# Peepoo Robot Language

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Keywords: Abstract, MSc thesis, LaTeX system.

# ON NOTATION AND TERMINOLOGY

Unless otherwise specified, we use the following notations and terminologies throughout the thesis.

| | |
|---|---|
| $\mathbb{R}$ | the set of real numbers, |
| $L$ | the loss function, |
| $\mathsf{T}$ | the transpose of a matrix, |
| $w$ | a member of a vocabulary (usually a word), |
| $V = \{w_1, w_2, \ldots, w_{\lvert V \rvert}\}$ | a (finite) vocabulary, |
| $\mathbf{w}$ | the vector representation of $w$, |
| $\mathbf{w}(w)$ | the vector representation of $w$, |

# Contents

# 1 Introduction

This section includes general despription of the field and the topic of the MSc thesis.

Note that if you have used AI in your writing, you must mention it. Check for guidelines https://utuguides.fi/artificialintelligence. Good place to mention that AI tolls have been used is here, for example, in the following form:

The ChatGPT AI has been used to enhance the language of the thesis.

# 2 Foundations of Statistical Learning for NLP

**Theorem 1.** *Assume that in interval* $[a, b]$ $F'(x) = f(x)$ *holds and that* $f$ *is continuous.*[1] *Then*

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

[1] The continuity is essential here.

# 3 Representation Learning

Machines need to understand each word first so as to master the sophisticated meanings of human languages. Hence, effective word representations are essential for natural language processing (NLP), and it is also a good start for introducing representation learning in NLP.

The form of a word representation can be divided into two categories: symbolic and distributed representations. In symbolic representation, each word is represented as a unique symbol or index, such as one-hot encoding. In distributed representation, each word is represented as a dense vector in a continuous vector space, where semantically similar words are mapped to nearby points.

Symbolic representations are easy to implement and interpret, but they suffer from the curse of dimensionality and cannot capture semantic relationships between words. The distributed word representation overcomes these problems by representing words as low-dimensional real-valued dense vectors.

In distributed word representation, each dimension in isolation is meaningless because semantics is distributed over all dimensions of the vector. Distributed representation can be obtained by factorizing the matrices of symbolic representations, such as in Latent Semantic Analysis LSA, or by optimizing the word vectors with gradient descent to predict the context words, such as in Word2Vec. Glove is a hybrid method that factorizes the co-occurrence matrix of words while also optimizing the word vectors to predict the co-occurrence counts.

## 3.1 Symbolic Representations

One-hot

$$\mathbf{w_j^{(i)}} = \begin{cases} 1, & \text{if } j = i, \\ 0, & \text{otherwise,} \end{cases}$$

Bag of Words (BoW) $\mathbf{W} \in \mathbb{R}^{|D| \times |V|}$ document x word matrix where $D$ is the set of documents, and V is the vocabulary.

$$\mathbf{W}_i = \sum_{k=0}^{|D_i|} \mathbf{w}(D_i^{(k)})$$

## 3.2 Distributed Word Representations

Although simple and interpretable, symbolic representations are not the best choice for computation. For example, the very sparse nature of the symbolic representation makes it difficult to compute word-to-word similarities.

The difficulty of symbolic representation is solved by the distributed representation. Distributed representation represents a subject (a word in our case) as a fixed-length real-valued vector, where no clear meaning is assigned to any single dimension of the vector. More specifically, semantics is scattered over all (or a large portion) of the dimensions of the representation, and one dimension contributes to the semantics of all (or a large proportion) of the words

### 3.2.1 Matrix Factorization-based Approaches

**Latent Semantic Analysis (LSA)** The idea of LSA is to perform singular value decomposition (SVD) on the word-document matrix $M = U\Sigma V^\intercal$, and use the rows of the left singular matrix $U$ as the word representations. We justify this by the following observation: the $i$-th row of $M$ is the symbolic representation of the word $w_i$. We get the dot product similarity between words $w_i$ and $w_j$ by the following calculation

$$\mathbf{M}_i(\mathbf{M}_j)^\intercal = (U\Sigma V^\intercal)_i \cdot (U\Sigma V^\intercal)_j^\intercal = U_i\Sigma^2 U_j^\intercal.$$

**Probabilistic LSA** plsaplsa

**Latent Dirichlet Allocation (LDA) IF APPLIED LATER**

### 3.2.2 Word2Vec and GloVe

**Word2Vec** Word2Vec adopts the distributional hypothesis but does not take a count-based approach. It directly uses gradient descent to optimize the representations of a word toward tis neighbors' representations. There are two main architectures for Word2Vec: Continuous Bag of Words (CBOW) and Skip-Gram. The difference is that CBOW predicts the target word based on multiple context words, while Skip-Gram predicts the context words based on the center word.

Formally, CBOW predicts the word $w_i$ as

$$P(w_i|w_{i-l},\ldots,w_{i-1},w_{i+1},\ldots,w_{i+l}) = \sigma\left(\mathbf{W}\sum_{\substack{j=i-l\\ j\neq i}}^{i+l}\mathbf{w}_j\right),$$

where $2l+1$ is the context size, $\sigma$ is the softmax function, $\mathbf{W}\in\mathbb{R}^{|V|\times m}$ is the weight matrix and $\mathbf{w}_j$ is the embedding of word $w_j$.

The CBOW model is optimized by minimizing the sum of the negative log probabilities:

$$\mathcal{L} = -\sum_{i=1}^{N}\log P(w_i|w_{i-l},\ldots,w_{i-1},w_{i+1},\ldots,w_{i+l}).$$

Contrary to CBOW, Skip-Gram predicts the context words based on the center word. Formally, given a word $w_i$, Skip-Gram predicts the context words as

$$P(w_j|w_i) = \sigma\left(\mathbf{W}\mathbf{w}_i^\intercal\right), \quad j\in\{i-l,\ldots,i-1,i+1,\ldots,i+l\}.$$

**GloVe**

# References

[1] Newton and Leibniz

[2] Riemann