第七次习题

（一） 一个自选正在按照下面的幺正矩阵

$$U_s(t)=\begin{pmatrix}\cos t & i\frac{\sqrt{2}}{2}\sin t & i\frac{\sqrt{2}}{2}\sin t\\ i\frac{\sqrt{2}}{2}\sin t & & \cos t+i\frac{\sqrt{2}}{2}\sin t\end{pmatrix}$$

进行动力学演化。

1. 初始的自旋态是$|u\rangle$，那么时刻t时，自旋处于什么态？假设在t_f时刻，自旋态演化成为$|b\rangle=(|u\rangle-|d\rangle)/\sqrt{2}$。请问$t_f$=?

   $$|\psi(t)\rangle=\begin{pmatrix}cost\ -\ i\frac{1}{\sqrt{2}}sint & i\frac{1}{\sqrt{2}}sint\\ i\frac{1}{\sqrt{2}}sint & cost+i\frac{1}{\sqrt{2}}sint\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}cost\ -\ i\frac{1}{\sqrt{2}}sint\\ i\frac{1}{\sqrt{2}}sint\end{pmatrix}.$$

   $$|\psi(t_f)\rangle=\begin{pmatrix}cost_f\ -\ i\frac{1}{\sqrt{2}}sint_f\\ i\frac{1}{\sqrt{2}}sint_f\end{pmatrix}=\frac{i}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}.$$

   $$\begin{cases}cost_f\ -\ i\frac{1}{\sqrt{2}}sint_f=\frac{1}{\sqrt{2}}i\\ i\frac{1}{\sqrt{2}}sint_f=-\frac{1}{\sqrt{2}}i\end{cases}.$$

   $cost_f=0, sint_f=-1.$

   $t_f=\frac{3}{2}\pi+2k\pi, k=0,1,2\ldots$

2. 初始的自旋态是$|d\rangle$，那么时刻t时，自旋处于什么态？在同样的t_f时刻，自旋处于什么态？

   $$|\psi(t)\rangle=\begin{pmatrix}cost\ -\ i\frac{1}{\sqrt{2}}sint & i\frac{1}{\sqrt{2}}sint\\ i\frac{1}{\sqrt{2}}sint & cost+i\frac{1}{\sqrt{2}}sint\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}i\frac{1}{\sqrt{2}}sint\\ cost+i\frac{1}{\sqrt{2}}sint\end{pmatrix}.$$

   $$|\psi(t_f)\rangle=\begin{pmatrix}i\frac{1}{\sqrt{2}}sint_f\\ cost_f+i\frac{1}{\sqrt{2}}sint_f\end{pmatrix}=-\frac{i}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}=|f\rangle.$$

3. 初始的自旋态是

   $$|\psi\rangle=\frac{\sqrt{3}}{2}|u\rangle+\frac{1}{2}|d\rangle$$

   那么在时刻t_f，自旋处于什么态？

   $$|\psi(t)\rangle=\begin{pmatrix}cost\ -\ i\frac{1}{\sqrt{2}}sint & i\frac{1}{\sqrt{2}}sint\\ i\frac{1}{\sqrt{2}}sint & cost+i\frac{1}{\sqrt{2}}sint\end{pmatrix}\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{3}}{2}cost\ -\ i\frac{\sqrt{6}}{4}sint+i\frac{\sqrt{2}}{4}sint\\ i\frac{\sqrt{6}}{4}sint+\frac{1}{2}cost+i\frac{\sqrt{2}}{4}sint\end{pmatrix}.$$

