Assignment 11

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Download all python codes from

https://github.com/ooharapolu/Assignment11/tree/main/CODES

and latex-tikz codes from

https://github.com/ooharapolu/Assignment11/blob/main/main.tex

1 Question No. 2.16

A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.

2 Solution

Item	Number	Machine A	Machine B	Profit
Screw A	x	4 minutes	6 minutes	Rs 7
Screw B	y	6 minutes	3 minutes	Rs 10
Max Available Time		4hours =240minutes	4hours =240minutes	

TABLE 2.1: Screw Requirements

Let the number of packages of screw A be x and the number of packages of screw B be y such that

$$x \geq 0 \tag{2.0.1}$$

$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$4x + 6y \leq 240 \tag{2.0.3}$$

$$\implies 2x + 3y \leq 120 \tag{2.0.4}$$

and,

$$6x + 3y \leq 240 \tag{2.0.5}$$

$$\implies 2x + y \leq 80 \tag{2.0.6}$$

$\therefore$ Our problem is

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 7 & 10 \end{pmatrix} \mathbf{x} \tag{2.0.7}$$

$$s.t. \quad \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 120 \\ 80 \end{pmatrix} \tag{2.0.8}$$

Lagrangian function is given by

$$\begin{aligned} & L(\mathbf{x}, \boldsymbol{\lambda}) \\ & = \begin{pmatrix} 7 & 10 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} + 120 \right] \right. \\ & + \left[\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} + 80 \right] \\ & + \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.9}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \tag{2.0.10}$$

Now,

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 7 + \begin{pmatrix} 2 & 3 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 10 + \begin{pmatrix} 2 & 1 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} + 120 \\ \begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} + 80 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.11}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 2 & 3 & -1 & 0 \\ 0 & 0 & 2 & 1 & 0 & -1 \\ 2 & 3 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -5 \\ -6 \\ 200 \\ 240 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.12}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 2 & 3 \\ 0 & 0 & 2 & 1 \\ 2 & 3 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -7 \\ -10 \\ 120 \\ 80 \end{pmatrix} \tag{2.0.13}$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 3 \\ 0 & 0 & 2 & 1 \\ 2 & 3 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -7 \\ -10 \\ 120 \\ 80 \end{pmatrix} \tag{2.0.14}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & 0 & \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{-1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} -7 \\ -10 \\ 120 \\ 80 \end{pmatrix} \tag{2.0.15}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 30 \\ 20 \\ \frac{-13}{4} \\ \frac{3}{2} \end{pmatrix} \tag{2.0.16}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix} \frac{-13}{4} \\ \frac{3}{2} \end{pmatrix} \succ \mathbf{0}$

$\therefore$ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 30 \\ 20 \end{pmatrix} \tag{2.0.17}$$

$$Z = \begin{pmatrix} 7 & 10 \end{pmatrix} \mathbf{x} \tag{2.0.18}$$

$$= \begin{pmatrix} 7 & 10 \end{pmatrix} \begin{pmatrix} 30 \\ 20 \end{pmatrix} \tag{2.0.19}$$

$$= 410 \tag{2.0.20}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 30.00000000 \\ 20.00000000 \end{pmatrix} \tag{2.0.21}$$

$$Z = 410.00000000 \tag{2.0.22}$$

Hence , $\boxed{x = 30}$ packages of screw A and $\boxed{y = 20}$ packages of screw B should be the factory owner produce in a day in order to maximise his profit is $\boxed{Z = 410}$.

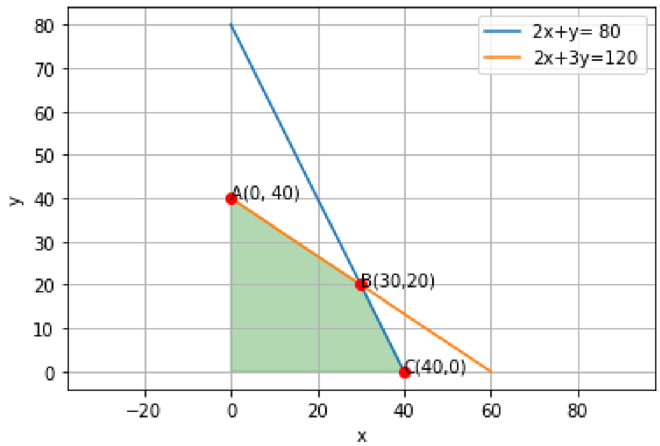

Fig. 2.1: graphical solution