香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

人工智能學院
School of Artificial Intelligence

Large Language Models for Social Simulations

(AIE1902)

Unit 3: Giant Catalytic Effect of Altruists in Schelling's Segregation Model

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Overview

Two Extensions:

1. Catalyst Effect of Altruists

- Even a very small number of altruists can trigger the system into the optimal state.
- *“Small perturbations → Big changes”* in social dynamics.

2. Urban Investment Intervention

- City planners can reshape utility functions to guide individual choices.
- Shows how **institutional design** can improve social welfare.

Part 1

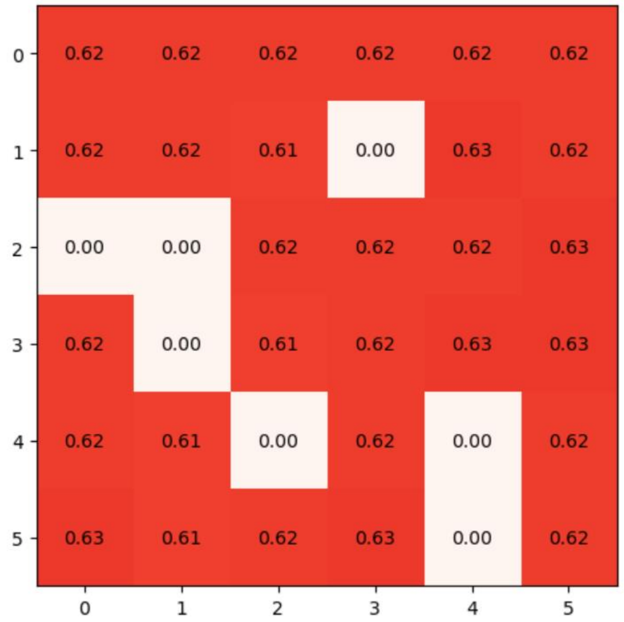
If only a small number of people are altruists🤗,

can they still improve the overall outcome?

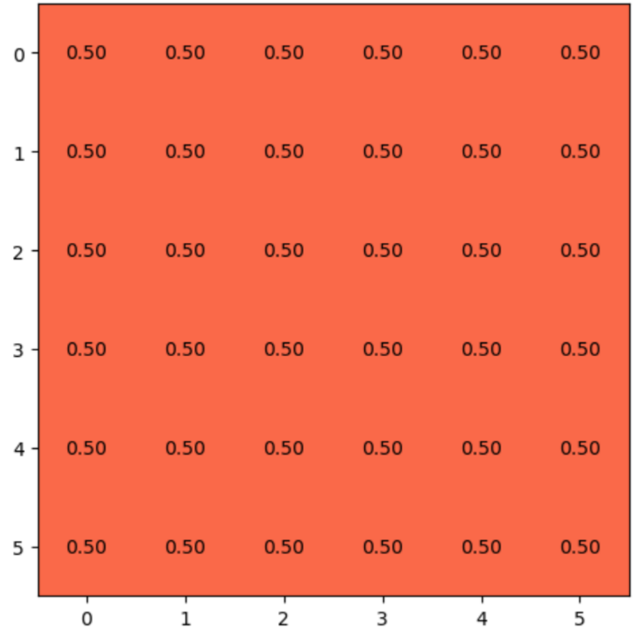
The "Catalyst Effect" of Altruists

From last class, we learned:

- If everyone is purely self-interested, the city tends to get crowded (low social welfare).



- If everyone cooperates to some extent (α > const), the city remains mid-level block density (high social welfare).



But here's the problem:

In reality, not everyone can be cooperative.

Research Question

We want to ask:

- When most people are still self-interested,
- What happens if we mix in a small group of altruists?

Intuition:

- It might help a little, but the effect should be limited.

Actual Finding:

Surprisingly, even a ***tiny*** *fraction* of altruists can ***catalyze*** the entire system into the optimal state! 🎉🎉🎉

Last Class: A Simplified City Model

The "city" is divided into Q blocks, each containing H housing units (cells).

Each housing unit can hold at most **one resident**.

