MIDDLE EAST TECHNICAL UNIVERSITY

IE407

Term Project Report

Assortment Selection and Shelf Space Allocation Problem

January 25, 2021

Students:
Ersel Hengirmen 2468015
Onat Özdemir 2310399
Alp Eren Yılmaz 2310662

Contents

1 Introduction

In this project, we have conducted a study on "Assortment Selection and Shelf Space Allocation Problem". Although in the literature, the problem has different variations, we studied a simplified version of the problem where,

- If a product is chosen for the assortment, then all the facings of the product have to be filled fully. This restriction was especially important since dealing with variable sized number of products per facing makes the problem more complex.
- The manager does not want to allocate more than four facings for a product. The importance of this constraint is explained in 2 in detail, basically we used this constraint to transform our non-linear model to a linear one.
- Other constraints and rules can be found in 4.2

The task we worked on was determining the optimal way to select products for the assortment and how these products placed in shelves, in order to gain maximum profit.

Initially, in the market, we had 25 products belong to a certain category and 5 shelves to place those products on, where for each product,

- π_i: Profit made by selling one unit of the product, this parameter was used to calculate total profit gained from the product
- b_i: Width of a facing for the product (in cm)
- dp_i : Depth of unit the product (in cm)
- β_i: Space elasticity factor for the product
- s_i^l : Lower bound on the shelf inventory of the product, if it is selected in the assortment
- s_i^u : Upper bound on the shelf inventory of the product, if it is selected in the assortment
- d_i: Coefficient for demand rate for the product per unit width and one facing

- β_i: Space elasticity factor for the product

parameters and for each shelve,

- w_k : Width of the shelf (in cm)
- ds_k : Depth of the shelf (in cm)
- γ_k : Shelf k's effect on demand

parameters were given. In line with these parameters, demand for a product was calculated as follows,

$$d_i(f_i * b_i)_i^{\beta}$$

Our approach to the given problem and the proposed mathematical model that best fits to our objective and constraints are given in 2

2 Project

Initially, we approached the problem by defining two decision variables:

X_{ik} : Will the product i be on shelf k. Such that,

$$X_{ik} = \begin{cases} 1, & if\ i\ is\ allocated\ in\ k \\ 0, & otherwise \end{cases}$$

f_{ik}: Number of allocated facings for product i on shelf k.

With these decision variables, the objective function (since our aim is maximizing the profit) became:

$$max \sum_{i=1}^{|I|} \sum_{k=1}^{|K|} \pi_i \gamma_k d_i (f_{ik} * b_i)^{\beta_i}$$

As can be confirmed, if we choose f_{ik} as a decision variable then the given objective function makes the model non-linear. Since the "opensolver" is not good at solving non-linear problems, we decided to modify the initial model to obtain a linear model.

The proposed model 2.1 has been built considering (with the help of) Rule 2. Since for a product we can not allocate more than 4 facings (i.e. $f_{ik} \in \{1, 2, 3, 4\}$, if the product is chosen for the assortment), defining a seperate decision variable for facings was unnecessary. Rather we defined new decision variables instead of the previous ones,

X_{ijk} : Will the product i have j allocations (facings) on shelf k. Such that,

$$X_{ijk} = \begin{cases} 1, & if\ i\ is\ allocated\ j\ times\ in\ k \\ 0, & otherwise \end{cases}$$

where $j \in \{1, 2, 3, 4\}$. With this new approach, we have achieved to develop a linear model which can be seen in 2.1.

2.1 Proposed Model

Parameters:
I : Set of products
I^1: Set of product pairs (i1, i2) that cannot be placed on the same shelf.
I^2: Set of product pairs (i1, i2) that will be included together in the assortment.
K : Set of shelves
π_i: Profit made by selling one unit of product i
w_k : Width of shelf k
γ_k : Shelf k's effect on demand
ds_k : Depth of shelf k
dp_i : Depth of unit product i
b_i: Width of a facing for product i
d_i: Coefficient for demand rate for product i per unit width and one facing
β_i: Space elasticity factor for product i
s_i^l : Lower bound on the shelf inventory of product, if i is selected in the assortment
s_i^u : Upper bound on the shelf inventory of product, if i is selected in the assortment

Notation:
$|I|$: Size of the set of products

$|K|$: Size of the set of shelves

Calculated Parameters:
N_{ik}: number of product i in shelf k per allocation (facing):
$N_{ik} = \lfloor ds_k/dp_i \rfloor$

As mentioned in the introduction part, since the we are expected to fill all facings fully, N_{ik} is a constant parameter calculated from the given data set rather than a decision variable which makes our life easier.

