Assignment 6

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Download all python codes from

https://github.com/ooharapolu/ASSIGNMNT6/Assignment6.py

and latex-tikz codes from

https://github.com/ooharapolu/ASSIGNMNT6/main.tex

1 Question No.2.70

In each of the following exercises,find the equation of the parabola that satisfies the following conditions:

e. vertex $\begin{pmatrix}0 & 0\end{pmatrix}$ passing through $\begin{pmatrix}2\\2\end{pmatrix}$ and axis is along the x-axis.

2 solution

Given that axis is along the x-axis.
So, equation of parabola is of the form,

$$\mathbf{y}^2 = 4\mathbf{a}\mathbf{x} \tag{2.0.1}$$

$$\implies \mathbf{y}^2 - 4\mathbf{a}\mathbf{x} = 0 \tag{2.0.2}$$

The equation (2.0.2) of the vector form is,

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-2\mathbf{a} & 0\end{pmatrix}\mathbf{x} + 0 = 0 \tag{2.0.3}$$

$$\mathbf{V} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}, \mathbf{u} = \begin{pmatrix}-2\mathbf{a}\\0\end{pmatrix}, f = 0 \tag{2.0.4}$$

$\because |\mathbf{V}| = 0$ and $\lambda_1 = 0$ i.e. it is in standard form

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p_1} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.5}$$

$$\eta = \mathbf{u}^T\mathbf{p_1} = -2\mathbf{a} \tag{2.0.6}$$

$\therefore \begin{pmatrix}2 & 2\end{pmatrix}$ satisfies it.

$$\begin{pmatrix}2 & 2\end{pmatrix}\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix} + 2\begin{pmatrix}-2\mathbf{a} & 0\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix} = 0 \tag{2.0.7}$$

$$\implies \begin{pmatrix}0 & 2\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix} + \begin{pmatrix}-4\mathbf{a} & 0\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix} = 0 \tag{2.0.8}$$

$$\implies \mathbf{4} - 8\mathbf{a} = 0 \tag{2.0.9}$$

$$\implies \mathbf{4} = 8\mathbf{a} \tag{2.0.10}$$

$$\implies \mathbf{a} = \frac{1}{2} \tag{2.0.11}$$

$\therefore$ The equation is,

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-1 & 0\end{pmatrix}\mathbf{x} + 0 = 0 \tag{2.0.12}$$

$$\implies \mathbf{y}^2 = \mathbf{2x} \tag{2.0.13}$$

Plot of given parabola

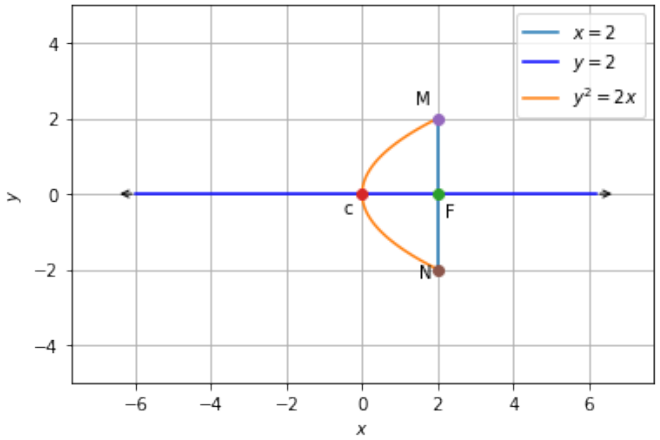

Fig. 2.1: Parabola $y^2 = 2x$