```
> IntersectGB := proc(I1, I2)
    local h1, h2, g1, g2, p_sol1, poly_y_1, poly_y_2, poly_y_3, poly_x_1, poly_x_2, poly_x_3, pre_1,
        pre_2, sol_poly_1, pre_3, pre_4, C, sol_poly_2, sol_poly_22;
    with(Groebner);
    Basis(I1, plex(x, y));
    Basis(I2, plex(x, y));
    h1 := I1[1];
    h2 := I2[1];
    g1 := I1[2];
    g2 := I2[2];
    p_sol1 := lcm(h1, h2);
```
$$poly\_y\_1 := simplify\left(\frac{p\_sol1}{h2}\right);$$
$$poly\_y\_2 := simplify\left(\frac{p\_sol1}{h1}\right);$$
```
    poly_y_3 := gcd(h1, h2);
    poly_x_1 := rem(g1, poly_y_1, y);
    poly_x_2 := rem(g2, poly_y_2, y);
    poly_x_3 := rem(g1, poly_y_3, y)·rem(g2, poly_y_3, y);
    gcdex(poly_y_1, poly_y_2, y,'u','v');
    pre_1 := u·poly_x_2·poly_y_1;
    pre_2 := v·poly_x_1·poly_y_2;
    sol_poly_1 := simplify(pre_1 + pre_2);
    gcdex(poly_y_1·poly_y_2, poly_y_3, y,'s','w');
    pre_3 := s·poly_x_3·(poly_y_1·poly_y_2);
    pre_4 := w·sol_poly_1·poly_y_3;
    C := Basis([p_sol1, poly_y_3·sol_poly_1], plex(x, y));
    sol_poly_2 := expand(pre_3 + pre_4);
    sol_poly_22 := NormalForm(sol_poly_2, C, plex(x, y));
    return [expand(p_sol1), expand(NormalForm(poly_y_3·sol_poly_1, [p_sol1], plex(x, y))),
        expand(sol_poly_22)]
    end proc;
```

$IntersectGB := \mathbf{proc}(I1, I2)$ **(1)**

$\quad$ **local** $h1, h2, g1, g2, p\_sol1, poly\_y\_1, poly\_y\_2, poly\_y\_3, poly\_x\_1, poly\_x\_2, poly\_x\_3,$

$\quad$ $pre\_1, pre\_2, sol\_poly\_1, pre\_3, pre\_4, C, sol\_poly\_2, sol\_poly\_22;$

$\quad$ $with(Groebner);$

$\quad$ $Groebner:\text{-}Basis(I1, plex(x, y));$

$\quad$ $Groebner:\text{-}Basis(I2, plex(x, y));$

$\quad$ $h1 := I1[1];$

$\quad$ $h2 := I2[1];$

$\quad$ $g1 := I1[2];$

$\quad$ $g2 := I2[2];$

$\quad$ $p\_sol1 := lcm(h1, h2);$

```
        poly_y_1 := simplify(p_sol1/h2);
        poly_y_2 := simplify(p_sol1/h1);
        poly_y_3 := gcd(h1, h2);
        poly_x_1 := rem(g1, poly_y_1, y);
        poly_x_2 := rem(g2, poly_y_2, y);
        poly_x_3 := rem(g1, poly_y_3, y) * rem(g2, poly_y_3, y);
        gcdex(poly_y_1, poly_y_2, y, 'u', 'v');
        pre_1 := u*poly_x_2*poly_y_1;
        pre_2 := v*poly_x_1*poly_y_2;
        sol_poly_1 := simplify(pre_1 + pre_2);
        gcdex(poly_y_1*poly_y_2, poly_y_3, y, 's', 'w');
        pre_3 := s*poly_x_3*poly_y_1*poly_y_2;
        pre_4 := w*sol_poly_1*poly_y_3;
        C := Groebner:-Basis([p_sol1, poly_y_3*sol_poly_1], plex(x, y));
        sol_poly_2 := expand(pre_3 + pre_4);
        sol_poly_22 := Groebner:-NormalForm(sol_poly_2, C, plex(x, y));
        return [expand(p_sol1), expand(Groebner:-NormalForm(poly_y_3*sol_poly_1, [p_sol1],
        plex(x, y))), expand(sol_poly_22)]
end proc
```

$G := [y^2 + 2y - 3, x - y + 1];$

$$G := [y^2 + 2y - 3, x - y + 1] \tag{2}$$

$F := [y^2 + 3y - 4, \ x - y + 3];$

$$F := [y^2 + 3y - 4, x - y + 3] \tag{3}$$

> $IntersectGB(G, F)$

$$[y^3 + 6y^2 + 5y - 12, xy - 3y^2 - x - 2y + 5, x^2 - 5y^2 + 2x - 8y + 13] \tag{4}$$