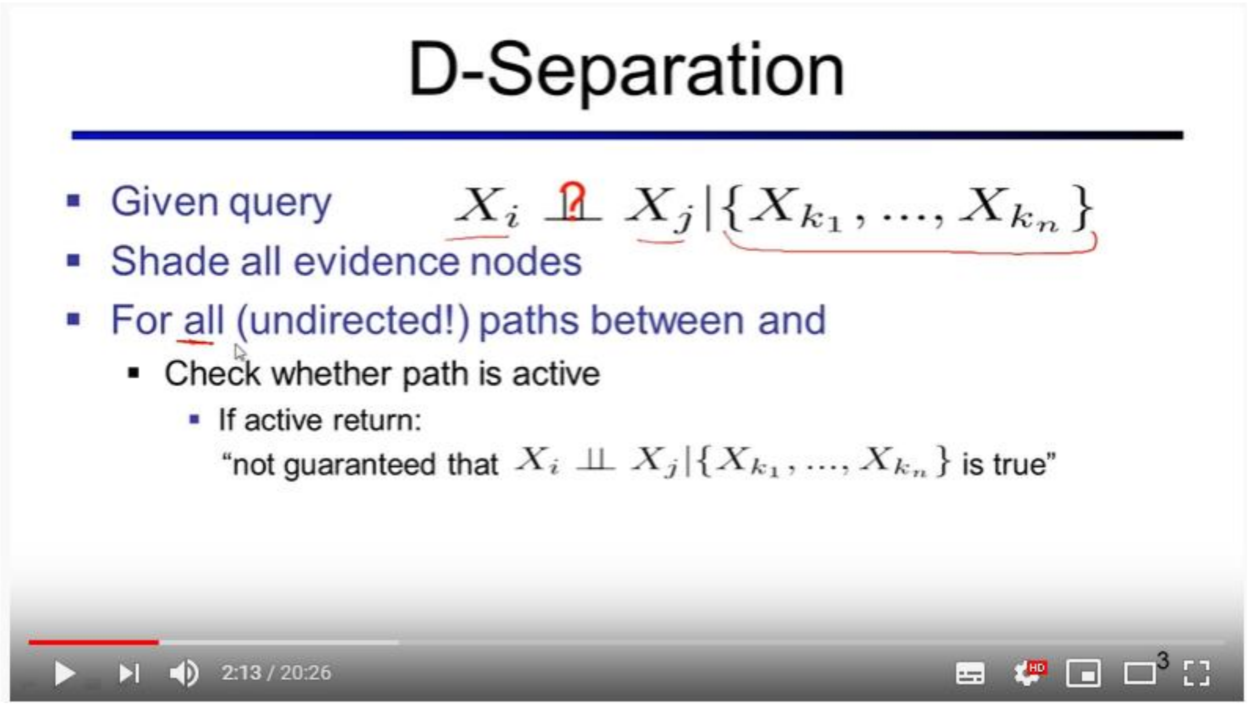
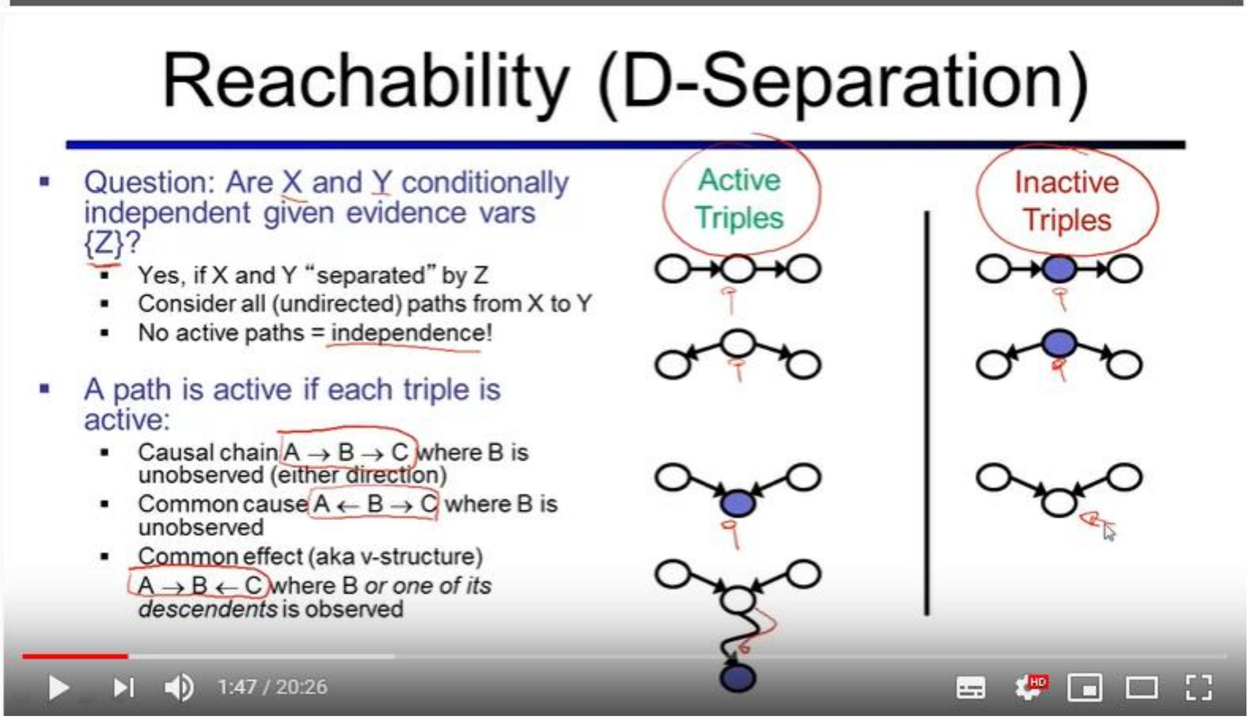




