Find the number of 4-digit codes that can be created using digits 1 through 9 if no digit is repeated and the code must be divisible by 5

To find the number of 4-digit codes that meet the conditions (using digits 1 through 9, no digit repeated, and divisible by 5), follow these steps:

Step 1: Determine divisibility by 5

A number is divisible by 5 if its last digit is 5. Therefore, the last digit of the 4-digit code must be 5.

Step 2: Arrange the remaining 3 digits

After choosing 5 as the last digit, there are 8 remaining digits ($1, 2, 3, 4, 6, 7, 8, 9$) to fill the first three positions. Since no digit can repeat, we calculate the number of permutations of these 3 digits:

$$P(8,3) = \frac{8!}{(8-3)!} = 8 \times 7 \times 6 = 336$$

Step 3: Total number of codes

Each of the 336 arrangements of the first three digits can pair with 5 as the last digit. Therefore, the total number of codes is:

$$336$$

Answer: 336 codes.

How many ways can 10 students be divided into 3 groups if the groups must have 4, 3, and 3 students each?

To determine the number of ways to divide 10 students into 3 groups of sizes 4, 3, and 3, we follow these steps:

Step 1: Choose 4 students for the first group

The number of ways to choose 4 students from 10 is:

$$\binom{10}{4} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

Step 2: Choose 3 students for the second group

After selecting the first group, 6 students remain. The number of ways to choose 3 students from these 6 is:

$$\binom{6}{3} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

Step 3: Assign the remaining 3 students to the third group

The final 3 students automatically form the third group, so there is only 1 way to assign them:

$$1$$

Step 4: Account for indistinguishable groups

Since the groups are **not labeled**, we must divide by the number of ways to arrange 3 groups (i.e., $3!$) to avoid overcounting:

$$3! = 6$$

Final Calculation:

Multiply the results and divide by $3!$:

$$\text{Total ways} = \frac{\binom{10}{4} \times \binom{6}{3} \times 1}{3!} = \frac{210 \times 20}{6} = 700$$

Answer: 700 ways.

How many 3-card hands can be formed from a deck of 52 cards if all three cards must be face cards?

There are 12 face cards in a standard deck of 52 cards (4 Jacks, 4 Queens, and 4 Kings). To form a 3-card hand where all cards are face cards, we need to choose 3 cards out of these 12.

Step 1: Use the combination formula

The number of ways to choose 3 cards from 12 is:

$$\binom{12}{3} = \frac{12!}{3!(12-3)!} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$$

Answer: 220 different 3-card hands.

In a group of 20 people, how many ways can 4 teams of 5 people each be formed?

To calculate the number of ways to form 4 teams of 5 people each from a group of 20 people, we need to account for the fact that the teams are **not labeled** (i.e., the teams are indistinguishable).

Step 1: Treat teams as labeled

First, if the teams were labeled (e.g., Team A, Team B, Team C, and Team D), the number of ways to divide the group is:

$$\frac{20!}{(5!) \times (5!) \times (5!) \times (5!)}$$

This accounts for dividing 20 people into four groups of 5 people each. Here, $5!$ accounts for the arrangements within each team.

$$\frac{20!}{(5!)^4} = \frac{20 \times 19 \times 18 \times \cdots \times 1}{(120)^4}$$

Step 2: Account for unlabeled teams

Since the teams are indistinguishable, we must divide the above result by the number of ways to arrange 4 teams, which is $4!$:

$$\text{Total ways} = \frac{\frac{20!}{(5!)^4}}{4!} = \frac{20!}{(5!)^4 \times 4!}$$

Final Calculation:

Substitute factorials:

$$20! = 20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \cdots 1$$

$$5! = 120 \quad \text{and} \quad 4! = 24$$

A 6-sided die is rolled 10 times. How many sequences are possible where the number 3 appears exactly four times?

To determine the number of possible sequences where the number 3 appears exactly four times when a 6-sided die is rolled 10 times:

Step 1: Choose the positions for the 4 occurrences of the number 3

There are 10 rolls, and we need to choose 4 of these to be the number 3. The number of ways to choose these positions is given by:

$$\binom{10}{4} = \frac{10!}{4!(10-4)!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

Step 2: Assign values to the remaining 6 rolls

For the remaining 6 rolls, any of the numbers $1, 2, 4, 5, 6$ (not 3) can appear. There are 5 possible outcomes for each of these rolls. Since there are 6 rolls, the total number of possibilities for these is:

$$5^6 = 15625$$

Step 3: Multiply the results

The total number of sequences is:

$$\binom{10}{4} \times 5^6 = 210 \times 15625 = 3281250$$

Answer: 3,281,250 sequences.

