Chapter 1

Demo problem: Large-amplitude shear deformation of a 3D elastic solid

Detailed documentation to be written. Here's the already fairly well documented driver code...

```
//LIC// ====================================================================
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
//LIC// at http://www.oomph-lib.org.
//LIC//
//LIC// Copyright (C) 2006-2026 Matthias Heil and Andrew Hazel
//LIC//
//LIC// This library is free software; you can redistribute it and/or
//LIC// modify it under the terms of the GNU Lesser General Public
//LIC// License as published by the Free Software Foundation; either
//LIC// version 2.1 of the License, or (at your option) any later version.
//LIC//
//LIC// This library is distributed in the hope that it will be useful,
//LIC// but WITHOUT ANY WARRANTY; without even the implied warranty of
//LIC// MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the GNU
//LIC// Lesser General Public License for more details.
//LIC//
//LIC// You should have received a copy of the GNU Lesser General Public
//LIC// License along with this library; if not, write to the Free Software
//LIC// Foundation, Inc., 51 Franklin Street, Fifth Floor, Boston, MA
//LIC// 02110-1301  USA.
//LIC//
//LIC// The authors may be contacted at oomph-lib@maths.man.ac.uk.
//LIC//
//LIC//====================================================================
// Driver for elastic deformation of a cuboidal domain
// The deformation is a simple shear in the x-z plane driven by
// motion of the top boundary, for exact solution see Green & Zerna
// Generic oomph-lib headers
#include "generic.h"

// Solid mechanics
#include "solid.h"

// The mesh
#include "meshes/simple_cubic_mesh.h"

using namespace std;

using namespace oomph;


///////////////////////////////////////////////////////////////////////
///////////////////////////////////////////////////////////////////////
///////////////////////////////////////////////////////////////////////

//=========================================================================
/// Simple cubic mesh upgraded to become a solid mesh
//=========================================================================
template<class ELEMENT>
class ElasticCubicMesh : public virtual SimpleCubicMesh<ELEMENT>,
                         public virtual SolidMesh
{

public:
```

```
 /// Constructor:
 ElasticCubicMesh(const unsigned &nx, const unsigned &ny, const unsigned &nz,
                  const double &a, const double &b, const double &c,
                  TimeStepper* time_stepper_pt=&Mesh::Default_TimeStepper) :
  SimpleCubicMesh<ELEMENT>(nx,ny,nz,-a,a,-b,b,-c,c,time_stepper_pt)
  {
   //Assign the initial lagrangian coordinates
   set_lagrangian_nodal_coordinates();
  }

 /// Empty Destructor
 virtual ~ElasticCubicMesh() { }

};



//////////////////////////////////////////////////////////////////////////
//////////////////////////////////////////////////////////////////////////
//////////////////////////////////////////////////////////////////////////



//=================================================================
/// Global variables
//=================================================================
namespace Global_Physical_Variables
{
 /// Pointer to strain energy function
 StrainEnergyFunction* Strain_energy_function_pt;

 /// Pointer to constitutive law
 ConstitutiveLaw* Constitutive_law_pt;

 /// Elastic modulus
 double E=1.0;

 /// Poisson's ratio
 double Nu=0.3;

 /// "Mooney Rivlin" coefficient for generalised Mooney Rivlin law
 double C1=1.3;

 /// Body force
 double Gravity=0.0;

 /// Body force vector: Vertically downwards with magnitude Gravity
 void body_force(const Vector<double>& xi,
                 const double& t,
                 Vector<double>& b)
 {
  b[0]=0.0;
  b[1]=-Gravity;
 }

}


//////////////////////////////////////////////////////////////////////////
//////////////////////////////////////////////////////////////////////////
//////////////////////////////////////////////////////////////////////////


//=======================================================================
/// Boundary-driven elastic deformation of fish-shaped domain.
