

Assignment 6

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Download all python codes from

<https://github.com/ooharapolu/ASSIGNMNT6/Assignment6.py>

and latex-tikz codes from

<https://github.com/ooharapolu/ASSIGNMNT6/main.tex>

1 QUESTION No.2.70

In each of the following exercises, find the equation of the parabola that satisfies the following conditions:

e. vertex $(0, 0)$ passing through $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and axis is along the x-axis.

2 SOLUTION

Given that axis is along the x-axis.

So, vector equation of the parabola is,

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -2a & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \quad (2.0.1)$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2a \\ 0 \end{pmatrix}, f = 0 \quad (2.0.2)$$

$\therefore |\mathbf{V}| = 0$ and $\lambda_1 = 0$ i.e. it is in standard form

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.3)$$

$$\eta = \mathbf{u}^T \mathbf{p}_1 = -2a \quad (2.0.4)$$

$\therefore \begin{pmatrix} 2 & 2 \end{pmatrix}$ satisfies it.

$$\begin{pmatrix} 2 & 2 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} -2a & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 0 \quad (2.0.5)$$

$$\implies \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} -4a & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 0 \quad (2.0.6)$$

$$\implies 4 - 8a = 0 \quad (2.0.7)$$

$$\implies 4 = 8a \quad (2.0.8)$$

$$\implies a = \frac{1}{2} \quad (2.0.9)$$

\therefore The equation is,

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \quad (2.0.10)$$

Plot of given parabola

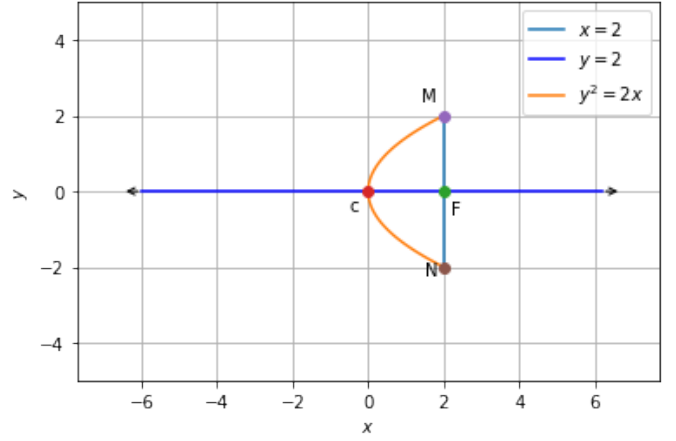


Fig. 2.1: Parabola $y^2 = 2x$