MATH1550
Practice Set 8 - Solutions

These exercises are suited to Chapter 4, from the beginning to Multivariate Expected Value.
Topics Covered:

- Definition of expected value for discrete and continuous random variables
- Properties of expected value
- Multivariate expected value

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1. (a) Write the definition of expected value both in the case of a discrete and a continuous random variable.
   (b) How can we interpret expected value; i.e. what does it tell us?
   (c) Give a real world example of where expected value may be used.
   (d) Let X be a discrete random variable with probability distribution $f(x)$, and let $Y = g(X)$ for some function g. How can we find the expected value for Y?
   (e) Let X be a random variable with $E(X) = 14$. Find $E(-5X + 10)$.
   (f) Let X and Y be joint continuous random variables with joint density $f(x, y)$, and let $Z = h(X, Y)$ for some function h. How can we find the expected value for Z?

   Solution.

   (a) If X is a discrete random variable with probability distribution $f(x)$, the *expected value* for X is

   $$E(X) = \sum_x xf(x)$$

   where the sum is taken over all x in the range of X. If X is a continuous random variable with probability density $f(x)$, the *expected value* for X is

   $$E(X) = \int_{-\infty}^{\infty} xf(x)\, dx.$$

   (b) Roughly speaking, the expected value tells us the average outcome of the probability experiment if it were to be repeated long term.

   (c) Expected value can be used to evaluate games of chance (e.g. casino games) or lotteries, for example which involve cash prizes. For example, suppose a person rolls a die and pays the number shown in dollars if the number is even, and wins the number shown in dollars if the number is odd. We can use expected value to determine whether we expect to win money or lose money by playing this game (perhaps repeatedly). If X is the amount won on a single roll, then range of X is $\{-6, -4, -2, 1, 3, 5\}$, with each outcome having equally likely probability $\frac{1}{6}$ (i.e. $f(x) = \frac{1}{6}$ for each $x \in \{-6, -4, -2, 1, 3, 5\}$). Therefore the expected winnings are

   $$E(X) = (-6) \cdot \frac{1}{6} + (-4) \cdot \frac{1}{6} + (-2) \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} = -\frac{1}{2}.$$

   i.e. we expect to lose (on average) \$0.50 per roll.

   (d) This can be computed as

   $$E(Y) = E(g(X)) = \sum_x g(x)f(x).$$

(e) This can be computed as

$$E(-5X+10) = (-5)E(X)+10 = (-5)(14)+10 = -60.$$

(f) This can be computed as

$$E(Z) = E(h(X,Y)) = \sum_x \sum_y h(x,y)f(x,y).$$

$\square$

2. For each probability distribution, find the expected value for the discrete random variable X.

(a)

x	0	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{64}$	$\frac{6}{64}$	$\frac{15}{64}$	$\frac{20}{64}$	$\frac{15}{64}$	$\frac{6}{64}$	$\frac{1}{64}$

(b)

x	-4	-2	0	2	3
$P(X=x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$

(c)

x	-3	-1	2	5
$P(X=x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{10}$

Solution.

(a)

$$E(X) = 0\cdot\frac{1}{64}+1\cdot\frac{6}{64}+2\cdot\frac{15}{64}+3\cdot\frac{20}{64}+4\cdot\frac{15}{64}+5\cdot\frac{6}{64}+6\cdot\frac{1}{64} = 3$$

(b)

$$E(X) = (-4)\cdot\frac{1}{9}+(-2)\cdot\frac{2}{9}+0\cdot\frac{3}{9}+2\cdot\frac{1}{9}+3\cdot\frac{2}{9} = 0$$

(c)

$$E(X) = (-3)\cdot\frac{4}{10}+(-1)\cdot\frac{1}{10}+2\cdot\frac{3}{10}+5\cdot\frac{2}{10} = \frac{3}{10}$$

$\square$

3. (a) Three apples are drawn from a bushel of 120 apples of which 9 are rotten. How many apples do you expect to be rotten?

(b) A regular 6-sided die is thrown and then a coin is tossed. If the coin toss is heads take the half the number shown on the roll of the die and if the coin toss is tails, multiply the die roll by 2. Let X be the number that you get. Find the expected value of X.

