Linear Independence Practice Problems

Determine whether the given sets of vectors in $\mathbb{R}^2$ are linearly independent or linearly dependent (over $\mathbb{R}$) by the row reduction method, and verify this using the determinant method.

1. $S_1 = \left\{ \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$
2. $S_2 = \left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\}$
3. $S_3 = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$
4. $S_4 = \left\{ \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 8 \end{pmatrix} \right\}$
5. $S_5 = \left\{ \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \end{pmatrix} \right\}$
6. $S_6 = \left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix} \right\}$

Determine whether the given sets in $\mathbb{R}^3$ are linearly independent by the row reduction method.

1. $S_7 = \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \right\}$
2. $S_8 = \left\{ \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix} \right\}$
3. $S_9 = \left\{ \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

Determine whether the given sets in $\mathbb{R}^3$ are linearly independent by determinant method.

1. $S_{10} = \left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} \right\}$
2. $S_{11} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \right\}$
3. $S_{12} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \right\}$

Determine whether the given sets of vectors are linearly independent or linearly dependent.

1. $S_{13} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \right\}$
2. $S_{14} = \left\{ \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \right\}$
3. $S_{15} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \\ 2 \end{pmatrix} \right\}$
4. $S_{16} = \left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \\ 3 \end{pmatrix} \right\}$
5. $S_{17} = \{x^2 - 3, 6x^2 + 2\}$
6. $S_{18} = \{2x^2 - x + 3, 4x^2 + x + 2, 8x^2 - x + 8\}$
7. $S_{19} = \{x^2 - x + 1, x + 1, -3\}$
8. $S_{20} = \{x^2 + 4x - 1, 3x^2 + x + 4, 2x^2 - 3x + 5, 5x^2 - 2x + 9\}$
9. $S_{21} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} \right\}$
10. $S_{22} = \left\{ \begin{pmatrix} -1 & 4 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$

Find a value of k such that the given vectors are linearly dependent.

1. $S_{23} = \left\{ \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ k \\ 1 \end{pmatrix} \right\}$
2. $S_{24} = \left\{ \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} k \\ -4 \end{pmatrix} \right\}$
3. $S_{25} = \left\{ \begin{pmatrix} 1 \\ k \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \\ k \end{pmatrix} \right\}$