ISTANBUL TECHNICAL UNIVERSITY

UUM 534

Aircraft Flight Control Systems Report I

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Contents

1 Introduction

In the second part of the assignment, we obtained the matrices A and B. In order to go further, longitudinal and lateral matrices were required. Longitudinal matrix for A is obtained using V_T, α, θ, and q while for B, δ_t and δ_e are used. Basically, the new matrices only include those values, and its component relations with each other. However, we unfortunately got the following matrix:

```
A_long =

   -0.0066    0.6461   -9.7524   -0.0000
   -0.0005   -0.7217   -0.0000    1.0000
         0         0         0    1.0000
    0.0000   -0.0448         0   -0.1119
```

Figure 1: Longitudinal Matrix Obtained Directly from 12×12 A matrix

and the eigenvalues:

```
A_long_eig =

  -0.6344 + 0.0000i
  -0.2094 + 0.0000i
   0.0018 + 0.0400i
   0.0018 - 0.0400i
```

Figure 2: Eigenvalues of the Longitudinal Matrix

As it can be seen, three of the roots are on the right side axis, which is not a satisfactory result. Since we couldn't figure out the problem, we opted for a different method to obtain the longitudinal matrix. In Steven's book, an alternative approach is luckily provided. The approach is given as

$$
\begin{aligned}
\mathrm{m}\dot{V}_T &= F_T \cos(\alpha + \alpha_T) - D - \mathrm{m}g_D \sin\gamma \\
\mathrm{m}\dot{\gamma}V_T &= F_T \sin(\alpha + \alpha_T) + L - \mathrm{m}g_D \cos\gamma \\
\dot{\alpha} &= Q - \dot{\gamma} \\
\dot{Q} &= \frac{m}{J_y}
\end{aligned} \tag{1}
$$

Using this set of equations, the new longitudinal matrix is (the algorithm used to obtain this is given in the Appendix)

```
A_matrix_lon =

   -0.0066    0.6461   -9.7524         0
   -0.0002   -0.7704    0.0488    1.0000
         0         0         0    1.0000
    0.0000   -0.0448         0   -0.1119
```

Figure 3: New Longitudinal Matrix

and hence, the new eigenvalues are

```
-0.6989 + 0.0000i
-0.1719 + 0.0000i
-0.0091 + 0.0304i
-0.0091 - 0.0304i
```

Figure 4: New Eigenvalues

As it is now obvious from the figures above, we know have a "stable" matrix that we will use in the remainder of the assignment. Before continuing, it is important that the problem be considered. After comparing the original longitudinal matrix and the one obtained from the new approach, it can be seen that only the second row of the matrices are different. This indicates that there is something wrong with the second and third 6-DoF force functions. Namely,

$$\dot{V} = -RU + PW + g_D \sin\phi \cos\theta + (Y_A + Y_T)/\mathrm{m}$$
$$\dot{W} = QU - PV + g_D \cos\phi \cos\theta + (Z_A + Z_T)/\mathrm{m}$$

Figure 5: New Longitudinal Matrix

Since it took a lot of time to analyse and figure out what went wrong with the original approach (which is yet to be resolved), and to find and implement a new approach, the rest of the assignment is not complete, but is going to be completed and implemented in the final report.

2 SAS Design

2.1 Pitch Rate Stability Augmentation

In previous sections, longitudinal A and B matrices were created. States of mentioned A matrix consisted of magnitude of velocity, angle of attack, pitch angle and pitch rate respectively. Also input matrix B is calculated in previous sections for 4 states and 2 inputs. So it can be said that size of matrix B is 4×2. Here, after obtaining longitudinal matrices, gains in feedback will be calculated. To realize this, C matrix that is an element matrix of state space matrices will be created. It must be noted that matrix D will be created from zero (or zeros) in design. Equation for calculation of raw transfer functions can be given as below.

$$T_{raw}(s) = C(sI - A)^{-1}B \tag{2}$$

After considerations, full state feedback method is decided to be use for design. It can be stated that, feedback matrix will be used in only inner loop. This matrix will be selected in size of 1×4. Also thrust ratio is decided to be constant. So B matrix can be shaped for only δ_e. It means that only first column of matrix B will be taken for new version and size of matrix B will be 4×1. Matrix C will be taken as below.

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

This means that for obtaining transfer function, only α and q will be taken in consideration. So in this situation, feedback matrix K will be shaped as below.

$$K = \begin{bmatrix} 0 & k_\alpha & 0 & k_q \end{bmatrix} \tag{4}$$

Here k_α and k_q are constants. So new transfer functions can be calculated with full state feedback design algorithm. Firstly, augmented A matrix must be calculated. Then same transfer function calculation method can be used.

$$A_{aug} = A - BK \tag{5}$$

Then transfer functions with full state feedback contraction can be written as below.

$$T_{fsf}(s) = C(sI - A_{aug})^{-1}B \tag{6}$$

Here k_α and k_q is selected as -5 and -7 respectively. After calculations that are handled with assist of Matlab, transfer functions are calculated. Here the step responses of these transfer functions are shared as below:

