MATH 1350, Winter 2025
Mini-Assignment 7

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1. Find the distance from the point $(4, 3)$ to line $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

   Answer: $2\sqrt{2}$

2. Find the general equation of the plane through the point $(1, 1, 0)$ with normal vector $\mathbf{n} = \begin{pmatrix} 3 \\ 0 \\ -5 \end{pmatrix}$.

   $$A : x + y = 3 \quad B : x + y + 3 = 0 \quad C : 3x - 5z = 3 \quad D : 3x - 5z + 3 = 0 \quad E : \text{Neither}$$

   Answer: $3x - 5z = 3$

3. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be vectors in a vector space V. Which of the following properties need not apply to V?

   $A : \mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ $\qquad B : \mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{v} + \mathbf{u}) + \mathbf{w}$

   C : Every $\mathbf{u} \in V$ has an additive inverse. $\qquad D : \mathbf{v} + (\mathbf{u} + \mathbf{w}) = (\mathbf{v} + \mathbf{u}) + \mathbf{w}$

   $\boxed{\text{E}}$: Every $\mathbf{u} \in V$ has a multiplicative inverse. $\qquad F$: There exists $\mathbf{x} \in V$ such that $\mathbf{x} + \mathbf{u} = \mathbf{u}$.

   (While E is a property of a field, it is not required in a vector space.)

4. Let $V = \mathcal{M}_{2\times 2}(\mathbb{R})$, the set of 2×2 matrices. Redefine addition and scalar multiplication in the following way:

   $$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a & x \\ y & d \end{pmatrix} \qquad r \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} r & r \\ r & r \end{pmatrix}$$

   Which of the following vector space axioms fail to hold true?

   VS1 The set V is closed under vector addition, that is, $\mathbf{u} + \mathbf{v} \in V$ for any $\mathbf{u}, \mathbf{v} \in V$

   $\boxed{\text{VS2}}$ Vector addition is commutative, $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

   VS3 Vector addition is associative, $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

   $\boxed{\text{VS4}}$ There is a zero vector (or additive identity element) $\mathbf{0} \in V$ such that $\mathbf{v} + \mathbf{0} = \mathbf{v}$ for all $\mathbf{v} \in V$.

   $\boxed{\text{VS5}}$ Each $\mathbf{v} \in V$ has an additive inverse $\mathbf{w} \in V$, so that $\mathbf{w} + \mathbf{v} = \mathbf{0}$

   VS6 The set V is closed under scalar multiplication, that is, $r \cdot \mathbf{v} \in V$

$\boxed{\text{VS7}}$ Addition of scalars distributes over scalar multiplication, $(r+s)\cdot\mathbf{v} = r\cdot\mathbf{v} + s\cdot\mathbf{v}$

VS8 Scalar multiplication distributes over vector addition, $r\cdot(\mathbf{v}+\mathbf{w}) = r\cdot\mathbf{v} + r\cdot\mathbf{w}$

$\boxed{\text{VS9}}$ Ordinary multiplication of scalars associates with scalar multiplication, $(rs)\cdot\mathbf{v} = r\cdot(s\cdot\mathbf{v})$

$\boxed{\text{VS10}}$ Multiplication by the scalar 1 is the identity operation, $1\cdot\mathbf{v} = \mathbf{v}$

5. Let $V = \mathbb{R}^3$. Redefine addition and scalar multiplication in the following way:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax \\ by \\ cz \end{pmatrix} \qquad r\cdot\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Is V a vector space under this new addition and scalar multiplication?

A : Yes $\boxed{\text{B}}$: No

(VS10 fails, also the zero vector here would have to be $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, which means vectors with zero entries have no additive inverse in this case)

6. Is the following subset of $\mathcal{M}_{2\times 2}(\mathbb{R})$ closed under the usual addition and scalar multiplication of matrices?

$$\left\{ M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \det(M) = 0 \right\}$$

A : Yes $\boxed{\text{B}}$: No

(While this subset is closed under scalar multiplication, it is not closed under vector addition. For example $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ each have determinant zero, but their sum, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, has determinant 1.)