SCIENCE 2

ASSIGNMENT-1

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Question 1: find the coefficient of polynomial $p(t) = a_1t^2 + a_2t + a_3 = y$

Set of 5 data points for the same are given below.

t	-1.0	-0.5	0.0	0.5	1.0
y	1	0.5	0.0	0.5	2.0

so we have, following equations

$$a_3 + a_2(-1) + a_1(-1)^2 = 1$$
$$a_3 + a_2(-0.5) + a_1(-0.5)^2 = 0.5$$
$$a_3 + a_2(0) + a_1(0)^2 = 0$$
$$a_3 + a_2(0.5) + a_1(0.5)^2 = 0.5$$
$$a_3 + a_2(1) + a_1(1)^2 = 2$$

can be seen as,

$$Ax = b$$

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \\ 1 & t_4 & t_4^2 \\ 1 & t_5 & t_5^2 \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}}_{b}$$

solution can be found using

$A^T A X = A^T B$.

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -0.5 & 0 & 0.5 & 1 \\ 1 & 0.25 & 0 & 0.25 & 1 \end{bmatrix}}_{A^T} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -0.5 & 0.25 \\ 1 & 0 & 0 \\ 1 & 0.5 & 0.25 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^T \begin{bmatrix} 1 \\ 0.5 \\ 0 \\ 0.5 \\ 2 \end{bmatrix}$$

A_dash X = B_dash

$$\begin{bmatrix} 5 & 0 & 2.5 \\ 0 & 2.5 & 0 \\ 2.5 & 0 & 2.125 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 3.25 \end{bmatrix}$$

Hence,

$5x_1 + 2.5x_3 = 4$ — (I)

$2.5x_2 = 1$ — (II)

$2.5x_1 + 2.125x_3 = 3.25$ — (III)

} solving

$x_1 = 0.0857$

$x_2 = 0.400$

$x_3 = 1.4286$