

# Chapter 1

## Example problem: Steady flow in a curved tube

The problem of steady flow in a curved tube is considered with a prescribed Poiseuille flow at the inlet and a traction-free outlet condition. It is not clear that the latter is appropriate, but the main aim of this example is to check that the TubeMesh works correctly.

A detailed comparison between the flow field and the Dean solution should be performed for validation purposes, but the qualitative features seem reasonable.

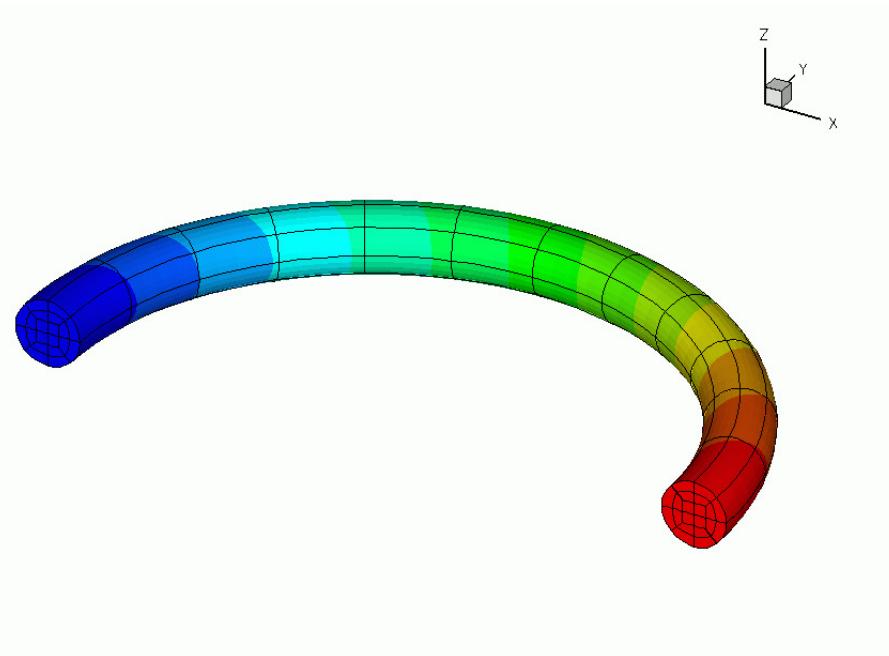


Figure 1.1 Sketch of the problem with pressure contours.

