generally from the table:

$P(\cancel{x}\, 0 \le X \le z)$

Standard Normal Table

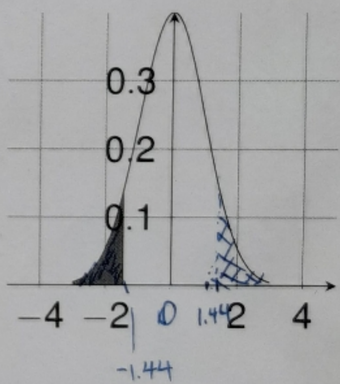
z negative

If $z < 0$, we find $P(X \le z)$ by finding $0.5 - P(X \le |z|)$.

Table III: Standard Normal Distribution

z	.00	.01	.02	.03	$\boxed{.04}$	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
$\boxed{1.4}$	.4192	.4207	.4222	.4236	$\boxed{.4251}$	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4988
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Find $P(X \le \underbrace{-1.44}_{z})$



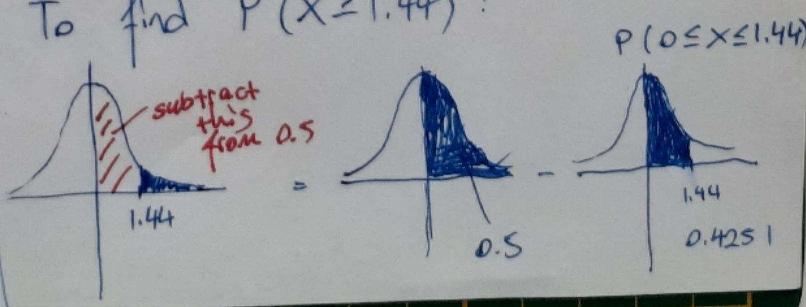
$$P(X \le -1.44)$$
$$= 0.5 - 0.4251$$
$$= 0.0749$$

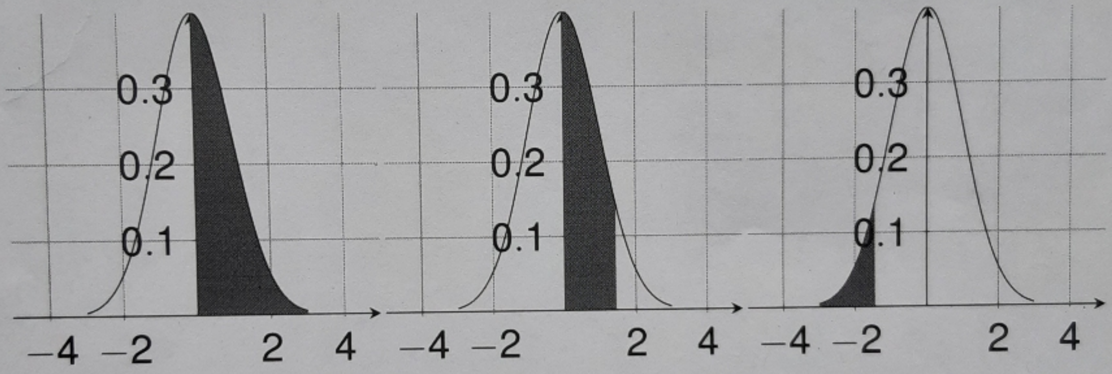
observe that by symmetry

$P(X \le -1.44) = P(X \ge 1.44)$

so, the desired prob. is $0.5 - 0.4251$
$= 0.0749$

To find $P(X \ge 1.44)$:





The difference of the first two areas is equal to the third.

neg.

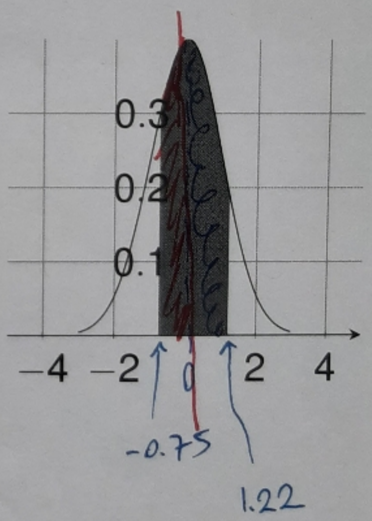
positive

Standard Normal Table

If X has standard normal distribution, find $P(-0.75 \leq X \leq 1.22)$

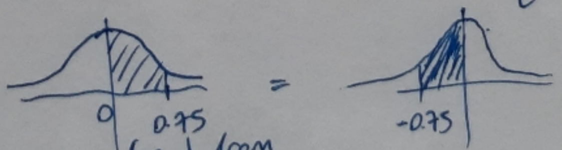
Table III: Standard Normal Distribution $P(0 \leq X \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4988
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990



