

Chapter 1

Demo problem: Boundary-driven elastic deformation of a fish-shaped domain

Detailed documentation to be written. Here's a plot of the result and the already fairly well documented driver code...

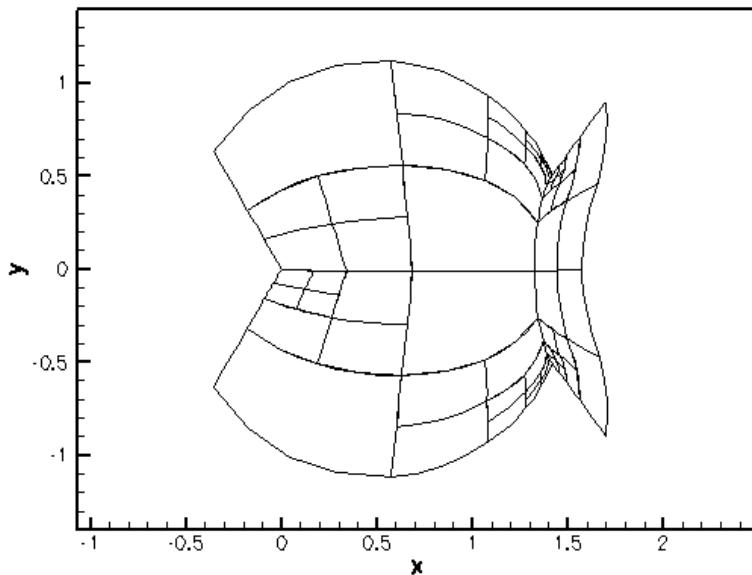


Figure 1.1 Boundary-driven elastic deformation of a fish-shaped domain.

```
//LIC// =====
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
//LIC// at http://www.oomph-lib.org.
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//LIC//
//LIC//=====
// Driver for elastic deformation of a fish-shaped domain with
// adaptivity -- deformation is driven by displacement of
// GeomObject-based boundary!

// Generic oomph-lib headers
#include "generic.h"

// Solid mechanics
#include "solid.h"

// The fish mesh
#include "meshes/fish_mesh.h"

using namespace std;
using namespace oomph;

/////////////////////////////////////////////////////////////////
/////////////////////////////////////////////////////////////////
/////////////////////////////////////////////////////////////////

//================================================================
// Refineable fish mesh upgraded to become a solid mesh
//================================================================
template<class ELEMENT>
class ElasticFishMesh : public virtual RefineableFishMesh<ELEMENT>,
                       public virtual SolidMesh
{
public:

  /// Constructor: Build underlying adaptive fish mesh and then
  /// set current Eulerian coordinates to be the Lagrangian ones.
  /// Pass pointer to geometric objects that specify the
  /// fish's back in the "current" and "undeformed" configurations,
  /// and pointer to timestepper (defaults to Static)
  // Note: FishMesh is virtual base and its constructor is automatically
  // called first! --> this is where we need to build the mesh;
  // the constructors of the derived meshes don't call the
  // base constructor again and simply add the extra functionality.
ElasticFishMesh(GeomObject* back_pt, GeomObject* undeformed_back_pt,
                TimeStepper* time stepper_pt=&Mesh::Default_TimeStepper) :
  FishMesh<ELEMENT>(back_pt,time stepper_pt),
  RefineableFishMesh<ELEMENT>(back_pt,time stepper_pt)
{
  // Mesh has been built, adaptivity etc has been set up -->
  // assign the Lagrangian coordinates so that the current
  // configuration becomes the stress-free initial configuration
  set_lagrangian_nodal_coordinates();

  // Build "undeformed" domain: This is a "deep" copy of the
  // Domain that we used to create set the Eulerian coordinates
  // in the initial mesh -- the original domain (accessible via
  // the private member data Domain_pt) will be used to update
  // the position of the boundary nodes; the copy that we're
  // creating here will be used to determine the Lagrangian coordinates
  // of any newly created SolidNodes during mesh refinement
  double xi_nose = this->Domain_pt->xi_nose();
  double xi_tail = this->Domain_pt->xi_tail();
  Undeformed_domain_pt=new FishDomain(undeformed_back_pt,xi_nose,xi_tail);

  // Loop over all elements and set the undeformed macro element pointer
  unsigned n_element=this->n_element();
  for (unsigned e=0;e<n_element;e++)
  {
    // Get pointer to full element type
    ELEMENT* el_pt=dynamic_cast<ELEMENT*>(this->element_pt(e));

    // Set pointer to macro element so the curvilinear boundaries
    // of the undeformed mesh/domain get picked up during adaptive
    // mesh refinement
    el_pt->set_undeformed_macro_elem_pt(
      Undeformed_domain_pt->macro_element_pt(e));

    // Use MacroElement representation for
    // Lagrangian coordinates of newly created
    // nodes
    el_pt->enable_use_of_undeformed_macro_element_for_new_lagrangian_coords()
  }
}

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}

/// Destructor: Kill "undeformed" Domain
virtual ~ElasticFishMesh()
{
    delete Undefomed_domain_pt;
}

private:

    /// Pointer to "undeformed" Domain -- used to determine the
    /// Lagrangian coordinates of any newly created SolidNodes during
    /// Mesh refinement
Domain* Undefomed_domain_pt;

};

////////////////////////////////////////////////////////////////

//=====================================================================
// Global variables
//=====================================================================