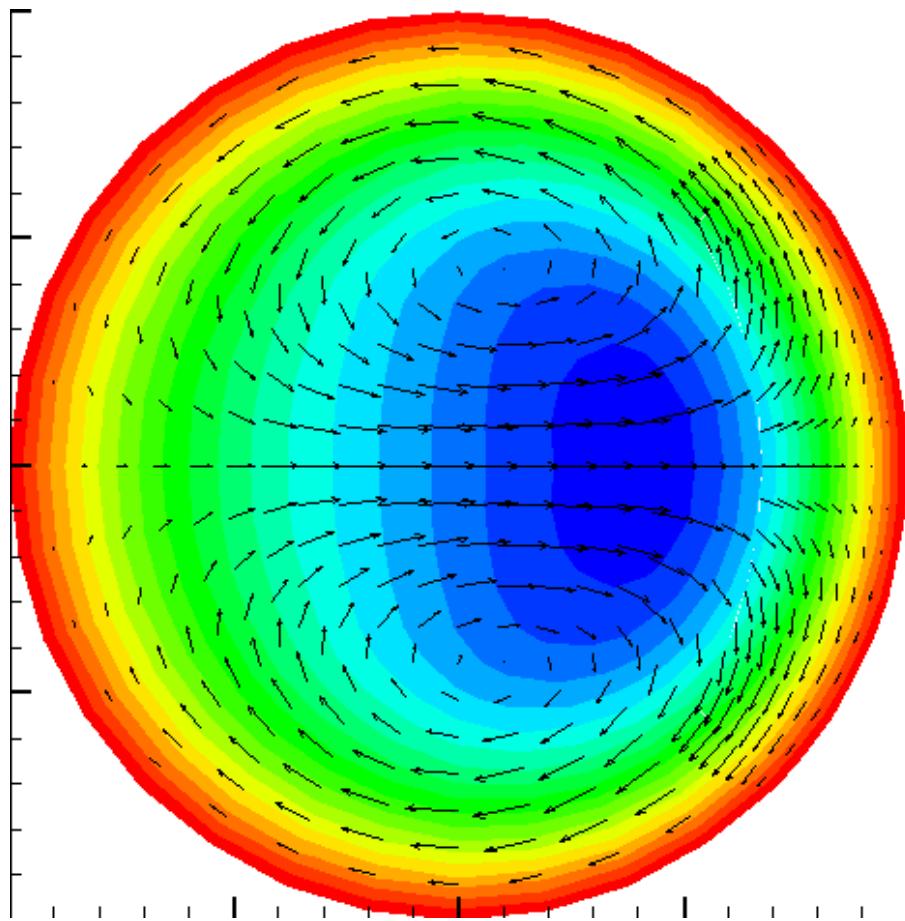


Figure 1.2 Contours of axial velocity and secondary streamlines.

Detailed documentation to be written. Here's the driver code...

```
//LIC// =====
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
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//LIC// The authors may be contacted at oomph-lib@maths.man.ac.uk.
//LIC// =====
/// Driver for a 3D navier stokes steady entry flow problem in a
/// uniformly curved tube

//Generic routines
#include "generic.h"
#include "navier_stokes.h"

// The mesh
```

```

#include "meshes/tube_mesh.h"

using namespace std;
using namespace oomph;

//==start_of_MyCurvedCylinder=====
//A geometric object that represents the geometry of the domain
//=====
class MyCurvedCylinder : public GeomObject
{
public:
    /// Constructor that takes the radius and curvature of the tube
    /// as its arguments
    MyCurvedCylinder(const double &radius, const double &delta) :
        GeomObject(3,3), Radius(radius), Delta(delta) { }

    /// Destructor
    virtual~MyCurvedCylinder() {}

    /// Lagrangian coordinate xi
    void position (const Vector<double>& xi, Vector<double>& r) const
    {
        r[0] = (1.0/Delta)*cos(xi[0]) + xi[2]*Radius*cos(xi[0])*cos(xi[1]);
        r[1] = (1.0/Delta)*sin(xi[0]) + xi[2]*Radius*sin(xi[0])*cos(xi[1]);
        r[2] = -xi[2]*Radius*sin(xi[1]);
    }

    /// Return the position of the tube as a function of time
    /// (doesn't move as a function of time)
    void position(const unsigned& t,
                  const Vector<double>& xi, Vector<double>& r) const
    {
        position(xi,r);
    }

private:
    /// Storage for the radius of the tube
    double Radius;

    //Storage for the curvature of the tube
    double Delta;
};

//==start_of_namespace=====
// Namespace for physical parameters
//=====
namespace Global_Physical_Variables
{
    /// Reynolds number
    double Re=50;

    /// The desired curvature of the pipe
    double Delta=0.1;
} // end_of_namespace

//==start_of_problem_class=====
// Entry flow problem in tapered tube domain
//=====
template<class ELEMENT>
class SteadyCurvedTubeProblem : public Problem
{

public:
    /// Constructor: Pass DocInfo object and target errors
    SteadyCurvedTubeProblem(DocInfo& doc_info, const double& min_error_target,
                           const double& max_error_target);

    /// Destructor (empty)
    ~SteadyCurvedTubeProblem() {}

    /// Update the problem specs before solve
    void actions_before_newton_solve();

    /// After adaptation: Pin redundant pressure dofs.
    void actions_after_adapt()
    {
        // Pin redundant pressure dofs
        RefineableNavierStokesEquations<3>::pin_redundant_nodal_pressures(mesh_pt() ->element_pt());
    }
}