(c) A coin is tossed until heads appears. How many tosses can one expect until heads appears? (This one can be tricky, take a guess at what you think the expected number of tosses will be.)

Solution.

(a) Let X be the number of rotten apples of the three drawn. The range of X is $\{0, 1, 2, 3\}$ with probability distribution

$$f(x) = \frac{\binom{9}{x}}{\binom{120}{3}}.$$

The expected value for X is

$$E(X) = 0 \cdot \frac{\binom{9}{0}}{\binom{120}{3}} + 1 \cdot \frac{\binom{9}{1}}{\binom{120}{3}} + 2 \cdot \frac{\binom{9}{2}}{\binom{120}{3}} + 3 \cdot \frac{\binom{9}{3}}{\binom{120}{3}} = \frac{1}{280840}(0 \cdot 1 + 1 \cdot 9 + 2 \cdot 36 + 3 \cdot 84) = \frac{333}{280840}$$

(b) The range of X is $\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, 4, 6, 8, 10, 12\}$ with probability distribution

$$f(x) = \begin{cases} \frac{1}{12} & x = \frac{1}{2}, 1, \frac{3}{2}, \frac{5}{2}, 3, 4, 6, 8, 10, 12 \\ \frac{1}{6} & x = 2 \end{cases}$$

The expected value for X is

$$E(X) = \left(\frac{1}{2} + 1 + \frac{3}{2} + \frac{5}{2} + 3 + 4 + 6 + 8 + 10 + 12\right) \cdot \frac{1}{12} + 2 \cdot \frac{1}{6} = \frac{35}{8}$$

(c) Let X be the number of tosses until heads is reached. The range of X is $\{1, 2, 3, \ldots\}$ with probability distribution

$$f(x) = \left(\frac{1}{2}\right)^x.$$

The expected value for X is

$$E(X) = \sum_{x=1}^{\infty} x \left(\frac{1}{2}\right)^x = \sum_{x=1}^{\infty} \frac{x}{2^x}.$$

Consider the partial sums of this infinite series.

$$S_n = \sum_{x=1}^{n} x \left(\frac{1}{2}\right)^x$$

Then

$$\begin{aligned}
\frac{1}{2} S_n &= \left(1 - \frac{1}{2}\right) S_n \\
&= \sum_{x=1}^{n} x \left(\frac{1}{2}\right)^x - \sum_{x=1}^{n} x \left(\frac{1}{2}\right)^{x+1} \\
&= \sum_{x=1}^{n} \frac{x}{2^x} - \sum_{x=2}^{n+1} \frac{x-1}{2^x} \\
&= \frac{1}{2} + \sum_{x=2}^{n} \frac{x}{2^x} - \sum_{x=2}^{n} \frac{x-1}{2^x} - \frac{n}{2^{n+1}} \\
&= \frac{1}{2} + \sum_{x=2}^{n} \frac{1}{2^x} - \frac{n}{2^{n+1}} \\
&= \sum_{x=1}^{n} \frac{1}{2^x} - \frac{n}{2^{n+1}}
\end{aligned}$$

and so

$$S_n = 2 - \frac{n}{2^n}$$

Thus

$$E(X) = \sum_{x=1}^{\infty} x \left(\frac{1}{2}\right)^x = \lim_{n\to\infty} S_n = \lim_{n\to\infty} 2 - \frac{n}{2^n} = 2.$$

□

4. Find the expected value for continuous random variable X given its probability density function $f(x)$.

(a)

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

(b)

$$f(x) = \begin{cases} 1 & 0 \le x \le 0.5 \\ 2 & 0.5 \le x \le 0.75 \\ 0 & \text{elsewhere} \end{cases}$$

(c)

$$f(x) = \begin{cases} \sin(x) & 0 \le x \le \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$

(d)

$$f(x) = \begin{cases} 0 & x \le 10 \\ \frac{10}{x^2} & 10 < x \end{cases}$$

Solution.