namespace Global_Physical_Variables
{
    /// Pointer to strain energy function
StrainEnergyFunction* Strain_energy_function_pt;

    /// Pointer to constitutive law
ConstitutiveLaw* Constitutive_law_pt;

    /// Elastic modulus
double E=1.0;

    /// Poisson's ratio
double Nu=0.3;

    /// "Mooney Rivlin" coefficient for generalised Mooney Rivlin law
double C1=1.3;

    /// Body force
double Gravity=0.0;

    /// Body force vector: Vertically downwards with magnitude Gravity
void body_force(const double& t,
                const Vector<double>& xi,
                Vector<double>& b)
{
    b[0]=0.0;
    b[1]=-Gravity;
}
}

////////////////////////////////////////////////////////////////

//=====================================================================
// Boundary-driven elastic deformation of fish-shaped domain.
//=====================================================================
template<class ELEMENT>
class ElasticFishProblem : public Problem
{

public:

    /// Constructor:
ElasticFishProblem();

    /// Run simulation.
void run();

    /// Access function for the mesh
ElasticFishMesh<ELEMENT>* mesh_pt()

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{return dynamic_cast<ElasticFishMesh<ELEMENT>*>(Problem::mesh_pt());}

/// Doc the solution
void doc_solution(DocInfo& doc_info);

/// Update function (empty)
void actions_after_newton_solve() {}

/// Update before solve: We're dealing with a static problem so
/// the nodal positions before the next solve merely serve as
/// initial conditions. For meshes that are very strongly refined
/// near the boundary, the update of the displacement boundary
/// conditions (which only moves the SolidNodes *on* the boundary),
/// can lead to strongly distorted meshes. This can cause the
/// Newton method to fail --> the overall method is actually more robust
/// if we use the nodal positions as determined by the Domain/MacroElement-
/// based mesh update as initial guesses.
void actions_before_newton_solve()
{
    bool update_all_solid_nodes=true;
    mesh_pt() ->node_update(update_all_solid_nodes);
}

/// Update after adapt: Pin all redundant pressure nodes (if required)
void actions_after_adapt()
{
    // Pin the redundant solid pressures (if any)
    PVDEquationsBase<2>::pin_redundant_nodal_solid_pressures(
        mesh_pt() ->element_pt());
}

private:
    // Geometric object that represents the deformable fish back
    Circle* Fish_back_pt;
};

//=====================================================================
// Constructor:
//=====================================================================
template<class ELEMENT>
ElasticFishProblem<ELEMENT>::ElasticFishProblem()
{
    // Set coordinates and radius for the circle that will become the fish back
    double x_c=0.5;
    double y_c=0.0;
    double r_back=1.0;

    // Build geometric object that will become the deformable fish back
    //GeomObject* fish_back_pt=new ElasticFishBackElement(x_c,y_c,r_back);
    Fish_back_pt=new Circle(x_c,y_c,r_back);

    // Build geometric object that specifies the fish back in the
    // undeformed configuration (basically a deep copy of the previous one)
    GeomObject* undeformed_fish_back_pt=new Circle(x_c,y_c,r_back);

    // Build fish mesh with geometric objects that specify the deformable
    // and undeformed fish back
    Problem::mesh_pt()=new ElasticFishMesh<ELEMENT>(Fish_back_pt,
                                                    undeformed_fish_back_pt);

    // Set error estimator
    Z2ErrorEstimator* error_estimator_pt=new Z2ErrorEstimator;
    mesh_pt() ->spatial_error_estimator_pt()=error_estimator_pt;

    // Change/doc targets for mesh adaptation
    mesh_pt() ->max_permitted_error()=0.05;
    mesh_pt() ->min_permitted_error()=0.005;
    mesh_pt() ->doc_adaptivity_targets(cout);

    // Pin all nodal positions apart from those on the tail
    unsigned num_bound = mesh_pt() ->nboundary();
    for(unsigned ibound=0;ibound<num_bound;ibound++)
    {
        if (ibound!=2)
        {
            unsigned num_nod=mesh_pt() ->nboundary_node(ibound);
            for (unsigned inod=0;inod<num_nod;inod++)
            {
                for (unsigned i=0;i<2;i++)
                {
                    mesh_pt() ->boundary_node_pt(ibound,inod) ->pin_position(i);
                }
            }
        }
    }
}

