

MATH1350, Winter 2025
Mini-Assignment 2

1. Which of the following matrices is in reduced row echelon form (RREF).

$$A: \left(\begin{array}{cccc|c} 1 & 0 & 3 & -1 & 0 \\ 0 & 1 & -4 & -7 & 3 \\ 0 & 1 & 0 & 0 & 0 \end{array} \right) \quad B: \left(\begin{array}{cccc|c} 1 & 0 & 10 & -7 & 0 \\ 0 & 1 & -4 & 2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right) \quad C: \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right) \quad D: \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 2 & 0 & -4 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Answer: B

2. Find the reduced row echelon form (RREF) for the following system.

$$\left(\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 0 \\ -2 & -5 & 0 & 4 & 0 \\ 1 & 4 & -6 & 1 & 0 \end{array} \right)$$

$$A: \left(\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 0 \\ 0 & 1 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad B: \left(\begin{array}{cccc|c} 1 & 0 & 10 & -7 & 0 \\ 0 & 1 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$C: \left(\begin{array}{cccc|c} 1 & 0 & 10 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right) \quad D: \left(\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Answer: B

(Solve this simply by putting the system into RREF, which is unique.)

3. Which of the following is the solution set for this system. $\left(\begin{array}{cccc|c} 1 & 0 & 10 & -7 & 0 \\ 0 & 1 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$

$$A: \left\{ \left(\begin{array}{c} -10 \\ 4 \\ 0 \end{array} \right) + t \left(\begin{array}{c} 7 \\ -2 \\ 1 \end{array} \right) \middle| t \in \mathbb{R} \right\} \quad B: \left\{ \left(\begin{array}{c} -10 \\ 4 \\ 0 \\ 0 \end{array} \right) + t \left(\begin{array}{c} 7 \\ -2 \\ 0 \\ 1 \end{array} \right) \middle| t \in \mathbb{R} \right\} \quad C: \left\{ \left(\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) \right\} \quad D: \text{Neither}$$

Answer: D

(The columns in A and C don't have enough entries as there are 4 variables in the system. There are 2 free variables in this system yet B only shows 1 free variable.)

4. Let

$$P_1 = \begin{pmatrix} 3 & 0 \\ -4 & 2 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 & 1 \\ -3 & -3 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 2 & 0 & 9 \\ -1 & -1 & 0 \end{pmatrix}, \quad P_4 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -3 & 12 \end{pmatrix}$$

Which of the following matrix operations is not defined?

$$A : 3P_1 + 6P_2 \quad B : (-1)P_4 - 2P_4 \quad C : (P_1 + P_2)P_3 \quad D : P_4P_3 \quad E : P_3P_4 \quad F : P_2P_4 \quad G : P_4P_2$$

Answer: F

(Recall that we may only sum matrices of the same size, and only define products AB where the number of columns of A equals the number of rows of B .)

5. Let

$$P_1 = \begin{pmatrix} 3 & 0 \\ -4 & 2 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 & 1 \\ -3 & -3 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 2 & 0 & 9 \\ -1 & -1 & 0 \end{pmatrix}, \quad P_4 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -3 & 12 \end{pmatrix}$$

What is the size of the matrix resulting from the calculation $((P_2 + 3P_1)(5P_3) - P_3)P_4)P_1$?

$$A : 2 \times 2 \quad B : 2 \times 3 \quad C : 3 \times 2 \quad D : 3 \times 3 \quad E : \text{This is not defined.}$$

Answer: A

(Here we must carefully follow the dimensions of the matrices in each step: $3P_1$ is 2×2 , $(P_2 + 3P_1)$ is 2×2 , $(P_2 + 3P_1)(5P_3)$ is 2×3 , $(P_2 + 3P_1)(5P_3) - P_3$ is 2×3 , $((P_2 + 3P_1)(5P_3) - P_3)P_4$ is 2×2 , $((P_2 + 3P_1)(5P_3) - P_3)P_4)P_1$ is 2×2 .)

6. Consider the product,

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$$

What is the 3,2-entry of the result?

Answer: -2

(We need only to consider the product of row 3 from the matrix on the left with column 2 of the matrix on the right: $1(2) + (-1)(2) + (-2)(1) = -2$.)