

# Chapter 1

## Demo problem: Large-amplitude shear deformation of a 3D elastic solid

Detailed documentation to be written. Here's the already fairly well documented driver code...

```
//LIC// =====
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
//LIC// at http://www.oomph-lib.org.
//LIC//
//LIC// Copyright (C) 2006-2025 Matthias Heil and Andrew Hazel
//LIC//
//LIC// This library is free software; you can redistribute it and/or
//LIC// modify it under the terms of the GNU Lesser General Public
//LIC// License as published by the Free Software Foundation; either
//LIC// version 2.1 of the License, or (at your option) any later version.
//LIC//
//LIC// This library is distributed in the hope that it will be useful,
//LIC// but WITHOUT ANY WARRANTY; without even the implied warranty of
//LIC// MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU
//LIC// Lesser General Public License for more details.
//LIC//
//LIC// You should have received a copy of the GNU Lesser General Public
//LIC// License along with this library; if not, write to the Free Software
//LIC// Foundation, Inc., 51 Franklin Street, Fifth Floor, Boston, MA
//LIC// 02110-1301 USA.
//LIC//
//LIC// The authors may be contacted at oomph-lib@maths.man.ac.uk.
//LIC//
//LIC//=====
// Driver for elastic deformation of a cuboidal domain
// The deformation is a simple shear in the x-z plane driven by
// motion of the top boundary, for exact solution see Green & Zerna
// Generic oomph-lib headers
#include "generic.h"

// Solid mechanics
#include "solid.h"

// The mesh
#include "meshes/simple_cubic_mesh.h"

using namespace std;

using namespace oomph;

////////////////////////////////////////////////////////////////////////
////////////////////////////////////////////////////////////////////////
////////////////////////////////////////////////////////////////////////

//=====
// Simple cubic mesh upgraded to become a solid mesh
//=====
template<class ELEMENT>
class ElasticCubicMesh : public virtual SimpleCubicMesh<ELEMENT>,
                        public virtual SolidMesh
{
public:
```

```

/// Constructor:
ElasticCubicMesh(const unsigned &nx, const unsigned &ny, const unsigned &nz,
                  const double &a, const double &b, const double &c,
                  TimeStepper* time stepper_pt=&Mesh::Default_TimeStepper) :
SimpleCubicMesh<ELEMENT>(nx,ny,nz,-a,a,-b,b,-c,c,time stepper_pt)
{
    //Assign the initial lagrangian coordinates
    set_lagrangian_nodal_coordinates();
}

/// Empty Destructor
virtual ~ElasticCubicMesh() { }

};

////////////////////////////////////////////////////////////////

//=====================================================================
// Global variables
//=====================================================================
namespace Global_Physical_Variables
{
    /// Pointer to strain energy function
    StrainEnergyFunction* Strain_energy_function_pt;

    /// Pointer to constitutive law
    ConstitutiveLaw* Constitutive_law_pt;

    /// Elastic modulus
    double E=1.0;

    /// Poisson's ratio
    double Nu=0.3;

    /// "Mooney Rivlin" coefficient for generalised Mooney Rivlin law
    double C1=1.3;

    /// Body force
    double Gravity=0.0;

    /// Body force vector: Vertically downwards with magnitude Gravity
    void body_force(const Vector<double>& xi,
                    const double& t,
                    Vector<double>& b)
    {
        b[0]=0.0;
        b[1]=-Gravity;
    }
}

////////////////////////////////////////////////////////////////

//=====================================================================
// Boundary-driven elastic deformation of fish-shaped domain.
//=====================================================================
template<class ELEMENT>
class SimpleShearProblem : public Problem
{
    double Shear;

    void set_incompressible(ELEMENT *el_pt,const bool &incompressible);

public:

    /// Constructor:
    SimpleShearProblem(const bool &incompressible);

    /// Run simulation.
    void run(const std::string &dirname);

    /// Access function for the mesh
    ElasticCubicMesh<ELEMENT>* mesh_pt()
    {return dynamic_cast<ElasticCubicMesh<ELEMENT>>(Problem::mesh_pt());}
}