Decision Variables:
X_{ijk} : Will the product i have j allocations (facings) on shelf k. Such that,

$$X_{ijk} = \begin{cases} 1, & if\ i\ is\ allocated\ j\ times\ (i.e.\ has\ j\ facings)\ in\ k \\ 0, & otherwise \end{cases}$$

Objective function:

$$max \sum_{i=1}^{|I|} \sum_{k=1}^{|K|} \sum_{j=1}^{4} \gamma_k * \pi_i * d_i * (b_i)^{\beta_i} * (j)^{\beta_i} * X_{ijk}$$

Subjected To:

$$1)\ \sum_{k=1}^{|K|} \sum_{j=1}^{4} X_{ijk} \leq 1 \quad \forall i \in I \qquad (Rule\ 1)$$

$$2)\ \sum_{k=1}^{|K|} (N_{ik} * (\sum_{j=1}^{4} X_{ijk} * (j)) - s_i^u * (\sum_{j=1}^{4} X_{ijk})) \leq 0 \quad \forall i \in I \tag{1}$$

$$3)\ \sum_{k=1}^{|K|} (N_{ik} * (\sum_{j=1}^{4} X_{ijk} * (j)) - s_i^l * (\sum_{j=1}^{4} X_{ijk})) \geq 0 \quad \forall i \in I \tag{2}$$

Constraint (1) and (2) come from Rule 3 and 4.

$$4)\ - w_k + \sum_{i=1}^{|I|} (b_i * (\sum_{j=1}^{4} X_{ijk} * (j))) \leq 0 \quad \forall k \in K \qquad (Rule\ 6)$$

$$5)\ (\sum_{k=1}^{|K|}\sum_{j=1}^{4} X_{i_1jk}) - (\sum_{k=1}^{|K|}\sum_{j=1}^{4} X_{i_2jk}) = 0 \quad \forall (i_1, i_2) \in I^2 \qquad (Rule\ 7)$$

$$6)\ (\sum_{j=1}^{4} X_{i_1jk}) + (\sum_{j=1}^{4} X_{i_2jk}) \leq 1 \qquad \forall k \in K \quad \forall (i_1, i_2) \in I^1 \qquad (Rule\ 8)$$

Restrictions:

$$X_{ijk} \in \{0,1\} \qquad \forall i \in I \quad \forall j \in \{1,2,3,4\} \quad \forall k \in K$$

2.1.1 Q2

In this section, the solution to the problem for given data set 4.1 obtained by using the given model, and our observations about the results can be found. We used below formulas in our excel files to simplify our calculations for Questions from 2 to 7:

$$1) \quad f(i,k)^{\beta_i} = \sum_{j=1}^{4} X_{ijk} * j^{\beta_i} \quad \forall i \in I \quad \forall k \in K$$

$$2) \quad f(i,k) = \sum_{j=1}^{4} X_{ijk} * j \quad \forall i \in I \quad \forall k \in K$$

$$3) \quad Z(i,k) = \sum_{j=1}^{4} X_{ijk} \quad \forall i \in I \quad \forall k \in K$$

Important Notes: 1st formula is used in objective function.
Second and third formulas are used repeatedly on constraints. 1st formula was selected like that to make our model linear.
The second one is used to determine the number of allocations (facings) of the product in shelf k while the third one is used to determining whether the product is placed in shelf k or not.

Objective function result was found as: **6945.73351921896**
Refer to 4.3.1 for decision varable results.

According to results from opensolver, the supermarket gained:
175.5006626 from products in shelf 1 which has γ_k of 0.25 and width of 50cm
1317.396698 from products in shelf 2 which has γ_k of 0.6 and width of 65cm
3512.138959 from products in shelf 3 which has γ_k of 1 and width of 80cm
1588.496409 from products in shelf 4 which has γ_k of 0.6 and width of 95cm
352.2007896 from products in shelf 5 which has γ_k of 0.25 and width of 110cm

The third shelf that has the highest γ_k value gained the most money. Also, the supermarket gained more from shelf 4 compared to shelf 2 and 5 compared to shelf 1 even though they had the same γ_k. The reason for this is the difference in their width. Shelves 4 and 5 had more space for products and facings. Because of that, those shelves gained more money even though they had the same γ_k.

Products 5,9,10,12,14,15,25 that has highest amount of β_i (0.8-0.9) had most number of facings(3-4) and made most money for the supermarket. Products 18 and 24 also had high β_i values, but they were not either selected for the assortment or only had 2 facing because their coefficient for demand (d_i) were much lower than others. Also, all of these products were in shelves 2,3 and 4, which has higher γ_k values than shelves 1 and 5.

Using the above information, the supermarket should choose products with high space elasticity factor (β_i) while considering coefficients like π_i, d_i and b_i and place them in most profitable shelves.

