MATH 1350,
Exercise Set 5

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1. Consider the line given by the equation $y = 5x - 3$ in $\mathbb{R}^2$. Give a vector equation for this line.

2. Consider the line in $\mathbb{R}^2$ which passes through the point $(1, 2)$ and has direction vector $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$. Give an equation for this line in:
   (a) Vector form,
   (b) Normal form,
   (c) General form,
   (d) Parametric form.

3. Consider the line in $\mathbb{R}^2$ which passes through the point $(2, 9)$ and has normal vector $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$. Give an equation for this line in:
   (a) Vector form,
   (b) Normal form,
   (c) General form,
   (d) Parametric form.

4. Give the vector form equation of the line in $\mathbb{R}^3$ which passes through the point $(5, 12, -7)$ and has direction vector $\begin{pmatrix} 3 \\ 11 \\ 0 \end{pmatrix}$.

5. Give the vector form equation of the line in $\mathbb{R}^3$ which passes through the points $(-4, 23, 6)$ and $(2, 0, 54)$.

6. Consider the line in $\mathbb{R}^3$ which passes through the point $(2, 0, -1)$ and has direction vector $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.
   (a) Give vector form and parametric form equations for this line.
   (b) Find two non-parallel vectors which are orthogonal to this line.
   (c) Give a system for the normal form equations of this line (i.e. express this line as the intersection of two planes in $\mathbb{R}^3$).

7. Find the general equation of the plane through the point $(1, 1, 0)$ with normal vector $\mathbf{n} = \begin{pmatrix} 3 \\ 0 \\ -5 \end{pmatrix}$

8. Find the general equation of the plane with the vector equation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$$

9. Calculate the cross product of $\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$.

10. Find the
    (a) Vector form,
    (b) Normal form,
    (c) General form,
    (d) Parametric form,

    of the plane which passes through the point $(3, -1, 3)$ with normal vector $\mathbf{n} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$.

11. Find the vector equation for the line of intersection of the two planes $x + y - 2z = 3$ and $2x - y + 3z = 6$.

12. Find the distance from the point $(0, 0)$ to the line

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

13. Find the distance from the point $(2, 3, 1)$ to the plane $3x - y + 4z = 5$.

14. Find the distance from the point $(2, 3, 2)$ to the line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$$

15. Find a nonzero vector $\mathbf{u}$ in $\mathbb{R}^4$ which is orthogonal to $\mathbf{v} = \begin{pmatrix} 2 \\ -2 \\ 4 \\ 1 \end{pmatrix}$.

16. Compute the cross products $\mathbf{u} \times \mathbf{v}$ and $\mathbf{v} \times \mathbf{u}$ for the following pairs:

(a)
$$\mathbf{u} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}.$$

(b)
$$\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

(c)
$$\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

(d)
$$\mathbf{u} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

(e)
$$\mathbf{u} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}.$$

(f)
$$\mathbf{u} = \begin{pmatrix} 0 \\ 6 \\ -7 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 0 \\ -12 \\ 14 \end{pmatrix}.$$

17. Let
$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

Verify that the cross product $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

18. Find a nonzero vector $\mathbf{w}$ in $\mathbb{R}^3$ which is orthogonal to both $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

19. Show that $\mathbf{u} \times \mathbf{u} = \mathbf{0}$ for any $\mathbf{u} \in \mathbb{R}^3$.

20. Let $r \in \mathbb{R}$ be a real number. Show that $(r\mathbf{u}) \times \mathbf{v} = r(\mathbf{u} \times \mathbf{v})$ for any $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$.

21. Show that $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$ for any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$.