$: $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix}$.

(b) Standard matrix for S: $\begin{pmatrix} 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & -2 \end{pmatrix}$.

Standard matrix for T: $\begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 2 \end{pmatrix}$.

$S \circ T$ is undefined.

Standard matrix for $T \circ S$: $\begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 & 0 \\ -1 & -3 & -2 & 0 \\ 0 & 0 & 2 & -4 \end{pmatrix}$.

(c) Standard matrix for S: $\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$.

Standard matrix for T: $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

Standard matrix for $S \circ T$: $\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -2 & -4 \end{pmatrix}$.

Standard matrix for $T \circ S$: $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 3 & -2 \end{pmatrix}$.

$\square$

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with

$$T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}.$$

Find the standard matrix for T.

solution:

To find the standard matrix for T we need to know how T maps the standard basis vectors. To find this we first express the standard basis vectors in terms of the vectors $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

where the action of T is known. Thus we solve the systems

$$x_1\begin{pmatrix}2\\3\end{pmatrix}+y_1\begin{pmatrix}3\\-1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix},$$

and

$$x_2\begin{pmatrix}2\\3\end{pmatrix}+y_2\begin{pmatrix}3\\-1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix},$$

for x_1, y_1, x_2 and y_2. We can solve these simultaneously with the following augmented matrix.

$$\left(\begin{array}{cc|c|c}2&3&1&0\\3&-1&0&1\end{array}\right)\xrightarrow{\frac{1}{2}R_1}\left(\begin{array}{cc|c|c}1&3/2&1/2&0\\3&-1&0&1\end{array}\right)\xrightarrow{R_2-3R_1}\left(\begin{array}{cc|c|c}1&3/2&1/2&0\\0&-11/2&-3/2&1\end{array}\right)$$

$$\xrightarrow{-\frac{2}{11}R_2}\left(\begin{array}{cc|c|c}1&3/2&1/2&0\\0&1&3/11&-2/11\end{array}\right)\xrightarrow{R_2-\frac{3}{2}R_2}\left(\begin{array}{cc|c|c}1&0&1/11&3/11\\0&1&3/11&-2/11\end{array}\right)$$

This yields

$$\frac{1}{11}\cdot\begin{pmatrix}2\\3\end{pmatrix}+\frac{3}{11}\cdot\begin{pmatrix}3\\-1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix},$$

and

$$\frac{3}{11}\cdot\begin{pmatrix}2\\3\end{pmatrix}+\left(-\frac{2}{11}\right)\cdot\begin{pmatrix}3\\-1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}.$$

Using this we have

$$T\begin{pmatrix}1\\0\end{pmatrix}=\frac{1}{11}\cdot T\begin{pmatrix}2\\3\end{pmatrix}+\frac{3}{11}\cdot T\begin{pmatrix}3\\-1\end{pmatrix}=\frac{1}{11}\cdot\begin{pmatrix}1\\-1\end{pmatrix}+\frac{3}{11}\cdot\begin{pmatrix}8\\8\end{pmatrix}=\begin{pmatrix}25/11\\7/11\end{pmatrix}$$
$$T\begin{pmatrix}0\\1\end{pmatrix}=\frac{3}{11}\cdot T\begin{pmatrix}2\\3\end{pmatrix}+\frac{-2}{11}\cdot T\begin{pmatrix}3\\-1\end{pmatrix}=\frac{3}{11}\cdot\begin{pmatrix}1\\-1\end{pmatrix}-\frac{2}{11}\cdot\begin{pmatrix}8\\8\end{pmatrix}=\begin{pmatrix}-13/11\\-19/11\end{pmatrix}$$

It follows that the standard matrix for T is

$$\begin{pmatrix}25/11&-13/11\\7/11&-19/11\end{pmatrix}.$$

□

8. Consider the maps,

$$T_1\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\-y\end{pmatrix}\quad\text{and}\quad T_2\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}y\\x\end{pmatrix}.$$

(a) Find the standard matrix for the map (T_1+T_2).

(b) Find the standard matrix for the map $(5T_1)$.

(c) Find the standard matrices for the maps $(T_2\circ T_1)$ and $(T_1\circ T_2)$.

solution:

(a) Standard matrix for T_1: $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

Standard matrix for T_2: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Standard matrix for $T_1 + T_2$: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

(b) Standard matrix for $5T_1$: $5 \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & -5 \end{pmatrix}$

(c) Standard matrix for $T_1 \circ T_2$: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Standard matrix for $T_2 \circ T_1$: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\square$