Assignment 5

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Download all python codes from

```
https://github.com/ooharapolu/ASSIGNMNT5/
    Assignment5.py
```

and latex-tikz codes from

```
https://github.com/ooharapolu/ASSIGNMNT5/main
    .tex
```

1 Question No.2.158

Find the area lying in the first quadrant and by the circle $\mathbf{x}^T\mathbf{x} = 4$ and the lines $\mathbf{x} = 0$ and $\mathbf{x} = 2$.

2 Solution

The given lines of the vector form is,

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.1}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2 \tag{2.0.2}$$

The general equation of the circle is,

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.3}$$

$$\|\mathbf{x}\|^2 + 2\mathbf{u_1}^T\mathbf{x} + f_1 = 0 \tag{2.0.4}$$

$$\mathbf{x}^T\mathbf{x} - 4 = 0 \tag{2.0.5}$$

$$\mathbf{u_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f_1 = -4 \tag{2.0.6}$$

To find points A,x and B
The parametric form of x-axis is,

$$\mathbf{B} = \mathbf{q} + \lambda\mathbf{m} \tag{2.0.7}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.8}$$

From the intersection of circle and line,the value of λ can be found by,

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} = \frac{4-0}{1} = 4 \tag{2.0.9}$$

$$\implies \lambda = \pm 2 \tag{2.0.10}$$

point of intersection sub equation (2.0.10) in (2.0.8)
Put, $\lambda = 2$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.12}$$

and, $\lambda = -2$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.0.13}$$

$$\implies \mathbf{B} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.0.14}$$

As given in question as first quadrant

$$\implies \mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.15}$$

The parametric form of y-axis is,

$$\mathbf{x} = \mathbf{q} + \lambda\mathbf{m} \tag{2.0.16}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.17}$$

From the intersection of circle and line,the value of λ can be found by,

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} = \frac{4-0}{1} = 4 \tag{2.0.18}$$

$$\implies \lambda = \pm 2 \tag{2.0.19}$$

Point of intersection Sub equation (2.0.19) in (2.0.17)
Put, $\lambda = 2$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.20}$$

$$\implies \mathbf{x} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.21}$$

and, $\lambda = -2$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{2.0.22}$$

$$\implies \mathbf{x} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{2.0.23}$$

As given in question as first quadrant

$$\implies \mathbf{x} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.24}$$

Similarly,to find point A ,The parametric form of the line is ,

$$\mathbf{A} = \mathbf{q} + \lambda\mathbf{m} \tag{2.0.25}$$

$$= \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.26}$$

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.27}$$

$$= \frac{4-4}{1} = 0 \tag{2.0.28}$$

$$\implies \lambda = 0 \tag{2.0.29}$$

Sub (2.0.29) in (2.0.26)

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.30}$$

$$\implies \mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.31}$$

The equation of a circle can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.32}$$

comparing equation (2.0.32) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.33}$$

$$\implies \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -4 \tag{2.0.34}$$

$$\implies \mathbf{r} = \sqrt{\mathbf{u}^T\mathbf{x} - f} = \sqrt{4} \tag{2.0.35}$$

$$\implies \mathbf{r} = 2 \tag{2.0.36}$$

Using equation (2.0.36) and,the area of the sector is obtained as,

$$\implies \frac{\theta}{360^\circ}\pi\mathbf{r}^2 = \frac{90^\circ}{360^\circ}\pi\left(2\right)^2 = \pi \tag{2.0.37}$$

Plot of the given -

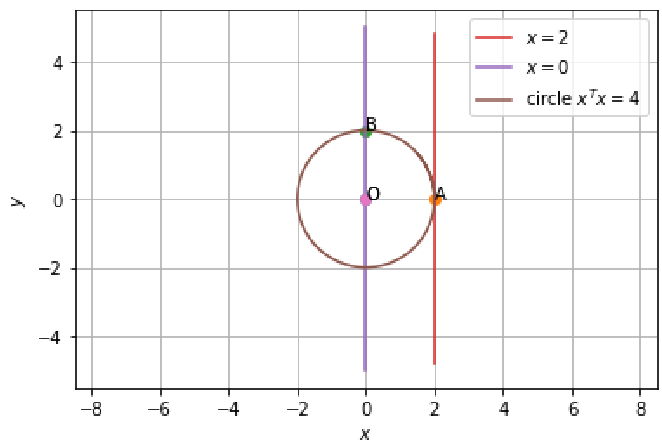

Fig. 2.1: The Circle of the lines