2024.3.15

$x_i = x_j$

§5.1 经验分布函数 $F_n(x)$

总体 $X \sim F(x)$

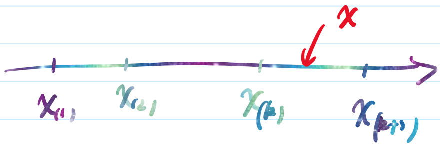
$X_1, \cdots, X_n$ (ω)

$X_1(\omega) = x_1$

$x_1, x_2, \cdots, x_n$

$x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)}$

(观测值) $F_n(x) = \begin{cases} 0, & x < x_{(1)} \\ \frac{k}{n}, & x_{(k)} \le x < x_{(k+1)} \\ 1, & x \ge x_{(n)} \end{cases}$



$X_1, X_2, \cdots, X_n$ 与 X 同分布.

$\{X \le x\}$ 发生了 k 次, 频率为 $\frac{k}{n}$

$\frac{\mu_n}{n} \xrightarrow{P} p$

$F_n(x) \xrightarrow{P} P(X \le x) = F(x)$

§5.2

统计量 $\bar{X}$、S_n^2、S_n^{*2}、A_k、B_k

μ σ^2

Th5.1 若总体的期望和方差存在, 则

$E(\bar{X}) = \mu$, $D(\bar{X}) = \frac{\sigma^2}{n}$, $E(S_n^2) = \frac{n-1}{n}\sigma^2$

$E(S_n^{*2}) = \sigma^2$

χ^2 分布: $X_1, \cdots, X_n$ 相互独立, 同分布, $X_1 \sim N(0,1)$

则 $\boxed{X_1^2 + \cdots + X_n^2 \sim \chi^2(n)} = \Gamma(\frac{n}{2}, \frac{1}{2})$

则 $X_1^2+\cdots+X_n^2 \sim \chi^2(n) = \Gamma(\frac{n}{2}, \frac{1}{2})$

$$f(x) = \begin{cases} \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} e^{-\frac{1}{2}x}, & x>0 \\ 0, & x\le 0. \end{cases}$$

$\Gamma(\alpha, \lambda)$ 分布，其密度函数为

$$f(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, & x>0 \\ 0, & x\le 0. \end{cases}$$

若 $X\sim\Gamma(\alpha,\lambda)$，$Y\sim\Gamma(\beta,\lambda)$，$X, Y$ 独立，则

$X+Y \sim \Gamma(\alpha+\beta, \lambda)$

若 $X\sim \underbrace{\chi^2(n_1)}_{=\Gamma(\frac{n_1}{2},\frac{1}{2})}$，$Y\sim \underbrace{\chi^2(n_2)}_{\Gamma(\frac{n_2}{2},\frac{1}{2})}$，$X$ 与 Y 相互独立，$\Gamma(\frac{n_1+n_2}{2},\frac{1}{2}) = \chi^2(n_1+n_2)$

则 $X+Y \sim \chi^2(n_1+n_2)$.

若 $X\sim\chi^2(n)$，$EX=?$ $DX=?$ $X_1\sim N(0,1)$

$X = X_1^2+\cdots+X_n^2$，从而 $EX = E(X_1^2+\cdots+X_n^2)$

$= n E(X_1^2) = n\left(DX_1+(EX_1)^2\right) = n(1+0) = n$

$\left(DX = E(X^2)-(EX)^2, \quad E(X^2) = DX+(EX)^2\right)$

$DX = D(X_1^2+\cdots+X_n^2) = nD(X_1^2) = n\left(E(X_1^4) - (\underbrace{EX_1^2}_{1})^2\right)$

$E(X_1^4) = \int_{-\infty}^{+\infty} x^4 f(x)dx$ $\qquad f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} x^4 e^{-\frac{x^2}{2}} dx$ $\qquad (e^{-\frac{x^2}{2}})' = -xe^{-\frac{x^2}{2}}$

$= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x^3 \, d(e^{-\frac{x^2}{2}})$

$= \cdots\cdots$

$$\boxed{T \sim t(n) \quad T^2 \sim ?}$$

$X \sim N(0,1)$ $Y \sim \chi^2(n)$, X、Y 独立

则 $\boxed{\dfrac{X}{\sqrt{Y/n}}} \sim t(n)$

稻盛和夫

人生成果 = 思维方式 × 热情 × 能力
利他 《活法》

$$T^2 = \left(\frac{X}{\sqrt{Y/n}}\right)^2 = \frac{X^2/1}{Y/n} \sim F(1,n)$$

$X^2/1 \to \chi^2(1)$

X^2 与 Y 独立.

$X_1, \cdots, X_n$ iid, $EX_i = \mu$, $DX_i = \sigma^2$ 有限.

则
$$\frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\,\sigma} \overset{\text{近似}}{\sim} N(0,1), \quad n \text{ 较大.}$$

$$\boxed{\sum_{i=1}^{n} X_i} \overset{\text{近似}}{\sim} N(n\mu, n\sigma^2)$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \ \underset{\sim}{\text{近似}} \ \boxed{N\left(\mu, \frac{\sigma^2}{n}\right)}$$