Q3

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```
N = 500;
Ds = [0, 1, 5, 10]; % four sets of values for D

for i = 1:length(Ds)
    D = Ds(i);
    M = randn(N, N); % generate a matrix with random elements from N(0,1)
    M = M - diag(diag(M)) + diag(-D * ones(N, 1)); % make the diagonal elements equal to -D

    [V, lambda] = eig(M); % compute the eigenvalues of the matrix
    lambda = diag(lambda); % extract the diagonal of the eigenvalue matrix

    figure;
    scatter(real(lambda), imag(lambda), '.'); % plot the eigenvalues in the complex plane
    xlabel('Real(\lambda)');
    ylabel('Imag(\lambda)');
    title(sprintf('D = %d', D));
end
```

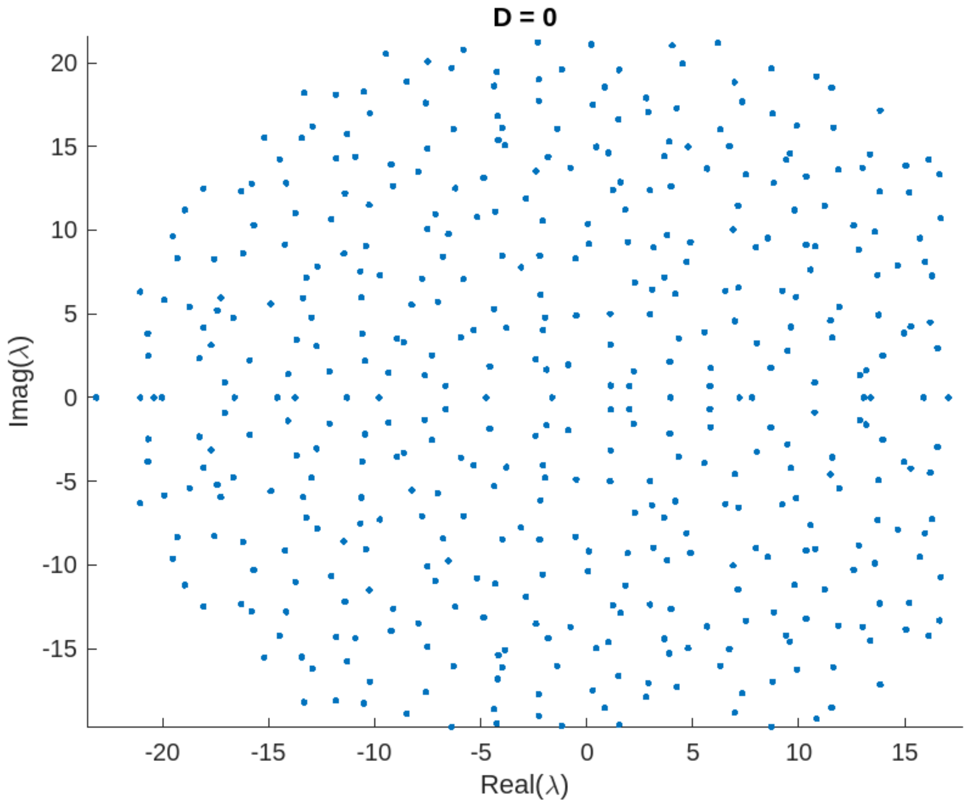
This Matlab code generates a scatter plot of the eigenvalues of a matrix in the complex plane. The elements of the matrix are drawn from a normal distribution with mean 0 and standard deviation 1. The size of the matrix is fixed at 500, and the diagonal elements are constant and fixed at -D, where D is a positive constant. The code generates a plot for four sets of D: 0, 1, 5, and 10.

