Practice with RREF

Put the following systems of eqtions in RREF, and write the solution in vector form.

$$\begin{aligned} x_1 + x_2 - 2x_3 &= 4 \\ x_1 + 3x_2 - x_3 &= 7 \\ 2x_1 + x_2 - 5x_3 &= 7 \end{aligned}$$

RREF:

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 2 \\ x_3 &= -1 \end{aligned}$$

Solution:

$$\left\{ \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\}$$

$$\left(\begin{array}{cccc|c} 0 & -2 & 2 & 2 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{array}\right)$$

RREF:

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Solution:

$$\left\{ s \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \middle| \, s, t \in \mathbb{R} \right\}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & -2 \end{array}\right)$$

RREF:

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -5 \end{array}\right)$$

Solution:

The system is inconsistent.

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 0 & -5 \\ 2 & 4 & -6 & 1 & -8 \\ 6 & 13 & -17 & 4 & -21 \end{array}\right)$$

RREF:

$$\left(\begin{array}{cccc|c} 1 & 0 & -5 & 0 & -7 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

Solution:

$$\left\{ \begin{pmatrix} -7 \\ 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 5 \\ -1 \\ 1 \\ 0 \end{pmatrix} \middle| \, t \in \mathbb{R} \right\}$$

The following systems are already in RREF. Write their solution sets in vector form.

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -5 & 2 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 7 \end{array}\right)$$

Solution:

$$\left\{ \begin{pmatrix} 2 \\ 0 \\ 7 \\ 0 \end{pmatrix} + t \begin{pmatrix} 5 \\ -3 \\ 0 \\ 1 \end{pmatrix} \middle| \, t \in \mathbb{R} \right\}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Solution:

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 0 \\ -3 \\ 1 \end{pmatrix} \middle| \, s, t \in \mathbb{R} \right\}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 4 & 0 \\ 0 & 1 & -3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Solution:

$$\left\{ +s \begin{pmatrix} 1 \\ 3 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4 \\ -5 \\ 0 \\ 1 \end{pmatrix} \middle| \, s, t \in \mathbb{R} \right\}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

Solution:

$$\left\{ + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -6 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \middle| \, s, t \in \mathbb{R} \right\}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 3 & 0 & 8 & 0 \\ 0 & 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Solution:

$$\left\{ s\begin{pmatrix} -3 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -8 \\ 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} \middle| s,t \in \mathbb{R} \right\}$$