

CSE 321 - Introduction to Algorithm Design

Homework 03

Deadline: 23:55 November 14th, 2016

1. a) If we measure the size of an instance of the problem of computing the greatest common divisor of m and n by the size of the second parameter n , by how much can the size decrease after one iteration of Euclid's algorithm?

b) Prove that an instance size will always decrease at least by a factor of 2 after two successive iterations of Euclid's algorithm. Let $r = m \bmod n$. Investigate two cases of r 's value relative to n 's value.

2. An algorithm which ensures that each new permutation is created by exchanging only two neighboring elements is called a minimal change algorithm. Design a minimal change decrease-by-one algorithm for generating permutations of a given set of integers $1, 2, \dots, n$.

3. Outline an algorithm for deleting a key from a binary search tree. Consider separately three cases: (1) the key's node is a leaf; (2) the key's node has one child; (3) the key's node has two children.

a) Would you classify this algorithm as a variable-size-decrease algorithm?

b) What is the time efficiency class of your algorithm?

4. Suppose that an array contains n numbers, each of which is -1 , 0 , or 1 . Then, the array can be sorted in $O(n)$ time in the worst case. Prove or disprove that statement.

5. Given the array $A[1..n]$ of sorted distinct integers, design a divide and conquer algorithm that finds an index i such that $A[i] = i$. Your algorithm should run in $O(\log n)$ time.