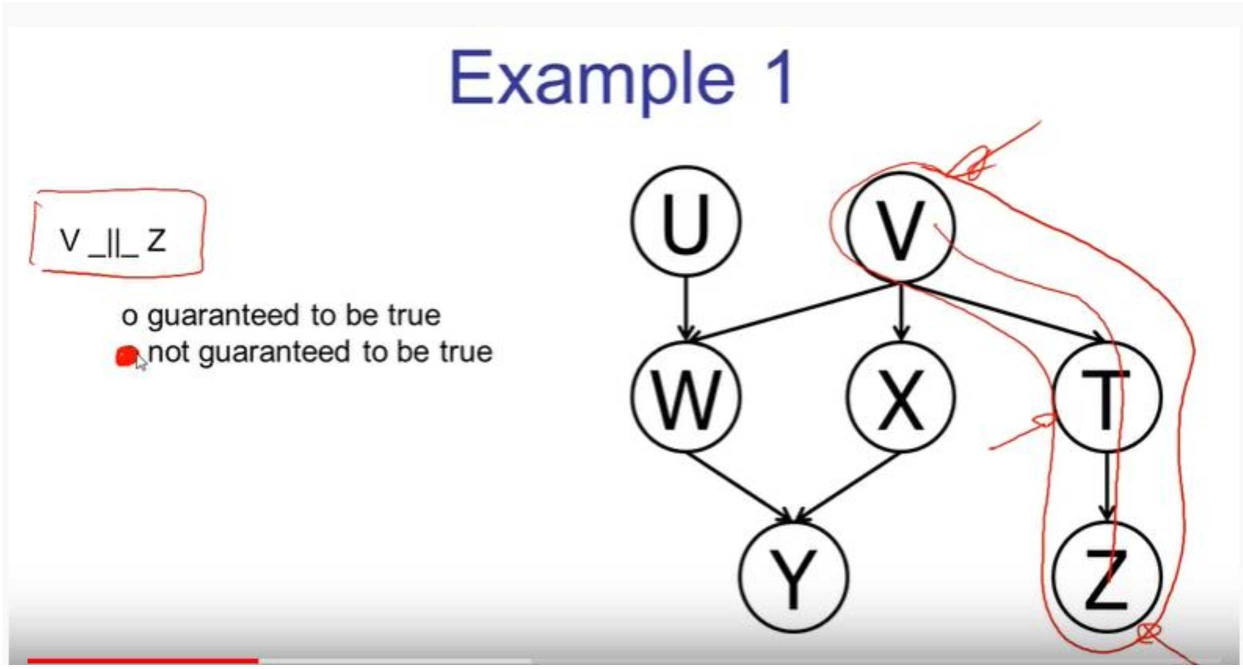
no active paths = independence. Active paths not guaranteed to be true.

Algorithm:

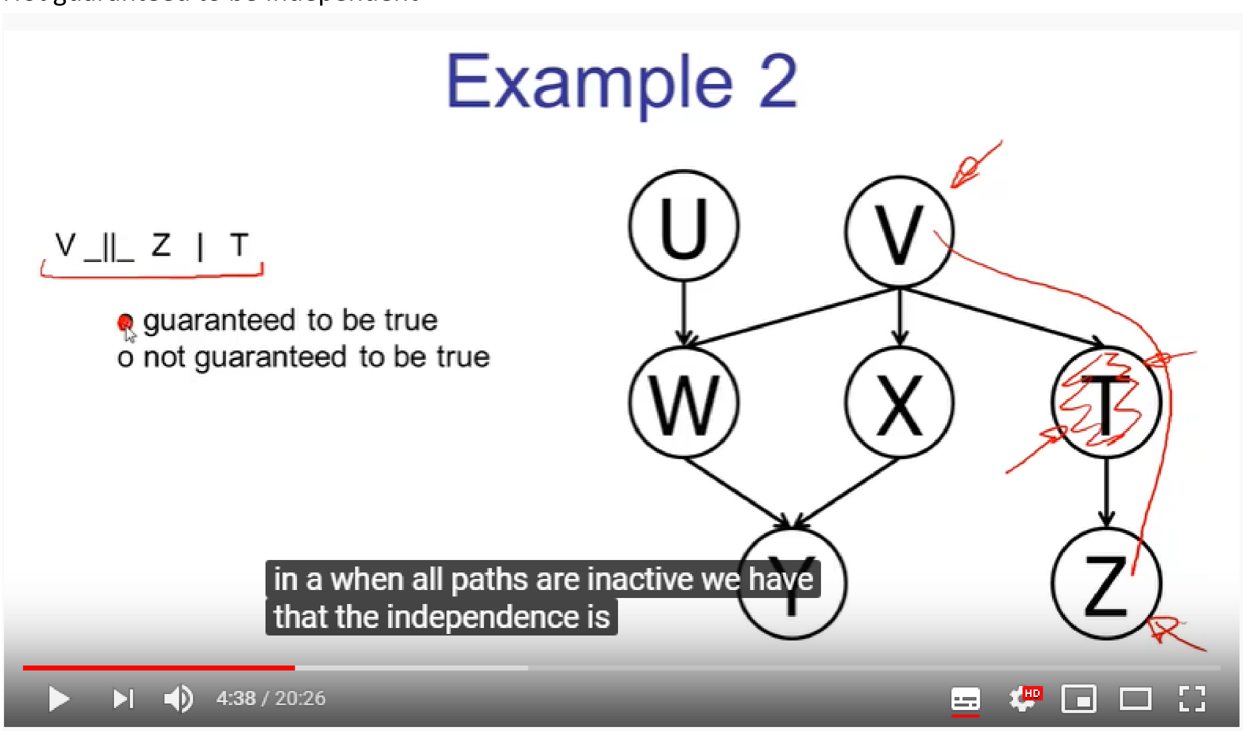
- Consider all (undirected) paths from x to y
- For all paths:
  - For every triple:
    - IF TRIPLE IS INACTIVE
      - Then all other triples are inactive (even you found active one before),
      - the path is inactive,
      - Continue, (go other path);
    - IF TRIPLE IS ACTIVE
      - If one triple only, its active
      - If there are more than one triple, go for other triples.