(a)

$$E(X) = \int_0^2 x\left(\frac{1}{2}x\right)\, dx = \frac{x^3}{6}\bigg|_0^2 = \frac{4}{3}$$

(b)

$$E(X) = \int_0^{0.5} x\,(1)\ dx + \int_{0.5}^{7.5} x\,(2)\ dx = \frac{x^2}{2}\bigg|_0^{0.5} + x^2\bigg|_{0.5}^{0.75} = \frac{1}{8} + \frac{5}{16} = \frac{7}{16}$$

(c)

$$E(X) = \int_0^{\pi/2} x\sin(x)\ dx = -x\cos x\Big|_0^{\pi/2} + \int_0^{\pi/2} \cos x\ dx = 0 + \sin(x)\Big|_0^{\pi/2} = 1.$$

(Integration by parts used here.)

(d)

$$E(X) = \int_{10}^{\infty} x\left(\frac{10}{x^2}\right)\ dx = \int_{10}^{\infty}\left(\frac{10}{x}\right)\ dx = 10\ln(x)\Big|_{10}^{\infty} = \infty$$

□

5. A company raffle gives out 500 tickets. The first ticket drawn wins \$100, the next three tickets drawn win \$50 each, and the next five tickets drawn with \$25 each.

(a) What are the expected winnings if tickets are drawn without replacement?

(b) What are the expected winnings if tickets are drawn with replacement? (The direct calculation is quite nasty! Start setting up the problem until you get the idea.)

Solution.

(a) Let X be the amount won on a single raffle ticket. The range of X is $\{0, 25, 50, 100\}$ with probability distribution

x	0	25	50	100
$f(x)$	$\frac{491}{500}$	$\frac{5}{500}$	$\frac{3}{500}$	$\frac{1}{500}$

The expected value of X is

$$E(X) = 0 \cdot \frac{491}{500} + 25 \cdot \frac{5}{500} + 50 \cdot \frac{3}{500} + 100 \cdot \frac{1}{500} = \frac{375}{500} = \frac{3}{4}$$

(b) Let X be the amount won on a single raffle ticket. The range of X is

$$\{0, 25, 50, 75, 100, 125, 150, 175, 200, 225, 250, 275, 300, 325, 350, 375\}$$

