Lines and Planes in $\mathbb{R}^2$ and $\mathbb{R}^3$

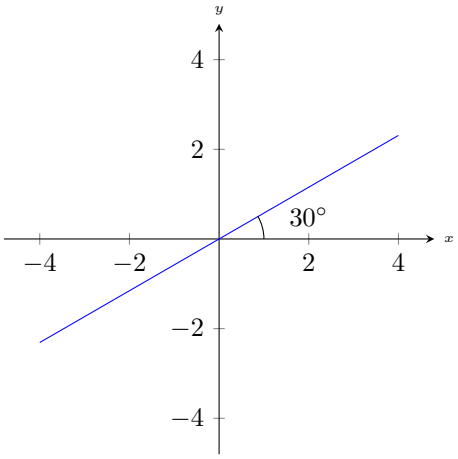
Equations of Lines:

For each of the lines described below, give the

(a) normal form,
(b) general form,
(c) vector form,
(d) parametric form,

equations.

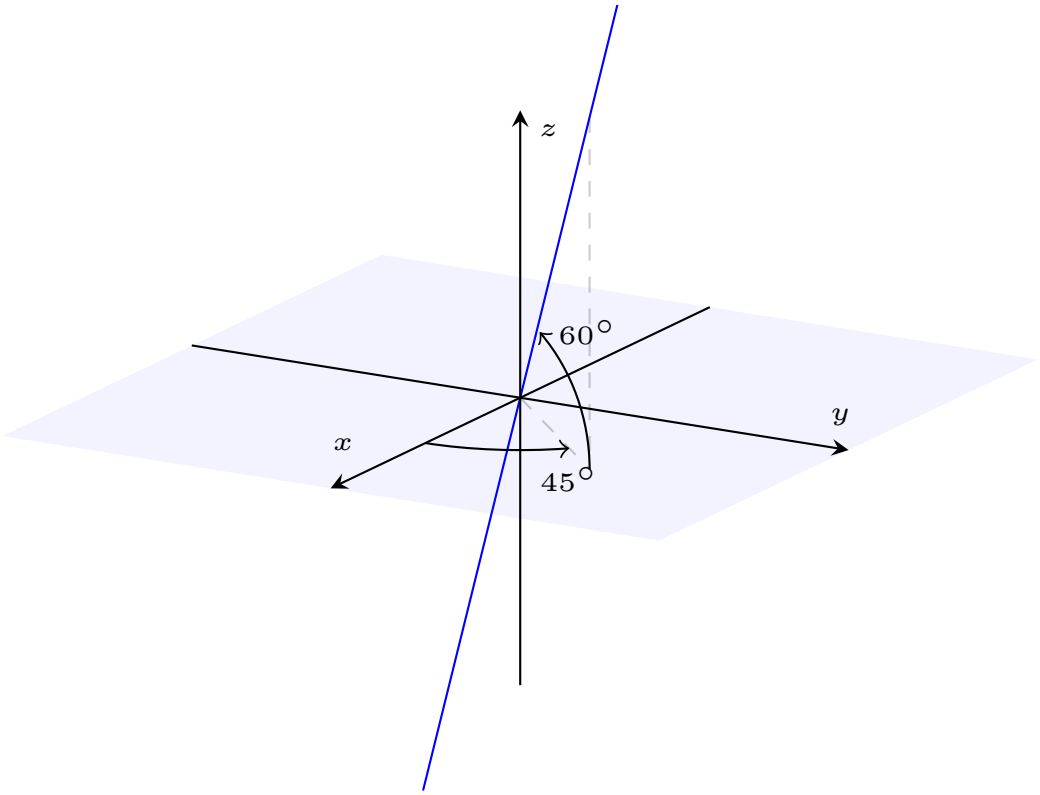
1. The line through the point $(2, -1)$ with direction vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$.
2. The line which passes through the points $(2, 4)$ and $(1, 2)$.
3. The line in $\mathbb{R}^2$ shown in this figure:



4. The line $y = x$ in $\mathbb{R}^2$.
5. The line through the point $(4, 4)$ that is perpendicular to $\begin{pmatrix} 7 \\ 9 \end{pmatrix}$.
6. The line through the point $(12, 0, 1)$ with direction vector $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.
7. The line which passes through the points $(0, 4, 5)$ and $(1, 6 - 3)$.
8. The line $y = x$ and $z = 2$ in $\mathbb{R}^3$.
9. The line of intersection of the planes $7x - 4y = 5$ and $-x + 4y + 3z = 2$.

10. The line through the origin which is perpendicular to both $\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$.

11. The (blue) line in $\mathbb{R}^3$ show in this figure:



Equations of Planes:

For each of the planes described below, give the

(a) normal form,
(b) general form,
(c) vector form,
(d) parametric form,

equations.

1. The plane through the point $(2, 2, 5)$ which is perpendicular to $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$.

2. The plane through $(-1, 0, 0)$ parallel to the vectors $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$.

3. The plane which passes through the points $(1, 2, 3)$, $(4, 5, 6)$, $(-3, 4, 5)$.

Distance from a point to a line or plane.

1. Find the distance from the point $(-2, 5)$ to the line with vector equation
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$
2. Find the distance from the origin to the line
$$2x - 3y - 7 = 0.$$
3. Find the distance from the point $(-1, 1, 1)$ to the line with direction vector equation
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ 7 \\ -1 \end{pmatrix}.$$
4. Find the distance from the point $(-1, 5, 6)$ to the plane
$$3x - 4y + 10z = 5.$$
5. Find the distance from the point $(2, 4, 4)$ to the plane
$$6x - 5y + z = 4.$$
6. Find the distance from the point $(0, 3, 6)$ to the plane through the origin with normal vector $\begin{pmatrix} 1 \\ 2 \\ -9 \end{pmatrix}$.
7. Find the distance from the point $(2, -10, 1)$ to the plane through the origin with vector equation
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 2 \\ 8 \end{pmatrix}.$$