```

```

/// Doc the solution
void doc_solution(DocInfo& doc_info);

/// Update function (empty)
void actions_after_newton_solve() {}

/// Update before solve: We're dealing with a static problem so
/// the nodal positions before the next solve merely serve as
/// initial conditions. For meshes that are very strongly refined
/// near the boundary, the update of the displacement boundary
/// conditions (which only moves the SolidNodes *on* the boundary),
/// can lead to strongly distorted meshes. This can cause the
/// Newton method to fail --> the overall method is actually more robust
/// if we use the nodal positions as determined by the Domain/MacroElement-
/// based mesh update as initial guesses.
void actions_before_newton_solve()
{
    apply_boundary_conditions();
    bool update_all_solid_nodes=true;
    mesh_pt() ->node_update(update_all_solid_nodes);
}

/// Shear the top
void apply_boundary_conditions()
{
    unsigned ibound = 5;
    unsigned num_nod=mesh_pt() ->nboundary_node(ibound);
    for (unsigned inod=0;inod<num_nod;inod++)
    {
        SolidNode *solid_nod_pt = static_cast<SolidNode*>(
            mesh_pt() ->boundary_node_pt(ibound,inod));

        solid_nod_pt->x(0) = solid_nod_pt->xi(0) + Shear*
            solid_nod_pt->xi(2);
    }
}

//=====
// Constructor:
//=====
template<class ELEMENT>
SimpleShearProblem<ELEMENT>::SimpleShearProblem(const bool &incompressible)
: Shear(0.0)
{
    double a = 1.0, b = 1.0, c = 1.0;
    unsigned nx = 5, ny = 5, nz = 5;

    // Build mesh
    Problem::mesh_pt() =new ElasticCubicMesh<ELEMENT>(nx,ny,nz,a,b,c);
    //Loop over all boundaries
    for(unsigned b=0;b<6;b++)
    {
        //Loop over nodes in the boundary
        unsigned n_node = mesh_pt() ->nboundary_node(b);
        for(unsigned n=0;n<n_node;n++)
        {
            //Pin all nodes in the y and z directions to keep the motion in plane
            for(unsigned i=1;i<3;i++)
            {
                mesh_pt() ->boundary_node_pt(b,n) ->pin_position(i);
            }
            //On the top and bottom pin the positions in x
            if((b==0) || (b==5))
            {
                mesh_pt() ->boundary_node_pt(b,n) ->pin_position(0);
            }
        }
    }
    //Loop over the elements in the mesh to set parameters/function pointers
    unsigned n_element =mesh_pt() ->nelement();
    for(unsigned i=0;i<n_element;i++)
    {
        //Cast to a solid element
        ELEMENT *el_pt = dynamic_cast<ELEMENT*>(mesh_pt() ->element_pt(i));

        // Set the constitutive law
        el_pt->constitutive_law_pt() =
            Global_Physical_Variables::Constitutive_law_pt;

        set_incompressible(el_pt,incompressible);

        // Set the body force
        //el_pt->body_force_fct_pt()=Global_Physical_Variables::body_force;
    }
}

