

Assignment 5

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Download all python codes from

<https://github.com/ooharapolu/ASSIGNMNT5/Assignment5.py>

and latex-tikz codes from

<https://github.com/ooharapolu/ASSIGNMNT5/main.tex>

1 QUESTION No.2.158

Find the area lying in the first quadrant and by the circle $\mathbf{x}^T \mathbf{x} = 4$ and the lines $\mathbf{x} = 0$ and $\mathbf{x} = 2$.

2 SOLUTION

The given lines of the vector form is,

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.1)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2 \quad (2.0.2)$$

The general equation of the circle is,

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.3)$$

$$\|\mathbf{x}\|^2 + 2\mathbf{u}_1^T \mathbf{x} + f_1 = 0 \quad (2.0.4)$$

$$\mathbf{x}^T \mathbf{x} - 4 = 0 \quad (2.0.5)$$

$$\mathbf{u}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f_1 = -4 \quad (2.0.6)$$

To find points A and B

The parametric form is,

$$\mathbf{B} = \mathbf{q} + \lambda \mathbf{m} \quad (2.0.7)$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.8)$$

From the intersection of circle and line, the value of λ can be found by,

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} = \frac{4 - 0}{1} = 4 \quad (2.0.9)$$

$$\Rightarrow \lambda = \pm 2 \quad (2.0.10)$$

They are parallel to y-axis. So they make 90 degrees
Point of intersection Sub equation (2.0.10) in (2.0.8)
Put, $\lambda = 2$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (2.0.11)$$

$$\Rightarrow \mathbf{B} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (2.0.12)$$

and, $\lambda = -2$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \mathbf{B} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad (2.0.14)$$

As given in question as first quadrant

$$\Rightarrow \mathbf{B} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (2.0.15)$$

Similarly, to find point A, The parametric form of the line is,

$$\mathbf{A} = \mathbf{q} + \lambda \mathbf{m} \quad (2.0.16)$$

$$= \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.17)$$

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \quad (2.0.18)$$

$$= \frac{4 - 4}{1} = 0 \quad (2.0.19)$$

$$\Rightarrow \lambda = 0 \quad (2.0.20)$$

Sub (2.0.20) in (2.0.17)

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (2.0.21)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (2.0.22)$$

The equation of a circle can be expressed as,

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.23)$$

comparing equation (2.0.23) with the circle equation given,

$$\mathbf{x}^T \mathbf{x} = 4 \quad (2.0.24)$$

$$\Rightarrow \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -4 \quad (2.0.25)$$

$$\Rightarrow \mathbf{r} = \sqrt{\mathbf{u}^T \mathbf{x} - f} = \sqrt{4} \quad (2.0.26)$$

$$\Rightarrow \mathbf{r} = 2 \quad (2.0.27)$$

Using equation (2.0.27) and, the area of the sector is obtained as,

$$\Rightarrow \frac{\theta}{360^\circ} \pi \mathbf{r}^2 = \frac{90^\circ}{360^\circ} \pi (2)^2 = \pi \quad (2.0.28)$$

Plot of the given -

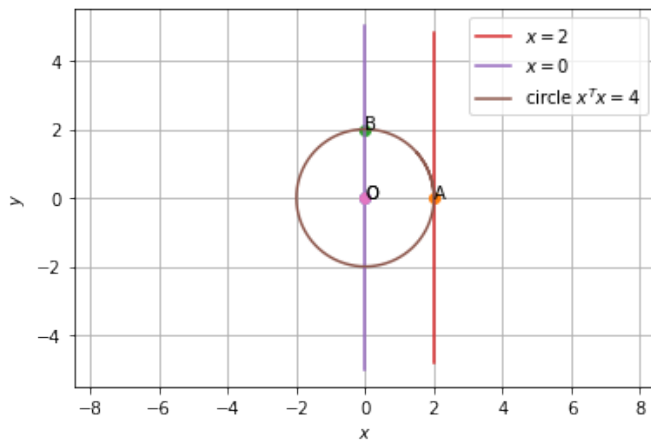


Fig. 2.1: The Circle of the lines