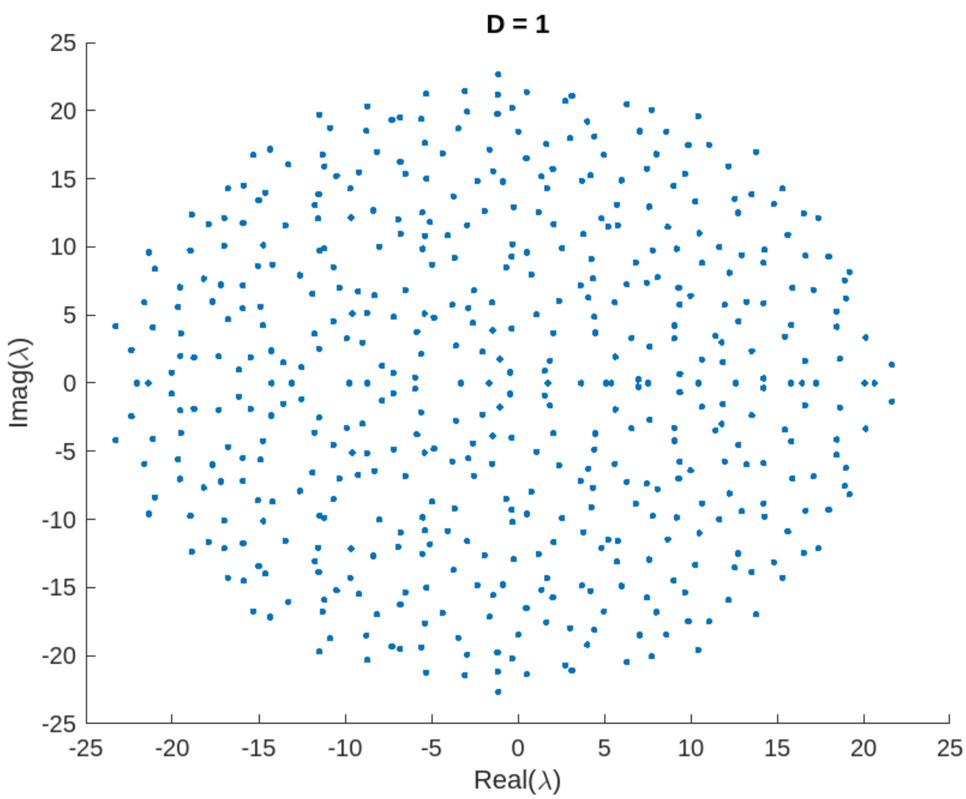
Code Explanation

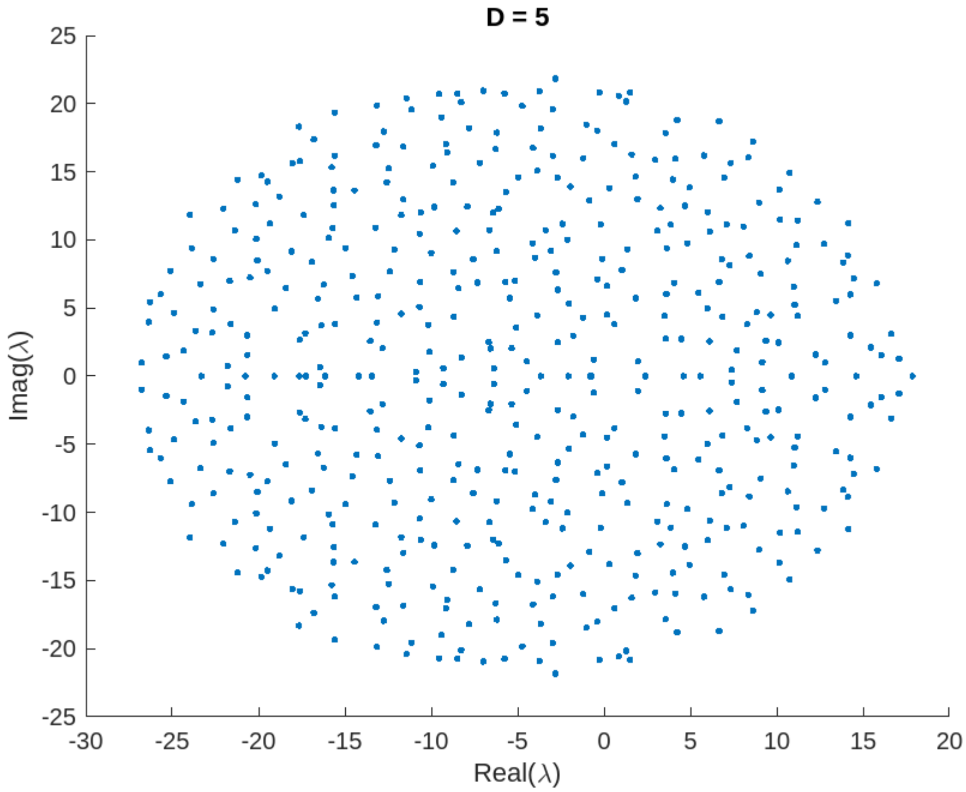
- N` is the size of the matrix `M`, which is fixed at 500.
- `Ds` is a vector that contains four values of `D`: 0, 1, 5, and 10. These values will be used to generate four different plots.
- The `for` loop iterates through the values of `D` in the vector `Ds`. In each iteration, the current value of `D` is assigned to the variable `D`.