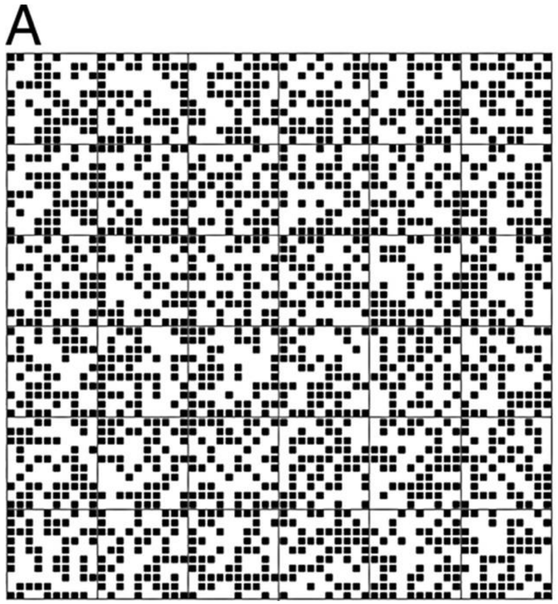
The **density** of block q is: $\rho_q = \dfrac{n_q}{H}$

where n_q, is the number of residents in block q.

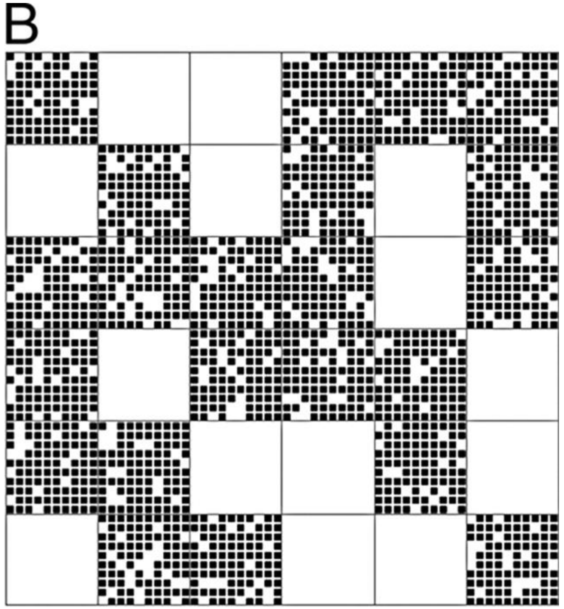
All residents share the **same utility function** $u(\rho_q)$,

which measures how satisfied they are with the density of their block.

Global utility / social welfare is the total satisfaction of all residents: $U = H \sum_q \rho_q \, u(\rho_q)$



Mixed state – residents evenly distributed across blocks.

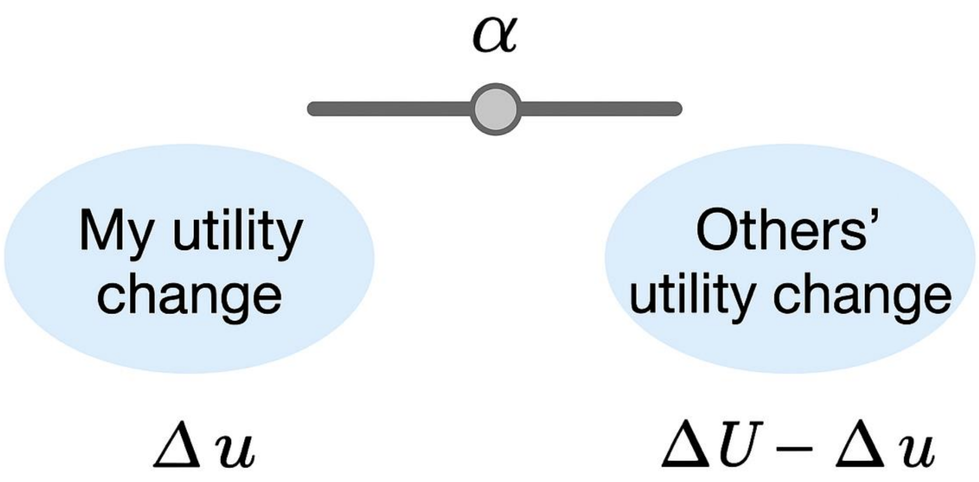


Segregated state – some blocks overcrowded, others empty.

