MATH 1550
Winter 2025
Test 2
2025-03-03
Time Limit: 50 Minutes

Name (Print): ____________________________

ID number: ____________________________

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1. (2 points) You will take a flight on Monday, Tuesday or Wednesday. You will fly with probability 0.2 on Monday, with probability 0.3 on Tuesday and with probability 0.5 on Wednesday. (These are the only days you are available.) During your flight, it will rain with probability 0.1 given you fly on Monday, with probability 0.4 given you fly on Tuesday, and with probability 0.2 given you fly on Wednesday.
   (a) (1 point) Find the probability that it will rain during your flight.
   (b) (1 point) Given that it rained during your flight, what is the probability that your flight was on Wednesday?

   (Show your work/formula and give a rounded answer to 4 decimal places if needed.)

   Solution:

   (a) Let R be the event that it will rain during your flight. Also, let F_M, F_T and F_W be the events that you are flying on Monday, Tuesday and Wednesday, respectively.

   We use the rule of total probability:

$$P(R) = P(F_M) \cdot P(R|F_M) + P(F_T) \cdot P(R|F_T) + P(F_W) \cdot P(R|F_W)$$

$$= 0.2 \cdot 0.1 + 0.3 \cdot 0.4 + 0.5 \cdot 0.2 = 0.24.$$

   (b) With events defined as in part (a), we use Bayes' Theorem:

$$P(F_W|R) = \frac{P(F_W) \cdot P(R|F_W)}{P(F_M) \cdot P(R|F_M) + P(F_T) \cdot P(R|F_T) + P(F_W) \cdot P(R|F_W)}$$

$$= \frac{0.10}{0.24} \approx 0.4166.$$

2. (4 points) Two balls are drawn randomly without replacement from an urn containing 3 white and 2 red balls. Suppose that we win \$5 for each white ball selected and we lose \$3 for each red ball selected. Let X denote our winnings.

   (a) (1 point) Find the range of X.

   (b) (2 points) Find the probability distribution of X.

   (c) (1 point) Find the cumulative distribution of X.

   (Show your work/formula and compute a final answer.)

   Solution:

   (a) The range of X is $\{-6, 2, 10\}$.

   (b) The probability of drawing i white balls and hence $2 - i$ red balls is

   $$\frac{\binom{3}{i}\binom{2}{2-i}}{\binom{5}{2}} \quad \text{for } i = 0, 1, 2.$$

   This gives the following probability distribution for X:

x	$P(X = x)$
-6	$\frac{1}{10}$
2	$\frac{6}{10}$
10	$\frac{3}{10}$

   (c)

   $$F(x) = \begin{cases} 0 & \text{for } x < -6 \\ \frac{1}{10} & \text{for } -6 \le x < 2 \\ \frac{7}{10} & \text{for } 2 \le x < 10 \\ 1 & \text{for } x \ge 10 \end{cases}$$

3. (1 point) The cumulative distribution for a discrete random variable X is given as follows.

$$F(x) = \begin{cases} 0 & \text{for } x < -4 \\ \frac{1}{8} & \text{for } -4 \leq x < 2 \\ \frac{3}{8} & \text{for } 2 \leq x < 7 \\ \frac{7}{8} & \text{for } 7 \leq x < 9 \\ 1 & \text{for } x \geq 9 \end{cases}$$

Find $P(X = 2)$.

(Show your work/formula and compute a final answer.)

Solution:

$P(X = 2) = F(2) - F(-4) = \frac{3}{8} - \frac{1}{8} = \frac{2}{8} = 0.25$.

4. (2 points) The following joint probability distribution is given for discrete random variables X and Y.

		x = 4	5	6
y	1	0.3	0.1	0
	2	0	0.2	0.1
	3	0.1	0	0.2

Find $P(X \geq 5 | Y \geq 2)$.

(Show your work and give a rounded answer to 4 decimal places if needed.)

Solution:

$$P(X \geq 5 | Y \geq 2) = \frac{P(X \geq 5, Y \geq 2)}{P(Y \geq 2)}, \text{ where}$$

$$P(X \geq 5, Y \geq 2) = P(X = 5, Y = 2) + P(X = 5, Y = 3) + P(X = 6, Y = 2) + P(X = 6, Y = 3)$$

$$= 0.2 + 0 + 0.1 + 0.2 = 0.5, \text{ and}$$

$$P(Y \geq 2) = P(Y = 2) + P(Y = 3) = (0 + 0.2 + 0.1) + (0.1 + 0 + 0.2) = 0.6$$

$$\text{Therefore, } P(X \geq 5 | Y \geq 2) = \frac{P(X \geq 5, Y \geq 2)}{P(Y \geq 2)} = \frac{0.5}{0.6} \approx 0.8333.$$

5. (2 points) A probability density function for two jointly continuous random variables X and Y is given as follows: $f(x,y) = \begin{cases} \frac{6}{5}(x+y^2) & \text{for } 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$

Find the marginal density $h(y)$ of Y.

(Show your work and give a rounded answer to 4 decimal places if needed.)

Solution:

For $0 \le y \le 1$,

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^1 \frac{6}{5}(x+y^2)\,dx = \frac{6}{5}(\frac{x^2}{2} + xy^2)\Big|_0^1$$

$$= \frac{6}{5}\left((\frac{1}{2}+y^2) - (0+0)\right) = \frac{3+6y^2}{5}$$

and $h(y) = 0$ elsewhere.

6. (3 points) A probability density function for two jointly continuous random variables X and Y is given as follows: $f(x,y) = \begin{cases} \frac{x+y}{3} & \text{for } 0 \le x \le 2, 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$

Find $P(1 \le X \le 2, 0 \le Y \le 1)$.

(Show your work and give a rounded answer to 4 decimal places if needed.)

Solution:

$$P(1 \le X \le 2, 0 \le Y \le 1) = \int_0^1 \int_1^2 \frac{x+y}{3}\,dx\,dy$$

$$= \int_0^1 (\frac{x^2}{6} + \frac{yx}{3})\Big|_1^2\,dy = \int_0^1 (\frac{4}{6} + \frac{2y}{3}) - (\frac{1}{6} + \frac{y}{3})\,dy = \int_0^1 \frac{2y+3}{6}\,dy$$

$$= \frac{1}{6}\int_0^1 2y+3\,dy = \frac{1}{6}(y^2+3y)\Big|_0^1 = \frac{1}{6}[(1+3)-(0+0)] = \frac{2}{3} \approx 0.6667$$