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Assignment 10

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Download all python codes from

https://github.com/ooharapolu/Matrix-Theory/tree/main/Assignment10/Codes

and latex-tikz codes from

https://github.com/ooharapolu/Matrix-Theory/tree/main/Assignment10

1 Question No. 2.53

Solve $x+y \le 9, y > x, x \ge 0$

2 Solution

The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -1 & -1 \\ -1 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{x} \ge \begin{pmatrix} -9 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.1}$$

Let $u_1 \ge 0$, $u_2 \ge 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \ge 0 \tag{2.0.2}$$

1) Now we have,

$$\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} \ge \begin{pmatrix} -9 \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\implies \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \mathbf{u} \qquad (2.0.4)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}^{-1} \mathbf{u}$$
(2.0.5)

$$\implies \mathbf{x} = \begin{pmatrix} \frac{9}{2} \\ \frac{1}{2} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{u}$$
 (2.0.6)

2) and we have,

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x} \ge \begin{pmatrix} -9 \\ 0 \end{pmatrix}$$
 (2.0.7)

$$\implies \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \mathbf{u} \qquad (2.0.8)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u}$$
(2.0.9)

$$\implies \mathbf{x} = \begin{pmatrix} 0 \\ 9 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \mathbf{u} \tag{2.0.10}$$

Thus, the solution of the system of inequalities can be determined graphically and the desired region is the shaded triangle which is represented in below fig.

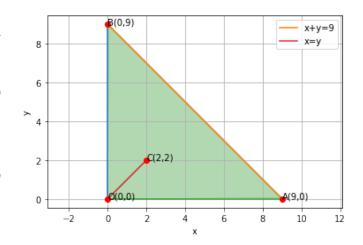


Fig. 2.1: Solution Region