MATH 1350
Winter 2025
Test 1
2025-01-21

Name (Print): _____

ID number: _____

1. For each of the following augmented matrices: If the matrix is in row-echelon form write "REF," and if not, write one elementary row operation which will put the matrix into REF. You do not need to perform the row operation in that case.

(a) $\left(\begin{array}{cccc|c} 1 & 1 & -2 & 4 & 0 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right)$ REF.

(b) $\left(\begin{array}{ccccc|c} 0 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$ REF

(c) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 2 & -3 & 1 \\ 0 & 2 & 3 & 3 & 4 \end{array} \right)$ $R_3 \leftrightarrow R_2$

(d) $\left(\begin{array}{ccccc|c} 1 & 1 & 2 & -1 & 1 & 4 \\ 1 & 3 & 1 & 4 & -2 & 4 \end{array} \right)$ $R_2 - R_1$

(e) $\left(\begin{array}{cccc|c} 2 & 2 & 4 & 6 & 0 \\ 3 & 5 & 5 & -1 & 7 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right)$ $R_2 - \frac{3}{2}R_1$

2. (1 point) Which of the following matrices is in RREF? (List all that apply.)

$$A : \begin{pmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 2 & 0 & | & 2 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \qquad B : \begin{pmatrix} 1 & 0 & -1 & 0 & | & -3 \\ 0 & 1 & 0 & 0 & | & 7 \\ 0 & 0 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 0 \end{pmatrix}$$

$$C : \begin{pmatrix} 1 & 0 & -1 & 0 & | & -3 \\ 0 & 1 & 0 & 0 & | & 7 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 2 \end{pmatrix} \qquad D : \begin{pmatrix} 1 & 0 & -1 & | & 3 \\ 0 & 1 & -1 & | & 2 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

Answer:_____D_____

3. (1 point) Which of the following column vectors is NOT a solution to

$$\begin{pmatrix} 1 & -3 & 2 & 2 & | & 1 \\ 0 & 3 & 6 & 0 & | & -2 \\ 0 & 3 & -6 & 0 & | & 2 \\ 3 & -3 & 6 & 6 & | & 3 \end{pmatrix}$$

$$A : \begin{pmatrix} 1 \\ 0 \\ -1/3 \\ 1/3 \end{pmatrix} \qquad B : \begin{pmatrix} 5/3 \\ 0 \\ -1/3 \\ 0 \end{pmatrix} \qquad C : \begin{pmatrix} -1 \\ -2/3 \\ 0 \\ 0 \end{pmatrix} \qquad D : \text{All three are solutions}$$

(List all that apply.)

Answer:_____C_____

4. (1 point) What is the correct solution set for the system whose augmented matrix is below?

$$\begin{pmatrix} 1 & 0 & -1 & | & 3 \\ 0 & 1 & -1 & | & 2 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$A : \left\{ \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \middle| t \in \mathbb{R} \right\} \qquad B : \left\{ \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \middle| t \in \mathbb{R} \right\} \qquad C : \left\{ \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| t \in \mathbb{R} \right\} \qquad D : \text{neither}$$

Answer:_____C_____

5. Given the linear system

$$
\begin{aligned}
2x_1 + 4x_3 + 4x_4 &= 2 \\
-x_1 + -x_3 &= 1 \\
-3x_1 + 6x_4 &= 9
\end{aligned}
$$

(a) (1 point) Write the linear system in augmented matrix form.

$$
\left(\begin{array}{cccc|c}
2 & 0 & 4 & 4 & 2 \\
-1 & 0 & -1 & 0 & 1 \\
-3 & 0 & 0 & 6 & 9
\end{array}\right)
$$

(b) (4 points) Put the matrix into reduced row-echelon form (RREF) and indicate all row operations in the process. (Show your work)

$$
\left(\begin{array}{cccc|c}
2 & 0 & 4 & 4 & 2 \\
-1 & 0 & -1 & 0 & 1 \\
-3 & 0 & 0 & 6 & 9
\end{array}\right)
\xrightarrow{\frac{1}{2}R_1}
\left(\begin{array}{cccc|c}
1 & 0 & 2 & 2 & 1 \\
-1 & 0 & -1 & 0 & 1 \\
-3 & 0 & 0 & 6 & 9
\end{array}\right)
$$

$$
\xrightarrow[R_3+3R_1]{R_2+R_1}
\left(\begin{array}{cccc|c}
1 & 0 & 2 & 2 & 1 \\
0 & 0 & 1 & 2 & 2 \\
0 & 0 & 6 & 12 & 12
\end{array}\right)
\xrightarrow[R_1-2R_2]{R_3-6R_2}
\left(\begin{array}{cccc|c}
1 & 0 & 0 & -2 & -3 \\
0 & 0 & 1 & 2 & 2 \\
0 & 0 & 0 & 0 & 0
\end{array}\right)
$$

(c) (2 points) Write the solution set in vector form.

$$
\left\{
\left.
\begin{pmatrix} -3 \\ 0 \\ 2 \\ 0 \end{pmatrix}
+ x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
+ x_4 \begin{pmatrix} 2 \\ 0 \\ -2 \\ 1 \end{pmatrix}
\right| x_2, x_4 \in \mathbb{R}
\right\}
$$