

后经计算得:

$$\lambda \approx 7.288,$$

$$V_k = (1, 0.5229, 0.2433)^T$$

ex 4.  $\lambda = 4$

$$\Rightarrow (\lambda E - A)x = 0$$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} x = 0$$

$$\Rightarrow x = \begin{pmatrix} k_1 \\ k_2 \\ k_2 \end{pmatrix}, \text{ 取 } k_1 = 1, k_2 = 1$$

$$\Rightarrow x_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + x_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x = C_1 x_1 + C_2 x_2, C_1, C_2 \text{ 取任意}$$

ex1.

$$A = \begin{pmatrix} 2 & -3 & 6 \\ 0 & 3 & -4 \\ 0 & 2 & -3 \end{pmatrix}$$

$$\Rightarrow |\lambda E - A| = \begin{vmatrix} \lambda-2 & 3 & -6 \\ 0 & \lambda-3 & 4 \\ 0 & -2 & \lambda+3 \end{vmatrix} = (\lambda^2-9)(\lambda-2) + 8(\lambda-2) \\ = (\lambda^2-1)(\lambda-2) = 0$$

$$\Rightarrow \lambda_1 = 2, \lambda_2 = -1, \lambda_3 = 1$$

$$\Rightarrow A \sim P \cdot \begin{pmatrix} 2 & & \\ & -1 & \\ & & 1 \end{pmatrix} P^T, P \text{ 为正交阵}$$

$\Rightarrow A$  相似于一个对角阵

ex2. 设  $v_0 = \text{random}(\mathbb{R}, 1)$

此题由幂法:  $v_n = A^n \cdot v_0 \rightarrow \text{the}$

$$\Rightarrow \lambda_n = \max \{ |v_n| \} = 9.6055$$

$$w_k = [1, 0.6055, -0.3944]$$

ex3. 取  $p=6 \Rightarrow$

$$B = A - pE = \begin{pmatrix} 0 & 2 & 1 \\ 2 & -3 & 1 \\ 1 & 1 & -5 \end{pmatrix}$$

对  $B$  作 LU 分解:

$$PB = LU \Rightarrow P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, L = \begin{pmatrix} 1 & & \\ 0.5 & 1 & \\ 0 & 0.8 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 2 & -3 & 1 \\ & 2.5 & -5.5 \\ & & 5.4 \end{pmatrix}$$