MATH1550
Practice Set 9

These exercises are suited to Chapter 4, from Moments to Moment Generating Functions.
Topics Covered:

- Moments about the origin and moments about the mean.
- Mean, variance and standard deviation.
- Chebyshev's Theorem.
- Moment generating functions.

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1. (a) Give the definition for the rth *moment about the origin* of a random variable.
   (b) What is the *mean* of a probability distribution? What symbol is used for the mean?
   (c) Give the definition for the rth *moment about the mean* of a random variable.
   (d) What are the *variance* and *standard deviation* of a random variable? What symbols are used for the variance and standard deviation?
   (e) Give the "shortcut" formula for finding the variance.
   (f) State Chebyshev's Theorem.
   (g) How do we find the *moment generating function* for a random variable?
   (h) How is the moment generating function used?

2. Let X be a discrete random variable with the given probability distribution.

   (a)

x	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{64}$	$\frac{6}{64}$	$\frac{15}{64}$	$\frac{20}{64}$	$\frac{15}{64}$	$\frac{6}{64}$	$\frac{1}{64}$

   (b)

x	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{6}{64}$	$\frac{6}{64}$	$\frac{15}{64}$	$\frac{15}{64}$	$\frac{20}{64}$

   (c)

x	0	1	2	3	4	5	6
$P(X = x)$	$\frac{20}{64}$	$\frac{15}{64}$	$\frac{15}{64}$	$\frac{6}{64}$	$\frac{6}{64}$	$\frac{1}{64}$	$\frac{1}{64}$

   In each case

   i. Sketch the probability histogram for X.
   ii. Find the mean of X.
   iii. Find the variance and standard deviation of X.
   iv. Find the third moment about the mean of X.

3. Let X be a continuous random variable with probability density

$$f(x) = \begin{cases} \frac{2(x-1)}{9} & -1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the mean, variance and standard deviation of X.

(b) Find the probability that X lies within 2.5 standard deviations of the mean, and compare with the lower bound given by Chebyshev's Theorem.

(c) Find the mean and variance $Y = X^2$.

(d) Find the probability density for $Y = X^2$.

4. In a certain manufacturing process, the (Fahrenheit) temperature never varies by more than $2°$ from $62°$. The temperature is a random variable F with distribution

x	60	61	62	63	64
$P(F = x)$	1/10	2/10	4/10	2/10	1/10

(a) Find the mean of and variance of F.

(b) To convert to the measurements to degrees Celsius we let $C = \frac{5}{9}(F - 32)$. Find the mean and variance of C.

5. For the given probability distribution, find the moment generating function for the discrete random variable X and use it to compute the mean and variance of X.

(a)

x	-3	-1	2	5
$P(X = x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{10}$

(b)

$$f(x) = \begin{cases} 3\left(\frac{1}{4}\right)^x & x = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

6. Suppose X is a continuous random variable with probability density

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

for all $x \in \mathbb{R}$. It can be shown that X has moment generating function

$$M_X(t) = e^{\frac{t^2}{2}}.$$

Find the mean and variance of X.

7. There are 1000 people applying for 70 new jobs opening up at a manufacturing plant. The company administers a test to select the best 70 applicants. The mean score turns out to be 60, and the scores have a standard deviation of 6. If a person scores 84 on the test are they guaranteed a job? To determine this, use Chebyshev's Theorem and assume that the probability distribution is symmetric about to mean.