MATH1550
Practice Set 8

These exercises are suited to Chapter 4, from the beginning to Multivariate Expected Value.
Topics Covered:

- Definition of expected value for discrete and continuous random variables
- Properties of expected value
- Multivariate expected value

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1. (a) Write the definition of expected value both in the case of a discrete and a continuous random variable.
   (b) How can we interpret expected value; i.e. what does it tell us?
   (c) Give a real world example of where expected value may be used.
   (d) Let X be a discrete random variable with probability distribution $f(x)$, and let $Y = g(X)$ for some function g. How can we find the expected value for Y?
   (e) Let X be a random variable with $E(X) = 14$. Find $E(-5X + 10)$.
   (f) Let X and Y be joint continuous random variables with joint density $f(x, y)$, and let $Z = h(X, Y)$ for some function h. How can we find the expected value for Z?

2. For each probability distribution, find the expected value for the discrete random variable X.

   (a)

x	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{64}$	$\frac{6}{64}$	$\frac{15}{64}$	$\frac{20}{64}$	$\frac{15}{64}$	$\frac{6}{64}$	$\frac{1}{64}$

   (b)

x	-4	-2	0	2	3
$P(X = x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$

   (c)

x	-3	-1	2	5
$P(X = x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{10}$

3. (a) Three apples are drawn from a bushel of 120 apples of which 9 are rotten. How many apples do you expect to be rotten?
   (b) A regular 6-sided die is thrown and then a coin is tossed. If the coin toss is heads take the half the number shown on the roll of the die and if the coin toss is tails, multiply the die roll by 2. Let X be the number that you get. Find the expected value of X.
   (c) A coin is tossed until heads appears. How many tosses can one expect until heads appears? (This one can be tricky, take a guess at what you think the expected number of tosses will be.)

4. Find the expected value for continuous random variable X given its probability density function $f(x)$.

   (a)
$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

   (b)
$$f(x) = \begin{cases} 1 & 0 \leq x \leq 0.5 \\ 2 & 0.5 \leq x \leq 0.75 \\ 0 & \text{elsewhere} \end{cases}$$

   (c)
$$f(x) = \begin{cases} \sin(x) & 0 \leq x \leq \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$

   (d)
$$f(x) = \begin{cases} 0 & x \leq 10 \\ \frac{10}{x^2} & 10 < x \end{cases}$$

5. A company raffle gives out 500 tickets. The first ticket drawn wins \$100, the next three tickets drawn win \$50 each, and the next five tickets drawn with \$25 each.

   (a) What are the expected winnings if tickets are drawn without replacement?
   (b) What are the expected winnings if tickets are drawn with replacement?

6. Let X and Y be jointly distributed random variables.

   (a) Show that $E(X + Y) = E(X) + E(Y)$.
   (b) Make use of this property to show that the expected value of a raffle equals the sum of the cash prizes divided by the number of tickets sold.

7. Let X and Y be jointly distributed discrete random variables with joint distribution given below.

		x		
		-5	0	10
y	1	0.12	0.3	0.18
	2	0.08	0.2	0.12

   Compute the following expected values.

   (a) $E(X)$
   (b) $E(X + Y)$
   (c) $E(XY)$
   (d) $E(3X + 4Y + 6)$