2.1.2 Q3

According to Opensolver objective function result was found as: **37906,15197**
Refer to 4.3.2 for decision varable results.
We also used formulas $f(i,k)^{\beta_i}, f(i,k), Z(i,k)$ from Question 2 for this part to simplify our calculations once again.
In our model we are using **4*ProductCount*ShelfCount** decision variables(500 for smaller, 2000 for bigger data set).
While the lesser data set worked in an instant, larger data set took 34 seconds to finish. This is because the number of decision variables increases, which increases the time it takes to solve the relaxation of a problem and the number of problems(increase in the number of problems is the main reason).

For a problem like this, the worst case is a complete enumeration of decision variables. Furthermore, all of our decision variables are binary which means for the worst case we would have needed to solve 2^{500} for lesser data set and 2^{2000} for a bigger data set. However, because of our first constraint for any i and k values, we would have 5 cases which are:

	X_{i1k}	X_{i2k}	X_{i3k}	X_{i4k}
case1:	0	0	0	0
case2:	1	0	0	0
case3:	0	1	0	0
case4:	0	0	1	0
case5:	0	0	0	1

That means we have only 5 different cases for every 4 decision variables instead of 2^4. Because of that, in the case of complete enumeration, we would have 5^{100} problems for smaller data set and 5^{400} for bigger data set (these numbers were only calculated using 1st group of constraints. By adding other constraints we can reduce it more). As we can see for the worst case (complete enumeration), the number of problems that we need to solve increases exponentially (There would have been 5^{300} times more problems which we would need to solve if we only had 1st group of constraints). Because of that, the total solution time will also increase exponentially.

Since we are using more constraints and decision variables, the matrices' size that we will need for the LP relaxation of a problem will also increase.

Let us calculate the sizes of these matrixes for each data set:

1- Smaller data set:
400 decision variables
25 constrains from 1st group of constraints. Which also adds 25 slack variables(<=)
25 constrains from 2nd group of constraints. Which also adds 25 excess variables(>=)
25 constrains from 3rd group of constraints. Which also adds 25 slack variables(<=)

5 constrains from 4th group of constraints. Which also adds 5 slack variables(<=)
4 constrains from 5th group of constraints. (=)
15 constrains from 6th group of constraints. Which also adds 15 slack variables(<=)
so we have 99 constrains
and 495 decision variables
495-99=396

That means size of our Matrices are:
B=99*99
N=99*396
x_B=99*1
x_N=396*1
c_B=99*1
c_N=396*1
b=99*1
B^{-1}=99*99

2- Larger data set:
2000 decision variables
100 constrains from rule 1st group of constraints. Which also adds 100 slack variables(<=)
100 constrains from rule 2nd group of constraints. Which also adds 100 excess variables(>=)
100 constrains from rule 3rd group of constraints. Which also adds 100 slack variables(<=)
5 constrains from rule 4th group of constraints. Which also adds 5 slack variables(<=)
13 constrains from rule 5th group of constraints. (=)
60 constrains from rule 6th group of constraints. Which also adds 60 slack variables(<=)
so we have 378 constrains
and 2365 decision variables
2365-378=1987

That means size of our matrices for larger data set is:
B=378*378
N=378*1987

x_B=378*1
x_N=1987*1
c_B=378*1
c_N=1987*1
b=378*1
B^{-1}=378*378

Note: Above numbers are calculated while ignoring artificial variables.

Since we are going to use the simplex method, we would need to make matrix multiplications, which are the most costly operations in big data sets (if you are making an operation: AxB for A matrix being a*b and B matrix b*c you would need to do a*b*c multiplications).

The costliest operation we are going to need to do in the simplex method is: $c_B^T * B^{-1} * N$ (its actually: $c_N^T - c_B^T * B^{-1} * N$(reduced cost vector) but matrix multiplication is much slower than matrix differention so it doesnt make almost any difference)

For the first data set the number of operations this multiplication will take is:
(1*99*99)*(1*99*396)=384238404 operations
For larger data set the number of operations this multiplication will take is:
(1*378*378)*(1*378*1987)=107318172024 operations

283910508/3881196=279.30100403

So as we can easily see, the time it takes to calculate multiplication of these 3 matrices has increased more than 275 times. However, since the difference between other operations will not be as big as this operation, the rate of time it takes to solve the group of operations for a given x_B and x_N will be a little less than this number. Nevertheless, the simplex method most probably will need to change more variables in the bigger data set since it has more variables. This would cause a great increase in solution time for an LP relaxation, but it is still nowhere near the increase in the number of problems.

So in the end, solution time of our problem will generally grow exponentially for larger data sets due to exponential increase in

problem count(for the most part) and increase in the time it takes to solve LP relaxation of those problems(this also affects the problem time but not as much as the increase in problem count).

