Assignment 5

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Download all python codes from

```
https://github.com/ooharapolu/ASSIGNMNT5/
Assignment5.py
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and latex-tikz codes from

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https://github.com/ooharapolu/ASSIGNMNT5/main
.tex
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1 Question No.2.158

Find the area lying in the first quadrant and by the circle $\mathbf{x}^T\mathbf{x} = \mathbf{4}$ and the lines $\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{2}$.

2 Solution

The given lines of the vector form is,

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0, \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2 \tag{2.0.1}$$

parametric form:

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.2}$$

Circle equation,

$$\mathbf{x}^T\mathbf{x} = \mathbf{4} \tag{2.0.3}$$

To find point of intersection.
First for circle,

$$\mathbf{x}^T\mathbf{x} = 4 \tag{2.0.4}$$

and line,

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.5}$$

Substitute parametric form in circle equation

$$\begin{pmatrix} 0 & \lambda \end{pmatrix} \begin{pmatrix} 0 \\ \lambda \end{pmatrix} = 4 \tag{2.0.6}$$

$$\implies \lambda^{\mathbf{2}} = 4 \tag{2.0.7}$$

$$\implies \lambda = +\mathbf{2} \tag{2.0.8}$$

$$\implies or, \lambda = -\mathbf{2} \tag{2.0.9}$$

Point of intersection

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.10}$$

$$\implies \mathbf{x} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{B} \begin{pmatrix} 0 & 2 \end{pmatrix} \tag{2.0.12}$$

Next, Circle and the line $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2$
Similarly we have,$\begin{pmatrix} 2 & \lambda \end{pmatrix} \begin{pmatrix} 2 \\ \lambda \end{pmatrix} = 4$

$$\mathbf{4} + \lambda^2 = 4 \tag{2.0.13}$$

$$\implies \lambda = 0 \tag{2.0.14}$$

Point of intersection,

$$\mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.15}$$

$$\implies \mathbf{A} \begin{pmatrix} 2 & 0 \end{pmatrix} \tag{2.0.16}$$

The equation of a circle can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u}^T\mathbf{x} + \mathbf{f} = 0 \tag{2.0.17}$$

comparing equation (2.0.17) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = \mathbf{4} \tag{2.0.18}$$

$$\implies \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{f} = -\mathbf{4} \tag{2.0.19}$$

$$\implies \mathbf{r} = \sqrt{\mathbf{u}^T\mathbf{x} - \mathbf{f}} = \sqrt{\mathbf{4}} \tag{2.0.20}$$

$$\implies r = 2 \tag{2.0.21}$$

The angle that the lines makes with the x-axis is given by,

$$\cos\theta = \frac{\begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{\left\| \begin{pmatrix} 0 & 0 \end{pmatrix} \right\| \left\| \begin{pmatrix} 2 & 0 \end{pmatrix} \right\|} = 0 \tag{2.0.22}$$

$$\implies \theta = 90^{\circ} \tag{2.0.23}$$

Using equation (2.0.21) and (2.0.23),the area of the sector is obtained as,

$$\implies \frac{\theta}{360^\circ}\pi\mathbf{r^2} = \frac{90^\circ}{360^\circ}\pi\left(2\right)^2 = \pi \qquad (2.0.24)$$

Plot of the given -

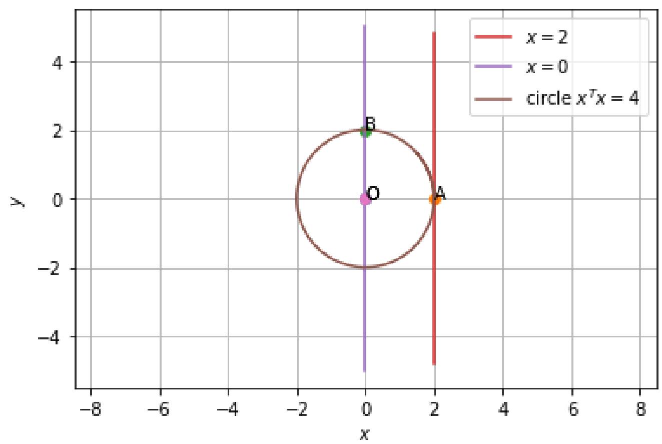

Fig. 2.1: The Circle of the lines