Assignment 2

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Download all python codes from

```
https://github.com/ooharapolu/ASSIGNMNT/Assignment1.py
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and latex-tikz codes from

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https://github.com/ooharapolu/ASSIGNMNT/main.tex
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1 Question No.2.8

Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution:

1)

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 6$$
$$\begin{pmatrix}4 & -2\end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

2)

$$\begin{pmatrix}2 & -2\end{pmatrix}\mathbf{x} = 2$$
$$\begin{pmatrix}4 & -4\end{pmatrix}\mathbf{x} = 5 \tag{1.0.2}$$

2 SOLUTION

1)

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 6$$
$$\begin{pmatrix}4 & -2\end{pmatrix}\mathbf{x} = 4 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}2 & 1\\4 & -2\end{pmatrix}\mathbf{x} = \begin{pmatrix}6\\4\end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}2 & 1 & 6\\4 & -2 & 4\end{pmatrix} \xleftrightarrow{R_2\to\frac{R_2}{4}} \begin{pmatrix}2 & 1 & 6\\1 & \frac{-1}{2} & 1\end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix}2 & 1 & 6\\1 & \frac{-1}{2} & 1\end{pmatrix} \xleftrightarrow{R_2\to-2R_2} \begin{pmatrix}2 & 1 & 6\\-2 & 1 & -2\end{pmatrix} \tag{2.0.4}$$

$$\begin{pmatrix}2 & 1 & 6\\-2 & 1 & -2\end{pmatrix} \xleftrightarrow{R_2\to R_2+R_1} \begin{pmatrix}2 & 1 & 6\\0 & 2 & 4\end{pmatrix} \tag{2.0.5}$$

$$\begin{pmatrix}2 & 1 & 6\\0 & 2 & 4\end{pmatrix} \xleftrightarrow{R_2\to\frac{R_2}{2}} \begin{pmatrix}2 & 1 & 6\\0 & 1 & 2\end{pmatrix} \tag{2.0.6}$$

$$\begin{pmatrix}2 & 1 & 6\\0 & 1 & 2\end{pmatrix} \xleftrightarrow{R_1\to\frac{R_1}{2}} \begin{pmatrix}1 & \frac{1}{2} & 3\\0 & 1 & 2\end{pmatrix} \tag{2.0.7}$$

$$\begin{pmatrix}1 & \frac{1}{2} & 3\\0 & 1 & 2\end{pmatrix} \xleftrightarrow{R_1\to R_1+\frac{R_2}{-2}} \begin{pmatrix}1 & 0 & 2\\0 & 1 & 2\end{pmatrix} \tag{2.0.8}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}2 & 1 & 6\\4 & -2 & 4\end{pmatrix} \tag{2.0.9}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}2 & 1\\4 & -2\end{pmatrix} \tag{2.0.10}$$

is also 2.

$$\begin{aligned}\because Rank\begin{pmatrix}2 & 1\\4 & -2\end{pmatrix} = Rank\begin{pmatrix}2 & 1 & 6\\4 & -2 & 4\end{pmatrix} = 2\\ = dim\begin{pmatrix}2 & 1\\4 & -2\end{pmatrix} = 2\end{aligned} \tag{2.0.11}$$

$\therefore$ Given lines 1.0.1 have unique solution so we can say they are intersect.The given lines are Consistent.

2)

$$\begin{pmatrix}2 & -2\end{pmatrix}\mathbf{x} = 2$$
$$\begin{pmatrix}4 & -4\end{pmatrix}\mathbf{x} = 5 \tag{2.0.12}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & -2 \\ 4 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \tag{2.0.13}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & -2 & 2 \\ 4 & -4 & 5 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 2 & -2 & 2 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.14}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 2 & -2 & 2 \\ 4 & -4 & 5 \end{pmatrix} \tag{2.0.15}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & -2 \\ 4 & -4 \end{pmatrix} \tag{2.0.16}$$

is 1

$$\begin{aligned} \because Rank \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} \neq Rank \begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix} = 1 \\ < dim \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} = 2 \end{aligned} \tag{2.0.17}$$

$\therefore$ Given lines 1.0.2 have no solution so we can say they are parallel.The given lines are inconsistent.

PLOT OF GIVEN LINES -

Plot of 1.0.1 -

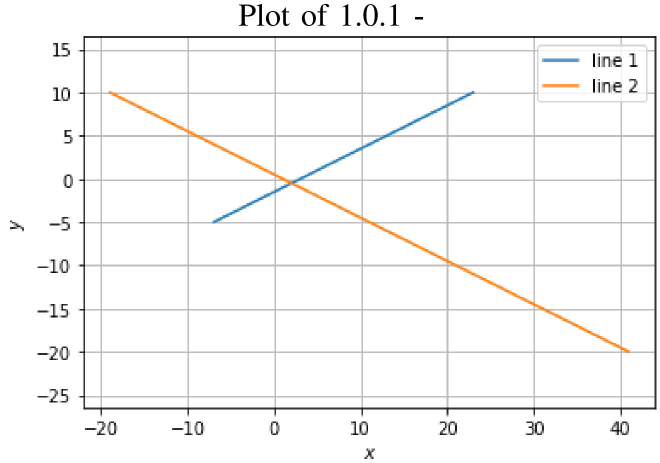

Fig. 2.1: INTERSECTING LINES.

Plot of 1.0.2 -

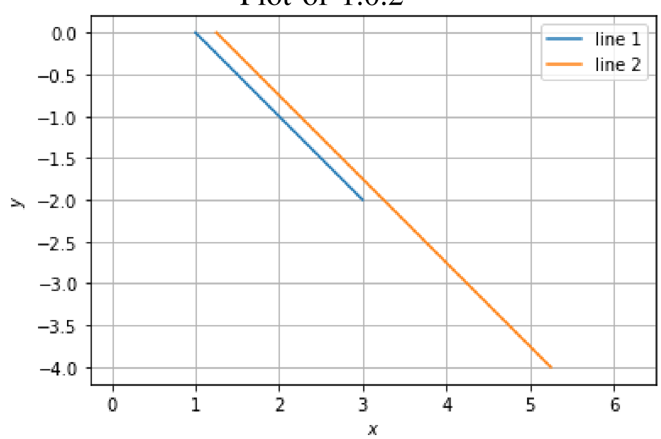

Fig. 2.2: PARALLEL LINES