2.1.3 Q4

Initial Profit	6945.73351921896
New Profit	7165.30316863258
Profit Change	+219.569649414
Decision Variables	Check 4.3.3

Table 1: Q4 Comparison Table

In this section, we analysed how the solution given in 2.1.1 changes if we add the sixth shelf with $w_6 = 40$, $ds_6 = 25$, $\gamma_6 = 0.45$ to the K given in 4.1.

Since we are adding more space to sell our products its given that we will either gain more money, or our profit will stay the same.
Even though the new shelf had less width than all other shelves, it gained us more money then what the first shelf gained us before we added the last shelf(175.5006626). Also, the profit gained per width for the new shelf is higher than the 5th shelf's profit per width before adding the new shelf, which is:

profit added per added width: $219.6/40 = 5.49$
5th shelf's profit per width(before adding the new shelf): $352/110 = 3.2$
1st shelf's profit per width(before adding the new shelf): $175.5/50 = 3.51$

This is because the $\gamma_k(\gamma_6)$ value of the new shelf(0.45) is higher than γ_k values of shelves 1 and 5(0.25 for both). Thus, adding the most profitable products on shelves 1 and 5 to the new shelf would be beneficial to us.

<u>**Changes in the excel model:**</u>
* First, we added the parameters of the new shelf to our parameters with given values.

Then, we added 100 new X_{ijk} decision variables since we need $|I|$*4 (25*4=100) variables for every shelf.
* After this is done, we added 25 N_{ik}(N_{ik}=Number of product i in shelf k per allocation) because of the added shelf(shelf 6).
* And for last; we added 25 new $f(i,k)^{\beta_i}, f(i,k), Z(i,k)$ values for shelf 6 as $f(i,6)^{\beta_i}, f(i,6), Z(i,6)$ (these derived values were used in every constrain and the objective function so we didnt need to change any other constrain)

2.1.4 Q5

Initial Profit	6945.73351921896
New Profit	6845.90079723537
Profit Change	-99.8327219836
Decision Variables	Check 4.3.4

Table 2: Q5 Comparison Table

In this section, we analysed how the solution and profit given in 2.1.1 change, if the shortest (in width) available shelf is allocated to some other category. The shortest shelf in 4.1 is the one with Shelf number 1. Since the shelves are allocated for only the products belong to the same category, we remove it from our data set by allocating it to some other category.

After removing the lowest in width shelf, supermarket's profit decreased by 99.8327219836 even though the shortest in width shelf(shelf 1) was gaining 175.5006626. This is because two products out of the three products that were in shelf 1 were moved to other shelves(product 8 was moved to shelf 5, and product 21(most profitable product on shelf 1 previously) was moved to shelf 4).

So we can easily conclude when removing a shelf we should consider to move the most profitable products in that shelf to other shelves even if our removed shelf has a low γ_k value.

<u>**Changes in the excel model:**</u>
We have added the below constrain to make the shortest in width shelf (first

shelf) unavailable:

$$\sum_{i=1}^{|I|} X_{ij1} = 0 \quad \forall j \in \{1, 2, 3, 4\}$$

2.1.5 Q6

Initial Profit	6945.73351921896
Profit after 5cm increase	6958.7030298688
Profit Change after 5cm increase	+12.9695106498
Profit after 10cm increase	6971.460542
Profit Change after 10cm increase	+25.72702278104
Decision Variables	Check 4.3.5 and 4.3.6

Table 3: Q6 Comparison Table

In this section, we analysed how the profit found in 2.1.1 changes and what are the basic variables when two separate cases happen:

- the width of Shelf 5 (w_5) is increased by 5 cm
- the width of Shelf 5 (w_5) is increased by 10 cm

Firstly, in both cases, we are relaxing constraints by increasing the shelf width. Therefore, we can say that our profit either will be a little higher or will be the same. For the first case, we see that the new profit is 6958.7. As a result of increasing shelf width, we gain a 12.96 profit increase. For the second case, we are relaxing constraints a bit more. As a result of increasing shelf width even more, our new profit is 6971.46, even 25.7 more than the first case, as we expected. As a result, increasing the width of Shelf 5 by 10cm is a preferable scenario when we compare it with 5cm increase.

<u>**Basic Variables:**</u>

Basic variables for a problem is dependent on its number of constraints. Since this question's model is same as Q2 we know that we have 99 constrains which means we have 99 basic variables. We have a total of 25 products and and every assorted product is 1 of those basic variables(It's actually not the product itself but the X_{ijk} variable of the product, if its value is 1). Since we have exactly 25 products, we will have at least 74 basic variables that are excess and slack variables(since our solution is feasible for both 5cm and 10 cm we won't have artificial variables as basic variables). Below you can find the basic variables which are neither excess nor slack variables.