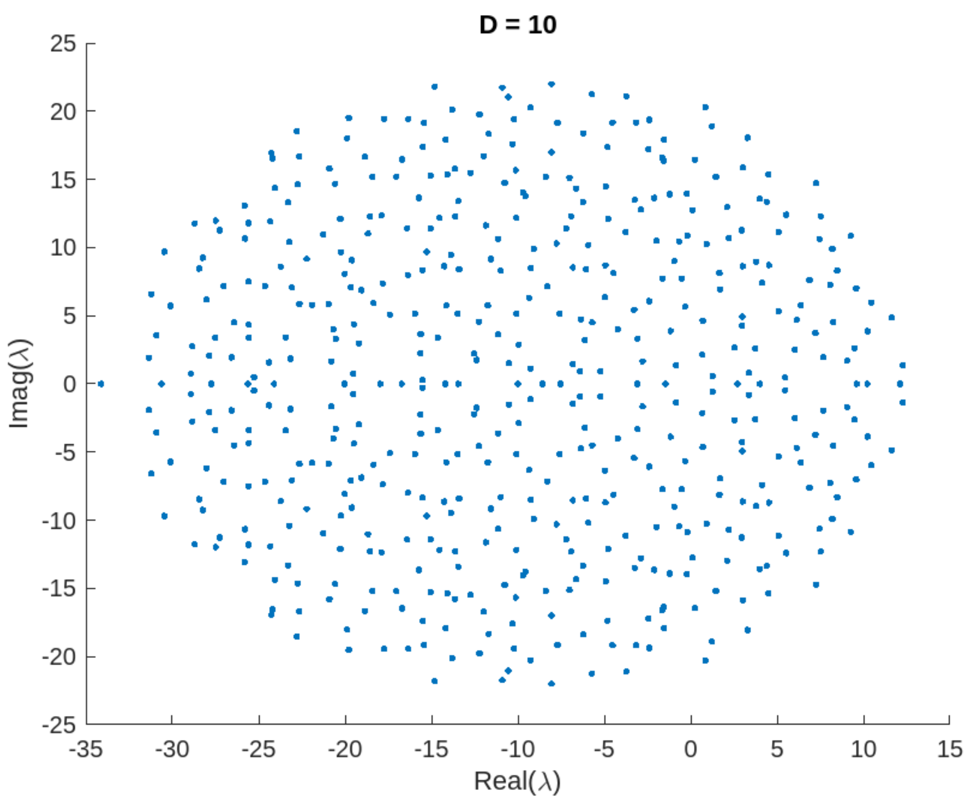
- `M` is a matrix of size `N` by `N` that is generated using the `randn` function. This function generates a matrix of random numbers from a standard normal distribution with mean 0 and standard deviation 1.
- The matrix `M` is modified by subtracting the diagonal elements of `M` and adding a diagonal matrix with elements equal to `-D`.
- The `eig` function is used to compute the eigenvalues of the matrix `M`. The output of the function are the eigenvalues `lambda` and the eigenvectors `V`.
- The `diag` function is used to extract the diagonal of the matrix `lambda`, which represents the eigenvalues.
- The `scatter` function is used to plot the eigenvalues in the complex plane. The `real` function is used to extract the real part of each eigenvalue, and the `imag` function is used to extract the imaginary part of each eigenvalue.
- The `xlabel`, `ylabel`, and `title` functions are used to add labels and a title to the plot.

PLOTS OBTAINED FOR DIFFERENT VALUES OF D


D = 0
Imag(λ)
Real(λ)
20
15
10
5
0
-5
-10
-15
-20
-15
-10
-5
0
5
10
15


D = 1
Imag(λ)
Real(λ)
25
20
15
10
5
0
-5
-10
-15
-20
-25
-25
-20
-15
-10
-5
0
5
10
15
20
25


D = 5
25
20
15
10
5
0
-5
-10
-15
-20
-25
Imag(λ)
-30
-25
-20
-15
-10
-5
0
5
10
15
20
Real(λ)


D = 10
25
20
15
10
5
0
-5
-10
-15
-20
-25
Imag(λ)
-35
-30
-25
-20
-15
-10
-5
0
5
10
15
Real(λ)

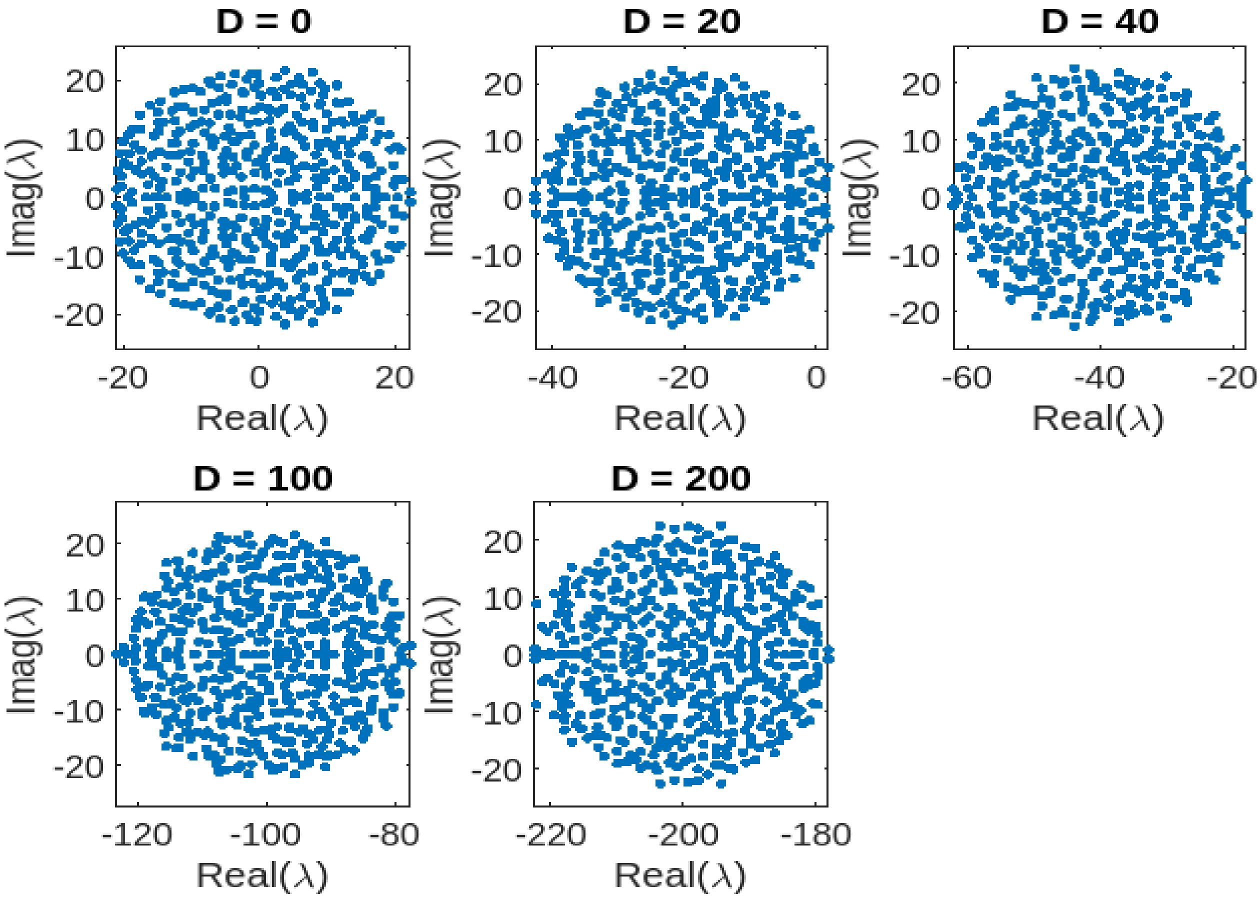
What shape do you see in the plot?

Distribution of eigenvalues in the complex plane is typically dense, and often appears as a cloud or a fog

Explain the effect of D on the plot.

For the matrix M generated by the code, the shape of the spectral density will depend on the value of D.as :