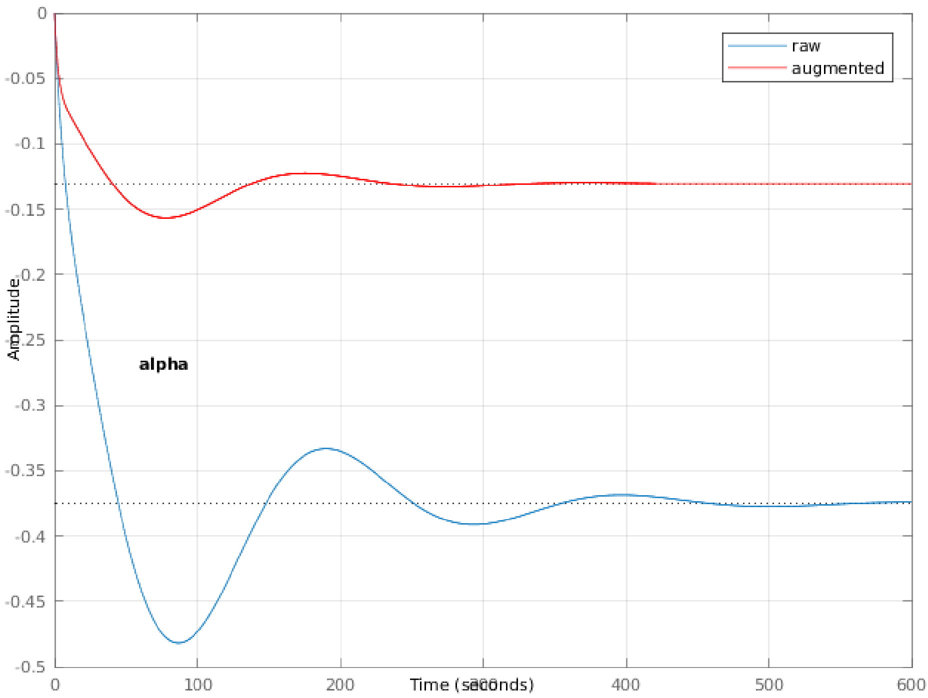

Figure 6: Step Response of α

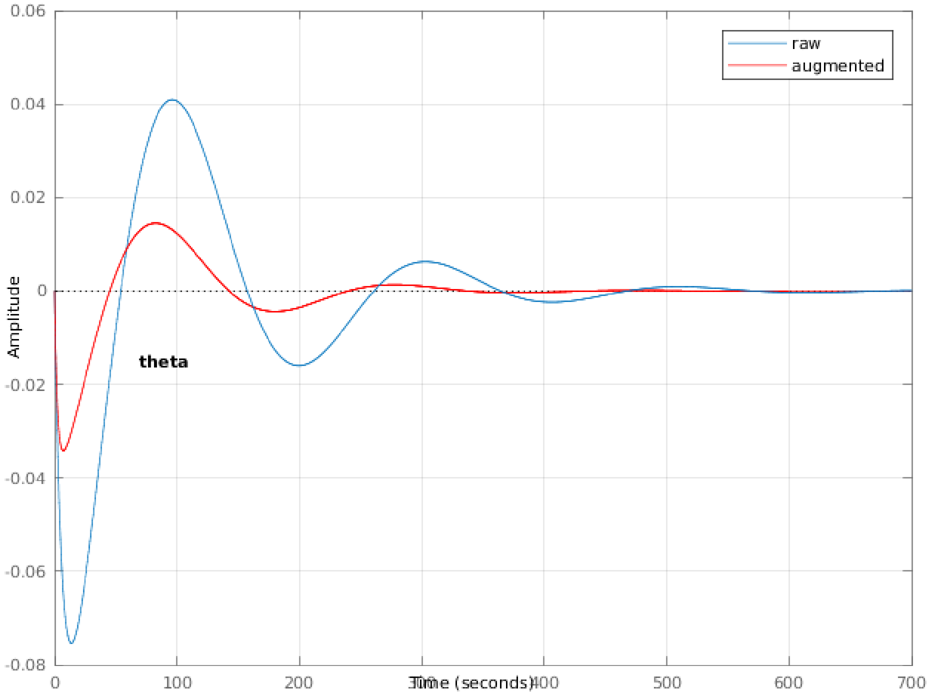

Figure 7: Step Response of θ

3 CAS Design

4 Autopilot Design

5 Gain Scheduling

6 Appendix

```
clear all ;
close all ;
clc ;
```

```
load longitudinal_matrices
s=tf('s') ;

%% Calculation for alpha
s=tf('s') ;
ka = -5 ;
kq = -7 ;

K_long =[ 0 ka 0 kq] ;

A_matrix_lon ;
B_matrix_lon ;
B_matrix_lon_dele = B_matrix_lon(:,1) ;
C_alpha_dele = [0 1 0 0 ; 0 0 0 1] ;

A_long_fb = (A_matrix_lon-B_matrix_lon_dele*K_long) ;
tf_pitch = C_alpha_dele*inv(s*eye(4)-A_long_fb)*B_matrix_lon_dele ;
tf_pitch = minreal(tf_pitch) ;

tf_pitch_ex = minreal(C_alpha_dele*inv(s*eye(4)-A_matrix_lon)*B_matrix_lo

figure
step(tf_pitch_ex(1))
hold on
step(tf_pitch(1),'r')
grid on
title('alpha')
legend('raw','augmented')

figure
step(tf_pitch_ex(2))
hold on
step(tf_pitch(2),'r')
grid on
title('theta')
legend('raw','augmented')
```

7 Bibliography

References

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