X_{ijk} basic variables for 5cm increase:
$X_{1,1,4}$ $X_{2,1,1}$ $X_{3,1,3}$ $X_{4,1,3}$ $X_{5,4,3}$ $X_{6,1,1}$ $X_{8,1,5}$ $X_{9,3,4}$ $X_{10,3,5}$ $X_{12,4,4}$ $X_{14,4,3}$ $X_{15,4,4}$ $X_{21,2,5}$ $X_{22,1,1}$ $X_{23,1,1}$ $X_{24,3,5}$ $X_{25,4,2}$

X_{ijk} basic variables for 10cm increase :
$X_{1,1,4}$ $X_{2,1,1}$ $X_{3,1,3}$ $X_{4,1,3}$ $X_{5,4,3}$ $X_{6,1,1}$ $X_{8,1,1}$ $X_{9,3,4}$ $X_{10,3,1}$ $X_{12,4,4}$ $X_{14,4,3}$ $X_{15,4,4}$ $X_{21,3,5}$ $X_{22,1,5}$ $X_{23,1,5}$ $X_{24,2,5}$ $X_{25,4,2}$

Comparison of the basic variables

As can be seen, in both of the scenarios 7th ,11th, 13th, 16th, 17th, 18th, 19th, 20th products weren't selected in the assortment. Hence,

$$\begin{aligned} X_{7,j,k} &= 0 \\ X_{11,j,k} &= 0 \\ X_{13,j,k} &= 0 \\ X_{16,j,k} &= 0 \\ X_{17,j,k} &= 0 \\ X_{18,j,k} &= 0 \\ X_{19,j,k} &= 0 \\ X_{20,j,k} &= 0 \end{aligned}$$

for all $i \in I \quad k \in K$, therefore these variables weren't basic variables.
In addition, you can find the basic variable differences between the two scenarios in the table below.

5cm increase	10cm increase
$X_{8,1,5}$	$X_{8,1,1}$
$X_{10,3,5}$	$X_{10,3,1}$
$X_{21,2,5}$	$X_{21,3,5}$
$X_{22,1,1}$	$X_{22,1,5}$
$X_{23,1,1}$	$X_{23,1,5}$
$X_{24,3,5}$	$X_{24,2,5}$

Table 4: Basic Variable Comparison Table

Changes in the excel model:
We changed w_5 to 115 and 120

2.1.6 Q7

Initial Profit	6945.73351921896
New Profit	6943.00439273943
Profit Change	-2.72912647953
Decision Variables	Check 4.3.7

Table 5: Q7 Comparison Table

In this section, we analysed how the solution found in 2.1.1 changes when the restriction requiring Product 3 and Product 8 to be placed on different shelves is removed, and the restriction requiring these two products to be assorted together and placed to the same shelf is added.

As a result of this change in constraints, we should see a change in profits probably. Nevertheless, this may not always happen. In the case of the products are never put in the shelves, or the constraint does not really constrain our optimal value, we should see that profit does not change. Otherwise, we still cannot say that this will change profit positively or negatively. The reason is we are removing one constraint, and adding a new constraint. Generally, what we can say is if we are removing a constraint, the profit will be at least as good as before, and vice versa. However, here two things happening, we are removing a constraint, and at the same time adding a new constraint. Before solving the model, it is almost impossible to say whether the profit will increase or decrease.

We removed the constraint in the model and saw that profit decreased a little bit. From that, we can conclude that it is better if we put product 3 and 8 separately.

Changes in the excel model:
We deleted the constrain that were making sure products 3 and 8 were not in the same shelf and added below constrain to make sure they were assorted together(if any one of them were assorted the other was also assorted) and were in the same shelf:

$$(\sum_{j=1}^{4} X_{3jk}) - (\sum_{j=1}^{4} X_{8jk}) = 0 \quad \forall k \in K$$

2.2 Discussion

By modelling the problem and using OpenSolver, we have found that under these circumstances the most important features of our model, and the ones that will add the most profit in case of addition, are space elasticity factors, shelves' effect on demand, coefficient for demand rate, and product profit per unit. We also want to note that since shelf effect on demand multiplies all the products on it, it is the best feature if we can only afford to increase one feature. We see that by changing the numbers in our model in OpenSolver, but we also see that in our objective function. All of the things are unique to a product, and one of them unique for every situation of products, except shelf effect on demand.

3 Conclusion and Recommendations

To summarize, in this project, we tried to solve Assortment Selection and Shelf Space Allocation Problem. We have designed a mathematical model that fits given constraints and achieved to obtain optimized objective function values and corresponding decision variables. As a result of our analysis on the dataset given in 4.1, we have found the objective function value as **6945.73351921896**. Furthermore, we got the highest profit of **7165.30316863258** among the given scenarios when we added a new shelf (2.1.3). In this scenario, we have achieved **+219.569649414** profit increase.

