第六次作业参考答案

1.

$$[\hat{\sigma}_x,\hat{\sigma}_y]=\hat{\sigma}_x\hat{\sigma}_y-\hat{\sigma}_y\hat{\sigma}_x=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix}-\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}=\begin{pmatrix}i&0\\0&-i\end{pmatrix}-\begin{pmatrix}-i&0\\0&i\end{pmatrix}=$$

$$2i\begin{pmatrix}1&0\\0&-1\end{pmatrix}=2i\hat{\sigma}_z;$$

2.

(1).我们知道$\langle u|d\rangle=0;\langle u|u\rangle=1;\langle d|d\rangle=1;$

$$\mathrm{P}_1(\mathrm{u})=|\langle u|\psi_1\rangle|^2=\frac{1}{4}$$

$$\mathrm{P}_1(\mathrm{d})=|\langle d|\psi_1\rangle|^2=\frac{3}{4}$$

(2).

$$\langle f|u\rangle=\begin{pmatrix}\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\frac{1}{\sqrt{2}};\langle f|d\rangle=\begin{pmatrix}\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=\frac{1}{\sqrt{2}};$$

$$\langle b|u\rangle=\begin{pmatrix}\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\frac{1}{\sqrt{2}};\langle b|u\rangle=\begin{pmatrix}\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=-\frac{1}{\sqrt{2}};$$

$$\mathrm{P}_1(\mathrm{f})=|\langle f|\psi_1\rangle|^2=\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)^2=\frac{2+\sqrt{3}}{4}$$

$$\mathrm{P}_1(\mathrm{b})=|\langle b|\psi_1\rangle|^2=\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)^2=\frac{2-\sqrt{3}}{4}$$

(3).

$$\vec{n}\cdot\hat{\sigma}=\begin{pmatrix}\frac{4}{5}&\frac{3}{5}\\\frac{3}{5}&-\frac{4}{5}\end{pmatrix}$$

$$\begin{vmatrix}\frac{4}{5}-\lambda&\frac{3}{5}\\\frac{3}{5}&-\frac{4}{5}-\lambda\end{vmatrix}=\lambda^2-1=0$$

所以本征值$\lambda=\pm1$；令$|n_+\rangle=\begin{pmatrix}x\\y\end{pmatrix},|n_-\rangle=\begin{pmatrix}m\\n\end{pmatrix}$;且$|\langle n_+|n_+\rangle|^2=|\langle n_-|n_-\rangle|^2=1$

$$\begin{cases}\frac{4}{5}x+\frac{3}{5}y=x\\x^2+y^2=1\end{cases};\quad\begin{cases}\frac{4}{5}m+\frac{3}{5}n=-m\\m^2+n^2=1\end{cases}$$

$$\begin{cases}x=\frac{3\sqrt{10}}{10}\\y=\frac{\sqrt{10}}{10}\end{cases};\quad\begin{cases}m=\frac{\sqrt{10}}{10}\\n=-\frac{3\sqrt{10}}{10}\end{cases}$$

$$\langle n_+|u\rangle = \begin{pmatrix}\frac{3\sqrt{10}}{10} & \frac{\sqrt{10}}{10}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{3\sqrt{10}}{10};\langle n_+|d\rangle = \begin{pmatrix}\frac{3\sqrt{10}}{10} & \frac{\sqrt{10}}{10}\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{\sqrt{10}}{10};$$

$$\langle n_-|u\rangle = \begin{pmatrix}\frac{\sqrt{10}}{10} & -\frac{3\sqrt{10}}{10}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{\sqrt{10}}{10};\langle n_-|d\rangle = \begin{pmatrix}\frac{\sqrt{10}}{10} & -\frac{3\sqrt{10}}{10}\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = -\frac{3\sqrt{10}}{10};$$

$$\mathrm{P}_1(n_+) = |\langle n_+|\psi_1\rangle|^2 = \left(\frac{3\sqrt{10}+\sqrt{30}}{20}\right)^2 = \frac{6+3\sqrt{3}}{20}$$

$$\mathrm{P}_1(n_-) = |\langle n_-|\psi_1\rangle|^2 = \left(\frac{\sqrt{10}-3\sqrt{30}}{20}\right)^2 = \frac{14-3\sqrt{3}}{20}$$

3.与 2 题相同的处理方法，这里给只给出答案：

(1).

$$\mathrm{P}_2(\mathrm{u}) = |\langle u|\psi_2\rangle|^2 = \frac{1}{4}$$

$$\mathrm{P}_2(\mathrm{d}) = |\langle d|\psi_2\rangle|^2 = \frac{3}{4}$$

(2).

$$\mathrm{P}_2(\mathrm{f}) = |\langle f|\psi_2\rangle|^2 = \frac{\sqrt{2}-\sqrt{6}i}{4}\cdot\frac{\sqrt{2}+\sqrt{6}i}{4} = \frac{1}{2}$$

$$\mathrm{P}_2(\mathrm{b}) = |\langle b|\psi_2\rangle|^2 = \frac{\sqrt{2}+\sqrt{6}i}{4}\cdot\frac{\sqrt{2}-\sqrt{6}i}{4} = \frac{1}{2}$$

(3).

$$\mathrm{P}_2(n_+) = |\langle n_+|\psi_2\rangle|^2 = \frac{3\sqrt{10}-\sqrt{30}i}{20}\cdot\frac{3\sqrt{10}+\sqrt{30}i}{20} = \frac{3}{10}$$

$$\mathrm{P}_2(n_-) = |\langle n_-|\psi_2\rangle|^2 = \frac{\sqrt{10}+3\sqrt{30}i}{20}\cdot\frac{\sqrt{10}-3\sqrt{30}i}{20} = \frac{7}{10}$$

(4).

$$\langle\psi_2|\vec{n}\cdot\hat{\sigma}|\psi_2\rangle = \begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}i\end{pmatrix}\begin{pmatrix}\frac{4}{5} & \frac{3}{5}\\ \frac{3}{5} & -\frac{4}{5}\end{pmatrix}\begin{pmatrix}\frac{1}{2}\\ -\frac{\sqrt{3}}{2}i\end{pmatrix} = \begin{pmatrix}\frac{4+3\sqrt{3}i}{10} & \frac{3-4\sqrt{3}i}{10}\end{pmatrix}\begin{pmatrix}\frac{1}{2}\\ -\frac{\sqrt{3}}{2}i\end{pmatrix} = -\frac{2}{5}$$