- IF ALL PATHS ARE INACTIVE
  - Independence
- IF EVEN ONE PATH IS ACTIVE:
  - Not guaranteed to be indepedent

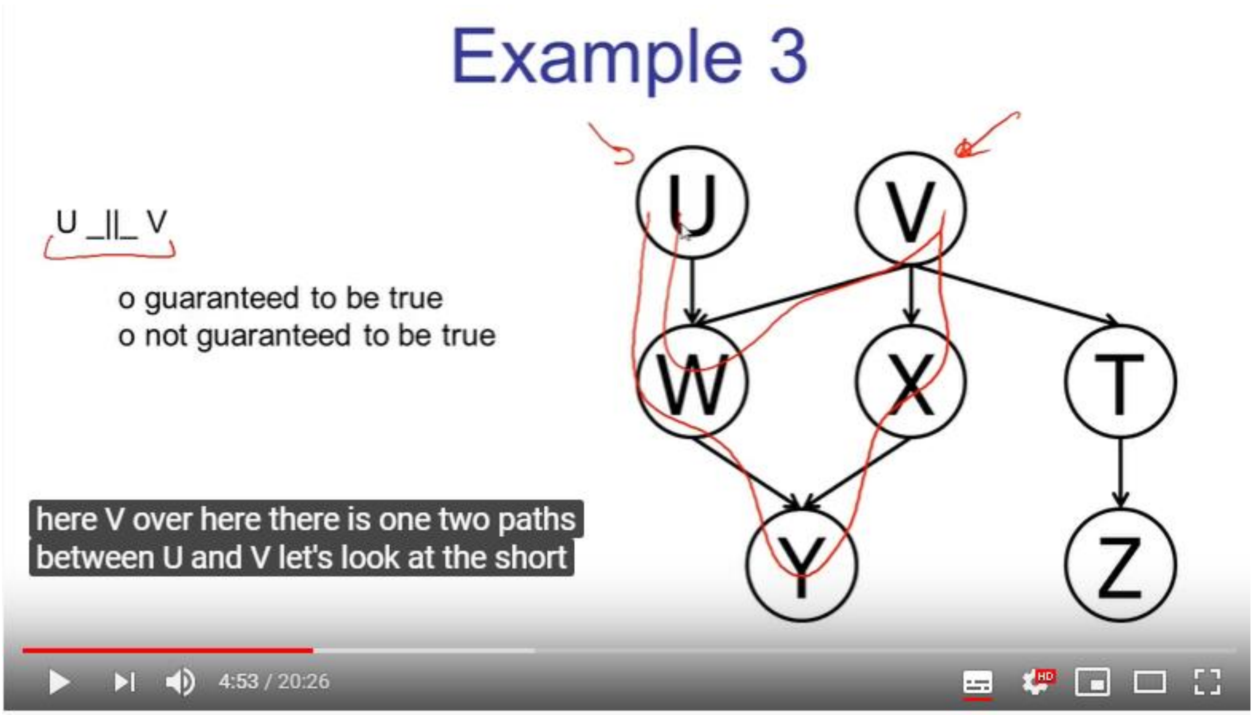
In V structure, the node or any of the descendants.
Active path = not guaranteed to be independent = not guaranteed to be true



Whether V is independent of Z
There is only one path, and one triple : V -> T -> Z
This triple is active, if triple is active, because there is only one triple the path is active
Not guaranteed to be independent



Whether V is independent of Z given T
There is only one path, and one triple : V -> T -> Z
This triple is inactive, if triple is inactive, all other triples are inacitive, that means path is inactive guaranteed to be independent



Whether U is independent of V

Two paths:

First Path

U->W->V

One triple which is V structure, W or the descended Y are not observed, that means triple is inactive. If triple is inactive, all other triples are inactive, that means path is inactive

Second Path

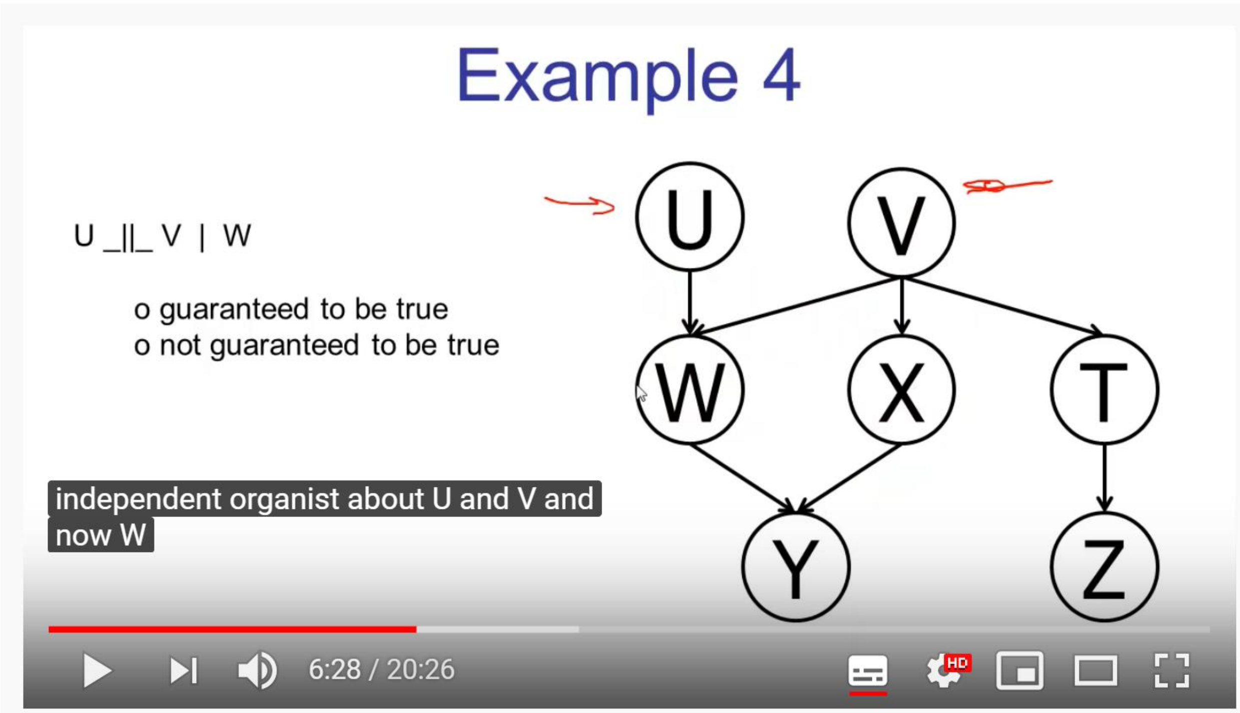
U->W->Y-X->V

There are several triples

U->W->Y : casual chain, active triple, triple is active, but we need to continue to look for other variables

W->Y->X : v-structure, inactive triple once a inactive triple found, the path also means inactive

guaranteed to be independent/true



Whether U is independent of V given W
Two paths:

First Path

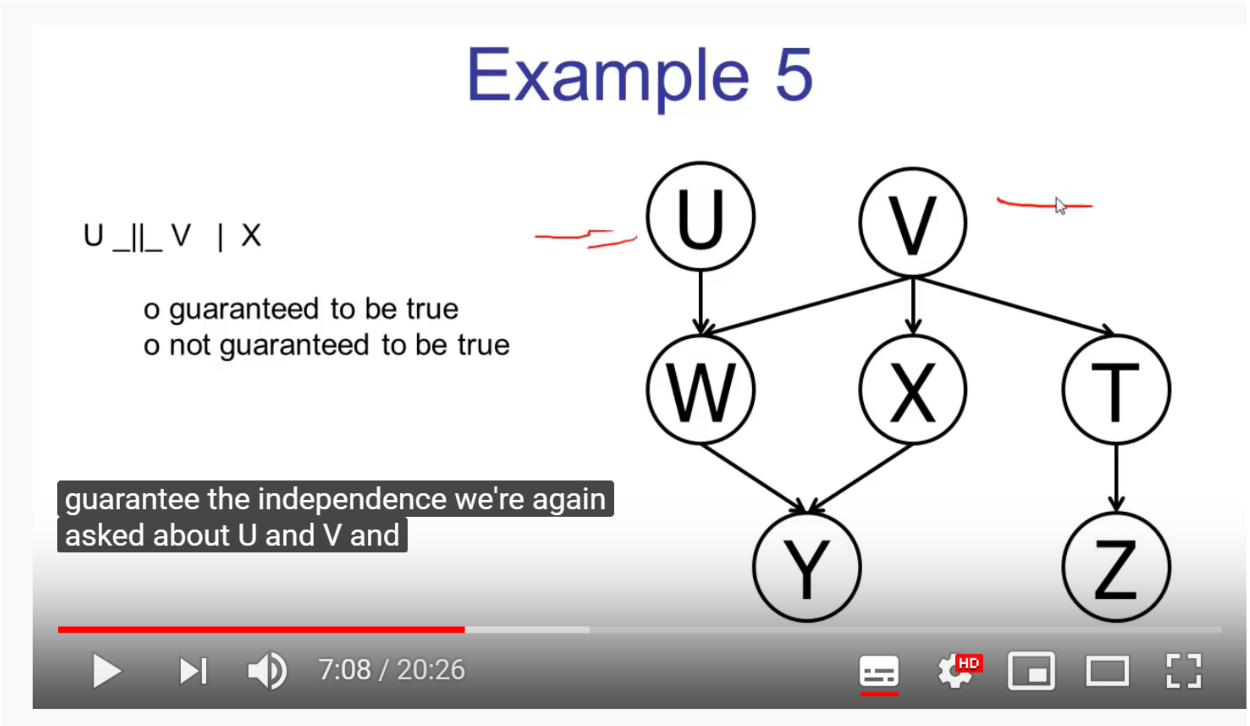
U->W->V

One triple which is V structure, W observed, that means triple is active. Because this is the only triple that path have, so the path is active

Second Path

U->W->Y-X->V

Because we find an path that is active, we now directly that not guaranteed to be independent



Whether U and V independent given X,

Two paths

Path 1

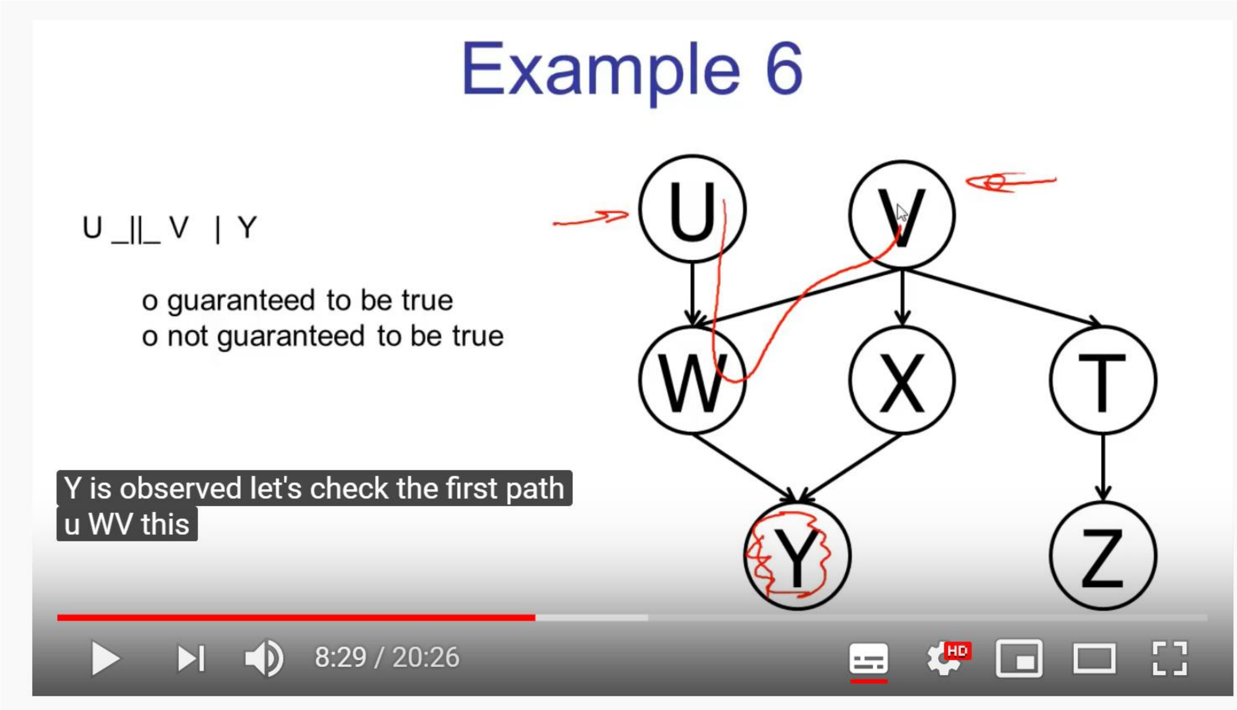
U-W-V: One triple which is V structure, W or the descended Y are not observed, that means triple is inactive. If triple is inactive, all other triples are inactive, that means path is inactive

Path 2:

U-W-Y : active triple, continue for other triples

W-Y-X : Inactive triple, if one triple is inactive, once we find inactive path then it means all path is inactive

guaranteed to be independent/true



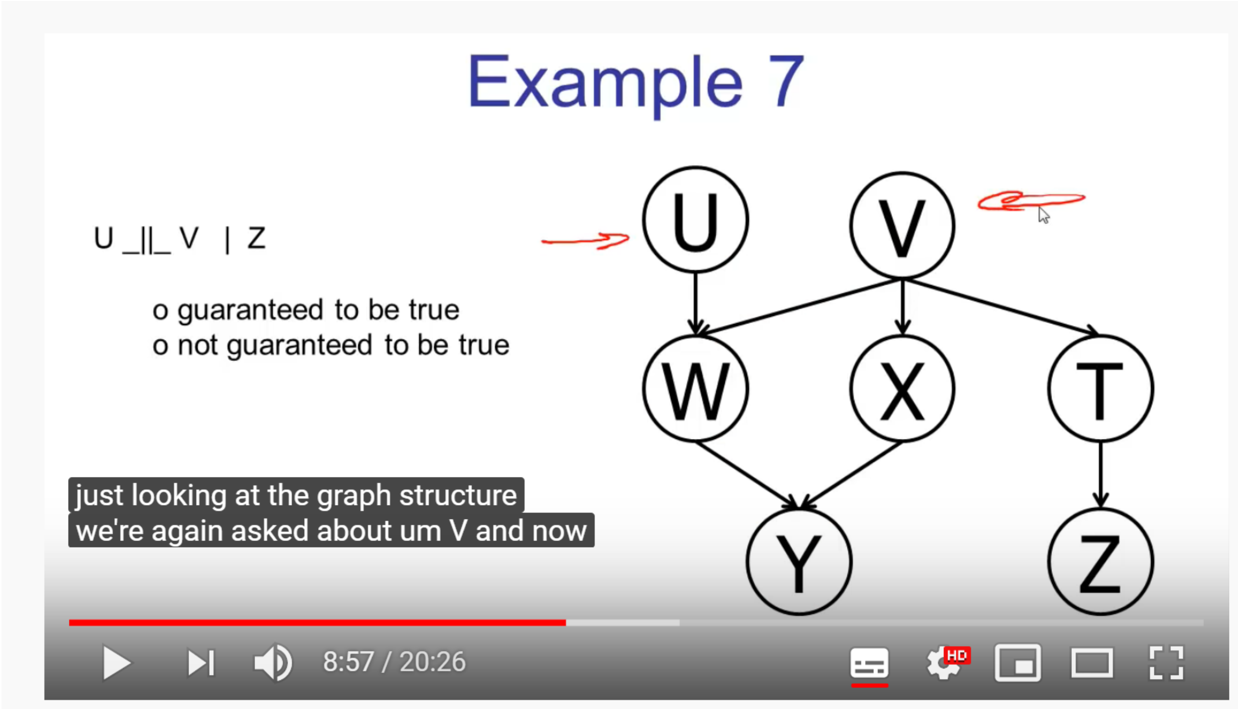
Whether U and V is independent given Y

Two path

Path 1

U-W-V: v structure, W unobserved but descended Y is observed, so its active. Because this is the only triple that path have the path is active.

Because Path 1 is active, no need to check for other paths, its not guaranteed to be independent.



Whether U and V independent given Z

Two Paths

Path 1

U-W-V : Inactive, Path is inactive

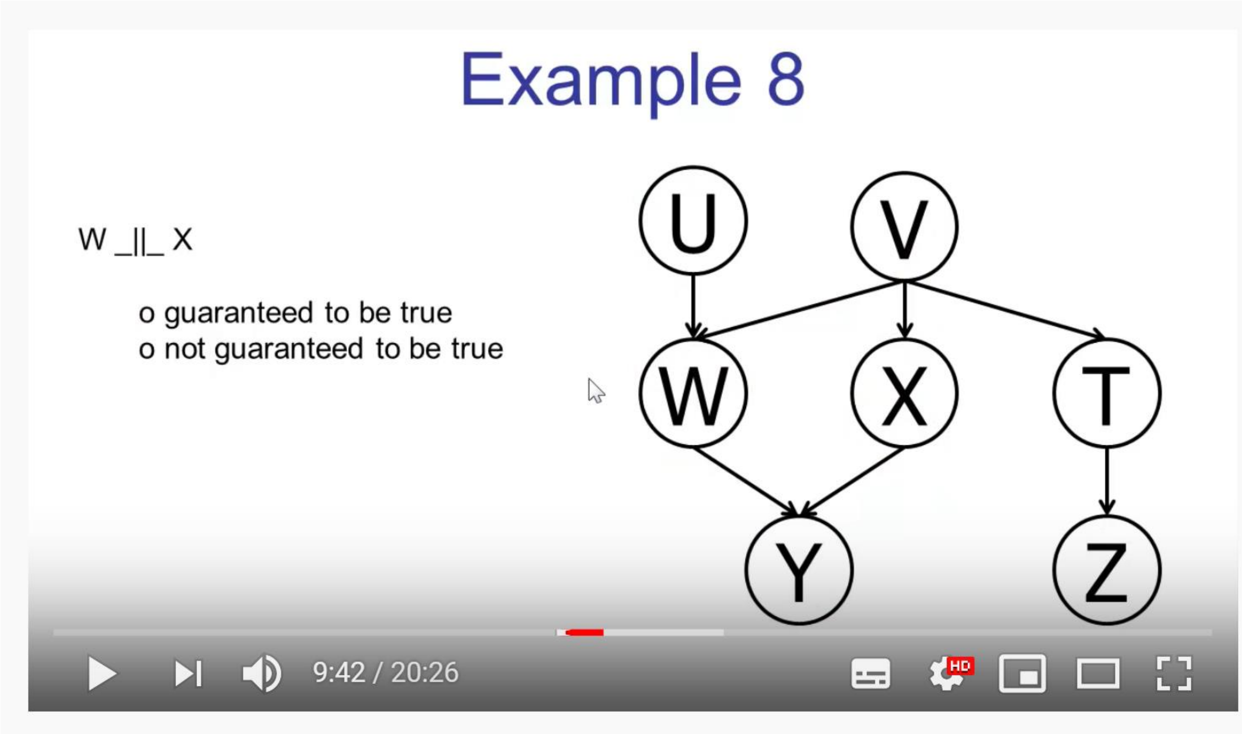
Path 2

U-W-Y-X-V:

U-W-Y (Triple) : Active go for other triples

W-Y-X (Triple) : Inactive : All path is inactive

Because both path are İndependent its guaranteed to be true



Whether W and X independent

Two Path

Path 1

W-Y-X : Inactive, path is inactive

Path 2

W-V-X : Active, this is the only triple that path is have, so the path is active.

W and X are not guaranteed independent

If you have time to do more please refer : https://www.youtube.com/watch?v=yDs_q6jKHb0

If you have time to do more please refer (2) : https://www.youtube.com/watch?v=i0CGsHhjISU

Independence

- Two events A and B are independent if and only if:

$$P(A, B) = P(A)P(B)$$

Mutually Exclusive Events

Mutually Exclusive: can't happen at the same time.

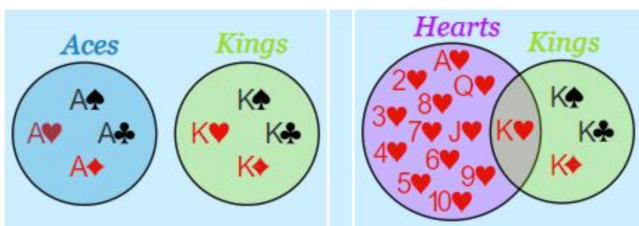
Examples:

- Turning left and turning right are Mutually Exclusive (you can't do both at the same time)
- Tossing a coin: Heads and Tails are Mutually Exclusive
- Cards: Kings and Aces are Mutually Exclusive

What is **not** Mutually Exclusive:

- Turning left and scratching your head can happen at the same time
- Kings and Hearts, because we can have a King of Hearts!

Like here:



Aces and Kings are **Mutually Exclusive** (can't be both)	Hearts and Kings are **not** Mutually Exclusive (can be both)