with probability distribution

x	$f(x)$
0	$\left(\frac{499}{500}\right)^9$
25	$\binom{5}{1}\left(\frac{499}{500}\right)^8\left(\frac{1}{500}\right)$
50	$\binom{3}{1}\left(\frac{499}{500}\right)^8\left(\frac{1}{500}\right) + \binom{5}{2}\left(\frac{499}{500}\right)^7\left(\frac{1}{500}\right)^2$
75	$\binom{3}{1}\binom{5}{1}\left(\frac{499}{500}\right)^7\left(\frac{1}{500}\right)^2 + \binom{5}{3}\left(\frac{499}{500}\right)^6\left(\frac{1}{500}\right)^3$
100	$\left(\frac{499}{500}\right)^8\left(\frac{1}{500}\right) + \binom{3}{2}\left(\frac{499}{500}\right)^7\left(\frac{1}{500}\right)^2 + \binom{3}{1}\binom{5}{2}\left(\frac{499}{500}\right)^6\left(\frac{1}{500}\right)^3 + \binom{5}{4}\left(\frac{499}{500}\right)^5\left(\frac{1}{500}\right)^4$
125	$\binom{5}{1}\left(\frac{499}{500}\right)^7\left(\frac{1}{500}\right)^2 + \binom{3}{2}\binom{5}{1}\left(\frac{499}{500}\right)^6\left(\frac{1}{500}\right)^3 + \binom{3}{1}\binom{5}{3}\left(\frac{499}{500}\right)^5\left(\frac{1}{500}\right)^4 + \binom{5}{5}\left(\frac{499}{500}\right)^4\left(\frac{1}{500}\right)^5$
150	$\binom{3}{1}\left(\frac{499}{500}\right)^7\left(\frac{1}{500}\right)^2 + \binom{5}{2}\left(\frac{499}{500}\right)^6\left(\frac{1}{500}\right)^3 + \binom{3}{3}\left(\frac{499}{500}\right)^6\left(\frac{1}{500}\right)^3 + \binom{3}{2}\binom{5}{2}\left(\frac{499}{500}\right)^5\left(\frac{1}{500}\right)^4 + \binom{3}{1}\binom{5}{4}\left(\frac{499}{500}\right)^4\left(\frac{1}{500}\right)^5$
175	$\binom{3}{1}\binom{5}{1}\left(\frac{499}{500}\right)^6\left(\frac{1}{500}\right)^3 + \binom{5}{3}\left(\frac{499}{500}\right)^5\left(\frac{1}{500}\right)^4 + \binom{3}{3}\binom{5}{1}\left(\frac{499}{500}\right)^5\left(\frac{1}{500}\right)^4 + \binom{3}{2}\binom{5}{3}\left(\frac{499}{500}\right)^4\left(\frac{1}{500}\right)^5 + \binom{3}{1}\binom{5}{5}\left(\frac{499}{500}\right)^3\left(\frac{1}{500}\right)^6$
200	$\binom{3}{2}\left(\frac{499}{500}\right)^6\left(\frac{1}{500}\right)^3 + \binom{3}{1}\binom{5}{2}\left(\frac{499}{500}\right)^5\left(\frac{1}{500}\right)^4 + \binom{5}{4}\left(\frac{499}{500}\right)^4\left(\frac{1}{500}\right)^5 + \binom{3}{3}\binom{5}{2}\left(\frac{499}{500}\right)^4\left(\frac{1}{500}\right)^5 + \binom{3}{2}\binom{5}{4}\left(\frac{499}{500}\right)^3\left(\frac{1}{500}\right)^6$
225	$\binom{3}{2}\binom{5}{1}\left(\frac{499}{500}\right)^5\left(\frac{1}{500}\right)^4 + \binom{3}{1}\binom{5}{3}\left(\frac{499}{500}\right)^4\left(\frac{1}{500}\right)^5 + \binom{5}{5}\left(\frac{499}{500}\right)^3\left(\frac{1}{500}\right)^6 + \binom{3}{3}\binom{5}{3}\left(\frac{499}{500}\right)^3\left(\frac{1}{500}\right)^6 + \binom{3}{2}\binom{5}{5}\left(\frac{499}{500}\right)^2\left(\frac{1}{500}\right)^7$
250	$\binom{3}{3}\left(\frac{499}{500}\right)^5\left(\frac{1}{500}\right)^4 + \binom{3}{2}\binom{5}{2}\left(\frac{499}{500}\right)^4\left(\frac{1}{500}\right)^5 + \binom{3}{1}\binom{5}{4}\left(\frac{499}{500}\right)^3\left(\frac{1}{500}\right)^6 + \binom{3}{3}\binom{5}{4}\left(\frac{499}{500}\right)^2\left(\frac{1}{500}\right)^7$
275	$\binom{3}{3}\binom{5}{1}\left(\frac{499}{500}\right)^4\left(\frac{1}{500}\right)^5 + \binom{3}{2}\binom{5}{3}\left(\frac{499}{500}\right)^3\left(\frac{1}{500}\right)^6 + \binom{3}{1}\binom{5}{5}\left(\frac{499}{500}\right)^2\left(\frac{1}{500}\right)^7 + \binom{3}{3}\binom{5}{5}\left(\frac{499}{500}\right)\left(\frac{1}{500}\right)^8$
300	$\binom{3}{3}\binom{5}{2}\left(\frac{499}{500}\right)^3\left(\frac{1}{500}\right)^6 + \binom{3}{2}\binom{5}{4}\left(\frac{499}{500}\right)^2\left(\frac{1}{500}\right)^7$

x	$f(x)$
325	$\binom{3}{3}\binom{5}{3}\left(\frac{499}{500}\right)^2\left(\frac{1}{500}\right)^7 + \binom{3}{2}\binom{5}{5}\left(\frac{499}{500}\right)\left(\frac{1}{500}\right)^8$
350	$\binom{3}{3}\binom{5}{4}\left(\frac{499}{500}\right)\left(\frac{1}{500}\right)^8$
375	$\binom{3}{3}\binom{5}{5}\left(\frac{1}{500}\right)^9$

