

Assignment 10

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Download all python codes from

<https://github.com/ooharapolu/Matrix-Theory/tree/main/Assignment10/Codes>

and latex-tikz codes from

<https://github.com/ooharapolu/Matrix-Theory/tree/main/Assignment10>

1 QUESTION No. 2.53

Solve $x+y \leq 9, y > x, x \geq 0$

2 SOLUTION

The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -1 & -1 \\ -1 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -9 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.1)$$

Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \quad (2.0.2)$$

1) Now we have,

$$\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -9 \\ 0 \end{pmatrix} \quad (2.0.3)$$

$$\Rightarrow \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \mathbf{u} \quad (2.0.4)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.5)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 0 \\ 9 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{u} \quad (2.0.6)$$

2) and we have,

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -9 \\ 0 \end{pmatrix} \quad (2.0.7)$$

$$\Rightarrow \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \mathbf{u} \quad (2.0.8)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.9)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 0 \\ 9 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \mathbf{u} \quad (2.0.10)$$

Thus , the solution of the system of inequalities can be determined graphically and the desired region is the shaded triangle which is represented in below fig .



Fig. 2.1: Solution Region