Last Class: Considering Myself and Others

At each step:

1. Randomly pick **one resident** and **one vacant unit**.
2. Calculate the "gain" G from moving.



Formula for gain: $G = \Delta u + \alpha(\Delta U - \Delta u)$

- Δu: Change in *my* own utility if I move.
- $\Delta U - \Delta u$: Change in *everyone else's* utility.
- A: Global *cooperation level* shared by all residents.

Special cases:

- $\alpha = 0$: Purely selfish – only care about my own utility.
- $\alpha = 1$: Fully cooperative – only care about total utility.

This setup allows us to smoothly adjust between **economic behavior** (self-interest) and **physical-system-like behavior** (global optimization).

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Model Recap

City setup:

•Q blocks, each with capacity H.

Utility function:

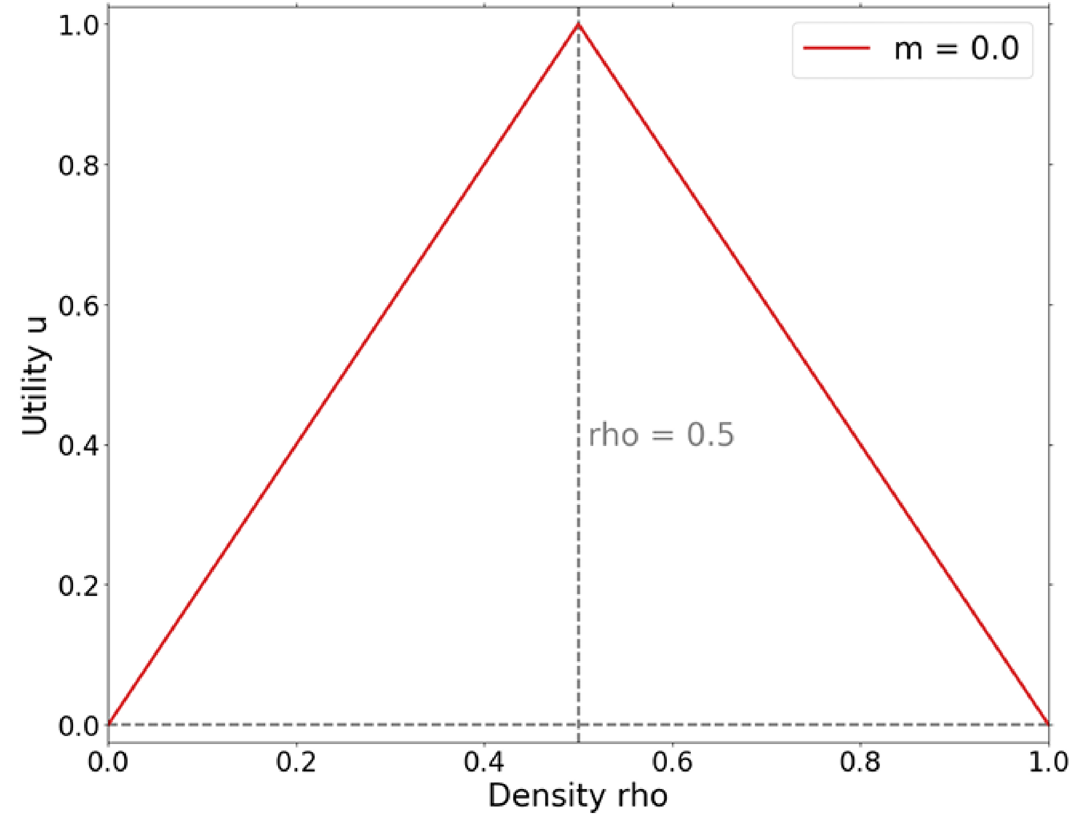
•Utility is highest (=1) when density ρ=0.5.

•Too sparse (ρ=0) or too crowded (ρ=1) → utility = 0

Two types of agents:

•**Self-interested:** Care only about their own Δu.

•**Altruists:** Care about the change in *total utility* ΔU.