```

```

/// Doc the solution
void doc_solution();

/// Overload generic access function by one that returns
/// a pointer to the specific mesh
RefineableTubeMesh<ELEMENT>* mesh_pt()
{
    return dynamic_cast<RefineableTubeMesh<ELEMENT>*>(Problem::mesh_pt());
}

private:

/// Doc info object
DocInfo Doc_info;

/// Pointer to GeomObject that specifies the domain volume
GeomObject *Volume_pt;

}; // end_of_problem_class

//==start_of_constructor=====
// Constructor: Pass DocInfo object and error targets
//=====
template<class ELEMENT>
SteadyCurvedTubeProblem<ELEMENT>::SteadyCurvedTubeProblem(DocInfo& doc_info,
    const double& min_error_target,
    const double& max_error_target)
: Doc_info(doc_info)
{

// Setup mesh:
//-----

// Create GeomObject that specifies the domain geometry
//The radius of the tube is one and the curvature is specified by
//the global variable Delta.
Volume_pt=new MyCurvedCylinder(1.0,Global_Physical_Variables::Delta);
//Define pi
const double pi = MathematicalConstants::Pi;
//Set the centerline coordinates spanning the mesh
Vector<double> centreline_limits(2);
centreline_limits[0] = 0.0;
centreline_limits[1] = pi;

//Set the positions of the angles that divide the outer ring
//These must be in the range -pi,pi, ordered from smallest to
//largest
Vector<double> theta_positions(4);
theta_positions[0] = -0.75*pi;
theta_positions[1] = -0.25*pi;
theta_positions[2] = 0.25*pi;
theta_positions[3] = 0.75*pi;

//Define the radial fraction of the central box (always halfway
//along the radius)
Vector<double> radial_frac(4,0.5);
// Number of layers in the initial mesh
unsigned nlayer=6;

// Build and assign mesh
Problem::mesh_pt ()= new RefineableTubeMesh<ELEMENT>(Volume_pt,
    centreline_limits,
    theta_positions,
    radial_frac,
    nlayer);

// Set error estimator
Z2ErrorEstimator* error_estimator_pt=new Z2ErrorEstimator;
mesh_pt()>>spatial_error_estimator_pt()=>error_estimator_pt;
// Error targets for adaptive refinement
mesh_pt()>>max_permitted_error()=max_error_target;
mesh_pt()>>min_permitted_error()=min_error_target;
// Set the boundary conditions for this problem: All nodal values are
// free by default -- just pin the ones that have Dirichlet conditions
// here.
//Choose the conventional form by setting gamma to zero
//The boundary conditions will be pseudo-traction free (d/dn = 0)
ELEMENT::Gamma[0] = 0.0;
ELEMENT::Gamma[1] = 0.0;
ELEMENT::Gamma[2] = 0.0;

//Loop over the boundaries
unsigned num_bound = mesh_pt ()>>nboundary ();
for(unsigned ibound=0;ibound<num_bound;ibound++)

