第八次习题

1. 一个自旋按照下面的幺正矩阵

$$\boldsymbol{U_s(t)}=\begin{pmatrix}\boldsymbol{cost\ -\ i\frac{1}{\sqrt{2}}sint} & \boldsymbol{i\frac{1}{\sqrt{2}}sint}\\ \boldsymbol{i\frac{1}{\sqrt{2}}sint} & \boldsymbol{cost+i\frac{1}{\sqrt{2}}sint}\end{pmatrix}$$

进行动力学演化。它的出事的自旋态是

$$|\boldsymbol{\psi}\rangle=\frac{\sqrt{3}}{2}|\boldsymbol{u}\rangle+\frac{1}{2}|\boldsymbol{d}\rangle$$

验证：$\boldsymbol{U_s(t)|\psi\rangle=\frac{\sqrt{3}}{2}U_s(t)|u\rangle+\frac{1}{2}U_s(t)|d\rangle}$.

左边 $=U_s(t)|\psi\rangle=\begin{pmatrix}cost\ -\ i\frac{1}{\sqrt{2}}sint & i\frac{1}{\sqrt{2}}sint\\ i\frac{1}{\sqrt{2}}sint & cost+i\frac{1}{\sqrt{2}}sint\end{pmatrix}\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{3}}{2}cost\ -\ i\frac{\sqrt{6}}{4}sint+i\frac{\sqrt{2}}{4}sint\\ i\frac{\sqrt{6}}{4}sint+\frac{1}{2}cost+i\frac{\sqrt{2}}{4}sint\end{pmatrix}$.

右边 $=\frac{\sqrt{3}}{2}\begin{pmatrix}cost\ -\ i\frac{1}{\sqrt{2}}sint & i\frac{1}{\sqrt{2}}sint\\ i\frac{1}{\sqrt{2}}sint & cost+i\frac{1}{\sqrt{2}}sint\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}+\frac{1}{2}\begin{pmatrix}cost\ -\ i\frac{1}{\sqrt{2}}sint & i\frac{1}{\sqrt{2}}sint\\ i\frac{1}{\sqrt{2}}sint & cost+i\frac{1}{\sqrt{2}}sint\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$.

$=\frac{\sqrt{3}}{2}\begin{pmatrix}cost\ -\ i\frac{1}{\sqrt{2}}sint\\ i\frac{1}{\sqrt{2}}sint\end{pmatrix}+\frac{1}{2}\begin{pmatrix}i\frac{1}{\sqrt{2}}sint\\ cost+i\frac{1}{\sqrt{2}}sint\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{3}}{2}cost\ -\ i\frac{\sqrt{6}}{4}sint+i\frac{\sqrt{2}}{4}sint\\ i\frac{\sqrt{6}}{4}sint+\frac{1}{2}cost+i\frac{\sqrt{2}}{4}sint\end{pmatrix}$.

由于“左边=右边”， $U_s(t)|\psi\rangle=\frac{\sqrt{3}}{2}U_s(t)|u\rangle+\frac{1}{2}U_s(t)|d\rangle$.

2. 二维旋转矩阵是

$$\mathbf{R}=\begin{pmatrix}\boldsymbol{cos\theta} & \boldsymbol{-sin\theta}\\ \boldsymbol{sin\theta} & \boldsymbol{cos\theta}\end{pmatrix}$$

对两个二维向量

$$|\boldsymbol{\varphi_1}\rangle=\begin{pmatrix}\boldsymbol{a_1}\\ \boldsymbol{a_2}\end{pmatrix},\ |\boldsymbol{\varphi_2}\rangle=\begin{pmatrix}\boldsymbol{b_1}\\ \boldsymbol{b_2}\end{pmatrix}$$

证明： $\boldsymbol{R(|\varphi_1\rangle+|\varphi_2\rangle)=R|\varphi_1\rangle+R|\varphi_2\rangle}$

左边 $=R(|\varphi_1\rangle+|\varphi_2\rangle)=\begin{pmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{pmatrix}\begin{pmatrix}a_1+b_1\\ a_2+b_2\end{pmatrix}=\begin{pmatrix}(a_1+b_1)cos\theta-(a_2+b_2)sin\theta\\ (a_1+b_1)sin\theta+(a_2+b_2)cos\theta\end{pmatrix}$.

右边 $=R|\varphi_1\rangle+R|\varphi_2\rangle=\begin{pmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{pmatrix}\begin{pmatrix}a_1\\ a_2\end{pmatrix}+\begin{pmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{pmatrix}\begin{pmatrix}b_1\\ b_2\end{pmatrix}$.

$=\begin{pmatrix}a_1cos\theta-a_2sin\theta\\ a_1sin\theta+a_2cos\theta\end{pmatrix}+\begin{pmatrix}b_1cos\theta-b_2sin\theta\\ b_1sin\theta+b_2cos\theta\end{pmatrix}=\begin{pmatrix}(a_1+b_1)cos\theta-(a_2+b_2)sin\theta\\ (a_1+b_1)sin\theta+(a_2+b_2)cos\theta\end{pmatrix}$.

由于“左边=右边”， $R(|\varphi_1\rangle+|\varphi_2\rangle)=R|\varphi_1\rangle+R|\varphi_2\rangle$.

3. **在课本图6.8描述的双缝干涉实验中，假设总共有2400个电子通过双缝。在线圈没有电流通过时，探测器$\boldsymbol{d_5}$上探测到了大约600个电子。现在线圈通电产生磁场，造成电子上下两部分波函数有一个π/3的相位差，即通过双缝以后，电子的波函数成为**

$$|\boldsymbol{\Phi}\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left(|\boldsymbol{\psi_1}\rangle + \boldsymbol{e^{i\pi/3}}|\boldsymbol{\psi_2}\rangle\right)$$

请问探测器$\boldsymbol{d_5}$上探测到了大约多少个电子。

由题知，没有电流通过时，d_5探测到的电子数与a_5、b_5满足下列方程：

$$\frac{N}{2}|a_5 + b_5|^2 = 2N|a_5|^2 = 4800|a_5|^2 = 600$$

解得

$$|a_5|^2 = \frac{1}{8}$$

当电流通过时，d_5探测到的电子数大约

$$\frac{N}{2}\left|a_5 + (\frac{1}{2} + i\frac{\sqrt{3}}{2})b_5\right|^2 = \frac{N}{2}\left|(\frac{3}{2} + i\frac{\sqrt{3}}{2})a_5\right|^2 = 1200 \cdot 3 \cdot \frac{1}{8} = 450$$