ex7、 $R_{Trapz} = -\frac{b-a}{12} \cdot h^2 \cdot f''(\eta)$

$= \cancel{I-Trap}\ \int_a^b f(x)dx - Trap$

$\because f''(\eta) > 0 \Rightarrow R_{Trapz} < 0 \Rightarrow \int_a^b \leq Trap$

$\Rightarrow \because f''(\eta) > 0 \Rightarrow f$ 下凸 $\Rightarrow$ 梯形 $>$ 曲边面积

ex10、两点 $\Rightarrow$ 对 3 阶多项式成立

设 $w(x) = (x-a)(x-b)$

$\cancel{\int_0^1 \frac{1}{\sqrt{x}} dx = 0}$

$\Rightarrow \int_0^1 \frac{1}{\sqrt{x}} \cdot w(x) \cdot 1 = 0$, $\int_0^1 \frac{1}{\sqrt{x}} w(x)\, x\, dx = 0$

$\Rightarrow \cancel{w(x) =} \begin{cases} a = \frac{1}{7}(3-2\sqrt{\frac{6}{5}}) \\ b = \frac{1}{7}(3+2\sqrt{\frac{6}{5}}) \end{cases}$

$\Rightarrow \int_0^1 \frac{1}{\sqrt{x}} dx = 2\sqrt{x} \Big|_0^1 = 2 = A_0 + A_1$

$\int_0^1 \frac{x}{\sqrt{x}} dx = \frac{2}{3}\sqrt{x^3} \Big|_0^1 = \frac{2}{3} = A_0 a + A_1 b$

$\Rightarrow$ 解得 $A_0 = 1 + \frac{1}{3}\sqrt{\frac{5}{6}}$, $A_1 = 1 - \frac{1}{3}\sqrt{\frac{5}{6}}$

$\Rightarrow \int_a^b f(x) dx = A_0 f(a) + A_1 f(b)$,

$a = \frac{1}{7}(3-2\sqrt{\frac{6}{5}})$, $b = \frac{1}{7}(3+2\sqrt{\frac{6}{5}})$

$A_0 = 1 + \sqrt{\frac{5}{6}}/3$, $A_1 = 1 - \sqrt{\frac{5}{6}}/3$

$\cancel{\int_a^b f(x)\,dx = \int_a^b f(a) +}$

(1) $f(x) = f(a) + f'(\xi)(x-a)$

$\Rightarrow \int_a^b f(x)\,dx = f(a)(b-a) + \frac{f'(\xi)}{2}(b-a)^2, \quad \xi \in (a,b)$

(2) $f(b) = f(b) + f'(\eta)(b-x)$

$\Rightarrow \int_a^b f(x)\,dx = f(b)(b-a) - \frac{f'(\eta)}{2}(b-a)^2$

(3) $f(x) = f\left(\frac{a+b}{2}\right) + f'\left(\frac{a+b}{2}\right)\left(x - \frac{a+b}{2}\right) + \frac{1}{2} f''(\xi)\left(x - \frac{a+b}{2}\right)^2$

$\int_a^b f(x)\,dx = (b-a) f\left(\frac{a+b}{2}\right) + \frac{1}{2} f''(\xi) \cdot (b-a)^3$

ex6. $f(x) = e^x, \quad f^{(n)}(x) = e^x$

$\Rightarrow R_{Trap} = -\frac{b-a}{12} \cdot h^2 \cdot f''(\eta) \le -\frac{1}{12} \cdot |e^x|_{max} h^2 \le \frac{1}{2} \times 10^{-5}$

$\Rightarrow \frac{e}{12} \cdot \frac{1}{n^2} \le \frac{1}{2} \times 10^{-5}$

$n^2 \geq \frac{e}{6} \times 10^5$

$\Rightarrow n = 213$

$R_{Simp} = \left| -\frac{b-a}{180} \cdot \left(\frac{h}{2}\right)^4 \cdot f''''(\eta) \right| \le \frac{1}{180} \cdot \left(\frac{1}{2n}\right)^4 \cdot e \le 0.5 \times 10^{-5}$

$n^4 \geq \frac{e}{144} \times 10^4$

$\Rightarrow n \geq 3.\sim$

$n = 4.$