Huizhen Li

HW5 - Probability Practice

Q1: chance that no student will answer more than 1 question.

$\binom{15}{8} \div (15)^8$

Q2: 00000 - 99999

$\_\ \_\ \_\ \_\ \_$ (only in even integers, starting with 2 odds)

(1, 3, 5, 7, 9) for the first two digits; 2, 4, 6, 8 for the last digit

Assume P is the probability that if we randomly generate 8 of these numbers, → start with odd, but is an even.

$$P = \frac{5 \times 5 \times 7 \times 6 \times 4}{100000} = \frac{21}{500}$$

$$\binom{8}{5} p^5 (1-p)^3$$

$$= 8 \times 7 \times 6 \left(\frac{21}{500}\right)^5 \left(1 - \frac{21}{500}\right)^3$$

Q3: 3 dices (all 6-sided).

A: 2 dices or more show 4, 5, 6.

B: 3 dices show the same value.

$\Big\}$ Independent?

$$P(A) = \binom{3}{2}\left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + \binom{3}{3}\left(\frac{1}{2}\right)^3 = \frac{1}{2}$$

$$P(B) = \frac{6}{6^3} = \frac{1}{36}$$

$$P(A \cap B) = \frac{3}{6^3} = \frac{1}{72}$$

$P(A \cup B) = \frac{1}{2} \cdot \frac{1}{36} = P(A \cap B)$, $\boxed{\text{therefore they are independent}}$

Q4: ♤ ♡ ♧ ♢ : any 5 from the same suit is a flush.

what's the expected number of hands of poker he has to play to get a flush → Default new set every time.

$$P(\text{flush}) \ \frac{\binom{13}{5}\cdot\binom{4}{1}}{\binom{52}{5}} \rightarrow \frac{13 \text{ types chose } 5 \ ; \ 4 \text{ suits choose } 1}{52 \text{ cards choose } 5.}$$

number of hands to play: $\frac{1}{P(\text{flush})} = \frac{\binom{52}{5}}{\binom{13}{5}\cdot\binom{4}{1}} = \boxed{\frac{16660}{33} = 505}$

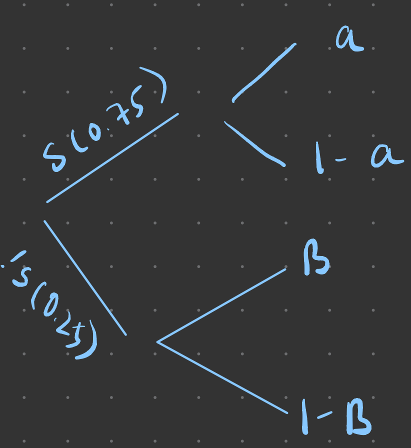
Q5: superstar.

if (s) → 70%
if (!s) → 50%
→ 1 2 3 4 5 $P(s) = 75\%$ → $4/5$ ✓.

chance of super star play $P(s) = \frac{3}{4}$.

$$P(w|s) = \frac{P(w \wedge s)}{P(s)} = \binom{5}{1}(0.7)^4 \cdot 0.3 = a$$

$$P(w|\bar{s}) = \binom{5}{1}\cdot 0.5^5 = B.$$



$$P(s) = \frac{0.75a}{0.75a + 0.25B} = \frac{\frac{3}{4}\binom{5}{1}\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right)}{\frac{3}{4}\binom{5}{1}\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right) + \frac{1}{4}\binom{5}{1}\left(\frac{1}{2}\right)^5}$$