There are 6 identical red balls and 4 identical blue balls. In how many ways can they be arranged in a row?

This is a classic problem of arranging **indistinguishable objects** in a row.

We are arranging 6 identical red balls and 4 identical blue balls. The formula for the number of arrangements is:

$$\text{Number of arrangements} = \frac{(n_r + n_b)!}{n_r! \times n_b!}$$

Where:

- $n_r = 6$ (red balls),
- $n_b = 4$ (blue balls).

Step 1: Apply the formula

$$\text{Number of arrangements} = \frac{(6+4)!}{6! \times 4!} = \frac{10!}{6! \times 4!}$$

Step 2: Simplify the factorials

$$10! = 10 \times 9 \times 8 \times 7 \times 6!$$

Cancel out $6!$ from numerator and denominator:

$$\frac{10!}{6! \times 4!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$$

Step 3: Calculate

$$\frac{10 \times 9 \times 8 \times 7}{24} = \frac{5040}{24} = 210$$

Answer: 210 ways.

In how many ways can a committee of 5 people be selected from a group of 12 if at least 2 members must be women (assume there are 7 men and 5 women)?

To solve this, we consider the cases where at least 2 members of the committee are women. We calculate the number of ways for each valid combination of women and men.

Total People:

- 7 men
- 5 women

Case 1: 2 women and 3 men

$$\binom{5}{2} \times \binom{7}{3} = \frac{5 \times 4}{2} \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 10 \times 35 = 350$$

Case 2: 3 women and 2 men

$$\binom{5}{3} \times \binom{7}{2} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \times \frac{7 \times 6}{2} = 10 \times 21 = 210$$

Case 3: 4 women and 1 man

$$\binom{5}{4} \times \binom{7}{1} = \frac{5}{1} \times 7 = 5 \times 7 = 35$$

Case 4: 5 women and 0 men

$$\binom{5}{5} \times \binom{7}{0} = 1 \times 1 = 1$$

Total Number of Ways:

Add all the cases:

$$350 + 210 + 35 + 1 = 596$$

Answer: 596 ways.

Seven different gifts are to be distributed among 10 children. How many distinct results are possible if no child is to receive more than one gift?

Since no child can receive more than one gift, we are assigning 7 distinct gifts to 7 out of the 10 children. This is a permutation problem:

1. **Choose 7 children out of 10 to receive the gifts:**

$$\binom{10}{7} = \frac{10!}{7!(10-7)!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

2. **Arrange the 7 gifts among the chosen 7 children:**

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

3. **Total distinct results:**

$$\binom{10}{7} \times 7! = 120 \times 5040 = 604800$$

Answer: 604,800 distinct results.

Find the number of ways that 9 toys can be distributed to 4 children if the youngest receives 3 toys and the others each receive 2

To solve this, we distribute 9 toys among 4 children with specific constraints:

Step 1: Assign 3 toys to the youngest child

The number of ways to choose 3 toys out of 9 for the youngest child is:

$$\binom{9}{3} = \frac{9!}{3!(9-3)!} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$$

Step 2: Distribute the remaining 6 toys among the other 3 children

Each of the other 3 children must receive 2 toys. This is a problem of distributing distinct items into groups of fixed size, so we use the multinomial coefficient:

$$\frac{6!}{2! \times 2! \times 2!} = \frac{720}{2 \times 2 \times 2} = 90$$

Step 3: Multiply the results

Combine the results from Step 1 and Step 2:

$$\text{Total ways} = \binom{9}{3} \times \frac{6!}{2! \times 2! \times 2!} = 84 \times 90 = 7560$$

Answer: 7560 ways.

9 students need to be placed into 3 rooms at a conference. The first room can accommodate 4 students, the second room can accommodate 3 students, and the third room can accommodate 2 students. How many arrangements are there?

To determine the number of ways to arrange 9 students into 3 rooms with specific capacities, we'll use the multinomial coefficient.

Steps:

1. **Choose 4 students for the first room:**

$$\binom{9}{4} = \frac{9!}{4!(9-4)!} = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126$$

2. **Choose 3 students for the second room from the remaining 5 students:**

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \times 4}{2 \times 1} = 10$$

3. **The remaining 2 students go into the third room:**

$$\binom{2}{2} = 1$$

Total Arrangements:

Multiply the results from each step:

$$126 \times 10 \times 1 = 1260$$

Answer: 1260 different arrangements.