$P(-0.75 \leq X \leq 1.22)$

$= \underbrace{P(-0.75 \leq X \leq 0)}_{\text{(by symmetry)} = 0.2734} + \underbrace{P(0 \leq X \leq 1.22)}_{\text{read from table} = 0.3888}$



read from the table $= 0.2734$

$= 0.2734 + 0.3888$

$= 0.6622$

midpoint of 1.35 & 1.36

$z = 1.355 \rightarrow$

0.4115 0.4131

average: 0.4123

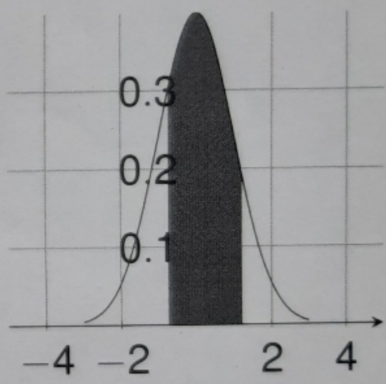
$z = 0.748 \approx 0.75$ $z = 0.9133 \approx 0.91$

Standard Normal Table

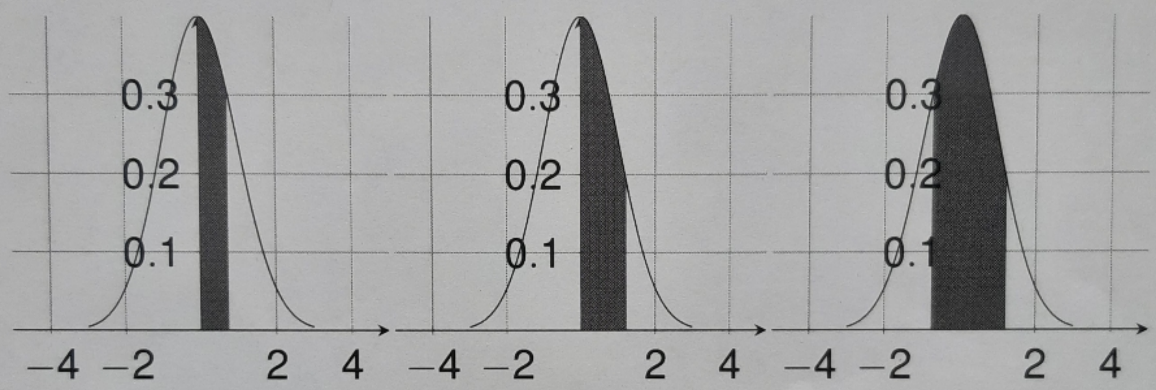
To find $P(-0.75 \leq X \leq 1.22)$, add $P(0 \leq X \leq 0.75)$ to $P(0 \leq X \leq 1.22)$

Table III: Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4988
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990



$P(-0.75 \leq X \leq 1.22)$
$= 0.2\overset{73}{\cancel{37}}4 + 0.3888$
$= \cancel{0.6262}$ 0.6622



\# $P(-0.75 \leq X \leq 0)$

$$P(0 \leq X \leq 0.75) + P(0 \leq X \leq 1.22) = P(-0.75 \leq X \leq 1.22)$$

The sum of the first two areas equals the third. (This plays on the symmetry of the graph)

Standard Normal Table

A couple of rules when using the table:

- For z values not found on the table we may simply choose the closest value. ✓
- If our z value is exactly the midpoint between two z values on the table, then we can average the two probabilities. ✓

standard normal distribution : $\mu = 0$, $\sigma = 1$

The Normal Distribution

Theorem

If X has a normal distribution with mean μ and standard deviation σ then

$$Z = \frac{X - \mu}{\sigma}$$

is a random variable having the standard normal distribution.

This allows us to compute probabilities for non-standard normal distributions with the standard normal table.

Proof (omit)

Let $Z = \frac{X-\mu}{\sigma}$. First note that

$$x_1 < X < x_2 \quad \Leftrightarrow \quad z_1 = \frac{x_1 - \mu}{\sigma} < Z < \frac{x_2 - \mu}{\sigma} = z_2$$

Then, using the substitution rule for integrals,

$$\begin{aligned} P(x_1 < X < x_2) &= \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \, dx \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{1}{2}(z)^2} \, dz \\ &= P(z_1 < Z < z_2). \end{aligned}$$

Therefore $P(x_1 < X < x_2) = P\left(\frac{x_1-\mu}{\sigma} < Z < \frac{x_2-\mu}{\sigma}\right)$, and we are able to look this up on the table.

