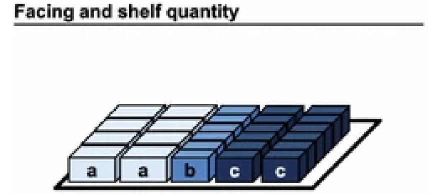
IE 407 FUNDAMENTALS OF OPERATIONAL RESEARCH, FALL 2020
TERM PROJECT
DEADLINE: 18th JANUARY 17.00

Groups of 3 people are expected

In this project, you will study an assortment selection and shelf space allocation problem for a category of products (i.e. soft drinks or chocolates) in a supermarket. In a supermarket there is a limited space of shelf for each category. In a given category, there are usually lots of products that can be displayed in the store and offered to customers. A supermarket of course wants to maximize its profit. Therefore, the manager has to make careful assortment planning (selecting a portfolio of products) and shelf allocation decisions. This is a complicated problem and lots of thought has been put into it. You will study a somewhat simplified version.

First of all, you will focus on a certain category of products and allocated shelf space to that category in a supermarket. There are a certain number of shelves allocated to this category. Each shelf may have a different width. The width of the shelf determines how many facings from which products can be placed in that shelf. An example allocation of space to products is given in Figure 1. In this example, in the given shelf, product a has two facings, product b has one facing and product c has two facings. For each product, the orientation of display is predetermined and hence the width of facing for each product is known in advance.

Figure 1. An example space allocation to goods in a shelf



Each shelf has a certain depth and this determines the quantity of products that can be placed in the shelf. For example, in the given shelf in Figure 1, we can place four packages of product a, if we have one facing for a. This quantity is six for product b. Since, the supermarket wants to have full shelves, it utilizes all the space in a facing, so, for example, always puts four packages of a in facing for a.

A well known fact in retail industry is that as the number of facings allocated to a product increases, the demand for that product increases, too. The forecasted demand with respect to number of facings is given below:

$$D_i = d_i(f_i \times b_i)^{\beta_i}$$

where D_i is the forecasted demand for product i. f_i is the number of allocated facings for product i. b_i is the width of a facing for product i. d_i is a coefficient for demand rate for product i per unit width and one facing. $\beta_i \in (0, 1)$ is a space elasticity factor, which represents the diminishing return effect, i.e. for a product as the number of facings increases, the demand increases at a diminishing rate. Also, the location of a shelf within the store, its height, etc. affect the demand of contained product by a factor of γ_k, i.e. $\gamma_k \times D_i$.

The following rules also must be satisfied in assortment selection and shelf space allocation decisions:

- If a product is selected in the assortment, than all facings for the product must be placed on the same shelf.
- The manager does not want to allocate more than four facings for a product.
- If a product is selected in the assortment, then a minimum shelf-inventory amount must be placed on the shelves. Similarly, for each product there is an upper bound on the shelf-inventory.
- If a product is selected in the assortment, lower and upper bounds on its facing number are calculated by using shelf depth, product depth and lower and upper bounds on the shelf-inventory.
- Each product provides a certain profit per unit sold.
- Each shelf has a certain width and the total width of the facings placed in the shelf cannot exceed its width.
- For some pairs of products, there is a restriction that if one is included in the assortment, the other product must also be included.
- For some pairs of products, there is a restriction that they cannot be on the same shelf.

Questions:

(1) Develop a mathematical model for the problem.
(2) Solve the problem for given data set below. Explain your solution, what are your observations on it?
(3) Another larger data set is provided in an Excel file, try to solve the problem for this data set also. Comment on how would the problem size affects solution time.

For questions 4, 5, 6 and 7 answer each independently.

(4) There is an empty space under the last shelf and it is planned to add a new shelf. Find the possible additional profit that can be made by adding a new shelf with 40cm width, 25 cm depth, 0.45 γ_k value? How does your solution change? Explain it in detail.

(5) Find how profit is affected if the shortest (in width) available shelf is allocated to some other category. How does your solution change? Explain it in detail.

(6) Find the additional profit that can be made by increasing the width of Shelf 5 by 5. What are your basic variables? Then find the additional profit that can be made by increasing the width of Shelf 5 by 10, and basic variables. Compare two scenarios.

(7) There has been a change for product 3 and product 8 due to some brand agreements and now these two products must be assorted together and must be placed to the same shelf, previous constraint of them being placed in different shelves is no longer active. How does your solution change? Explain it in detail.

Data Sets:

I	:	Set of products
I^1	:	Set of product pairs (i_1, i_2) that cannot be placed on the same shelf.
I^2	:	Set of product pairs (i_1, i_2) that will be included together in the assortment.
K	:	Set of shelves
π_i	:	Profit made by selling one unit of product i
w_k	:	Width of shelf k
γ_k	:	Shelf k's effect on demand
ds_k	:	Depth of shelf k
dp_i	:	Depth of unit product i
b_i	:	Width of a facing for product i
d_i	:	Coefficient for demand rate for product i per unit width and one facing
β_i	:	Space elasticity factor for product i
$s_i^l(s_i^u)$	:	Lower (upper) bound on the shelf inventory of product, if i is selected in the assortment

Table 1. Sets of product pairs

I^1	:	$(2,5)$, $(3,8)$, $(16,20)$
I^2	:	$(1,12)$, $(3,8)$, $(9,15)$, $(16,20)$

TABLE 2. Product based data

Product #	π_i	b_i	dp_i	d_i	β_i	s_i^l	s_i^u
1	15	10	7	4	0.5	2	15
2	8	9	6	10	0.2	1	19
3	12	5	10	10	0.3	1	23
4	6	7	7	7	0.2	1	9
5	11	9	9	5	0.9	1	16
6	14	6	8	2	0.4	1	21
7	14	9	6	1	0.5	1	11
8	6	5	9	6	0.3	1	18
9	5	9	9	7	0.8	2	11
10	11	10	9	3	0.8	1	11
11	12	7	5	4	0.1	2	17
12	8	5	6	7	0.8	2	22
13	11	7	6	2	0.1	1	12
14	13	8	9	9	0.8	3	12
15	7	9	8	11	0.8	3	19
16	14	23	5	2	0.1	2	16
17	9	25	6	9	0.1	1	10
18	10	17	8	1	0.8	2	16
19	13	15	9	4	0.2	2	20
20	5	23	6	2	0.6	3	19
21	11	19	8	6	0.6	2	24
22	11	19	9	6	0.4	1	16
23	7	16	7	8	0.5	2	13
24	10	14	5	2	0.8	1	16
25	13	16	10	4	0.9	2	14

TABLE 3. Shelf based data

Shelf #	w_k	ds_k	γ_k
1	50	34	0.25
2	65	30	0.60
3	80	26	1
4	95	27	0.60
5	110	29	0.25