An Automatic Jigsaw Puzzle Solver

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Abstract

A computer vision system to automatically analyze and assemble an image of the pieces of a jigsaw puzzle is presented. The system, called Automatic Puzzle Solver (APS), derives a new set of features based on the shape and color characteristics of the puzzle pieces. A combination of the shape dependent features and color cues is used to match the puzzle pieces. Matching is performed using a modified iterative labeling procedure in order to reconstruct the original picture represented by the jigsaw puzzle. Algorithms for obtaining shape description and matching are explained with experimental results.

1: Introduction

Jigsaw puzzles contain a number of problems endemic to machine vision, like shape description, partial boundary matching, pattern recognition, feature extraction, and heuristic matching. Most of the past research involving jigsaw puzzles ignores the color information contained on the individual pieces. Freeman and Gardner [1], Hirota and Ohto [2], Nagura, *et al.* [3], and Radack and Badler [4] use critical points extracted from piece boundaries as their basis for matching. Webster, *et al.* [5] use a method based on identifying isthmus points on the pieces and compute the distance function from the isthmus point to the edge of the puzzle pieces, while Wolfson *et al.* [6] employ curve matching techniques combined with an optimal search algorithm.

While neglecting color may seem attractive at first, it is simply a waste of valuable information. Humans make use of all possible information to constrain their search for matching pieces, and when dealing with large search spaces, there is no reason for computers to attack the problem any differently. Therefore, the APS system uses a combination of shape features along with the color information contained on the surface of the individual jigsaw puzzle pieces as its matching criteria.

2: Problem definition

The problem of solving a cardboard puzzle consists of reconstructing a known image from a set of interlocking pieces for which a unique solution exists. It is necessary not only to match the pieces, but also to rotate, translate, and assemble them into the original image that is usually found on the box cover of the puzzle. To this end, several assumptions have been made as part of an understood set of "puzzle rules":

- A jigsaw puzzle is defined as a set of pieces that, when properly assembled, fit together into one region.
- A puzzle piece is defined as a connected planar region.
- Two pieces that mate share a common border segment.
- The corners of two adjacent matching pieces approximately coincide when assembled.
- The contours can be segmented into individual sides separated by the corners.
- The individual sides are either flat, or contain a protrusion or a hole. (This can be generalized to conform to any piece shape.)
- There are no gaps between correctly matching pieces of the puzzle, and the solution to the problem is unique (i.e., no two pieces can fit perfectly with the same side of a third piece).

The matching process which is described in this paper makes use of several important features of puzzle pieces. The features which are extracted consist of color samples along the edges of the pieces, parameters which describe the curvature of the edges of the pieces, convexity or concavity of the pieces, and a measure of the goodness of fit between pieces.

3: Image acquisition and preprocessing

The initial image is obtained using a vision system consisting of a Mustek 24-bit color flatbed scanner, set to a scanning resolution of 150 dpi. All the puzzle pieces are scanned at once. Processing of the image, shape analysis, matching and assembling are all done on a SUN Sparcstation-IPX. The image is segmented into disjoint pieces, and a connected component analysis is performed on the image. A simple boundary tracking algorithm [7] is applied for each piece in the labeled image.

4: Feature extraction / shape representation

Following the segmentation procedure, the APS system isolates the boundaries of the idividual puzzle pieces so that neighboring edges can be correctly matched. In order to do this, relevant features are extracted from the individual puzzle pieces.

Since this is a problem in partial boundary matching, the first goal is to separate the individual sides of each puzzle piece. In order to accomplish this task, the locations of the corners of each puzzle piece must be obtained. Once the corners are located, the piece contours are separated into their constituent sides.

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4.1: Corner detection

The objective of this process is to locate the corners of the individual puzzle pieces. In the general case, the puzzle pieces can be oriented in any direction in the original image (i.e. rotated by any angle θ). A *dual-space* approach is chosen to detect corners as well as to obtain an estimate of the angle of rotation of each piece.

