PDEV6800Z: Introduction to Teaching and Learning in Higher Education
Teaching Demo Handout
T08 Group 6

MATH1013: Calculus IB
NOV 2022

MATH1013 Tutorial

Limit Laws

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1 Revision

Suppose c is a constant and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to} g(x)$ exits. Then the following laws are valid:

(1) Constant Law:

$$\lim_{x\to a} c = c$$

(2) Sum Law (Law of Addition)

$$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

(3) Difference Law (Law of Subtraction)

$$\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$

(4) Constant Multiple Law

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

(5) Product Law (Law of Multiplication)

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$$

(6) Quotient Law (Law of Division)

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \text{ if } \lim_{x\to a} g(x) \neq 0$$

(7) Special Limit

$$\lim_{x\to a} x = a$$

(8) Power Law

$$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$$

if n is a positive integer.

(9) Power Special Limit

$$\lim_{x\to a} x^n = a^n$$

if n is positive.

(10) Root Special Limit

$$\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$$

where n is a positive integer. If n is positive, then it is assumed that $a > 0$.

(11) Root Law

$$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$$

where n is positive integer. It is assumed that $\lim_{x\to a} f(x) > 0$ if n is even.

2 Examples

Example 1

Use the Limit Laws and the graphs of f and g in Figure 1 to evaluate the following limits, if they exist.

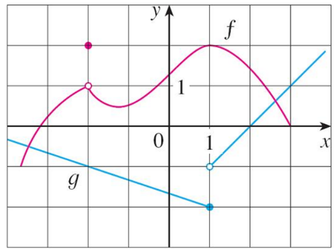

Figure 1: Example 1

(a) $\lim_{x\to-2}[f(x)+5g(x)]$

(b) $\lim_{x\to1}[f(x)g(x)]$; $\lim_{x\to1^+}[f(x)g(x)]$; $\lim_{x\to1^-}[f(x)g(x)]$

(c) $\lim_{x\to-2}\frac{f(x)}{g(x)}$

SOLUTION

(a) From the graph and the definition of limit, we know that:

$$\lim_{x\to-2} f(x) = 1 \textbf{ and } \lim_{x\to-2} g(x) = -1$$

By applying Sum Law (Law of Addition) and Constant Multiple Law, we can get:

$$\begin{aligned}\lim_{x\to-2}[f(x)+5g(x)] &= \lim_{x\to-2} f(x) + \lim_{x\to-2} 5g(x) && \text{Apply Sum Law}\\ &= \lim_{x\to-2} f(x) + 5\lim_{x\to-2} g(x) && \text{Apply Constant Multiple Law}\\ &= 1+5(-1)\\ &= -4\end{aligned}$$

(b) From the graph and the definition of limit, we know that:

$$\lim_{x\to1} f(x) = 2 \textbf{ and } \lim_{x\to1} g(x) \text{ does not exist}$$

Therefore $\lim_{x\to1}[f(x)g(x)]$ does not exist. But we can also indicate from the graph that

$$\lim_{x\to1^+} g(x) = -2 \textbf{ and } \lim_{x\to1^-} g(x) = -1$$

So one-side limits can be calculated using Product Law (Law of Multiplication):

$$\begin{aligned}\lim_{x\to1^+}[f(x)g(x)] &= \lim_{x\to1^+} f(x) \lim_{x\to1^+} g(x) && \text{Apply Product Law}\\ &= 2\times(-1)\\ &= -2\end{aligned}$$

$$\begin{aligned}\lim_{x\to1^-}[f(x)g(x)] &= \lim_{x\to1^-} f(x) \lim_{x\to1^-} g(x) && \text{Apply Product Law}\\ &= 2\times(-2)\\ &= -4\end{aligned}$$

If we recall, The left and right limits aren't equal, so $\lim_{x\to1}[f(x)g(x)]$ does not exist.

(c) From the graph and the definition of limit, we know that:

$$\lim_{x\to -2} f(x) = 1 \textbf{ and } \lim_{x\to -2} g(x) = -1$$

By applying Quotient Law (Law of Division), we can get:

$$\begin{aligned} \lim_{x\to -2} \frac{f(x)}{g(x)} =& \frac{\lim_{x\to -2} f(x)}{\lim_{x\to -2} g(x)} && \text{Apply Quotient Law} \\ =& 1 \div -1 \\ =& -1 \end{aligned}$$

3 Practices

Practice 1

Use the condition and graph in Example 1 and calculate:

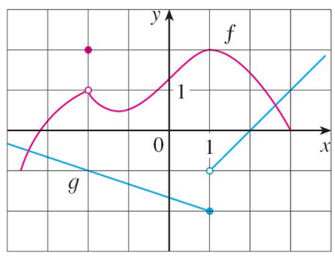

Figure 2: Example 1

(a) $\lim_{x\to 1}[8f(x) - g(x)]$; $\lim_{x\to 1^+}[8f(x) - g(x)]$; $\lim_{x\to 1^-}[8f(x) - g(x)]$
Answer: Does not exist; 17 ; 18

(b) $\lim_{x\to -2}[3f(x)g(x)]$
Answer: -6

(c) $\lim_{x\to 2} \frac{f(x)}{g(x)}$
Answer: Does not exist

Practice 2

Compute the following limits:

(a) $\lim_{x\to-3} \frac{x^2+5x+6}{x+3}$
Answer: Does not exist

(b) $\lim_{x\to42} 1337$
Answer: 1337

(c) $\lim_{x\to-1} \frac{1}{|x|}$
Answer: Does not exist

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