- For a small value of D, the eigenvalues of the matrix would be distributed randomly in the complex plane, with no particular clustering.
- For a larger value of D, spread of eigen values in the complex plane would decrease minutely.
- So, as the value of D increases, the shape and the spread almost remain same, but the location of the graph wrt origin shifts, graph tends to move away from origin as evident below.



However, the exact effect of changing the value of D on the distribution of eigenvalues can vary depending on the specific matrix and its properties.

What will happen if the matrix is real and symmetric?

If the matrix M is real and symmetric, the shape of the plot of the eigenvalues will be a line along the real axis. This is because all the eigenvalues will be real and will not have any imaginary component.

What will happen if the random elements in the matrix are correlated? E.g if Mij>0 then Mji<0.

If the random elements in the matrix are correlated, such as if Mij > 0 then Mji < 0, the eigenvalues of the matrix may be clustered in a certain region in the complex plane, as opposed to being distributed randomly as in the uncorrelated case. This clustering of eigenvalues would be more pronounced for larger values of D.

Code would now include following portion if Mij=-Mij

```
M = randn(N, N);
for i = 1:N
    for j = i+1:N
        if M(i,j) > 0
            M(j,i) = -M(i,j);
        end
    end
end
M = M - diag(D(d)*ones(1,N));
```

Along with observation : In this case, the plot of the eigenvalues in the complex plane would be a set of points along the imaginary axis, with no real component.