```

```

// Pin the redundant solid pressures (if any)
// PVEDEquationsBase<2>::pin_redundant_nodal_solid_pressures()
// mesh_pt() ->element_pt());

// Attach the boundary conditions to the mesh
omph_info << "Number of equations: " << assign_eqn_numbers() << std::endl;
}

//=====================================================================
/// Doc the solution
//=====================================================================
template<class ELEMENT>
void SimpleShearProblem<ELEMENT>::doc_solution(DocInfo& doc_info)
{
    ofstream some_file;
    char filename[100];

    // Number of plot points
    unsigned npts = 5;

    // Output shape of deformed body
    sprintf(filename, sizeof(filename), "%s/soln%u.dat", doc_info.directory().c_str(),
            doc_info.number());
    some_file.open(filename);
    mesh_pt() ->output(some_file,npts);
    some_file.close();
    sprintf(filename, sizeof(filename), "%s/stress%u.dat", doc_info.directory().c_str(),
            doc_info.number());
    some_file.open(filename);
    //Output the appropriate stress at the centre of each element
    Vector<double> s(3,0.0);
    Vector<double> x(3);
    DenseMatrix<double> sigma(3,3);
    unsigned n_element = mesh_pt() ->nElement();
    for(unsigned e=0;e<n_element;e++)
    {
        ELEMENT* el_pt = dynamic_cast<ELEMENT*>(mesh_pt() ->element_pt(e));
        el_pt->interpolated_x(s,x);
        el_pt->get_stress(s,sigma);

        //Output
        for(unsigned i=0;i<3;i++)
        {
            some_file << x[i] << " ";
        }
        for(unsigned i=0;i<3;i++)
        {
            for(unsigned j=0;j<3;j++)
            {
                some_file << sigma(i,j) << " ";
            }
        }
        some_file << std::endl;
    }
    some_file.close();
}

//=====================================================================
/// Run the problem
//=====================================================================
template<class ELEMENT>
void SimpleShearProblem<ELEMENT>::run(const std::string &dirname)
{
    // Output
    DocInfo doc_info;
    // Set output directory
    doc_info.set_directory(dirname);

    // Step number
    doc_info.number()=0;
    // Initial parameter values
    // Gravity:
    Global_Physical_Variables::Gravity=0.1;
    //Parameter incrementation
    unsigned nstep=2;
    for(unsigned i=0;i<nstep;i++)
    {
        //Solve the problem with Newton's method, allowing for up to 5
        //rounds of adaptation
        newton_solve();

        // Doc solution
        doc_solution(doc_info);
}

```

```

doc_info.number()++;
//Increase the shear
Shear += 0.5;
}

}

template<>
void SimpleShearProblem<QPVDElement<3,3> >::set_incompressible(
  QPVDElement<3,3> *el_pt, const bool &incompressible)
{
  //Does nothing
}

template<>
void SimpleShearProblem<QPVDElementWithPressure<3> >::set_incompressible(
  QPVDElementWithPressure<3> *el_pt, const bool &incompressible)
{
  if(incompressible) {el_pt->set_incompressible();}
  else {el_pt->set_compressible();}
}

template<>
void SimpleShearProblem<QPVDElementWithContinuousPressure<3> >::
set_incompressible(
  QPVDElementWithContinuousPressure<3> *el_pt, const bool &incompressible)
{
  if(incompressible) {el_pt->set_incompressible();}
  else {el_pt->set_compressible();}
}

//=====================================================================
/// Driver for simple elastic problem
//=====================================================================
int main()
{
  //Initialise physical parameters
  Global_Physical_Variables::E = 2.1;
  Global_Physical_Variables::Nu = 0.4;
  Global_Physical_Variables::C1 = 1.3;
  for (unsigned i=0;i<2;i++)
  {

    // Define a strain energy function: Generalised Mooney Rivlin
    Global_Physical_Variables::Strain_energy_function_pt =
      new GeneralisedMooneyRivlin(&Global_Physical_Variables::Nu,
                                  &Global_Physical_Variables::C1,
                                  &Global_Physical_Variables::E);
    // Define a constitutive law (based on strain energy function)
    Global_Physical_Variables::Constitutive_law_pt =
      new IsotropicStrainEnergyFunctionConstitutiveLaw(
        Global_Physical_Variables::Strain_energy_function_pt);

    {
      //Set up the problem with pure displacement formulation
      SimpleShearProblem<QPVDElement<3,3> > problem(false);
      problem.run("RESLT");
    }

    //Discontinuous pressure
    {
      //Set up the problem with pure displacement formulation
      SimpleShearProblem<QPVDElementWithPressure<3> > problem(false);
      problem.run("RESLT_pres");
    }

    /*{
      //Set up the problem with pure displacement formulation
      SimpleShearProblem<QPVDElementWithPressure<3> > problem(true);
      problem.run("RESLT_pres_incomp");
    }*/

    {
      //Set up the problem with pure displacement formulation
      SimpleShearProblem<QPVDElementWithContinuousPressure<3> > problem(false);
      problem.run("RESLT_cont_pres");
    }

    /*{
      //Set up the problem with pure displacement formulation
      SimpleShearProblem<QPVDElementWithContinuousPressure<3> > problem(true);
      problem.run("RESLT_cont_pres_incomp");
    }*/
}

```

```
 } */  
}  
}
```

---

## 1.1 PDF file

A [pdf version](#) of this document is available. \