MATH 1550
Winter 2025
Test 1
2025-01-27
Time Limit: 50 Minutes

Name (Print): ____________________

ID number: ____________________

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1. (5 points) A class of 10 students consists of 4 chemistry majors 3 mathematics majors and 3 physics majors.
   (a) (1 point) How many groups of 5 can be formed containing exactly 3 mathematics majors?
   (b) (1 point) How many groups of 4 can be formed with 2 chemistry majors and 2 physics majors?
   (c) (2 points) How many groups of 4 can be formed if the group should contain at least 1 student of each major?
   (d) (1 point) What is the probability that a randomly chosen group of 4 contains no mathematics majors?

   (Show your work/formula and compute a final answer.)

   Solution:

   (a) Choose all 3 of the 3 mathematics majors. Then choose any 2 of the remaining 7 students. The multiplication rule yields
   $$\binom{3}{3} \cdot \binom{7}{2} = (1)(21) = 21.$$
   (b) Choose 2 of the 4 chemistry majors. Then choose 2 of the 3 physics majors. The multiplication rule yields
   $$\binom{4}{2} \cdot \binom{3}{2} = (6)(3) = 18.$$
   (c) To have one student of each major, we can have (a) 2 chemistry majors, 1 mathematics major and 1 physics major; or (b) 1 chemistry major, 2 mathematics majors and 1 physics major; or (c) 1 chemistry major, 1 mathematics major and 2 physics majors. Therefore, we obtain
   $$\binom{4}{2} \cdot \binom{3}{1} \cdot \binom{3}{1} + \binom{4}{1} \cdot \binom{3}{2} \cdot \binom{3}{1} + \binom{4}{1} \cdot \binom{3}{1} \cdot \binom{3}{2}$$
   $$= (6)(3)(3) + (4)(3)(3) + (4)(3)(3) = 54 + 36 + 36 = 126.$$
   (d) There are $\binom{10}{4} = 210$ groups of 4 that are all equally like to be formed. Disregarding mathematics majors, there are 7 students in the class; therefore $\binom{7}{4} = 35$ groups of 4 can be formed that contain no mathematics majors. Hence, the probability is $\frac{35}{210} = \frac{1}{6} \approx 0.1667$.

$\square$

2. (2 points) 12 players will be assigned to four different teams A, B, C, D. 5 of them will join Team A, 4 of them will join Team B, 2 of them will join Team C and 1 of them will join Team D. In how many ways can this be done?

   (Show your work/formula and compute a final answer.)

   Solution:

   There are 12 players of four types regarding their teams, with 5 of the first type, 4 of the second type, 2 of the third type and 1 of the fourth type. Thus there are

   $$\frac{12!}{5! \cdot 4! \cdot 2! \cdot 1!} = \frac{479001600}{(120)(24)(2)(1)} = 83160$$

   different ways this can be done.

   $\square$

3. (3 points) Three cards are drawn randomly from a regular deck of 52 cards. What is the probability that the cards belong to exactly two different suits? (Suits in a deck are $\heartsuit, \clubsuit, \spadesuit, \diamondsuit$.) For example $\boxed{2\heartsuit}$ $\boxed{5\heartsuit}$ $\boxed{Q\clubsuit}$ is one such example.

   (Show your work/formula, give an exact fractional answer, and give a rounded answer to 4 decimal places.)

   Solution:

   In order for three cards to belong to two suits, one suit must appear on two cards and the other suit on the remaining card. There are 4 suits. Choose one and pick two of the 13 cards of that suit in $4 \cdot \binom{13}{2} = 312$ ways. The remaining card will be any one of the 39 cards that is not of the same suit. Therefore, there are $312 \cdot 39 = 12168$ such 3-card hands.

   The total number of 3 card hands is

   $$\binom{52}{3} = 22100.$$

   Therefore, the probability of drawing three cards belonging to exactly two different suits is

   $$\frac{12168}{22100} \approx 0.5506.$$

   $\square$

4. (4 points) A survey of 50 students indicates that 20 students play basketball every week, 15 students play volleyball every week and 8 students play both basketball and volleyball every week. One of these 50 students is chosen randomly.

   (a) (1 point) What is the probability that the student plays volleyball every week?

   (b) (1 point) What is the probability that the student plays at least one of basketball or volleyball every week?

   (c) (1 point) What is the probability that the student plays neither basketball nor volleyball every week?

   (d) (1 point) Given that the student plays basketball every week, what is the probability that they play volleyball every week?

   (Show your work and give a rounded answer to 4 decimal places if needed.)

   Solution:

   Let B be the event that a randomly selected student plays basketball every week, and V the event that a randomly selected student plays volleyball every week.

   (a) $P(V) = \frac{15}{50} = 0.3$.

   (b) By the inclusion-exclusion principle

   $$P(B \cup V) = P(B) + P(V) - P(B \cap V) = 0.4 + 0.3 - 0.16 = 0.54.$$

   (c)

   $$P((B \cup V)') = 1 - P(B \cup V) = 1 - 0.54 = 0.46.$$

   (d)

   $$P(V|B) = \frac{P(V \cap B)}{P(B)} = \frac{0.16}{0.4} = \frac{2}{5} = 0.4.$$

   $\square$