//=======================================================================
template<class ELEMENT>
class SimpleShearProblem : public Problem
{
 double Shear;

 void set_incompressible(ELEMENT *el_pt,const bool &incompressible);

public:

 /// Constructor:
 SimpleShearProblem(const bool &incompressible);

 /// Run simulation.
 void run(const std::string &dirname);

 /// Access function for the mesh
 ElasticCubicMesh<ELEMENT>* mesh_pt()
  {return dynamic_cast<ElasticCubicMesh<ELEMENT>*>(Problem::mesh_pt());}
```

```
/// Doc the solution
void doc_solution(DocInfo& doc_info);

/// Update function (empty)
void actions_after_newton_solve() {}

/// Update before solve: We're dealing with a static problem so
/// the nodal positions before the next solve merely serve as
/// initial conditions. For meshes that are very strongly refined
/// near the boundary, the update of the displacement boundary
/// conditions (which only moves the SolidNodes *on* the boundary),
/// can lead to strongly distorted meshes. This can cause the
/// Newton method to fail --> the overall method is actually more robust
/// if we use the nodal positions as determined by the Domain/MacroElement-
/// based mesh update as initial guesses.
void actions_before_newton_solve()
 {
  apply_boundary_conditions();
  bool update_all_solid_nodes=true;
  mesh_pt()->node_update(update_all_solid_nodes);
 }

/// Shear the top
void apply_boundary_conditions()
 {
  unsigned ibound = 5;
  unsigned num_nod=mesh_pt()->nboundary_node(ibound);
  for (unsigned inod=0;inod<num_nod;inod++)
   {
    SolidNode *solid_nod_pt = static_cast<SolidNode*>(
    mesh_pt()->boundary_node_pt(ibound,inod));

    solid_nod_pt->x(0) = solid_nod_pt->xi(0) + Shear*
     solid_nod_pt->xi(2);
   }
 }

};

//=======================================================================
/// Constructor:
//=======================================================================
template<class ELEMENT>
SimpleShearProblem<ELEMENT>::SimpleShearProblem(const bool &incompressible)
 : Shear(0.0)
{
 double a = 1.0, b = 1.0, c = 1.0;
 unsigned nx = 5, ny = 5, nz = 5;

 // Build mesh
 Problem::mesh_pt()=new ElasticCubicMesh<ELEMENT>(nx,ny,nz,a,b,c);
 //Loop over all boundaries
 for(unsigned b=0;b<6;b++)
  {
   //Loop over nodes in the boundary
   unsigned n_node = mesh_pt()->nboundary_node(b);
   for(unsigned n=0;n<n_node;n++)
    {
     //Pin all nodes in the y and z directions to keep the motion in plane
     for(unsigned i=1;i<3;i++)
      {
       mesh_pt()->boundary_node_pt(b,n)->pin_position(i);
      }
     //On the top and bottom pin the positions in x
     if((b==0) || (b==5))
      {
       mesh_pt()->boundary_node_pt(b,n)->pin_position(0);
      }
    }
  }
 //Loop over the elements in the mesh to set parameters/function pointers
 unsigned  n_element =mesh_pt()->nelement();
 for(unsigned i=0;i<n_element;i++)
  {
   //Cast to a solid element
   ELEMENT *el_pt = dynamic_cast<ELEMENT*>(mesh_pt()->element_pt(i));

   // Set the constitutive law
   el_pt->constitutive_law_pt() =
    Global_Physical_Variables::Constitutive_law_pt;

   set_incompressible(el_pt,incompressible);

   // Set the body force
   //el_pt->body_force_fct_pt()=Global_Physical_Variables::body_force;
  }
```

```
 // Pin the redundant solid pressures (if any)
 //PVDEquationsBase<2>::pin_redundant_nodal_solid_pressures(
 // mesh_pt()->element_pt());

  // Attach the boundary conditions to the mesh
  oomph_info « "Number of equations: " « assign_eqn_numbers() « std::endl;
}


//==================================================================
/// Doc the solution
//==================================================================
template<class ELEMENT>
void SimpleShearProblem<ELEMENT>::doc_solution(DocInfo& doc_info)
{

 ofstream some_file;
 char filename[100];

 // Number of plot points
 unsigned npts = 5;

 // Output shape of deformed body
 snprintf(filename, sizeof(filename), "%s/soln%i.dat",doc_info.directory().c_str(),
         doc_info.number());
 some_file.open(filename);
 mesh_pt()->output(some_file,npts);
