MATH1350, Winter 2025
Mini-Assignment 3

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1. Find <u>all</u> values of k for which the matrix $\begin{pmatrix} 14 & 7 \\ 2k & -3 \end{pmatrix}$ is invertible?

   $A: k = 0$ $\quad B: k \neq 0$ $\quad C: k = -3$ $\quad \boxed{\text{D}}: k \neq -3$ $\quad E: k = 7$ $\quad F: k \neq 7$ $\quad G: k \in \mathbb{R}$

   (Non-invertible if determinant is zero: $0 = 14(-3) - 7(2k) = -42 - 14k \Rightarrow k = 3$)

2. Which of the following matrices is singular?

   $A: \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$ $\quad B: \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ $\quad C: \begin{pmatrix} 12 & 3 \\ 4 & -1 \end{pmatrix}$ $\quad D: \begin{pmatrix} 0 & 2 \\ 2 & -1 \end{pmatrix}$ $\quad \boxed{\text{E}}: \begin{pmatrix} 12 & 6 \\ -2 & -1 \end{pmatrix}$

   (Check the 2×2 determinants)

3. Is the following matrix invertible?

   $$\begin{pmatrix} 1 & 3 & -2 \\ 3 & 7 & 7 \\ -2 & -6 & 4 \end{pmatrix}$$

   A : Yes $\quad \boxed{\text{B}}$: No

   Notice that $\begin{pmatrix} 1 & 3 & -2 \\ 3 & 7 & 7 \\ -2 & -6 & 4 \end{pmatrix} \xrightarrow{R_3+2R_1} \begin{pmatrix} 1 & 3 & -2 \\ 3 & 7 & 7 \\ 0 & 0 & 0 \end{pmatrix}$. This matrix does not have RREF equal to I_3 and hence is not invertible.

4. Which of the following is not an elementary matrix?

   $$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix},$$

   $$\boxed{\text{D}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, E = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \boxed{\text{F}} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{pmatrix}.$$

   (In D, row 3 has been multiplied by zero, which is not an ERO, F has 2 EROS)

5. The matrix $A = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$ is invertible. What is 2,2-entry of A^{-1}?

   (Enter your answer as a decimal number and round to 4 decimal places if needed.)

   Answer: $\frac{3}{2} = 1.5$

Recall that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

6. The matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ is invertible. What is 3,3-entry of A^{-1}? (For practice find A^{-1}.)

(Enter your answer as a decimal number and round to 4 decimal places if needed.)

Answer: 3

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 1 & 1 & 0 & 0 \\ -15 & 6 & -5 & 0 & 1 & 0 \\ 5 & -2 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{\frac{1}{3}R_1} \left(\begin{array}{ccc|ccc} 1 & -1/3 & 1/3 & 1/3 & 0 & 0 \\ -15 & 6 & -5 & 0 & 1 & 0 \\ 5 & -2 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3-5R_1]{R_2+15R_1} \left(\begin{array}{ccc|ccc} 1 & -1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1 & 0 & 5 & 1 & 0 \\ 0 & -1/3 & 1/3 & -5/3 & 0 & 1 \end{array}\right) \xrightarrow[R_3+\frac{1}{3}R_2]{R_1+\frac{1}{3}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 1/3 & 2 & 1/3 & 0 \\ 0 & 1 & 0 & 5 & 1 & 0 \\ 0 & 0 & 1/3 & 0 & 1/3 & 1 \end{array}\right)$$

$$\xrightarrow{R_1-R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 5 & 1 & 0 \\ 0 & 0 & 1/3 & 0 & 1/3 & 1 \end{array}\right) \xrightarrow{3R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 5 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 3 \end{array}\right)$$