Akin to the Hough Transform, this method maps from the Euclidean plane to its dual ρ–θ plane. Every directed line maps to a unique point in ρ–θ space. By taking the polygonal approximation [8] of the piece contours, we obtain a contour representation consisting of many small line segments, as shown in Figure 1. By transforming these segments into dual-space using the equations below, we obtain four peaks in ρ–θ space corresponding to the four dominant edge directions. Since the direction of the

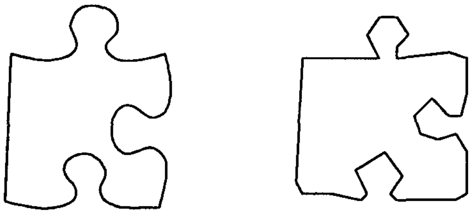

Figure 1: Original piece contour and its polygonal approximation.

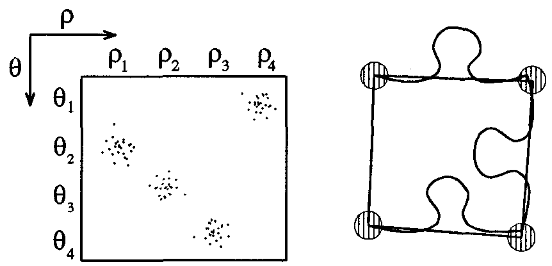

Figure 2: (a) The dual-space of the above approximation, and (b) the original contour with enclosing box and regions of possible corner location.

line is taken into consideration, the range of this dual-space is twice that of the conventionally used Hough Transform. The origin of the Euclidean plane is taken to be the approximate center of mass of the puzzle piece. The angle θ varies from -180 to 180 degrees and ρ can be positive or negative. A line segment going from (x_1,y_1) to (x_2,y_2) is represented in dual space by

$$\theta = \tan^{-1}(y_2 - y_1, x_2 - x_1) + \frac{\pi}{2} \qquad (1)$$

$$\rho = x_1 \cos\theta + x_2 \sin\theta \qquad (2)$$

where the four quadrant arc-tangent is used and θ is taken modulo 360° (Figure 2a). Since the original puzzle pieces are generally square, the θ values are approximately 90 degrees apart.

The discrete array of ρ–θ space is then filtered, and non-maximal suppression is performed to obtain the local maxima corresponding to the dominant direction of each edge. These maxima are then transformed back into the Euclidean plane and their intersections are taken to be the candidate locations for the corners of the individual puzzle pieces. The corners are found by locating the points of maximum curvature along the original piece contour in the neighborhood of the candidate corner locations, thus effectively partitioning the contour into four segments (Figure 2b). Along with the locations of the corners, this method also gives an approximate enclosing box and the general orientation for each piece.

4.2: The feature set

The next step is to calculate the set of features to be used in the matching of the puzzle pieces. The first, and most important, feature is the *convexity/concavity* of each individual side of each puzzle piece. Using simple curvature analysis, we are able to determine whether a side of a puzzle piece is flat, or contains a protrusion (convexity), or a hole (concavity). The next feature is the *Euclidean distance* between adjacent corners of each puzzle piece. Again, this is a very coarse shape description and like above, is used to eliminate many initial candidate matches.

We use two specialized path-length functions and two generalized distance measures as the primary features for partial boundary matching. The value of the first path-length function is taken to be the Euclidean distance from the corner point (A) to every point (P) along the contour (Figure 3a). The result, in the general case, is a 5th order polynomial. For each of these functions, the regression coefficients are calculated using a method described in [9]. The second path-length function is concerned only with distances DQ for points Q on the contour that lie between the two inflection points (B and C) along the concavity/convexity of the piece (Figure 3a).

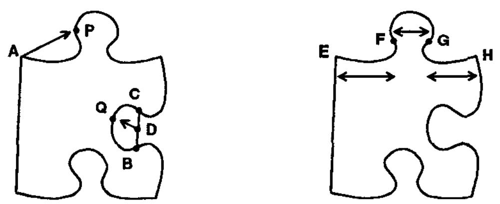

Figure 3: (a) The measures used to generate the distance functions and (b) the general distance features used in the APS.

