MATH 1350, Winter 2025
Mini-Assignment 6

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1. Choose the vector with the same direction as $\mathbf{v}_1 + \mathbf{v}_2$ where

$$\mathbf{v}_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

A: $\uparrow$ B: $\downarrow$ $\boxed{\text{C}}$: $\leftarrow$ D: $\rightarrow$

E: $\nearrow$ F: $\nwarrow$ G: $\searrow$ H: $\swarrow$

Since $\mathbf{v}_1 + \mathbf{v}_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, the resulting vector has x-component -1 and y component 0. Represented geometrically, $\mathbf{v}_1 + \mathbf{v}_2$ is a vector parallel to the x-axis and pointing to the left.

2. Assuming that each of the four vectors $\mathbf{u} = \uparrow$, $\mathbf{d} = \downarrow$, $\mathbf{l} = \leftarrow$, $\mathbf{r} = \rightarrow$, has a length (or magnitude) of 1, find the length of the vector sum

$$\mathbf{u} + \mathbf{u} + \mathbf{r} + \mathbf{r} + \mathbf{d} + \mathbf{u} + \mathbf{r} + \mathbf{l}$$

Write your answer: 2.83

We may represent each of these four vectors with column matrices as

$$\mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \quad \mathbf{l} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \quad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

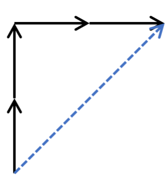
Thus

$$\mathbf{u} + \mathbf{u} + \mathbf{r} + \mathbf{r} + \mathbf{d} + \mathbf{u} + \mathbf{r} + \mathbf{l} = \begin{pmatrix} 2 \\ 2 \end{pmatrix},$$

and hence

$$||\mathbf{u} + \mathbf{u} + \mathbf{r} + \mathbf{r} + \mathbf{d} + \mathbf{u} + \mathbf{r} + \mathbf{l}|| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2} \approx 2.8284.$$

This can be solved with a picture. Notice that the last two pairs of vectors in the sum cancel with each other and so $\mathbf{u} + \mathbf{u} + \mathbf{r} + \mathbf{r} + \mathbf{d} + \mathbf{u} + \mathbf{r} + \mathbf{l} = \mathbf{u} + \mathbf{u} + \mathbf{r} + \mathbf{r}$. Adding geometrically yields the right triangle below.



The vertical and horizontal sides each have length 2. Use the Pythagorean Theorem to find the length of the result: $\sqrt{2^2 + 2^2} = 2\sqrt{2}$.

3. Find a unit vector in the same direction as $\mathbf{w} = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$

A: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ B: $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ C: $\begin{pmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ -1/\sqrt{3} \end{pmatrix}$ D: $\begin{pmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ -3/\sqrt{6} \end{pmatrix}$ $\boxed{\text{E}}$: $\begin{pmatrix} 1/\sqrt{14} \\ -2/\sqrt{14} \\ -3/\sqrt{14} \end{pmatrix}$

A unit vector in the same direction as a given vector is obtained by dividing said vector by its length. In this case

$$||\mathbf{w}|| = \sqrt{1^2 + (-2)^2 + (-3)^2} = \sqrt{14}.$$

Therefore a unit vector with the same direction as $\mathbf{w}$ is

$$\frac{1}{||\mathbf{w}||}\mathbf{w} = \begin{pmatrix} 1/\sqrt{14} \\ -2/\sqrt{14} \\ -3/\sqrt{14} \end{pmatrix}.$$

Notice that A and C are both unit vectors but they do not point in the same direction as $\mathbf{w}$. Neither B nor D are unit vectors.

4. Find the angle between vectors $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

(Express the angle in degrees and round to 1 decimal place)

Write your answer: $\cos^{-1}\left(\frac{1}{\sqrt{30}}\right) \approx 79.5$ degrees.

The angle θ between vectors $\mathbf{u}$ and $\mathbf{v}$ is given by

$$\theta = \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}|| \, ||\mathbf{v}||}\right).$$

In this case

$$\mathbf{u} \cdot \mathbf{v} = 1 \cdot 2 + (-1) \cdot 1 + 2 \cdot 0 = 1, \quad ||\mathbf{u}|| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}, \quad ||\mathbf{v}|| = \sqrt{2^2 + 1^2 + 0^2} = \sqrt{5}$$

5. Find $||\text{proj}_{\mathbf{u}}(\mathbf{v})||$, the magnitude of the projection of vector $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$ onto $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(Round to 2 decimal places)

Write your answer: 1.15

The projection of $\mathbf{v}$ onto $\mathbf{u}$ is given by

$$\text{proj}_{\mathbf{u}}(\mathbf{v}) = \frac{\mathbf{v}\cdot\mathbf{u}}{||\mathbf{u}||^2}\mathbf{u}.$$

In this case

$$\mathbf{v}\cdot\mathbf{u} = 3\cdot 1 + 1\cdot(-1) + (-4)\cdot 1 = -2, \quad ||\mathbf{u}||^2 = \mathbf{u}\cdot\mathbf{u} = 1^2 + (-1)^2 + 1^2 = 3,$$

and so

$$\text{proj}_{\mathbf{u}}(\mathbf{v}) = \frac{-2}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}-2/3\\2/3\\-2/3\end{pmatrix}.$$

Then

$$||\text{proj}_{\mathbf{u}}(\mathbf{v})|| = \sqrt{\left(-\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2} = \sqrt{\frac{12}{9}} = \frac{2}{\sqrt{3}} \approx 1.1547$$

6. Find the general equation of a line in $\mathbb{R}^2$ passing through the point $(1,3)$ and parallel to $\begin{pmatrix}2\\-7\end{pmatrix}$.

$\boxed{\text{A}}$: $7x+2y=13$ B: $7x-2y=13$ C: $x+3y=-5$ D: $2x-7y=4$ E: Neither

A line parallel to $\begin{pmatrix}2\\-7\end{pmatrix}$ will have slope $m = \frac{-7}{2}$ (rise over run). Using point slope form, $y - y_0 = m(x - x_0)$, with $(x_0, y_0) = (1,3)$ we have

$$y - 3 = -\frac{7}{2}(x-1).$$

Rearrange to get

$$\frac{7}{2}x + y = \frac{7}{2} + 3 \quad \Rightarrow \quad 7x + 2y = 13.$$

We can find the general form in another way. Since this line has direction vector $\mathbf{d} = \begin{pmatrix}2\\-7\end{pmatrix}$, we can see that it has normal vector $\mathbf{n} = \begin{pmatrix}7\\2\end{pmatrix}$ (check that $\mathbf{d}\cdot\mathbf{n} = 0$). The normal form of the equation for the line is

$$\mathbf{n}\cdot(\mathbf{x}-\mathbf{p}) = 0 \quad \text{or} \quad \mathbf{n}\cdot\mathbf{x} = \mathbf{n}\cdot\mathbf{p}$$

where $\mathbf{x} = \begin{pmatrix}x\\y\end{pmatrix}$ and $\mathbf{p} = \begin{pmatrix}1\\3\end{pmatrix}$. Computing the dot product on both sides gives the general form

$$7x + 2y = 13.$$

Notice that the vector form for the equation of the line is

$$\mathbf{x} = \mathbf{p} + t\mathbf{d}, \quad t\in\mathbb{R} \quad \text{or} \quad \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix} + t\begin{pmatrix}2\\-7\end{pmatrix}, \quad t\in\mathbb{R},$$

from which we can obtain the parametric equations for the line

$$x = 1 + 2t, \quad y = 3 - 7t, \quad t \in \mathbb{R}.$$