Key Result 1: Threshold Effect

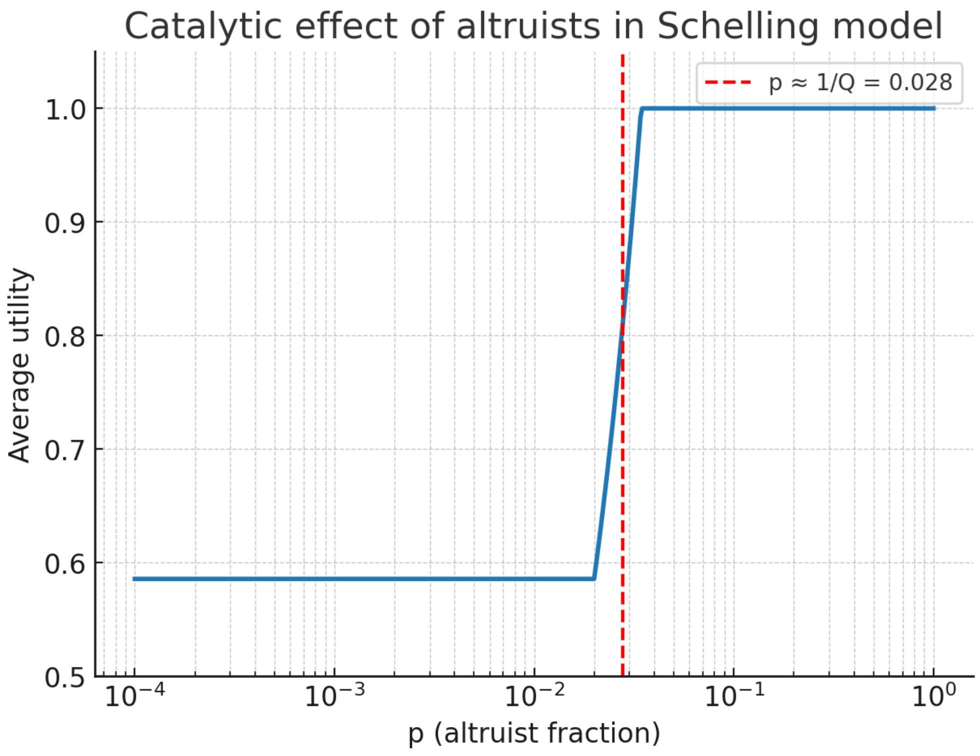
If the fraction of altruists in the population is about $p \approx \frac{1}{Q}$,

that is already enough to ***trigger*** the whole system into the optimal mixed state.

Example:
In a city with 36 blocks, only about **3% altruists** are needed.

This is **not** a smooth, linear improvement—

it's a sudden *jump* to high social welfare!



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Key Result 2: The Mechanism

Without altruists:

•Self-interested agents cluster together → blocks become overcrowded → low efficiency / social welfare.

With altruists ("sacrifice" behavior):

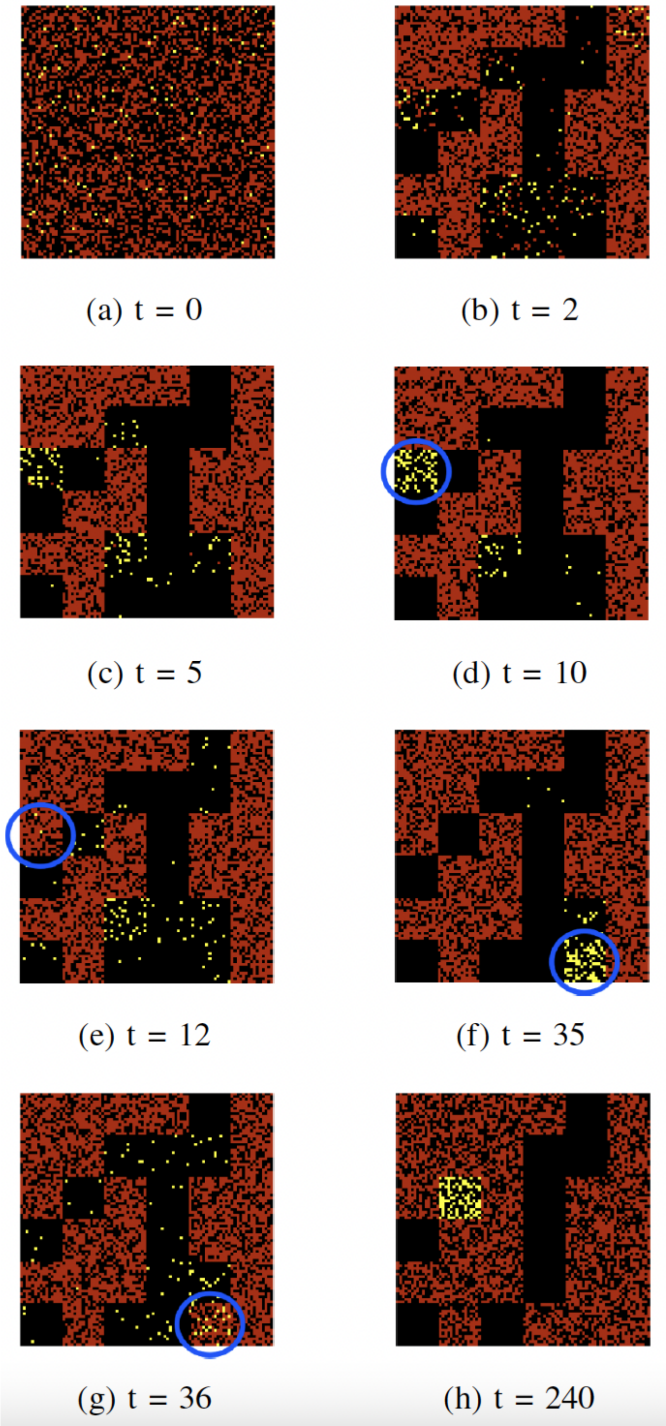
•They *choose* to leave crowded blocks and move to sparser ones.