The expected value is therefore

$$\begin{aligned}
E(X) = &\frac{1}{500^9}\,[375 + (350\cdot 5 + 325\cdot 3 + 275\cdot 1)\cdot 499 \\
&+ (325\cdot 10 + 300\cdot 15 + 275\cdot 3 + 250\cdot 5 + 225\cdot 3)\cdot 499^2 \\
&+ (300\cdot 10 + 275\cdot 30 + 250\cdot 15 + 225\cdot 11 + 200\cdot 15 + 175\cdot 3)\cdot 499^3 \\
&+ (275\cdot 5 + 250\cdot 30 + 225\cdot 30 + 200\cdot 15 + 175\cdot 30 + 150\cdot 15 + 125\cdot 1)\cdot 499^4 \\
&+ (250\cdot 1 + 225\cdot 15 + 200\cdot 30 + 175\cdot 15 + 150\cdot 30 + 125\cdot 30 + 100\cdot 5)\cdot 499^5 \\
&+ (200\cdot 3 + 175\cdot 15 + 150\cdot 11 + 125\cdot 15 + 100\cdot 30 + 75\cdot 10)\cdot 499^6 \\
&+ (150\cdot 3 + 125\cdot 5 + 100\cdot 3 + 75\cdot 15 + 50\cdot 10)\cdot 499^7 \\
&+ (100\cdot 1 + 50\cdot 3 + 25\cdot 5)\cdot 499^8 \\
&+(0)\cdot 499^9] \\
= &\frac{1}{500^9}\left[375 + (3000)\cdot 499 + (10500)\cdot 499^2 + (21000)\cdot 499^3 + (26250)\cdot 499^4\right. \\
&\left.+(21000)\cdot 499^5 + (10500)\cdot 499^6 + (3000)\cdot 499^7 + (375)\cdot 499^8\right] \\
= &\frac{1,464,843,750,000,000,000,000,000}{1,953,125,000,000,000,000,000,000} \\
= &\frac{3}{4}
\end{aligned}$$

The expected value calculation $E(X) = \sum_x xf(x)$ is quite tedious here since it involves powers of large numbers. Calculating expected value in this way for a more complicated lottery may not be feasible without some approximations. We will see a clever shortcut in the next problem.

□

6. Let X and Y be jointly distributed random variables.
   (a) Show that $E(X+Y) = E(X) + E(Y)$.
   (b) Make use of this property to show that the expected value of a raffle equals the sum of the cash prizes divided by the number of tickets sold.

Solution.

(a) in the case that X and Y are discrete random variables, let $f(x, y)$ be their joint distribution, and let $g(x)$ and $h(y)$ be their respective marginal distributions. Then

$$\begin{aligned}
E(X+Y) &= \sum_x \sum_y (x+y)f(x,y) \\
&= \sum_x \sum_y (xf(x,y) + yf(x,y)) \\
&= \sum_x \sum_y xf(x,y) + \sum_x \sum_y yf(x,y) \\
&= \sum_x x \sum_y f(x,y) + \sum_y y \sum_x f(x,y) \\
&= \sum_x xg(x) + \sum_y yh(y) \\
&= E(X) + E(Y)
\end{aligned}$$

This property can be extended to any finite number of joint random variables so that

$$E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n).$$

(b) Suppose a raffle distributes N tickets, where there are k prizes to be won on k different draws, and draws are done without replacement. Denote the prize values by $C_1, C_2, \ldots, C_k$. If X is the amount won on a given ticket, then the range of X is $\{0, C_1, C_2, \ldots, C_k\}$ with probability distribution $P(X = C_i) = \frac{1}{N}$ and $P(X = 0) = \frac{N-1}{N}$. The expected value of a ticket is

$$E(X) = 0 \cdot \frac{N-1}{N} + C_1 \frac{1}{N} + C_2 \frac{1}{N} + \cdots + C_k \frac{1}{N} = \frac{\sum_{i=1}^{k} C_i}{N}.$$

i.e. the expected value of a raffle ticket is the sum of the cash prizes divided by the number of tickets sold.

Now consider the case when raffle tickets are drawn with replacement. Let X_i be the amount a ticket wins on the ith draw, for each $i \in \{1, \ldots, k\}$. The range of X_i is $\{0, C_i\}$ with probability distribution $P(X_i = C_i) = \frac{1}{N}$ and $P(X_i = 0) = \frac{N-1}{N}$. Thus

$$E(X_i) = 0 \cdot \frac{N-1}{N} + C_i \cdot \frac{1}{N} = \frac{C_i}{N}.$$

If X is the total amount that a given ticket wins, then $X = X_1 + X_2 + \cdots + X_k$. Thus the expected value of a ticket is

$$E(X) = E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n) = \frac{C_1}{N} + \frac{C_2}{N} + \cdots + \frac{C_k}{N} = \frac{\sum_{i=1}^{k} C_i}{N}.$$

i.e. the expected value of a raffle ticket is the sum of the cash prizes divided by the number of tickets sold.

