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HW5: Counting Problem sets.

Q1: u n u s u a l

a. unique set: u n s a l Because there are 3 "n"s, So there will be 1 unique set.

b. different strings made _ _ _ _ _ , out of 5 of those 7 letters.

therefore, it should be $\frac{7!}{3!}$, 7! because there are 7 letters in total, and then since there are 3 repeated letter. therefore:

$$\frac{7!}{3!} = \frac{7\times6\times5\times4\times3\times2\times1}{3\times2\times1} = 7\times6\times5\times4 = 840.$$

Q2: $\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}$

Q3: 16 songs, 1 hour, 7 couples. Since 1 couple wants at most 1 song to be played, therefore the songs that are distributed:

Since there are 7 couples, there'll be 6 separation between.

$$\frac{(16+6)!}{16!\cdot 6!} = \frac{22\times21\times20\times19\times17}{6!} = 74613$$

Q4. BST: 12 nodes, value varies from 1-12

If we make g(x) be the number of BST for x nodes

① 2 node tree: $g(2) = 2$

② 3 node tree: $g(3) = g(2)g(0) + g(1)g(1) + g(0)g(2) = 5$

③ 4 node tree $g(4) = g(3)g(0) + g(2)g(1) + g(1)g(2) + g(0)g(3)$
$= 5 + 2 + 2 + 5 = 14$

④ 5 node tree: $g(5) = g(4)g(0) + g(3)g(1) + g(2)g(2) + g(1)g(3) + g(0)g(4) = 14 + 5 + 4 + 5 + 14 = 42$

since left child for 9 could only be 4~8, because it's smaller than 9; and for the right child can only be any from 10~12. f(3)=5; and for the left child is only f(5)=42.

Therefore the number of BST that can be formed:

$$2 \times 5 \times 42 = 420$$

Q5: 10 → 4 nurses, 1 of them might take a break.

a. if no nurse takes a break,

(1,1,1,7), (1,1,2,6), (1,1,3,5), (1,1,4,4), (1,2,2,5)
(1,2,3,4), (1,3,3,3), (2,2,2,4), (2,2,3,3) } 9

b. if there's a nurse who wants to take a break.

(1,1,8), (1,2,7), (1,3,6), (1,4,5),
(2,2,6), (2,3,5), (2,4,4), (3,3,4). } 8

Therefore the total number is 8+9 = 17