•This makes sparse blocks gradually more attractive → self-interested agents move in.

•Overcrowded blocks get diluted → overall utility increases.

Bottom line:

Altruists keep repeating this process, acting like a *catalyst* that drives the whole system toward efficiency / high social welfare.



(a) t = 0 (b) t = 2 (c) t = 5 (d) t = 10 (e) t = 12 (f) t = 35 (g) t = 36 (h) t = 240

Why Call It "Catalysis"?

In chemistry:

- A catalyst is *not consumed* but can *accelerate* a reaction.

In the model:

- Altruists don't disappear or "get tired,"
- Yet they continuously guide self-interested agents toward better configurations.

Impact of a single altruist:

- As the system size Q increases,
  the *marginal effect* of each altruist becomes even larger.

Summary

Core Finding: A very small number of altruists can *catalyze* the whole system toward the optimal state.

Key Insight:

- Individual self-interest may cause collective failure.
- But just a few "pioneers" — or clever institutional design — can shift the entire system.

Real-World Connections:

- Public policy
- Green behaviors
- Volunteer actions

👉 **The altruism of a few may be the *spark* for large-scale social transformation.**

Part 2

What if city planners could invest in certain blocks?

Extension: Urban Investment 💴

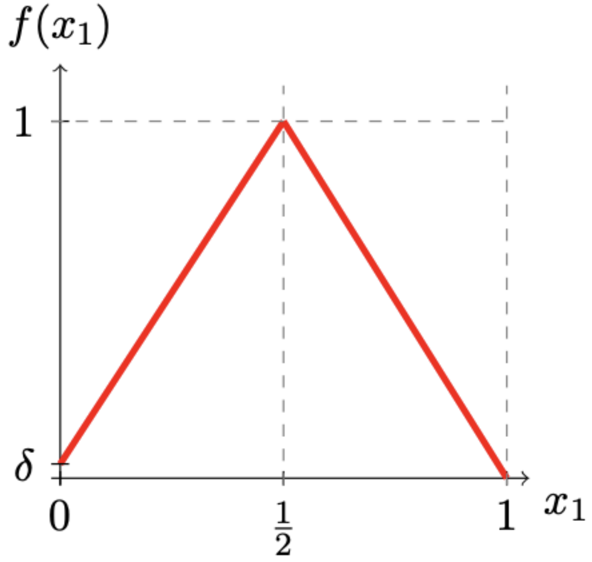
Planner intervention (city-level):

