

MATH1350, Winter 2025
Mini-Assignment 1

1. Which of the following is a row echelon form (REF) of the system below.

$$\begin{array}{rcl} x_1 - 2x_2 + x_3 & = & 0 \\ 2x_2 - 8x_3 & = & 8 \\ -4x_1 + 5x_2 + 9x_3 & = & -9 \end{array}$$

$$A : \begin{array}{rcl} x_1 - 2x_2 + x_3 & = & 0 \\ x_3 & = & 2 \\ 0 & = & 0 \end{array} \quad \boxed{B} : \begin{array}{rcl} x_1 - 2x_2 + x_3 & = & 0 \\ x_2 - 4x_3 & = & 4 \\ x_3 & = & 3 \end{array} \quad C : \begin{array}{rcl} x_1 - 2x_2 + x_3 & = & 0 \\ 2x_2 - 8x_3 & = & 8 \\ 9x_3 & = & -9 \end{array}$$

$$D : \begin{array}{rcl} x_1 - 2x_2 + x_3 & = & 0 \\ 2x_2 - 8x_3 & = & 8 \\ -3x_2 + 13x_3 & = & -9 \end{array} \quad E : \text{Neither}$$

2. The following system is in REF. How many free variables are there?

$$\begin{array}{rcl} x_1 + 2x_2 + 3x_3 - x_4 + 2x_5 + x_7 + 10x_9 & = & 0 \\ \frac{1}{2}x_3 - x_4 + 3x_5 - 2x_6 + x_7 - x_8 & = & 0 \\ x_4 + 2x_6 + 3x_7 - x_8 - 4x_9 & = & 0 \\ x_5 - 3x_6 - x_7 - x_9 & = & 0 \\ x_8 + 2x_9 & = & 0 \\ 0 & = & 0 \end{array}$$

Answer: 4. The leading variables are x_1, x_3, x_4, x_5, x_8 , while x_2, x_6, x_7 and x_9 are free.

3. Find the solution set for this system.

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array} \right)$$

$$A : \{(-24, -7, 4)\} \quad B : \{(-24-2s+3t, -7-2s+2t, 4) | s, t \in \mathbb{R}\} \quad C : \{(-24+2s-3t, -7+2s-2t, 4) | s, t \in \mathbb{R}\}$$

$$D : \{(-24+2s-3t, -7+2s-2t, 4, s, t) | s, t \in \mathbb{R}\} \quad E : \{(-24-2s+3t, -7-2s+2t, s, t, 4) | s, t \in \mathbb{R}\}$$

$$F : \{(-24+2s-3t, -7+2s-2t, t, s, 4) | s, t \in \mathbb{R}\} \quad \boxed{G} : \{(-24+2s-3t, -7+2s-2t, s, t, 4) | s, t \in \mathbb{R}\}$$

$H : \text{Neither}$

4. Which (if any) of the following tuples is a solution to the system below.

$$\begin{array}{rcl} x_1 - 2x_2 + 3x_3 + x_4 & = & -3 \\ 2x_1 - x_2 + 3x_3 - x_4 & = & 0 \end{array}$$

$$\begin{array}{llllll}
 A : (0, 0, 0, 0) & B : (0, 0, 0, -3) & \boxed{C} : (1, 2, 0, 0) & \boxed{D} : (1, 4, 1, 1) & E : (2, 3, 0, 0) \\
 F : (0, 0, -1, 0) & \boxed{G} : (2, 5, 1, 2) & H : (2, 4, 0, 0) & I : (1, 1, 1, 1) & J : \text{Neither}
 \end{array}$$

5. Find all values for k such that the following system has only one solution.
- $$\begin{array}{rcl}
 x_1 - 2x_2 & = & 4 \\
 x_2 & = & k
 \end{array}$$

$$A : k = 0 \quad B : k = 4 \quad C : \text{Any } k \neq 4 \quad D : \text{Any } k \neq 0 \quad \boxed{E} : \text{Any } k \in \mathbb{R} \quad F : \text{Neither}$$

(It doesn't matter what the value for k is, this determines x_2 which we plug in to row 1 get a value for x_1 .)

6. Find all values for n such that the following system has a free variable.
- $$\begin{array}{rcl}
 x_1 + x_2 & = & 0 \\
 x_1 + nx_2 & = & 0
 \end{array}$$

$$A : n = 0 \quad \boxed{B} : n = 1 \quad C : \text{Any } n \neq 0 \quad D : \text{Any } n \neq 1 \quad E : \text{Any } n \in \mathbb{R} \quad F : \text{Neither}$$

(Put into REF to see this; i.e. do the row operation $R_2 - R_1$. If $n \neq 1$ we get one solution $(x_1, x_2) = (0, 0)$, and if $n = 1$ we have that x_2 is a free variable.)