A dance class consists of 22 students, of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?

To solve this, we break it into two steps:

Step 1: Choose 5 men and 5 women

1. **Choose 5 men out of 12 men:**

$$\binom{12}{5} = \frac{12!}{5!(12-5)!} = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$

2. **Choose 5 women out of 10 women:**

$$\binom{10}{5} = \frac{10!}{5!(10-5)!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$$

Step 2: Pair off the 5 men and 5 women

The number of ways to pair 5 men with 5 women is $5!$:

$$5! = 120$$

Total Results

Multiply the results from the two steps:

$$\text{Total results} = \binom{12}{5} \times \binom{10}{5} \times 5! = 792 \times 252 \times 120 = 23990400$$

Answer: 23,990,400 results.

In a race with 10 runners, how many ways can the gold, silver, and bronze medals be awarded?

The gold, silver, and bronze medals are awarded to 3 distinct runners, and the order matters. This is a permutation problem:

$$\text{Total ways} = P(10, 3) = \frac{10!}{(10-3)!} = \frac{10 \times 9 \times 8}{1} = 720$$

Answer: 720 ways

How many 4-card hands can be made from a deck of 52 cards if exactly two of the four cards are 7s?

To create a 4-card hand with exactly two 7s, follow these steps:

1. **Choose 2 7s out of the 4 available 7s:**

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \times 3}{2} = 6$$

2. **Choose 2 non-7s from the remaining 48 cards (since there are 52 cards total, and 4 are 7s):**

$$\binom{48}{2} = \frac{48!}{2!(48-2)!} = \frac{48 \times 47}{2} = 1128$$

3. **Multiply the results to get the total number of hands:**

$$6 \times 1128 = 6768$$

Answer: 6768 4-card hands.

In how many ways can 8 people be seated in a row if:

- (a) There are no restrictions on the seating arrangement.

- (b) Persons A and B must sit beside each other.

- (c) 5 men must sit next to one another.

- (d) There are 4 married couples and each couple must sit together.

(a) No restrictions on the seating arrangement:

All 8 people can be arranged in $8!$ ways.

$$8! = 40320$$

Answer: 40320 ways

(b) Persons A and B must sit beside each other:

Treat A and B as a single "block." This gives 7 "blocks" to arrange (6 individuals + 1 block). These blocks can be arranged in $7!$ ways. Additionally, A and B can switch places within their block in $2!$ ways.

$$7! \times 2 = 5040 \times 2 = 10080$$

Answer: 10080 ways

(c) 5 men must sit next to one another:

Treat the 5 men as a single "block." This gives 4 "blocks" to arrange (3 individuals + 1 block). These blocks can be arranged in $4!$ ways. The 5 men can be arranged among themselves in $5!$ ways.

$$4! \times 5! = 24 \times 120 = 2880$$

Answer: 2880 ways

(d) 4 married couples, each couple must sit together:

Treat each couple as a single "block," giving 4 "blocks" to arrange. These blocks can be arranged in $4!$ ways. Within each block, the two people can swap places in $2!$ ways. Since there are 4 couples, this gives 2^4 arrangements for the couples.

$$4! \times 2^4 = 24 \times 16 = 384$$

Answer: 384 ways

The number of different 8-digit numbers that can be made using the digits 2, 2, 2, 3, 3, 4, 4, 4

To find the number of different 8-digit numbers that can be made using the digits $2, 2, 2, 3, 3, 4, 4, 4$, we calculate the number of permutations of these digits accounting for repeated elements:

$$\text{Total permutations} = \frac{8!}{3! \times 2! \times 3!}$$

Where:

- $8!$ is the factorial of the total digits.
- $3!$ accounts for the 3 repeated 2s.
- $2!$ accounts for the 2 repeated 3s.
- $3!$ accounts for the 3 repeated 4s.

$$\text{Total permutations} = \frac{40320}{6 \times 2 \times 6} = \frac{40320}{72} = 560$$

So, there are **560 different 8-digit numbers**.

A test consists of 12 true-false questions. How many different ways can a student mark the test paper with one answer to each question?

Each question has 2 possible answers (true or false). For 12 questions:

$$\text{Total ways} = 2^{12} = 4096$$

So, there are **4096 different ways** to mark the test paper.

The number of 3-digit pass codes using digits 0 through 9

Each digit in the passcode can be any of the 10 digits (0 to 9). For a 3-digit passcode:

$$\text{Total passcodes} = 10 \times 10 \times 10 = 10^3 = 1000$$

So, there are **1000 possible passcodes**.