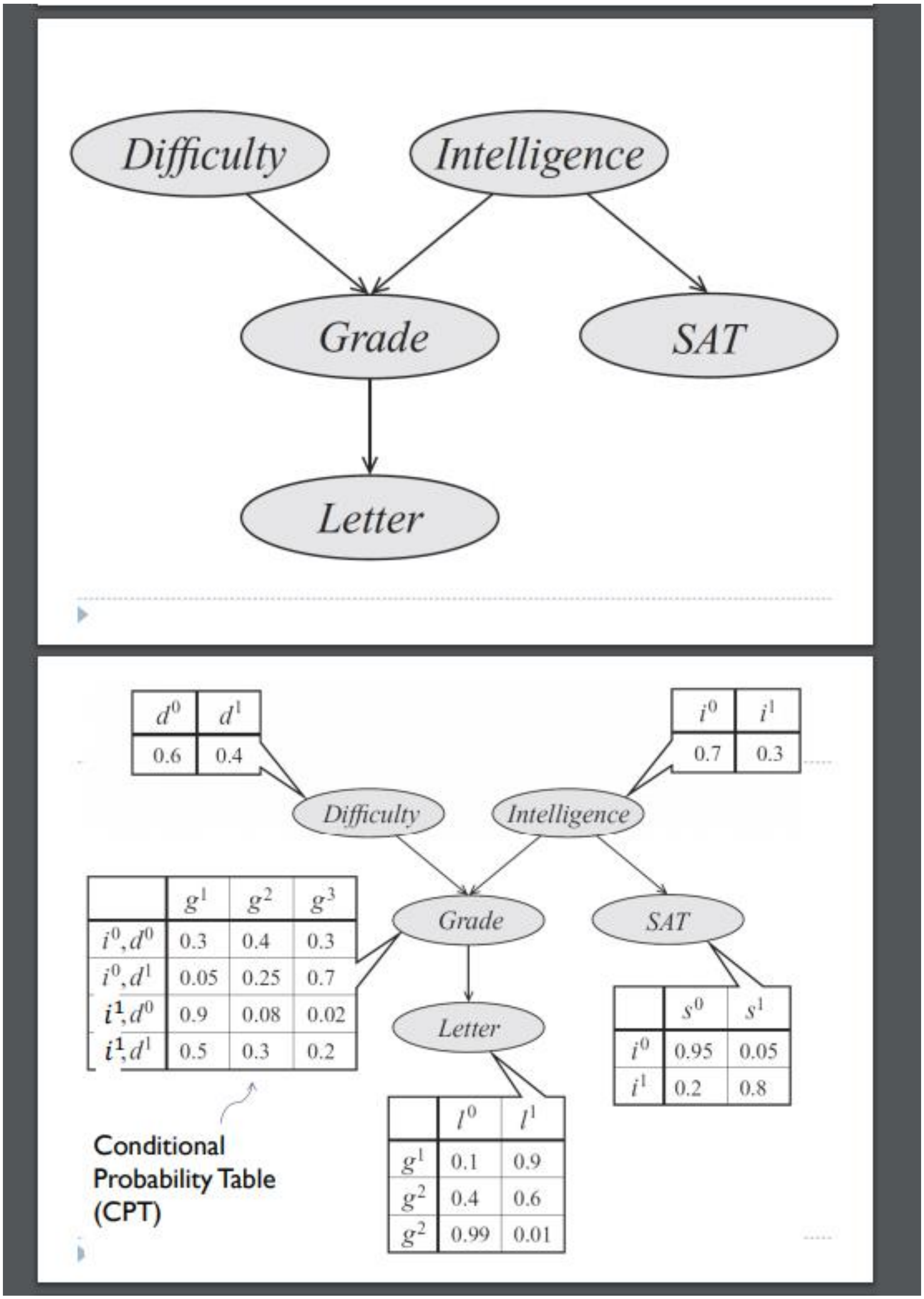
Independence

- Two events A and B are independent if and only if:

$$P(A, B) = P(A)P(B)$$

A Bayesian network is directed acyclic graph

A Directed Acyclic Graph (DAG) is a type of graph in which it's impossible to come back to the same node by traversing the edges.



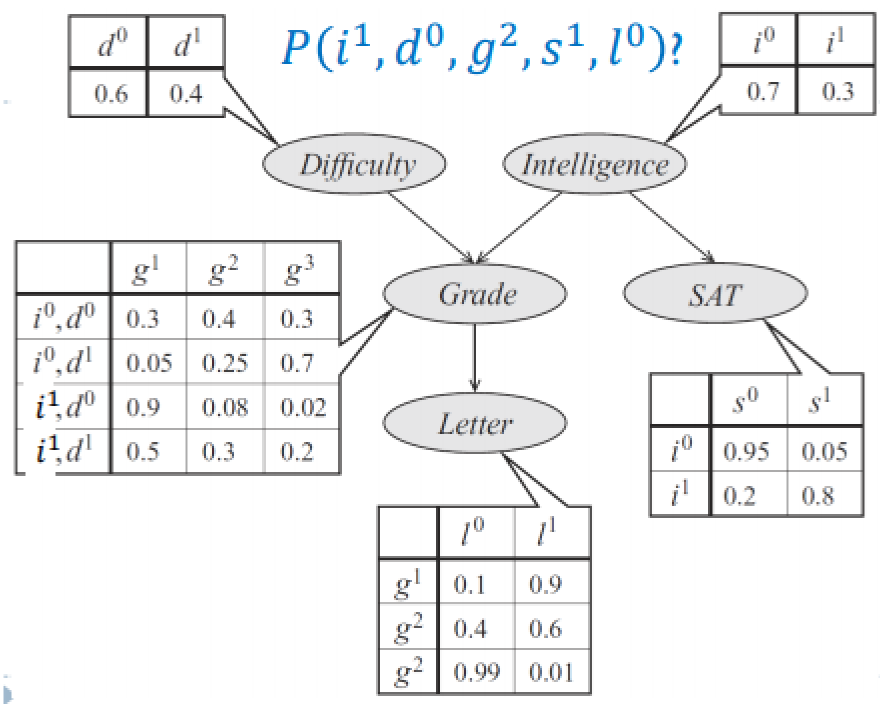
What does this wacky thing do?

- BNs represent the joint distribution compactly

- You can obtain the BN's probabilities for an event by multiplying the relevant values from each CPT:

$$P(i^1, d^0, g^2, s^1, l^0) = \cdots$$

$P(i^1, d^0, g^2, s^1, l^0)$?



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	g^1	g^2	g^3
i^0, d^0	0.3	0.4	0.3
i^0, d^1	0.05	0.25	0.7
i^1, d^0	0.9	0.08	0.02
i^1, d^1	0.5	0.3	0.2

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^2	0.99	0.01

What does this wacky thing do?

- BNs represent the joint distribution compactly
- You can obtain the BN's probabilities for an event by multiplying the relevant values from each CPT:

$$
\begin{aligned}
&P(i^1, d^0, g^2, s^1, l^0) \\
&= P(i^1)P(d^0)P(g^2|i^1, d^0)P(s^1|i^1)P(l^0|g^2) \\
&= 0.3 \cdot 0.6 \cdot 0.08 \cdot 0.8 \cdot 0.4 = 0.004608
\end{aligned}
$$

For instance, if we're asked to figure out "Is P(A|BDF) = P(A|DF)?", we can convert it into an independence question like this: "Are A and B independent, given D and F?"

to a smaller form. After simplification, the joint distribution for a Bayesian network is equal to the product of P(node|parents(node)) for all nodes, stated below:

$$P(X_1,\ldots,X_n)=\prod_{i=1}^{n}P(X_i \mid X_1,\ldots,X_{i-1})=\prod_{i=1}^{n}P(X_i \mid Parents(X_i))$$

What Is a Joint Probability?

Joint probability is a statistical measure that calculates the likelihood of two events occurring together and at the same point in time. Joint probability is the probability of event Y occurring at the same time that event X occurs.