```
> ppcm_list := proc(list_elem)
        local start, i;
      start := list_elem[1];
      for i in list_elem do
         start := lcm(start, i);
      end do;
       return start;
   end proc;
```

```
ppcm_list := proc(list_elem)
    local start, i;
    start := list_elem[1]; for i in list_elem do start := lcm(start, i) end do; return start
end proc
```

(1)

```
lcm([5, 3, 8, 12])
```

$$[5, 3, 8, 12]$$

(2)

```
pgcd_list := proc(list_elem)
      local start, i;
    start := list_elem[1];
    for i in list_elem do
       start := gcd(start, i);
    end do;
     return start;
end proc;

crt_list := proc(list_xj, list_yj)
   local l_1, l_2, i, g;
   l_1 := [list_xj[1], list_xj[2]];
   l_2 := [list_yj[1], list_yj[2]];
  for i from 3 to nops(list_xj) do
    gcdex(l_2[1], l_2[2], y,'u','v');
    g := simplify(l_2[1]·u·l_1[2] + l_2[2]·v·l_1[1]);
    l_1 := [g, list_xj[i]];
    l_2 := [l_2[1]·l_2[2], list_yj[i]];
  end do;
return [l_1[1], l_2[1]] ;
end proc;
pgcd_list := proc(list_elem)
    local start, i;
    start := list_elem[1]; for i in list_elem do start := gcd(start, i) end do; return start
end proc
```

```
crt_list := proc(list_xj, list_yj)
    local l_1, l_2, i, g;
    l_1 := [list_xj[1], list_xj[2]];
```

(3)

```
    l_2 := [list_yj[1], list_yj[2]];
    for i from 3 to nops(list_xj) do
        gcdex(l_2[1], l_2[2], y, 'u', 'v');
        g := simplify(u*l_1[2]*l_2[1] + v*l_1[1]*l_2[2]);
        l_1 := [g, list_xj[i]];
        l_2 := [l_2[1]*l_2[2], list_yj[i]]
    end do;
    return [l_1[1], l_2[1]]
end proc
lcm(5, 3, 8, 12)
```

120 **(4)**

```
ppcm_all_but_i := proc(list_of_elem, i)
     local res, j; res := [ ];
    res := 1;
    for j from 1 to nops(list_of_elem) do
       if i ≠ j then
          res := [op(res), list_of_elem[j]];
       end if;
    end do;
    return ppcm_list(res);
end proc;
ppcm_all_but_i := proc(list_of_elem, i)
    local res, j;
    res := [ ];
    res := 1;
    for j to nops(list_of_elem) do if i<>j then res := [op(res), list_of_elem[j]] end if end do;
    return ppcm_list(res)
end proc

 ppcm_list([3, 5, 8, 12])
```

120 **(5)**

**(6)**

```
 IntersectGB := proc(list_of_ideals)
 local i, h1, h2, g1, g2, p_sol1, poly_y_1, poly_y_2, poly_y_3, poly_x_1, poly_x_2, poly_x_3, pre_1,
     pre_2, sol_poly_1, pre_3, pre_4, C, sol_poly_2, sol_poly_22, hi := [ ], v_yj := [ ], xj := [ ];
  with(Groebner);
 for i in list_of_ideals do
    Basis(i, plex(x, y));
 end do;
 for i in list_of_ideals do
   hi := [op(hi), i[1]]
 end do;
  p_sol1 := ppcm_list(hi);
 for i from 1 to nops(hi) do
```

```
    v_yj := [op(v_yj), simplify(p_sol1/ppcm_all_but_i(hi, i))];
  end do;
  y_star := pgcd_list(hi);
  for i from 1 to nops(v_yj) do
    xj := [op(xj), rem(list_of_ideals[i][2], v_yj[i], y)];
  end do;
    res_crt := crt_list(xj, v_yj);
    p_sol2 := NormalForm(expand(res_crt[1]·y_star), Basis([p_sol1], plex(x, y)), plex(x, y));
   end proc;
```

```
Warning, (in IntersectGB) `y_star` is implicitly declared local
Warning, (in IntersectGB) `res_crt` is implicitly declared local
Warning, (in IntersectGB) `p_sol2` is implicitly declared local
```

```
IntersectGB := proc(list_of_ideals)
    local i, h1, h2, g1, g2, p_sol1, poly_y_1, poly_y_2, poly_y_3, poly_x_1, poly_x_2, poly_x_3,
    pre_1, pre_2, sol_poly_1, pre_3, pre_4, C, sol_poly_2, sol_poly_22, hi, v_yj, xj, y_star, res_crt,
    p_sol2;
    hi := [ ];
    v_yj := [ ];
    xj := [ ];
    with(Groebner);
    for i in list_of_ideals do Groebner:-Basis(i, plex(x, y)) end do;
    for i in list_of_ideals do hi := [op(hi), i[1]] end do;
    p_sol1 := ppcm_list(hi);
    for i to nops(hi) do v_yj := [op(v_yj), simplify(p_sol1/ppcm_all_but_i(hi, i))] end do;
    y_star := pgcd_list(hi);
    for i to nops(v_yj) do xj := [op(xj), rem(list_of_ideals[i][2], v_yj[i], y)] end do;
    res_crt := crt_list(xj, v_yj);
    p_sol2 := Groebner:-NormalForm(expand(res_crt[1]*y_star), Groebner:-Basis([p_sol1],
    plex(x, y)), plex(x, y))
end proc
```

**(7)**

```
G := [expand((y + 1)·(y + 2)), x − y + 1];
```

$$G := [y^2 + 3y + 2, x - y + 1] \quad \textbf{(8)}$$

```
H := [expand((y + 1)·(y + 5)), x − y + 3];
```

$$H := [y^2 + 6y + 5, x - y + 3] \quad \textbf{(9)}$$

```
V := [expand((y + 1)·(y + 3)), x − y + 2];
```

$$V := [y^2 + 4y + 3, x - y + 2] \quad \textbf{(10)}$$

*IntersectGB*( [*G*, *H*, *V*] )

$$x\,y + x - \frac{5}{3}\,y^2 - 2\,y - \frac{1}{3} \tag{11}$$