

# ASSIGNMENT 2

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Download all python codes from

<https://github.com/ooharapolu/ASSIGNMENT2/tree/main/ASSIGNMENT2/CODES>

and latex-tikz codes from

<https://github.com/ooharapolu/ASSIGNMENT2/tree/main/ASSIGNMENT2>

## 1 QUESTION No 2.8

Which of the following pairs of linear equations are consistent/inconsistent, obtain the solution:

1)

$$\begin{aligned} (2 \ 1)\mathbf{x} &= 6 \\ (4 \ -2)\mathbf{x} &= 4 \end{aligned} \quad (1.0.1)$$

2)

$$\begin{aligned} (2 \ -2)\mathbf{x} &= 2 \\ (4 \ -4)\mathbf{x} &= 5 \end{aligned} \quad (1.0.2)$$

## 2 SOLUTION

1)

$$\begin{aligned} (2 \ 1)\mathbf{x} &= 6 \\ (4 \ -2)\mathbf{x} &= 4 \end{aligned} \quad (2.0.1)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \quad (2.0.2)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & 1 & 6 \\ 4 & -2 & 4 \end{pmatrix} \xrightarrow{R_2 \rightarrow \frac{R_2}{4}} \begin{pmatrix} 2 & 1 & 6 \\ 1 & -\frac{1}{2} & 1 \end{pmatrix} \quad (2.0.3)$$

$$\begin{pmatrix} 2 & 1 & 6 \\ 1 & -\frac{1}{2} & 1 \end{pmatrix} \xrightarrow{R_2 \rightarrow -2R_2} \begin{pmatrix} 2 & 1 & 6 \\ -2 & 1 & -2 \end{pmatrix} \quad (2.0.4)$$

$$\begin{pmatrix} 2 & 1 & 6 \\ -2 & 1 & -2 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 + R_1} \begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 4 \end{pmatrix} \quad (2.0.5)$$

$$\begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 4 \end{pmatrix} \xrightarrow{R_2 \rightarrow \frac{R_2}{2}} \begin{pmatrix} 2 & 1 & 6 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.0.6)$$

$$\begin{pmatrix} 2 & 1 & 6 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{R_1 \rightarrow \frac{R_1}{2}} \begin{pmatrix} 1 & \frac{1}{2} & 3 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.0.7)$$

$$\begin{pmatrix} 1 & \frac{1}{2} & 3 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 + \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.0.8)$$

$\therefore$  row reduction of the  $2 \times 3$  matrix

$$\begin{pmatrix} 2 & 1 & 6 \\ 4 & -2 & 4 \end{pmatrix} \quad (2.0.9)$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & 1 \\ 4 & -2 \end{pmatrix} \quad (2.0.10)$$

is also 2.

$$\begin{aligned} \therefore \text{Rank} \begin{pmatrix} 2 & 1 \\ 4 & -2 \end{pmatrix} &= \text{Rank} \begin{pmatrix} 2 & 1 & 6 \\ 4 & -2 & 4 \end{pmatrix} = 2 \\ &= \dim \begin{pmatrix} 2 & 1 \\ 4 & -2 \end{pmatrix} = 2 \end{aligned} \quad (2.0.11)$$

$\therefore$  Given lines (1.0.1) have unique solution so we can say they intersect. The given lines are consistent.

2)

$$\begin{aligned} (2 \ -2)\mathbf{x} &= 2 \\ (4 \ -4)\mathbf{x} &= 5 \end{aligned} \quad (2.0.12)$$

The above equations can be expressed as the

matrix equation

$$\begin{pmatrix} 2 & -2 \\ 4 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \quad (2.0.13)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & -2 & 2 \\ 4 & -4 & 5 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - 2R_1} \begin{pmatrix} 2 & -2 & 2 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.14)$$

$\therefore$  row reduction of the  $2 \times 3$  matrix

$$\begin{pmatrix} 2 & -2 & 2 \\ 4 & -4 & 5 \end{pmatrix} \quad (2.0.15)$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix} \quad (2.0.16)$$

is also 1.

$$\begin{aligned} \therefore \text{Rank} \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} &\neq \text{Rank} \begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix} = 1 \\ &< \dim \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} = 2 \end{aligned} \quad (2.0.17)$$

$\therefore$  Given lines (1.0.2) have no solution so we say they are parallel. The given lines are inconsistent. PLOT OF GIVEN LINES -

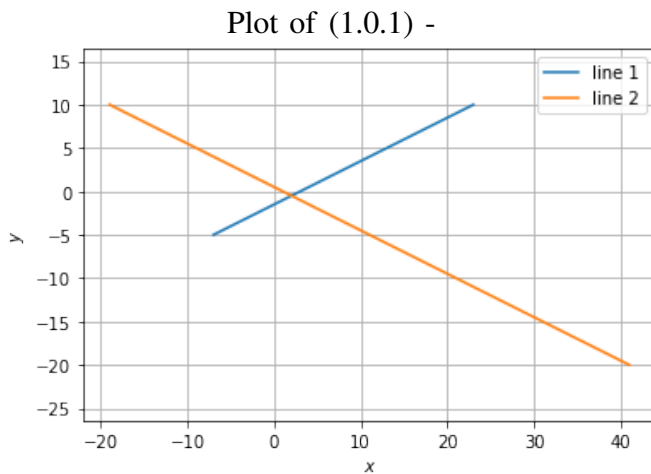


Fig. 2.1: INTERSECTING LINES

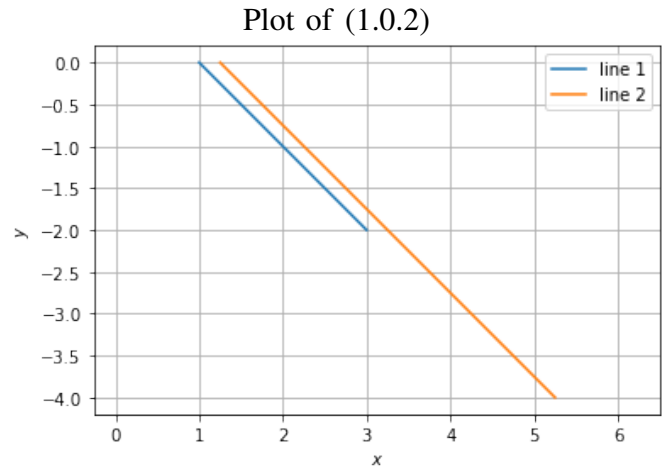


Fig. 2.2: PARALLEL LINES