```

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        }

    }

//Loop over the elements in the mesh to set parameters/function pointers
unsigned n_element =mesh_pt ()->n element ();
for(unsigned i=0;i<n_element;i++)
{
    //Cast to a solid element
ELEMENT *el_pt = dynamic_cast<ELEMENT*>(mesh_pt ()->element_pt (i));

    // Set the constitutive law
el_pt->constitutive_law_pt () =
Global_Physical_Variables::Constitutive_law_pt;

    // Set the body force
el_pt->body_force_fct_pt ()=Global_Physical_Variables::body_force;

}

// Pin the redundant solid pressures (if any)
PVDEquationsBase<2>::pin_redundant_nodal_solid_pressures(
mesh_pt ()->element_pt ());

//Attach the boundary conditions to the mesh
cout << assign_eqn_numbers() << std::endl;

// Refine the problem uniformly (this automatically passes the
// function pointers/parameters to the finer elements
refine_uniformly();

// The non-pinned positions of the newly SolidNodes will have been
// determined by interpolation. Update all solid nodes based on
// the Mesh's Domain/MacroElement representation.
bool update_all_solid_nodes=true;
mesh_pt ()->node_update(update_all_solid_nodes);

// Now set the Eulerian equal to the Lagrangian coordinates
mesh_pt ()->set_lagrangian_nodal_coordinates();
}

//=====================================================================
/// Doc the solution
//=====================================================================
template<class ELEMENT>
void ElasticFishProblem<ELEMENT>::doc_solution(DocInfo& doc_info)
{
    ofstream some_file;
    char filename[100];

    // Number of plot points
    unsigned npts = 5;

    // Output shape of deformed body
    snprintf(filename, sizeof(filename), "%s/soln%u.dat", doc_info.directory().c_str(),
    doc_info.number());
    some_file.open(filename);
    mesh_pt ()->output(some_file,npts);
    some_file.close();

    // removed until Jacobi eigensolver is re-instated
    // // Output principal stress vectors at the centre of all elements
    // SolidHelpers::doc_2D_principal_stress<ELEMENT>(doc_info,mesh_pt ());

}

//=====================================================================
/// Run the problem
//=====================================================================
template<class ELEMENT>
void ElasticFishProblem<ELEMENT>::run()
{
    // Output
DocInfo doc_info;

    // Set output directory
doc_info.set_directory("RESLT");

    // Step number
doc_info.number ()=0;
    // Initial parameter values
    // Gravity:
Global_Physical_Variables::Gravity=0.1;
    //Parameter incrementation
unsigned nstep=5;
}

```

```

for(unsigned i=0;i<nstep;i++)
{
  //Solve the problem with Newton's method, allowing for up to 5
  //rounds of adaptation
  newton_solve(5);

  // Doc solution
  doc_solution(doc_info);
  doc_info.number()++;

  // Increment width
  Fish_back_pt->y_c()+=0.03;
}

//=====
/// Driver for simple elastic problem
//=====
int main()
{
  //Initialise physical parameters
  Global_Physical_Variables::E = 2.1;
  Global_Physical_Variables::Nu = 0.4;
  Global_Physical_Variables::C1 = 1.3;
  // Define a strain energy function: Generalised Mooney Rivlin
  Global_Physical_Variables::Strain_energy_function_pt =
    new GeneralisedMooneyRivlin(&Global_Physical_Variables::Nu,
                                 &Global_Physical_Variables::C1,
                                 &Global_Physical_Variables::E);
  // Define a constitutive law (based on strain energy function)
  Global_Physical_Variables::Constitutive_law_pt =
    new IsotropicStrainEnergyFunctionConstitutiveLaw(
      Global_Physical_Variables::Strain_energy_function_pt);
  //Set up the problem with pure displacement formulation
  ElasticFishProblem<RefineableQPVDElement<2,3> > problem;
  problem.run();
}

```

1.1 PDF file

A [pdf version](#) of this document is available. \