Assignment 5

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Download all python codes from

https://github.com/ooharapolu/ASSIGNMNT5/Assignment5.py

and latex-tikz codes from

https://github.com/ooharapolu/ASSIGNMNT5/main.tex

1 Question No.2.158

Find the area lying in the first quadrant and by the circle $\mathbf{x}^T\mathbf{x} = \mathbf{4}$ and the lines $\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{2}$.

2 Solution

The equation of a circle can be expressed as,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u}^T\mathbf{x} + \mathbf{f} = 0 \tag{2.0.1}$$

where c is the center.
comparing equation (2.0.1) with the circle equation given,

$$\mathbf{x}^T\mathbf{x} = \mathbf{4} \tag{2.0.2}$$

$$\implies \mathbf{c} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{f} = -\mathbf{4} \tag{2.0.3}$$

$$\implies \mathbf{r} = \sqrt{\mathbf{c}^T\mathbf{x} - \mathbf{f}} = \sqrt{\mathbf{4}} \tag{2.0.4}$$

$$\implies r = 2 \tag{2.0.5}$$

From equation (2.0.5), the point at which circle touches x-axis is $\begin{pmatrix}2\\0\end{pmatrix}$.
The area bounded by the circle and the lines ,$\mathbf{x} = \mathbf{0}, \mathbf{x} = \mathbf{2}$,in the first quadrant is represented as shaded region in the plot.
The angle that the line makes with the x-axis is given by,

$$\cos\theta = \frac{\begin{pmatrix}0 & 0\end{pmatrix}\begin{pmatrix}2\\0\end{pmatrix}}{\left\|\begin{pmatrix}0 & 0\end{pmatrix}\right\|\left\|\begin{pmatrix}2 & 0\end{pmatrix}\right\|} = 0 \tag{2.0.6}$$

$$\implies \theta = 90^\circ \tag{2.0.7}$$

Using equation (2.0.5) and (2.0.7),the area of the sector is obtained as,

$$\implies \frac{\theta}{360^\circ}\pi\mathbf{r^2} = \frac{90^\circ}{360^\circ}\pi\left(2\right)^2 = \pi \tag{2.0.8}$$

Plot of the given -

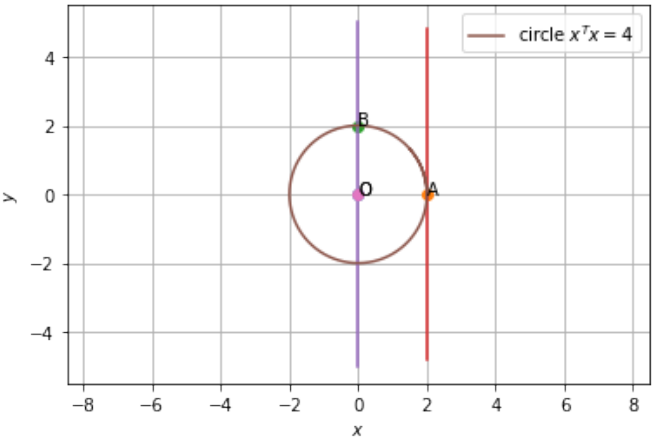

Fig. 2.1: The Circle of the lines