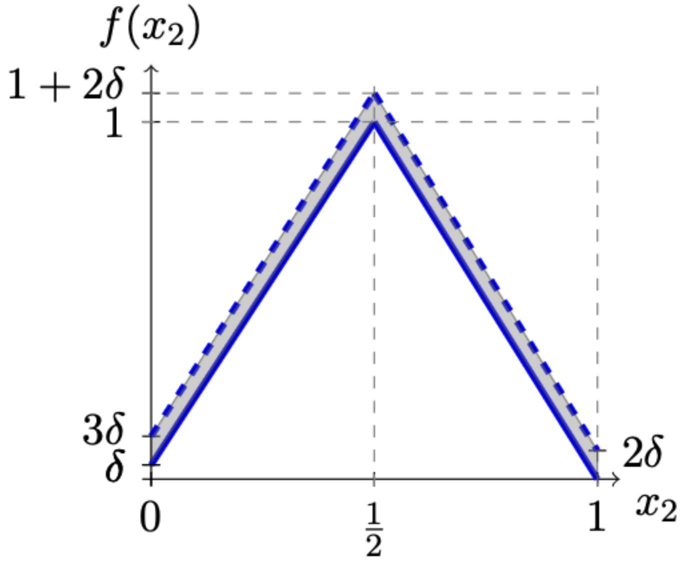
- The planner can change the utility function of certain blocks.
- This makes certain blocks *q more attractive*.

Mathematical expression:

$$g_q(\rho) \geq f_q(\rho)$$

Investment raises the attractiveness of block q.





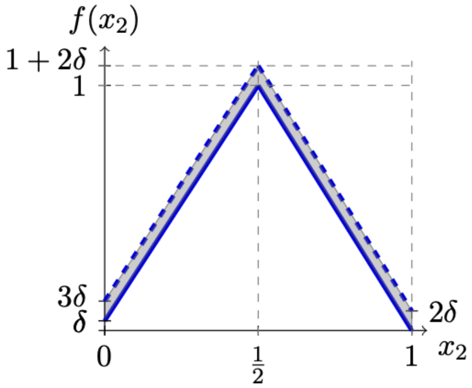
Effects of Investment

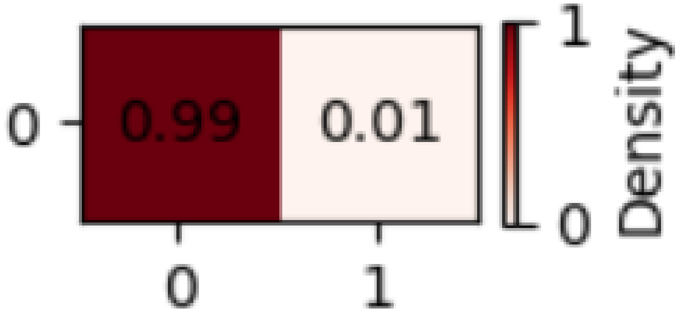
Individuals remain self-interested: They only respor to utility differences.

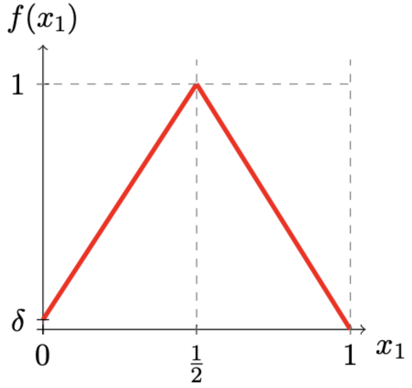
But with investment: Some blocks become more attractive.

Result:

- Migration paths of individuals are indirectly altered.
- At the macro level, social welfare gets changed