 some_file.close();
 snprintf(filename, sizeof(filename), "%s/stress%i.dat", doc_info.directory().c_str(),
         doc_info.number());
 some_file.open(filename);
 //Output the appropriate stress at the centre of each element
 Vector<double> s(3,0.0);
 Vector<double> x(3);
 DenseMatrix<double> sigma(3,3);
 unsigned n_element = mesh_pt()->nelement();
 for(unsigned e=0;e<n_element;e++)
  {
   ELEMENT* el_pt = dynamic_cast<ELEMENT*>(mesh_pt()->element_pt(e));
   el_pt->interpolated_x(s,x);
   el_pt->get_stress(s,sigma);

   //Output
   for(unsigned i=0;i<3;i++)
    {
     some_file « x[i] « " ";
    }
   for(unsigned i=0;i<3;i++)
    {
     for(unsigned j=0;j<3;j++)
      {
       some_file « sigma(i,j) « " ";
      }
    }
   some_file « std::endl;
  }
 some_file.close();

}

//==================================================================
/// Run the problem
//==================================================================
template<class ELEMENT>
void SimpleShearProblem<ELEMENT>::run(const std::string &dirname)
{

 // Output
 DocInfo doc_info;
 // Set output directory
 doc_info.set_directory(dirname);

 // Step number
 doc_info.number()=0;
 // Initial parameter values
 // Gravity:
 Global_Physical_Variables::Gravity=0.1;
 //Parameter incrementation
 unsigned nstep=2;
 for(unsigned i=0;i<nstep;i++)
  {
   //Solve the problem with Newton's method, allowing for up to 5
   //rounds of adaptation
   newton_solve();

   // Doc solution
   doc_solution(doc_info);
```

```
  doc_info.number()++;
  //Increase the shear
  Shear += 0.5;
 }

}

template<>
void SimpleShearProblem<QPVDElement<3,3> >::set_incompressible(
 QPVDElement<3,3> *el_pt, const bool &incompressible)
{
 //Does nothing
}


template<>
void SimpleShearProblem<QPVDElementWithPressure<3> >::set_incompressible(
 QPVDElementWithPressure<3> *el_pt, const bool &incompressible)
{
 if(incompressible) {el_pt->set_incompressible();}
 else {el_pt->set_compressible();}
}

template<>
void SimpleShearProblem<QPVDElementWithContinuousPressure<3> >::
set_incompressible(
 QPVDElementWithContinuousPressure<3> *el_pt, const bool &incompressible)
{
 if(incompressible) {el_pt->set_incompressible();}
 else {el_pt->set_compressible();}
}


//=======================================================================
/// Driver for simple elastic problem
//=======================================================================
int main()
{
 //Initialise physical parameters
 Global_Physical_Variables::E  = 2.1;
 Global_Physical_Variables::Nu = 0.4;
 Global_Physical_Variables::C1 = 1.3;
  for (unsigned i=0;i<2;i++)
  {

 // Define a strain energy function: Generalised Mooney Rivlin
 Global_Physical_Variables::Strain_energy_function_pt =
  new GeneralisedMooneyRivlin(&Global_Physical_Variables::Nu,
                              &Global_Physical_Variables::C1,
                              &Global_Physical_Variables::E);
 // Define a constitutive law (based on strain energy function)
 Global_Physical_Variables::Constitutive_law_pt =
  new IsotropicStrainEnergyFunctionConstitutiveLaw(
   Global_Physical_Variables::Strain_energy_function_pt);

 {
  //Set up the problem with pure displacement formulation
  SimpleShearProblem<QPVDElement<3,3> > problem(false);
  problem.run("RESLT");
 }

 //Discontinuous pressure
 {
  //Set up the problem with pure displacement formulation
  SimpleShearProblem<QPVDElementWithPressure<3> > problem(false);
  problem.run("RESLT_pres");
 }

 /*{
  //Set up the problem with pure displacement formulation
  SimpleShearProblem<QPVDElementWithPressure<3> > problem(true);
  problem.run("RESLT_pres_incomp");
  }*/


 {
  //Set up the problem with pure displacement formulation
  SimpleShearProblem<QPVDElementWithContinuousPressure<3> > problem(false);
  problem.run("RESLT_cont_pres");
 }

 /*{
  //Set up the problem with pure displacement formulation
  SimpleShearProblem<QPVDElementWithContinuousPressure<3> > problem(true);
  problem.run("RESLT_cont_pres_incomp");
```

```
  }*/
  }
}
```

1.1 PDF file

A pdf version of this document is available. \