In conclusion, the employees should place the most profitable products on shelves with the highest shelf effect on demand. When adding a shelf that has shelf effect on-demand value(γ_k) of let us say A, the employees should move some of the most profitable products from shelves that has lower shelf effect on-demand value(γ_k) then A to the new shelf. Also, when removing a shelf, we should consider moving the most profitable products on that shelf to other shelves even if the shelf we are removing has the lowest shelf effect on-demand value(γ_k). Lastly, if a product is going to be assorted the employees should choose how many of facings a product will have based on its β_i value(for the most part) and other coefficients(π_i, b_i, d_i).

4 Appendix

4.1 Data Set for Q1,Q2

Table 6: Sets of product pairs

I^1 :	(2, 5) , (3, 8), (16, 20)
I^2 :	(1, 12), (3, 8), (9, 15), (16, 20)

Table 7: Product based data

Product number	π_i	b_i	dp_i	d_i	β_i	s_i^l	s_i^U
1	15	10	7	4	0.5	2	15
2	8	9	6	10	0.2	1	19
3	12	5	10	10	0.3	1	23
4	6	7	7	7	0.2	1	9
5	11	9	9	5	0.9	1	16
6	14	6	8	2	0.4	1	21
7	14	9	6	1	0.5	1	11
8	6	5	9	6	0.3	1	18
9	5	9	9	7	0.8	2	11
10	11	10	9	3	0.8	1	11
11	12	7	5	4	0.1	2	17
12	8	5	6	7	0.8	2	22
13	11	7	6	2	0.1	1	12
14	13	8	9	9	0.8	3	12
15	7	9	8	11	0.8	3	19
16	14	23	5	2	0.1	2	16
17	9	25	6	9	0.1	1	10
18	10	17	8	1	0.8	2	16
19	13	15	9	4	0.2	2	20
20	5	23	6	2	0.6	3	19
21	11	19	8	6	0.6	2	24
22	11	19	9	6	0.4	1	16
23	7	16	7	8	0.5	2	13
24	10	14	5	2	0.8	1	16
25	13	16	10	4	0.9	2	14

Table 8: Shelf based data

Shelf number	w_k	ds_k	γk
1	50	34	0.25
2	65	30	0.60
3	80	26	1
4	95	27	0.60
5	110	29	0.25

4.2 Rules

1- **If a product is selected in the assortment, than all facings for the product must be placed on the same shelf.**$\surd$
2- **The manager does not want to allocate more than four facings for a product.** $\surd$
3- **If a product is selected in the assortment, then a minimum shelf inventory amount must be placed on the shelves. Similarly, for each product there is an upper bound on the shelf-inventory.**$\surd$
4- **If a product is selected in the assortment, lower and upper bounds on its facing number are calculated by using shelf depth, product depth and lower and upper bounds on the shelf-inventory.**$\surd$
5- *Each product provides a certain profit per unit sold.*
6- **Each shelf has a certain width and the total width of the facings placed in the shelf cannot exceed its width.**$\surd$
7- **For some pairs of products, there is a restriction that if one is included in the assortment, the other product must also be included.**$\surd$
8- **For some pairs of products, there is a restriction that they cannot be on the same shelf**$\surd$

4.3 Opensolver Results

Objective function results for the scenarios are already given and discussed in the 2 section. In this section, the tables for the decision variable values calculated by opensolver for the corresponding objective function results are given.

$$X_{ijk} = \begin{cases} 1, & if\ i\ is\ allocated\ j\ times\ (i.e.\ has\ j\ facings)\ in\ k \\ 0, & otherwise \end{cases}$$

Tables given below represent X_{i1k}, X_{i2k}, X_{i3k}, X_{i4k} from left to right, respectively. For each table,

- Column k represents kth shelf,
- Row i represents ith product

For instance, if the value in the table 3, row 4 and column 2 equals to 1, that denotes that to obtain the optimum value of the objective function, product 4 would be placed in 2nd shelf with 3 allocations (facings).

4.3.1 Q2 Result

Figure 1: Q2 Result

DECISION VARIABLES

x(i,j,k)= Is there going to be j facings of product i at shelf k.

x(i,1,k)

i\k	1	2	3	4	5
	0	0	0	1	0
	0	0	0	0	1
	0	0	1	0	0
	0	0	1	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
j=	1				

x(i,2,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	0
j=	2				

x(i,3,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	1	0
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
j=	3				

x(i,4,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	1	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	1	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	1	0	0	0
j=	4				