Non-standard Normal Distribution

$Z = \dfrac{x-\mu}{\sigma}$ is a standard normal distribution

Example

Let X be a continuous random variable with normal distribution $n(x, 70, 4)$; i.e. $\mu = 70, \sigma = 4$. Find

observe that X is a non-standard normal distribution

1. $P(\underbrace{68}_{x_1} \leq X \leq \underbrace{74}_{x_2})$

$x_1 = 68 \qquad z_1 = \dfrac{x_1-\mu}{\sigma} = \dfrac{68-70}{4} = -\dfrac{1}{2} = -0.5$

$x_2 = 74 \qquad z_2 = \dfrac{x_2-\mu}{\sigma} = \dfrac{74-70}{4} = 1$

$P(68 \leq X \leq 74) = P\left(\dfrac{68-70}{4} \leq Z \leq \dfrac{74-70}{4}\right) = P(-0.5 \leq Z \leq 1)$

standard normal distrib.

Then, $P(-0.5 \leq Z \leq 1) = P(-0.5 \leq Z \leq 0) + P(0 \leq Z \leq 1)$

$= P(0 \leq Z \leq 0.5) + P(0 \leq Z \leq 1)$

$= 0.1915 + 0.3413 = \underline{\underline{0.5328}}$

Non-standard Normal Distribution

By the theorem

$$P(68 \leq X \leq 74) = P(\frac{68-70}{4} \leq Z \leq \frac{74-70}{4}) = P(-0.5 \leq Z \leq 1)$$

Then, by the symmetry in the graph,

$$P(-0.5 \leq Z \leq 1) = P(\overset{0\leq}{Z} \leq 0.5) + P(\overset{0 \leq Z \leq 1}{\cancel{Z \leq 1}})$$

$$= \cancel{0.1915 + 0.3413 = 0.5328}$$

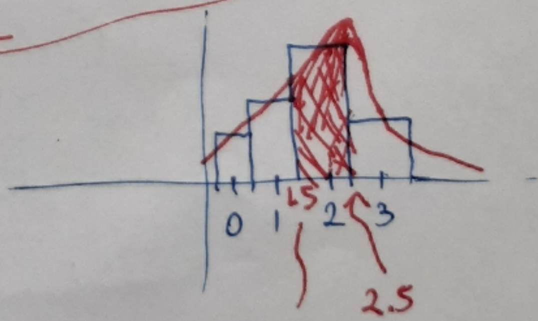
$0.1915 + 0.3413$
$= 0.5328$

The Normal Approximation of the Binomial Distribution

If X is a random variable with binomial distribution $b(x; n, \theta)$, then the normal distribution $n(x; n\theta, \sqrt{n\theta(1-\theta)})$, with mean $n\theta$ and standard deviation $\sqrt{n\theta(1-\theta)}$, gives an approximation of the binomial distribution.

See ~~the~~ a text book for how this is derived using the moment generating functions as $n \to \infty$.

To use this approximation, we need to "convert" the discrete binomial random variable to the continuous case. Here $P(X = k)$ will be a approximated with the normal distribution by integrating from $k - 0.5$ to $k + 0.5$.



The Normal Approximation of the Binomial Distribution

Example

Find the probability of getting 6 heads in 16 flips of a balanced coin. (binomial distribution) $b(6; 16, 0.5) = \binom{16}{6} \cdot (0.5)^{6} \cdot (1-0.5)^{16-6}$

$= 0.1222\ldots$

Approximate this with the normal distribution.

$x = 6 \rightarrow x_1 = 6 - 0.5 = 5.5$

$\rightarrow x_2 = 6 + 0.5 = 6.5$

mean $\mu = n\theta = 16 \cdot 0.5 = 8$

st. dev. $\sigma = \sqrt{n\theta(1-\theta)} = \sqrt{16 \cdot 0.5 \cdot 0.5} = 2$

switching to standard normal distribution (Z): $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{5.5-8}{2} = \frac{-2.5}{2} = -1.25$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{6.5-8}{2} = \frac{-1.5}{2} = -0.75$

30230

So the desired probability can be approximated as $P(\underset{z_1}{-1.25} \le Z \le \underset{z_2}{-0.75})$. By symmetry, this is equal to $P(0.75 \le Z \le 1.25)$

$= P(0 \le Z \le 1.25) - P(0 \le Z \le 0.75)$

$= 0.3944 - 0.2734 = 0.1210$