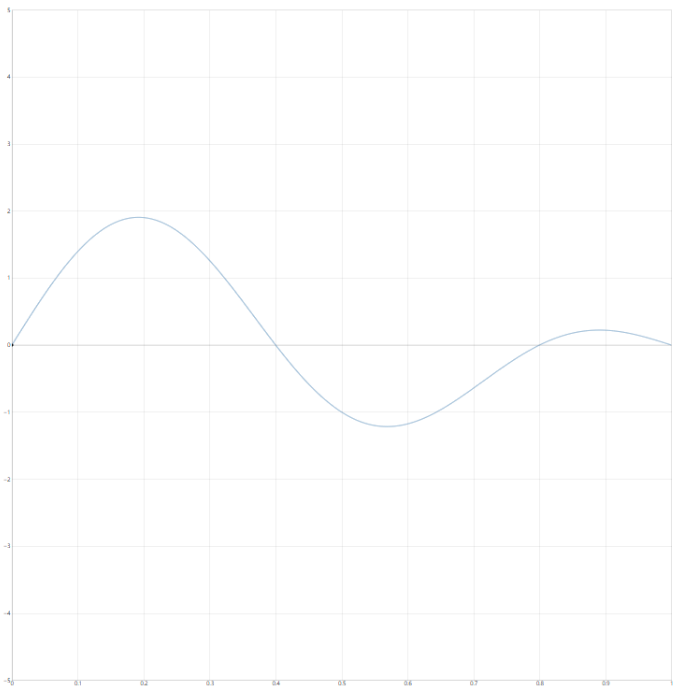
   $$|\psi(t_f)\rangle=\begin{pmatrix}\frac{\sqrt{3}}{2}cost_f\ -\ i\frac{\sqrt{6}}{4}sint_f+i\frac{\sqrt{2}}{4}sint_f\\ i\frac{\sqrt{6}}{4}sint_f+\frac{1}{2}cost_f+i\frac{\sqrt{2}}{4}sint_f\end{pmatrix}=\begin{pmatrix}i\frac{\sqrt{6}}{4}-i\frac{\sqrt{2}}{4}\\ -i\frac{\sqrt{6}}{4}-i\frac{\sqrt{2}}{4}\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{2}-\sqrt{6}}{4}\\ \frac{\sqrt{2}+\sqrt{6}}{4}\end{pmatrix}.$$

（二） 一个长度为a=1的一维盒子里，粒子处于两个能量本征态的叠加态

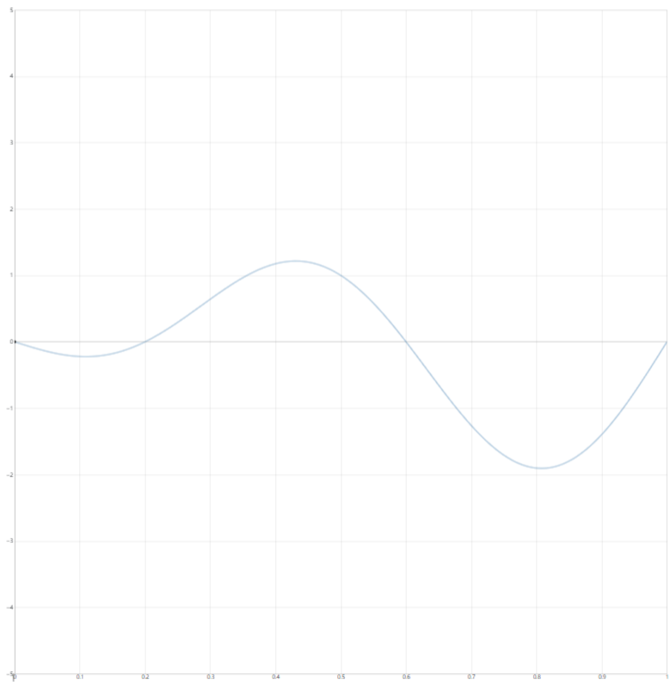
$$\psi_+(x) \quad = \quad \frac{1}{\sqrt{2}}\left[\psi_2(x)+\psi_3(x)\right] = \sin(2\pi x)+\sin(3\pi x) \qquad (1)$$

$$\psi\ \ (x) \quad = \quad \frac{1}{\sqrt{2}}\left[\psi_2 x) \quad \psi_3(x)\right] = \sin(2\pi x) \quad \sin(3\pi x) \qquad (2)$$

请画出这两个波函数ψ_+和ψ_-。



ψ_+



ψ_-