第八次作业参考答案

1.待验证式左边：

$$U_S(t)|\psi\rangle = \begin{pmatrix} \cos t - i\frac{\sqrt{2}}{2}\sin t & i\frac{\sqrt{2}}{2}\sin t \\ i\frac{\sqrt{2}}{2}\sin t & \cos t + i\frac{\sqrt{2}}{2}\sin t \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{3}}{2}\cos t - i\frac{\sqrt{6}-\sqrt{2}}{4}\sin t \\ \frac{1}{2}\cos t + i\frac{\sqrt{6}+\sqrt{2}}{4}\sin t \end{pmatrix}$$

待验证式右边：

$$\frac{\sqrt{3}}{2}U_S(t)|u\rangle + \frac{1}{2}U_S(t)|d\rangle$$

$$= \frac{\sqrt{3}}{2}\begin{pmatrix} \cos t - i\frac{\sqrt{2}}{2}\sin t & i\frac{\sqrt{2}}{2}\sin t \\ i\frac{\sqrt{2}}{2}\sin t & \cos t + i\frac{\sqrt{2}}{2}\sin t \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$+ \frac{1}{2}\begin{pmatrix} \cos t - i\frac{\sqrt{2}}{2}\sin t & i\frac{\sqrt{2}}{2}\sin t \\ i\frac{\sqrt{2}}{2}\sin t & \cos t + i\frac{\sqrt{2}}{2}\sin t \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{3}}{2}\cos t - i\frac{\sqrt{6}}{4}\sin t \\ i\frac{\sqrt{6}}{4}\sin t \end{pmatrix} + \begin{pmatrix} i\frac{\sqrt{2}}{4}\sin t \\ \frac{1}{2}\cos t + i\frac{\sqrt{2}}{4}\sin t \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{3}}{2}\cos t - i\frac{\sqrt{6}-\sqrt{2}}{4}\sin t \\ \frac{1}{2}\cos t + i\frac{\sqrt{6}+\sqrt{2}}{4}\sin t \end{pmatrix}$$

所以左边等于右边。

2.左边：

$$R(|\varphi_1\rangle + |\varphi_2\rangle) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta \cdot (a_1 + b_1) - \sin\theta \cdot (a_2 + b_2) \\ \cos\theta \cdot (a_2 + b_2) + \sin\theta \cdot (a_1 + b_1) \end{pmatrix}$$

右边：

$$R|\varphi_1\rangle + R|\varphi_2\rangle = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta\, a_1 - \sin\theta\, a_2 \\ \cos\theta\, a_2 + \sin\theta\, a_1 \end{pmatrix} + \begin{pmatrix} \cos\theta\, b_1 - \sin\theta\, b_2 \\ \cos\theta\, b_2 + \sin\theta\, b_1 \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta \cdot (a_1 + b_1) - \sin\theta\, (a_2 + b_2) \\ \cos\theta \cdot (a_2 + b_2) + \sin\theta \cdot (a_1 + b_1) \end{pmatrix}$$

所以左边等于右边。

3.根据课本 6.36 式，我们知道没有磁场干涉时，电子处于：$|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$

经过双缝干涉后变为 9 个探测器上量子态的叠加：$\frac{1}{\sqrt{2}}\sum_{j=1}^{9}(a_j + b_j)\,|d_{\mathrm{j}}\rangle$

所以第 j 个探测器探测到的电子的概率为：$\frac{1}{2}|a_j + b_j|^2$

由于d_5探测器的对称性，$a_5 = b_5$，所以：$\frac{600}{2400} = \frac{1}{2}|a_5 + b_5|^2 = 2|a_5|^2$

即：$|a_5|^2 = \frac{1}{8}$

当电子波函数变为：

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + \mathrm{e}^{\mathrm{i}\pi/3}|\psi_2\rangle)$$

经过双缝干涉后变为 9 个探测器上量子态的叠加：

$$\frac{1}{\sqrt{2}}\sum_{j=1}^{9}(a_j + \mathrm{e}^{\mathrm{i}\pi/3} b_j)\,|d_{\mathrm{j}}\rangle$$

所以第 5 个探测器探测到的电子数为：

$$\frac{1}{2}|a_5 + \mathrm{e}^{\mathrm{i}\pi/3} b_5|^2 \cdot 2400 = \frac{1}{2} \cdot |a_5|^2 \cdot |1 + \mathrm{e}^{\mathrm{i}\pi/3}|^2 \cdot 2400 = 150 \cdot |\frac{3}{2} + \mathrm{i}\frac{\sqrt{3}}{2}|^2$$
$$= 150 \cdot 3 = 450$$