4.3.2 Q3 Result

Figure 2: Q3 Result

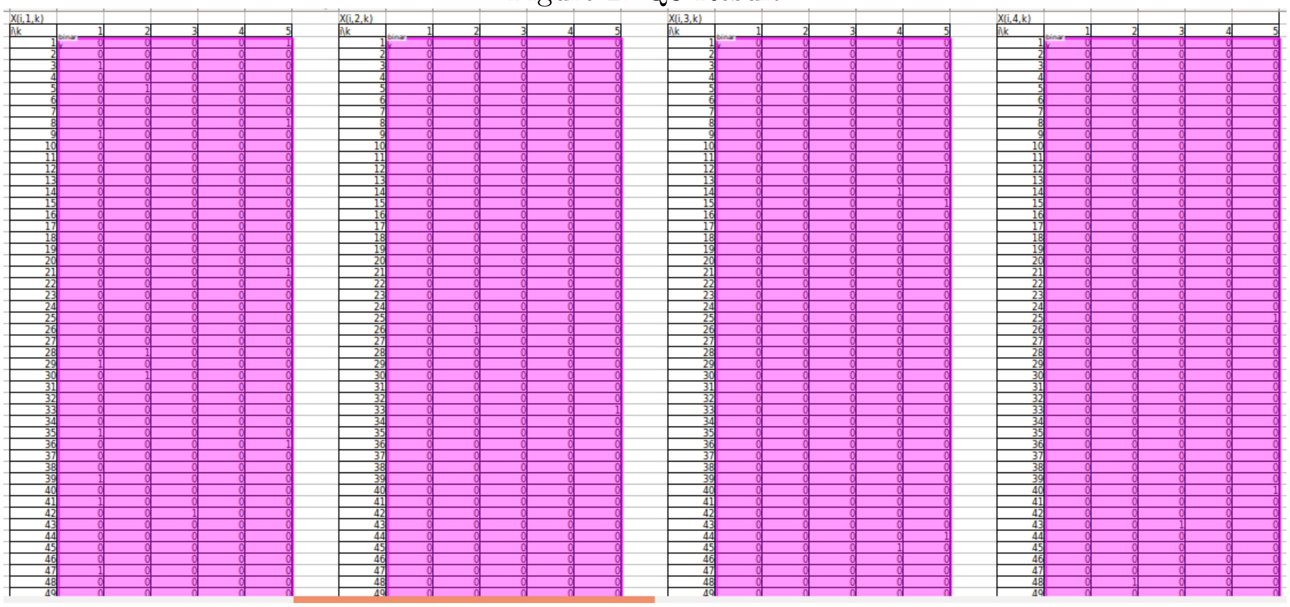
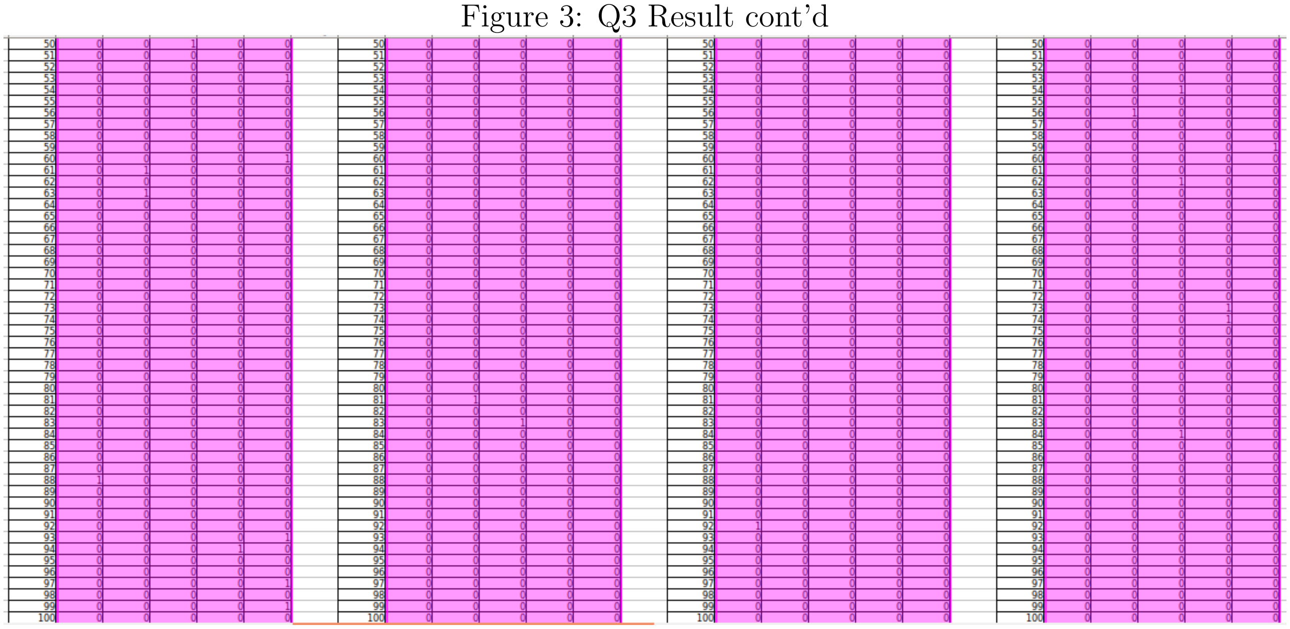
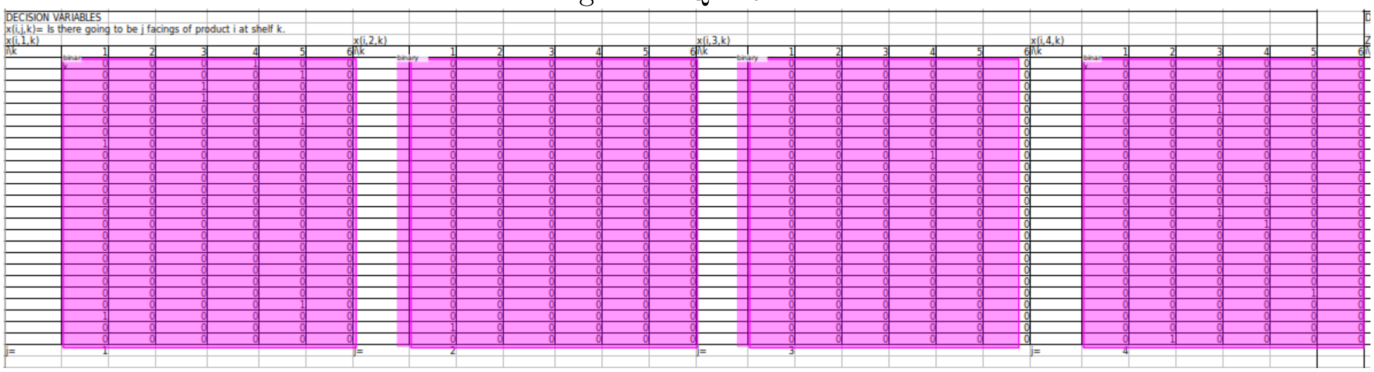


