MATH1350: Linear Algebra I — **Seminar 4: Vector basics**

Warm-up question

Consider the set of matrices of the form $M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ where $\theta \in \mathbb{R}$.
Sketch the vector $\mathbf{x}$ and the matrix product $M\mathbf{x}$ where:

(a) $\theta = \frac{\pi}{4}$ and $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(b) $\theta = \frac{\pi}{4}$ and $x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

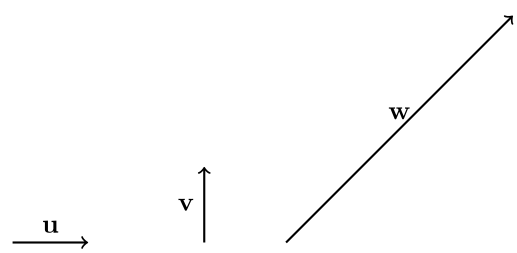
(c) $\theta = \frac{\pi}{2}$ and $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(d) $\theta = \frac{\pi}{2}$ and $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Seminar questions

1. Using the cosine law from lecture, verify that the angle between $\mathbf{x}$ and $M\mathbf{x}$ is θ for parts (a) and (c) of the Warm-up question.

2. Three vectors, $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are given below. Show, geometrically, how the vector $\mathbf{w}$ can be written as a linear combination of $\mathbf{u}$ and $\mathbf{v}$; that is, show that $\mathbf{w} = C_1\mathbf{u} + C_2\mathbf{v}$ for some $C_1, C_2 \in \mathbb{R}$. Do this by finding the required values for C_1 and C_2 (use a ruler if you have one, or give your best estimate).



3. Create one vector that runs parallel to $\begin{pmatrix} 2 \\ -1 \\ 1 \\ 2 \end{pmatrix}$ and one vector that runs perpendicular to it. Find the magnitude of each.

4. Create a line that passes through point $(0, -2, 2)$ and runs perpendicular to the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Write the equation of this line in vector, parametric, normal, and general forms.

5. Verify the Cauchy-Schwarz inequality and the triangle inequality for the vectors
$$\mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}.$$

Additional practice problem

6. What is the distance between the vectors $\begin{pmatrix} 1 \\ 1 \\ 4 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 5 \\ -1 \\ 1 \end{pmatrix}$?