MATH 1350, Winter 2025
Mini-Assignment 4

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1. Find the determinant of $\begin{pmatrix} 12 & 1 & 2 & 2 \\ 0 & 2 & -7 & 0 \\ 0 & 0 & 1/2 & 4 \\ 0 & 0 & 0 & 5/2 \end{pmatrix}$

   Answer: 30

   (Since the matrix is in REF, its determinant is the product down the diagonal)

2. Consider the following row reduction of the matrix M to the 5×5 identity matrix I_5.

$$M \xrightarrow{R_3-4R_1} M_1 \xrightarrow{R_1 \leftrightarrow R_4} M_2 \xrightarrow{R_4-7R_2} M_3 \xrightarrow{2R_4} M_4 \xrightarrow{-4R_5} M_5 \xrightarrow{R_3 \leftrightarrow R_5} M_6 \xrightarrow{\frac{1}{3}R_2} I_5$$

   Find the determinant of M.

   Answer: $-\frac{3}{8}$

   Working backwards from I_5 we have:

$$1 = \det I_5 = \frac{1}{3}\det M_6 = \frac{1}{3}(-\det M_5) = \frac{1}{3}(-(-4\det M_4)) = \frac{1}{3}(-(-4(2\det M_3)))$$

$$= \frac{1}{3}(-(-4(2\det M_2))) = \frac{1}{3}(-(-4(2(-\det M_1)))) = \frac{1}{3}(-(-4(2(-\det M)))) = -\frac{8}{3}\det M$$

   This means

$$\det M = -\frac{3}{8}\det I_5 = -\frac{3}{8}.$$

3. Find all values of h such that $\begin{pmatrix} 3 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & h & 4 \end{pmatrix}$ is a singular matrix.

   Answer: $h = 4$

   Consider the row reduction:

$$\begin{vmatrix} 3 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & h & 4 \end{vmatrix} \xrightarrow{R_3-hR_2} \begin{vmatrix} 3 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4-h \end{vmatrix}$$

The determinant (which is the product down the diagonal) is $3(4-h)$, and is zero precisely when $h=4$. So matrix is singular when $h=4$ and nonsingular otherwise. Alternatively, by Laplace expansion

$$\begin{vmatrix} 3 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & h & 4 \end{vmatrix} = 3\begin{vmatrix} 1 & 1 \\ h & 4 \end{vmatrix} = 3(4-h)$$

we see that the determinant is zero (i.e. the matrix is singular) if and only if $h=4$.

4. Find the determinant of $\begin{pmatrix} 1 & 3 & -2 \\ 3 & 7 & 7 \\ -2 & -6 & 4 \end{pmatrix}$ using Gauss' method.

Answer: 0

Consider the row reduction:

$$\begin{pmatrix} 1 & 3 & -2 \\ 3 & 7 & 7 \\ -2 & -6 & 4 \end{pmatrix} \xrightarrow[R_3+2R_1]{R_2-3R_1} \begin{pmatrix} 1 & 3 & -2 \\ 0 & -2 & 13 \\ 0 & 0 & 0 \end{pmatrix}$$

The two matrices above have the same determinant (row combination operations do no change the determinant) and the second matrix has determinant 0 since it has a row of zeros.

5. Find the determinant of this product of matrices:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Answer: 2

The determinant of this product is product of the individual determinants. These are elementary matrices, whose determinants are easy to find. The first one has determinant 2 (since it corresponds to $2R_2$) the second and third matrices have determinant 1 (same as the identity matrix) and the fourth and fifth matrices each have determinant -1 (since they are row swaps).

6. Which of the following statements are not true in general?
   - A singular square matrix has determinant zero. *True*
   - A singular square matrix has nonzero determinant. *False*
   - A singular square matrix has no inverse. *True*
   - A nonsingular square matrix must always have an inverse. *True*
   - The identity matrix is invertible. *True*
   - If A is a square matrix, then $A\mathbf{x} = \mathbf{0}$ has only one solution. *False*
   - If B is any square matrix, then $\det(3B) = 3\det B$. *False*
   - If C is any square matrix, then $\det(C^3) = (\det C)^3$. *True*