Broader Connections: Braess's Paradox

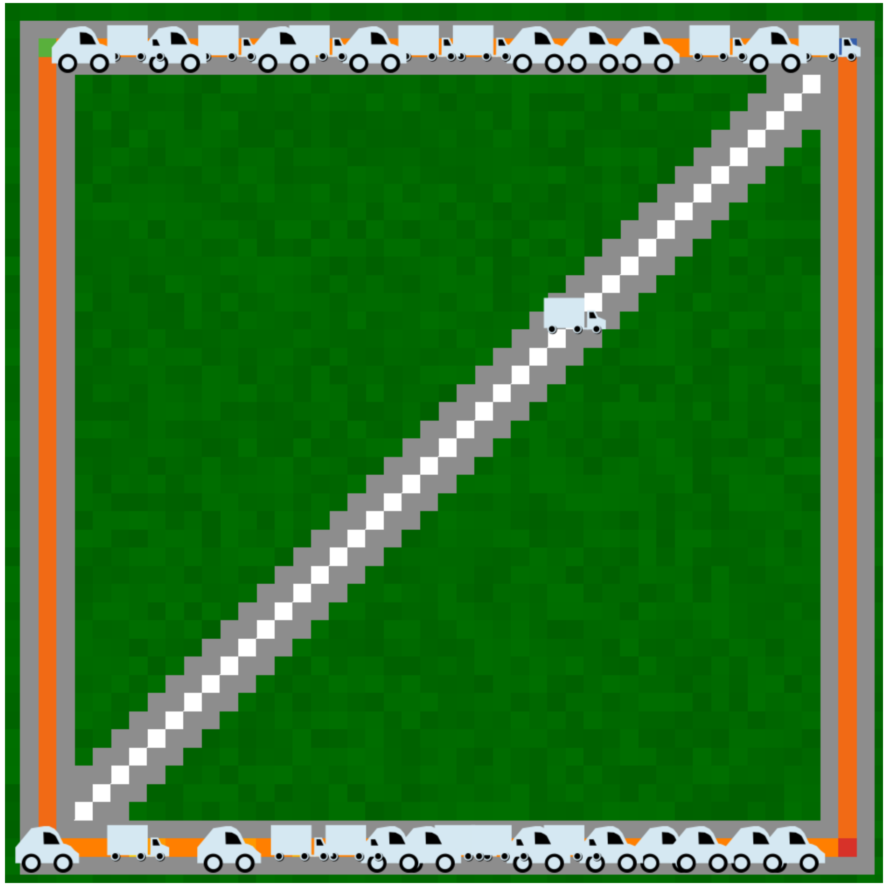
Sometimes, *"better conditions"* can actually make outcomes worse.

Example:

Adding an extra road can make traffic jams *worse*.

Why?

- Each driver chooses what seems best for themselves.
- But collectively, this creates a *less efficient* traffic pattern.



Broader Connections: **Congestion Games**

Each individual pursues their own *shortest path*.

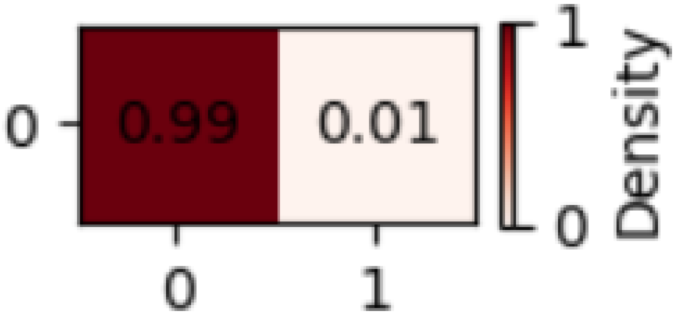
But when everyone acts this way, the overall outcome is *inefficient (low social welfare)*.

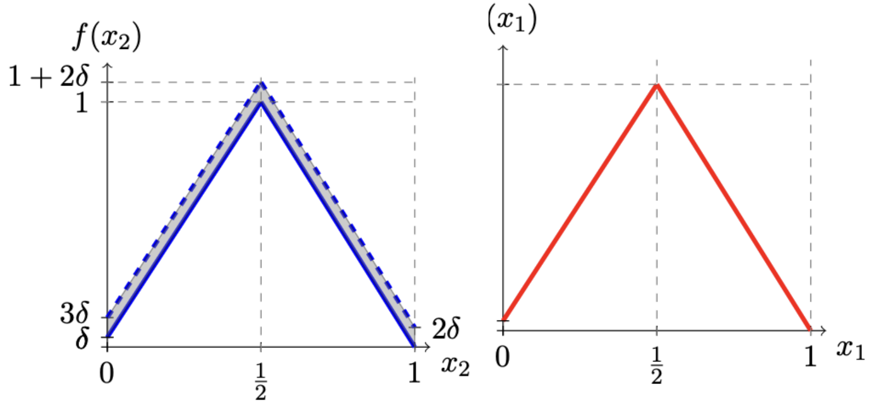
Lesson:

What is *best for individuals* is not always *best for the group*.

Question to think about:

Can we make smart investments to improve social welfare despite egoists (selfish individuals)?





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Thank You!

Thank you for your attention and participation.

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