第十次习题

1. 在课本中我们举例说明了$|S\rangle = \frac{1}{\sqrt{2}}(|ud\rangle - |du\rangle)$会违反贝尔不等式，其中用到了三个方向$\vec{n}_1, \vec{n}_2, \vec{n}_3$。它们之间夹角 θ是$60°$。

（1）夹角为$\frac{\Pi}{6}$时，

$$p(A, \neg B) = p(\vec{n}_1, \vec{n}_2) = \frac{1}{2} sin^2 \frac{\Pi}{12} = \frac{2 - \sqrt{3}}{8}$$

$$p(B, \neg C) = p(\vec{n}_2, \vec{n}_3) = \frac{1}{2} sin^2 \frac{\Pi}{12} = \frac{2 - \sqrt{3}}{8}$$

$$p(A, \neg C) = p(\vec{n}_1, \vec{n}_3) = \frac{1}{2} sin^2 \frac{\Pi}{6} = \frac{1}{8}$$

$$p(A, \neg B) + p(B, \neg C) = \frac{2 - \sqrt{3}}{4} < \frac{1}{8} = p(A, \neg C)$$

违反贝尔不等式

（2）夹角为$\frac{\Pi}{2}$时

$$p(A, \neg B) = p(\vec{n}_1, \vec{n}_2) = \frac{1}{2} sin^2 \frac{\Pi}{4} = \frac{1}{4}$$

$$p(B, \neg C) = p(\vec{n}_2, \vec{n}_3) = \frac{1}{2} sin^2 \frac{\Pi}{4} = \frac{1}{4}$$

$$p(A, \neg C) = p(\vec{n}_1, \vec{n}_3) = \frac{1}{2} sin^2 \frac{\Pi}{2} = \frac{1}{2}$$

$$p(A, \neg B) + p(B, \neg C) = \frac{1}{2} = p(A, \neg C)$$

满足贝尔不等式

2.

$|\psi\rangle = \frac{\sqrt{3}}{2}|u\rangle - \frac{1}{2}i|d\rangle$

① $\langle\psi|\hat{\sigma}_z|\psi\rangle = \left(\frac{\sqrt{3}}{2}\langle u| + \frac{1}{2}i\langle d|\right)\hat{\sigma}_z\left(\frac{\sqrt{3}}{2}|u\rangle - \frac{1}{2}i|d\rangle\right)$

$= \left(\frac{\sqrt{3}}{2}\langle u| + \frac{1}{2}i\langle d|\right)\left(\frac{\sqrt{3}}{2}|u\rangle + \frac{1}{2}i|d\rangle\right)$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

$\langle\psi|\hat{\sigma}_y|\psi\rangle = \left(\frac{\sqrt{3}}{2}\langle u| + \frac{1}{2}i\langle d|\right)\hat{\sigma}_y\left(\frac{\sqrt{3}}{2}|u\rangle - \frac{1}{2}i|d\rangle\right)$

$= \left(\frac{\sqrt{3}}{2}\langle u| + \frac{1}{2}i\langle d|\right)\left(\frac{\sqrt{3}}{2}i|d\rangle - \frac{1}{2}|u\rangle\right)$

$= -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$

③ $\Delta\hat{\sigma}_z^2 = 1 - \bar{\sigma}_z^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\Delta\hat{\sigma}_y^2 = 1 - \bar{\sigma}_y^2 = 1 - \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}$

$\therefore \Delta\hat{\sigma}_z^2 + \Delta\hat{\sigma}_y^2 = \frac{3}{4} + \frac{1}{4} = 1 \geq 1$

∴满足这个等式 $\Delta\hat{\sigma}_z^2 + \Delta\hat{\sigma}_y^2 \geq 1$