Assignment 8

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1 Question No.VECTORS-2.4

Show that the points $\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$ are the vertices of a right angle triangle

2 SOLUTION

The direction vectors of **AC** and **BC** are

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \tag{2.0.2}$$

If **A**,**B**,**C** form a line,then,**AC** and **BC** should have the same direction vector.Hence,there exists a k such that

$$\mathbf{A} - \mathbf{C} = k\left(\mathbf{B} - \mathbf{A}\right) \tag{2.0.3}$$

$$\implies \mathbf{A} = \frac{k\mathbf{B} + \mathbf{C}}{k+1} \tag{2.0.4}$$

Since

$$\mathbf{B} - \mathbf{A} \neq k\left(\mathbf{B} - \mathbf{C}\right) \tag{2.0.5}$$

the points are not colinear and form a traingle.An alternative method is to create the matrix.

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} - \mathbf{C} & \mathbf{B} - \mathbf{C} \end{pmatrix}^\top \tag{2.0.6}$$

If $rank\left(\mathbf{M}\right) = 1$,the points are colinear. The rank of a matrix is the number of nonzero rows left after doing row operations.In this problem,

$$\mathbf{M} = \begin{pmatrix} -1 & 3 & 5 \\ -2 & 1 & -1 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} -1 & 3 & 5 \\ 0 & -5 & -11 \end{pmatrix} \tag{2.0.7}$$

$$\implies rank\left(\mathbf{M}\right) = 2 \tag{2.0.8}$$

as the number of non zero rows is 2 and the following figure represents the triangle formed by given points A,B and C

PLOT OF GIVEN - From the figure,it appears

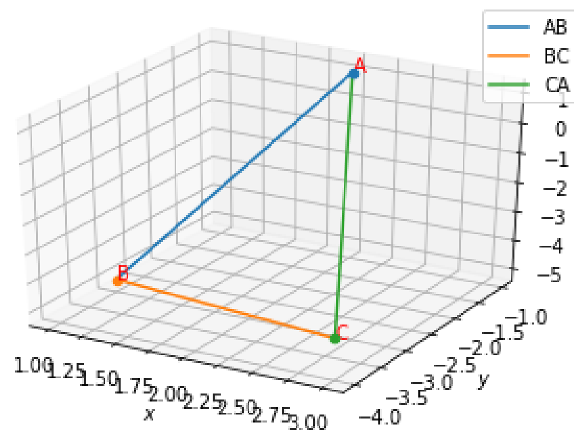

Fig. 0: The right angle triangle

that $\triangle\mathbf{ABC}$ is right angled,with **AB** as the hypotenuse.From Baudhayana's theorem,this would be true if

$$\|\mathbf{A} - \mathbf{C}\|^2 + \|\mathbf{B} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{B}\|^2 \tag{2.0.9}$$

which can be expressed as

$$\|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^\top\mathbf{C} + \|\mathbf{B}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{B}^\top\mathbf{C} = \|\mathbf{A}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{A}^\top\mathbf{B} \tag{2.0.10}$$

to obtain

$$\left(\mathbf{A} - \mathbf{C}\right)^\top\left(\mathbf{B} - \mathbf{C}\right) = 0 \tag{2.0.11}$$

after simplification.From (2.0.1) and (2.0.2),it is easy to verify that

$$\left(\mathbf{A} - \mathbf{C}\right)^\top\left(\mathbf{B} - \mathbf{C}\right) = \begin{pmatrix} -1 & 3 & 5 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \tag{2.0.12}$$

$$= 0 \tag{2.0.13}$$

satisfying (2.0.11).Thus,$\triangle\mathbf{ABC}$ is right angled at **C**