Assignment 5

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Download all python codes from

```
https://github.com/ooharapolu/ASSIGNMNT5/
    Assignment5.py
```

and latex-tikz codes from

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https://github.com/ooharapolu/ASSIGNMNT5/main
    .tex
```

1 Question No.2.158

Find the area lying in the first quadrant and by the circle $\mathbf{x}^T\mathbf{x} = \mathbf{4}$ and the lines $\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{2}$.

2 Solution

The given lines of the vector form is,

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = 0 \tag{2.0.1}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = 2 \tag{2.0.2}$$

To find points A and B
The parametric form is,

$$\mathbf{B} = \mathbf{q} + \lambda\mathbf{m} \tag{2.0.3}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.4}$$

From the intersection of circle and line,the value of λ can be found by,

$$\lambda^2 = \frac{-\mathbf{f_1} - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} = \frac{\mathbf{4-0}}{\mathbf{1}} = 4 \tag{2.0.5}$$

$$\implies \lambda = \pm\mathbf{2} \tag{2.0.6}$$

They are parallel to y-axis.So they make 90 degrees
Point of intersection Sub equation (2.0.6) in (2.0.4)
Put, $\lambda = \mathbf{2}$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{B} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.8}$$

and, $\lambda = -\mathbf{2}$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{2.0.9}$$

$$\implies \mathbf{B} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{2.0.10}$$

As given in question as first quadrant

$$\implies \mathbf{B} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.11}$$

Similarly,to find point A ,The parametric form of the line is ,

$$\mathbf{A} = \mathbf{q} + \lambda\mathbf{m} \tag{2.0.12}$$

$$= \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.13}$$

$$\lambda^2 = \frac{-\mathbf{f_1} - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.14}$$

$$= \frac{\mathbf{4-4}}{\mathbf{1}} = 0 \tag{2.0.15}$$

$$\implies \lambda = 0 \tag{2.0.16}$$

Sub (2.0.16) in (2.0.13)

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.17}$$

$$\implies \mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.18}$$

The equation of a circle can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u}^T\mathbf{x} + \mathbf{f} = 0 \tag{2.0.19}$$

comparing equation (2.0.19) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = \mathbf{4} \tag{2.0.20}$$

$$\implies \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{f} = -\mathbf{4} \tag{2.0.21}$$

$$\implies \mathbf{r} = \sqrt{\mathbf{u}^T\mathbf{x} - \mathbf{f}} = \sqrt{\mathbf{4}} \tag{2.0.22}$$

$$\implies r = 2 \tag{2.0.23}$$

Using equation (2.0.23) and,the area of the sector is obtained as,

$$\implies \frac{\theta}{360^{\circ}}\pi \mathbf{r^2} = \frac{90^{\circ}}{360^{\circ}}\pi\left(2\right)^2 = \pi \tag{2.0.24}$$

Plot of the given -

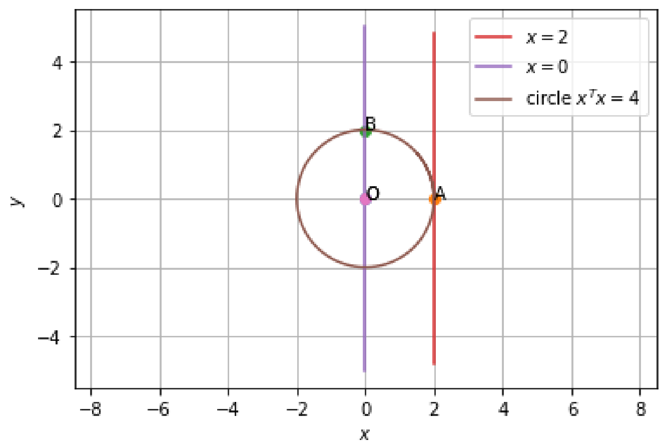

Fig. 2.1: The Circle of the lines