Figure 3: Q3 Result cont'd



4.3.3 Q4 Result

Figure 4: Q4 Result



DECISION VARIABLES

x(i,j,k)= Is there going to be j facings of product i at shelf k.

x(i,1,k)

i\k	1	2	3	4	5	6
	0	0	0	1	0	0
	0	0	0	0	1	0
	0	0	1	0	0	0
	0	0	1	0	0	0
	0	0	0	0	0	0
	0	0	0	0	1	0
	0	0	0	0	0	0
	1	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	1	0
	1	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
j=	1					

x(i,2,k)

x(i,3,k)

x(i,4,k)

4.3.4 Q5 Result

Figure 5: Q5 Result

DECISION VARIABLES

x(i,j,k)= Is there going to be j facings of product i at shelf k.

x(i,1,k)

x(i,2,k)

x(i,3,k)

x(i,4,k)

4.3.5 Q6 Result - 5cm increase

Figure 6: Q6 Result after 5cm increase

DECISION VARIABLES

x(i,j,k)= Is there going to be j facings of product i at shelf k.

x(i,1,k)

i\k	1	2	3	4	5
	0	0	0	1	0
	1	0	0	0	0
	0	0	1	0	0
	0	0	1	0	0
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	1	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
j=	1				

x(i,2,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
j=	2				

x(i,3,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	1	0
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	0
j=	3				

x(i,4,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	1	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	1	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	1	0	0	0
j=	4				

4.3.6 Q6 Result - 10cm increase

Figure 7: Q6 Result after 10cm increase

DECISION VARIABLES

x(i,j,k)= Is there going to be j facings of product i at shelf k.

x(i,1,k)

i\k	1	2	3	4	5
	0	0	0	1	0
	1	0	0	0	0
	0	0	1	0	0
	0	0	1	0	0
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
j=	1				

x(i,2,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	0
j=	2				

x(i,3,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	1	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
j=	3				

x(i,4,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	1	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	1	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	1	0	0	0
j=	4				

4.3.7 Q7 Result

Figure 8: Q7 Result

DECISION VARIABLES

x(i,j,k)= Is there going to be j facings of product i at shelf k.

x(i,1,k)

i\k	1	2	3	4	5
	0	0	0	1	0
	1	0	0	0	0
	0	0	1	0	0
	0	0	0	0	1
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	0	0
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
j=	1				

x(i,2,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
	1	0	0	0	0
	0	0	0	0	0
j=	2				

x(i,3,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	1	0
	0	0	0	0	1
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
j=	3				

x(i,4,k)

i\k	1	2	3	4	5
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	1	0
	0	0	0	0	0
	0	0	1	0	0
	0	0	0	1	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	1	0	0	0
j=	4				