Applying this to the problem above, we have $k = 9$ with $C_1 = 100$, $C_2 = 50$, $C_3 = 50$, $C_4 = 50$, $C_5 = 25$, $C_6 = 25$, $C_7 = 25$, $C_8 = 25$ and $C_9 = 25$. Therefore the expected value of a raffle ticket is

$$\frac{100 + 50 + 50 + 50 + 25 + 25 + 25 + 25 + 25}{500} = \frac{375}{500} = \frac{3}{4}.$$

□

7. Let X and Y be jointly distributed discrete random variables with joint distribution given below.

		x		
		-5	0	10
y	1	0.12	0.3	0.18
	2	0.08	0.2	0.12

Compute the following expected values.

(a) $E(X)$

(b) $E(X+Y)$

(c) $E(XY)$

(d) $E(3X+4Y+6)$

Solution.

(a) We can calculate this directly as,

$$E(X) = \sum_x \sum_y x f(x,y) = (-5)\cdot((0.12)+(0.08)) + 0\cdot(0.3+0.2) + 10\cdot(0.18+0.12) = 2$$

Or, we can use the marginal distribution

x	-5	0	10
$g(x)$	0.2	0.5	0.3

to obtain

$$E(X) = \sum_x x g(x) = (-5)\cdot(0.2) + 0\cdot(0.5) + 10\cdot(0.3) = 2.$$

(b) We can calculate this directly as,

$$\begin{aligned} E(X+Y) &= \sum_x \sum_y (x+y) f(x,y) \\ &= (-5+1)\cdot(0.12) + (-5+2)\cdot(0.08) + (0+1)\cdot(0.3) \\ &\quad + (0+2)\cdot(0.2) + (10+1)\cdot(0.18) + (10+2)\cdot(0.12) \\ &= 3.4. \end{aligned}$$

Or, we can use the that fact that $E(X) = 2$, and marginal distribution

y	1	2
$h(y)$	0.6	0.4

to obtain

$$E(Y) = \sum_y y h(y) = 1\cdot(0.6) + 2\cdot(0.4) = 1.4,$$

and so

$$E(X+Y) = E(X) + E(Y) = 2 + 1.4 = 3.4.$$

(c) We can calculate this directly as

$$\begin{aligned}
E(XY) &= \sum_x \sum_y (xy) f(x,y) \\
&= (-5 \cdot 1) \cdot (0.12) + (-5 \cdot 2) \cdot (0.08) + (0 \cdot 1) \cdot (0.3) \\
&\quad + (0 \cdot 2) \cdot (0.2) + (10 \cdot 1) \cdot (0.18) + (10 \cdot 2) \cdot (0.12) \\
&= 2.8.
\end{aligned}$$

However, it can shown that X and Y are independent (this was done in a previous exercise), and so

$$E(XY) = E(X)E(Y) = 2 \cdot (1.4) = 2.8.$$

(d) We can calculate this directly as

$$\begin{aligned}
E(3X + 4Y + 6) &= \sum_x \sum_y (3x + 4y + 6) f(x,y) \\
&= (3(-5) + 4(1) + 6) \cdot (0.12) + (3(-5) + 4(2) + 6) \cdot (0.08) \\
&\quad + (3(0) + 4(1) + 6) \cdot (0.3) + (3(0) + 4(2) + 6) \cdot (0.2) \\
&\quad + (3(10) + 4(1) + 6) \cdot (0.18) + (3(10) + 4(2) + 6) \cdot (0.12) \\
&= 17.6.
\end{aligned}$$

Or, using $E(X) = 2$ and $E(Y) = 1.4$) we have

$$E(3X + 4Y + 6) = 3E(X) + 4E(Y) + 6 = 3(2) + 4(1.4) + 6 = 17.6.$$

$\square$