The first general distance feature is similar to the use of isthmus critical points [5]. However, instead of taking the minimum distance along the isthmus, we take the distance of the line FG connecting the two inflection points that lie along the boundary (Figure 3b). The last distance feature used is the distances from the piece corners to the inflection points along the contour flanking the concavity/convexity on either side (Figure 3b).

The next feature used, which distinguishes this puzzle solution from others, is *color*. Color is utilized in two ways: The first method is used only if the original photograph from the lid of the jigsaw puzzle box is available. In this case, we calculate the overall color characteristics for each puzzle piece, namely the mean and variance of the hue, saturation, and intensity values. These are used when comparing pieces with the color characteristics of various regions in the original photograph. When matching using this feature, an attempt is made to reassemble the puzzle in the same orientation as the original photograph. The second use of color is as follows: Points at regular intervals along the contour are isolated, and the direction towards the interior of the piece is determined. A window about a point along this direction, just in the interior of the piece, is taken and the color characteristics are calculated (Figure 4).

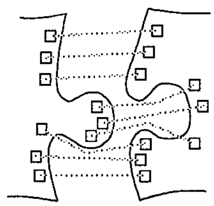

Figure 4: Calculation of color sampling windows at regular intervals along contours of both pieces.

The last feature in the set is the actual fit of the assembled pieces. This is not checked until the assembly stage. When the pieces are assembled, they are oriented so that their corners align. Then, the amount of overlap and gap, in number of pixels, between the two pieces is checked. If this value exceeds some threshold, the match is rejected and a new candidate match is considered.

5: Matching and assembly

When properly assembled, the puzzle pieces must fit together to form a single region. Matching is carried out in two stages [6]. First, all of the pieces in the outside border of the puzzle are matched. Then, the pieces which lie in the interior of the puzzle are matched.

Each pixel belonging to the matching pieces is translated and rotated so that the matching edges are correctly aligned. This process is repeated until all the pieces have been assembled correctly. The amount of rotation required is determined by comparing the angle (with respect to the horizontal) made by the matching side and the angle made by the side it is expected to fit. The amount of translation required is determined by comparing the corner locations of the matching side and the side of the puzzle it has to fit.

6: Experimental results

Experiments were carried out using six, nine, twenty-five and fifty-four pieces of the puzzle. Obviously, the results of the system when matching and assembling six and nine pieces were good, even when using just one or two features from the feature set. Ambiguous multiple matches were obtained when the number of pieces was increased to twenty-five and higher. However, by increasing the number of features used for matching, satisfactory results were obtained. Since a comparative study of the performance of the system when using an extensive feature set (as opposed to using just a few features) has not been done at this time, accurate predictions on the performance of this system when dealing with large puzzles is difficult.

The result of the matching and assembly routine can be seen in Figure 5, where a section of the entire puzzle has been correctly matched and assembled.

7: Conclusions

This paper has presented some very promising approaches to solving the problem of assembling cardboard jigsaw puzzles. Refinement of the methods proposed herein may lead to some improved results, and further developments in the area of color matching could result in a very effective system.

It should be noted that, when neglecting color information, the level of success of the APS system depends greatly on the initial segmentation of the puzzle pieces. This is where the use of color becomes significant. The color features are calculated from areas within the individual puzzle pieces, and are independent of the quality of the boundary representation.

Possible directions of future research include the use of different color spaces, the use of color combined with only primitive shape descriptions, and the initial assembly of subregions of like color (as opposed to the initial construction of the border), along with an extensive evaluation of the currently used feature set.

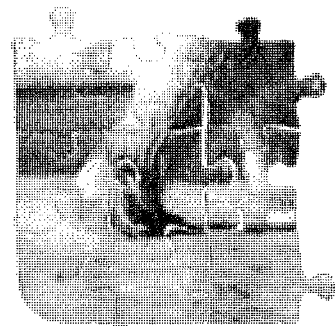

Figure 5: Assembled puzzle section.

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