```

```

{
  unsigned num_nod= mesh_pt()->nboundary_node(ibound);
  for (unsigned inod=0;inod<num_nod;inod++)
  {
    // Boundary 0 is the inlet symmetry boundary:
    // Boundary 1 is the tube wall
    // Pin all values
    if((ibound==0) || (ibound==1))
    {
      mesh_pt()->boundary_node_pt(ibound,inod)->pin(0);
      mesh_pt()->boundary_node_pt(ibound,inod)->pin(1);
      mesh_pt()->boundary_node_pt(ibound,inod)->pin(2);
    }
  }
} // end loop over boundaries

// Loop over the elements to set up element-specific
// things that cannot be handled by constructor
unsigned n_element = mesh_pt()->nelement();
for(unsigned i=0;i<n_element;i++)
{
  // Upcast from GeneralisedElement to the present element
  ELEMENT* el_pt = dynamic_cast<ELEMENT*>(mesh_pt()->element_pt(i));

  //Set the Reynolds number, etc
  el_pt->re_pt() = &Global_Physical_Variables::Re;
}

// Pin redundant pressure dofs
RefineableNavierStokesEquations<3>::
pin_redundant_nodal_pressures(mesh_pt()->element_pt());

//Attach the boundary conditions to the mesh
cout << "Number of equations: " << assign_eqn_numbers() << std::endl;

} // end_of_constructor

//=====================================================================
// Set the inflow boundary conditions
//=====================================================================
template<class ELEMENT>
void SteadyCurvedTubeProblem<ELEMENT>::actions_before_newton_solve()
{

  // (Re-)assign velocity profile at inflow values
  //-----
  unsigned ibound=0;
  unsigned num_nod= mesh_pt()->nboundary_node(ibound);
  for (unsigned inod=0;inod<num_nod;inod++)
  {
    // Recover coordinates of tube relative to centre position
    double x=mesh_pt()->boundary_node_pt(ibound,inod)->x(0) -
      1.0/Global_Physical_Variables::Delta;
    double z=mesh_pt()->boundary_node_pt(ibound,inod)->x(2);
    //Calculate the radius
    double r=sqrt(x*x+z*z);

    // Poiseuille-type profile for axial velocity (component 1 at the inlet)
    mesh_pt()->boundary_node_pt(ibound,inod)->
      set_value(1,(1.0-pow(r,2.0)));
  }
} // end_of_actions_before_newton_solve

//=====================================================================
// Doc the solution
//=====================================================================
template<class ELEMENT>
void SteadyCurvedTubeProblem<ELEMENT>::doc_solution()
{
  //Output file stream
  ofstream some_file;
  char filename[100];

  // Number of plot points
  unsigned npts;
  npts=5;

  //Need high precision for large radii of curvature
  //some_file.precision(10);
  // Output solution labelled by the Reynolds number
  sprintf(filename, sizeof(filename), "%s/soln_Re%g.dat", Doc_info.directory().c_str(),
  Global_Physical_Variables::Re);
  some_file.open(filename);
  mesh_pt()->output(some_file,npts);
}

```

```

some_file.close();

} // end_of_doc_solution

////////////////////////////////////////////////////////////////
////////////////////////////////////////////////////////////////
////////////////////////////////////////////////////////////////

//==start_of_main=====
/// Driver for 3D entry flow into a curved tube. If there are
/// any command line arguments, we regard this as a validation run
/// and perform only a single adaptation
//=====
int main(int argc, char* argv[])
{
    // Store command line arguments
    CommandLineArgs::setup(argc,argv);

    // Allow (up to) two rounds of fully automatic adaption in response to
    // -----
    // error estimate
    // -----
    unsigned max_adapt;
    double max_error_target,min_error_target;

    // Set max number of adaptations in black-box Newton solver and
    // error targets for adaptation
    if (CommandLineArgs::Argc==1)
    {
        // Up to two adaptions
        max_adapt=2;

        // Error targets for adaptive refinement
        max_error_target=0.001;
        min_error_target=0.00001;
    }
    // Validation run: Only one adaptation. Relax error targets
    // for faster solution
    else
    {
        // Validation run: Just one round of adaptation
        max_adapt=1;

        // Error targets for adaptive refinement
        max_error_target=0.02;
        min_error_target=0.002;
    }
    // end max_adapt setup

    // Set up doc info
    DocInfo doc_info;
    // Do Taylor-Hood elements
    // -----
    {
        // Set output directory
        doc_info.set_directory("RESLT_TH");

        // Step number
        doc_info.number()=0;

        // Build problem
        SteadyCurvedTubeProblem<RefineableQTaylorHoodElement<3> >
        problem(doc_info,min_error_target,max_error_target);

        cout << " Doing Taylor-Hood elements " << std::endl;

        // Solve the problem
        problem.newton_solve(max_adapt);
        // Doc solution after solving
        problem.doc_solution();
    }

    // Do Crouzeix-Raviart elements
    // -----
    {
        // Set output directory
        doc_info.set_directory("RESLT_CR");

        // Step number
        doc_info.number()=0;

        // Build problem
        SteadyCurvedTubeProblem<RefineableQCrouzeixRaviartElement<3> >

```

```
problem(doc_info,min_error_target,max_error_target);  
cout << " Doing Crouzeix-Raviart elements " << std::endl;  
  
// Solve the problem  
problem.newton_solve(max_adapt);  
// Doc solution after solving  
problem.doc_solution();  
}  
  
} // end_of_main
```

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## 1.1 PDF file

A [pdf version](#) of this document is available. \