

# MATH1013 Tutorial

## Limit Laws

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### 1 Revision

Suppose  $c$  is a constant and the limits  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  exists. Then the following laws are valid:

- (1) Constant Law:

$$\lim_{x \rightarrow a} c = c$$

- (2) Sum Law (Law of Addition)

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

- (3) Difference Law (Law of Subtraction)

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

- (4) Constant Multiple Law

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$$

- (5) Product Law (Law of Multiplication)

$$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$$

- (6) Quotient Law (Law of Division)

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \text{ if } \lim_{x \rightarrow a} g(x) \neq 0$$

- (7) Special Limit

$$\lim_{x \rightarrow a} x = a$$

- (8) Power Law

$$\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$$

if  $n$  is a positive integer.

- (9) Power Special Limit

$$\lim_{x \rightarrow a} x^n = a^n$$

if  $n$  is positive.

- (10) Root Special Limit

$$\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$$

where  $n$  is a positive integer. If  $n$  is positive, then it is assumed that  $a > 0$ .

- (11) Root Law

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$$

where  $n$  is positive integer. It is assumed that  $\lim_{x \rightarrow a} f(x) > 0$  if  $n$  is even.

## 2 Examples

### Example 1

Use the Limit Laws and the graphs of  $f$  and  $g$  in Figure 1 to evaluate the following limits, if they exist.

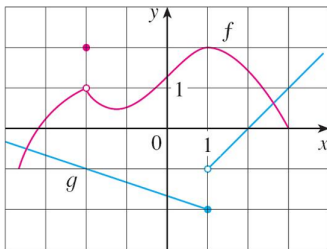


Figure 1: Example 1

- (a)  $\lim_{x \rightarrow -2} [f(x) + 5g(x)]$   
 (b)  $\lim_{x \rightarrow 1} [f(x)g(x)]$  ;  $\lim_{x \rightarrow 1^+} [f(x)g(x)]$  ;  $\lim_{x \rightarrow 1^-} [f(x)g(x)]$   
 (c)  $\lim_{x \rightarrow -2} \frac{f(x)}{g(x)}$

#### SOLUTION

- (a) From the graph and the definition of limit, we know that:

$$\lim_{x \rightarrow -2} f(x) = 1 \text{ and } \lim_{x \rightarrow -2} g(x) = -1$$

By applying Sum Law (Law of Addition) and Constant Multiple Law, we can get:

$$\begin{aligned} \lim_{x \rightarrow -2} [f(x) + 5g(x)] &= \lim_{x \rightarrow -2} f(x) + \lim_{x \rightarrow -2} 5g(x) && \text{Apply Sum Law} \\ &= \lim_{x \rightarrow -2} f(x) + 5 \lim_{x \rightarrow -2} g(x) && \text{Apply Constant Multiple Law} \\ &= 1 + 5(-1) \\ &= -4 \end{aligned}$$

- (b) From the graph and the definition of limit, we know that:

$$\lim_{x \rightarrow 1} f(x) = 2 \text{ and } \lim_{x \rightarrow 1} g(x) \text{ does not exist}$$

Therefore  $\lim_{x \rightarrow 1} [f(x)g(x)]$  does not exist. But we can also indicate from the graph that

$$\lim_{x \rightarrow 1^+} g(x) = -2 \text{ and } \lim_{x \rightarrow 1^-} g(x) = -1$$

So one-side limits can be calculated using Product Law (Law of Multiplication):

$$\begin{aligned} \lim_{x \rightarrow 1^+} [f(x)g(x)] &= \lim_{x \rightarrow 1^+} f(x) \lim_{x \rightarrow 1^+} g(x) && \text{Apply Product Law} \\ &= 2 \times (-2) \\ &= -4 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 1^-} [f(x)g(x)] &= \lim_{x \rightarrow 1^-} f(x) \lim_{x \rightarrow 1^-} g(x) && \text{Apply Product Law} \\ &= 2 \times (-1) \\ &= -2 \end{aligned}$$

If we recall, The left and right limits aren't equal, so  $\lim_{x \rightarrow 1} [f(x)g(x)]$  does not exist.

(c) From the graph and the definition of limit, we know that:

$$\lim_{x \rightarrow -2} f(x) = 1 \text{ and } \lim_{x \rightarrow -2} g(x) = -1$$

By applying Quotient Law (Law of Division), we can get:

$$\begin{aligned} \lim_{x \rightarrow -2} \frac{f(x)}{g(x)} &= \frac{\lim_{x \rightarrow -2} f(x)}{\lim_{x \rightarrow -2} g(x)} && \text{Apply Quotient Law} \\ &= 1 \div -1 \\ &= -1 \end{aligned}$$

### 3 Practices

#### Practice 1

Use the condition and graph in Example 1 and calculate:

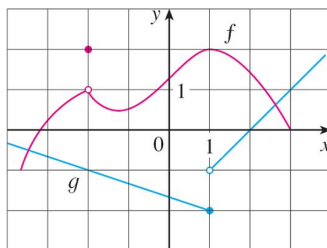


Figure 2: Example 1

(a)  $\lim_{x \rightarrow 1} [8f(x) - g(x)]$  ;  $\lim_{x \rightarrow 1^+} [8f(x) - g(x)]$  ;  $\lim_{x \rightarrow 1^-} [8f(x) - g(x)]$

(b)  $\lim_{x \rightarrow -2} [3f(x)g(x)]$

(c)  $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)}$

## Practice 2

Compute the following limits:

(a)  $\lim_{x \rightarrow -3} \frac{x^2+5x+6}{x+3}$

(b)  $\lim_{x \rightarrow 42} 1337$

(c)  $\lim_{x \rightarrow -1} \frac{1}{|x|}$

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