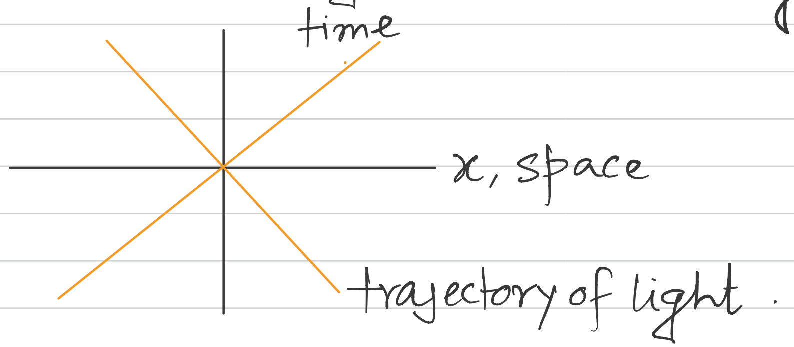
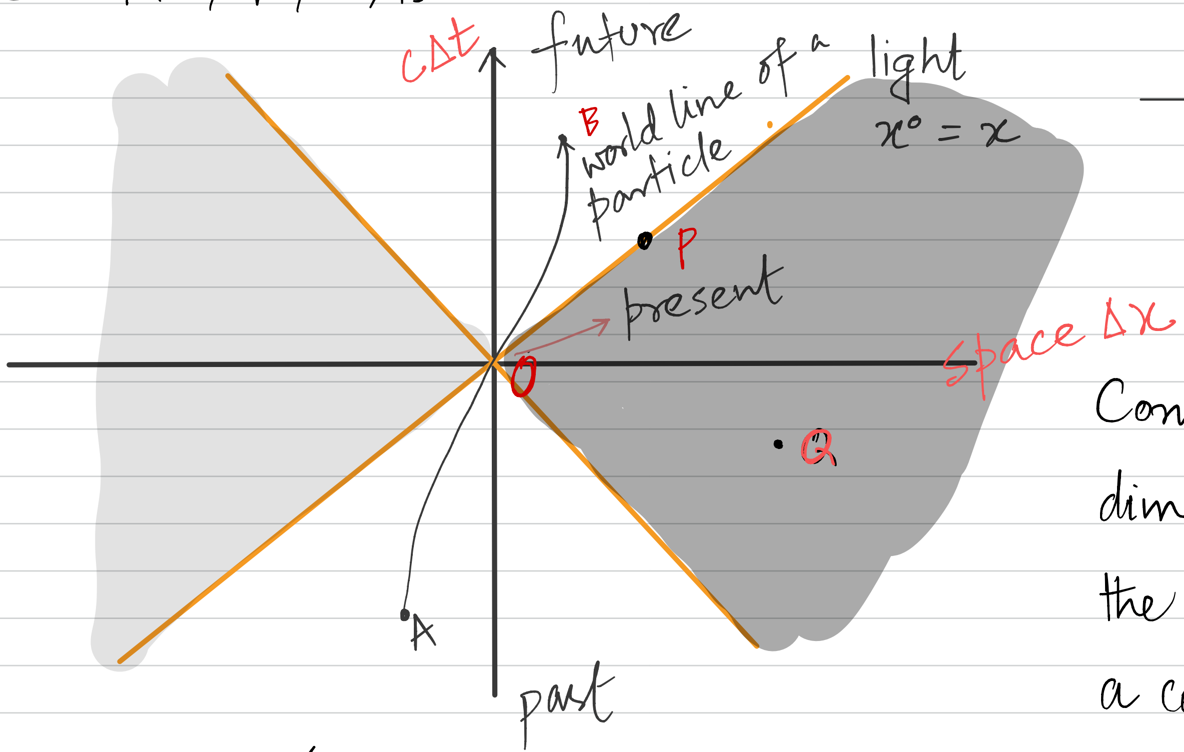
Light Cone: Light ray follows the trajectory $(\Delta\tau)^2 = 0$ Class 4.

$$\Rightarrow (c\Delta t)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

In the spacetime diagram trajectories of light are straight lines making 45° angle with X & Y axis.





Considering two space dimension & one time axis, the paths of the light rays form a cone called the light cone.

Event A in the 'past' light cone. Event B in the 'future' light cone.
Event Q is 'outside light cone'. Event P is 'on the light cone'.

Relativistic Dynamics:

Relativistic Momentum: In Newtonian mechanics the linear momentum is defined as $\vec{p} = m_0 \vec{v}$, where m_0 is the mass of a particle when it is at rest. In relativity the linear momentum is given by

$$\vec{p} = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \left(\frac{m_0}{\sqrt{1 - v^2/c^2}} \right) v = m v$$

In the last line we have defined $m \equiv \frac{m_0}{\sqrt{1 - v^2/c^2}}$, called the relativistic mass to distinguish from rest mass.

* Relativistic 2nd law of motion: Relativistic 2nd law can be written as

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt} \left(\frac{m_0}{\sqrt{1 - v^2/c^2}} v \right)$$

* Mass & Energy Relation: Einstein's mass-energy relation is the most famous and one of the most powerful equation in all of science. To derive it

We note that in Newtonian mechanics work done on a object by a force $\vec{F}$ is , $dW = \vec{F}\cdot d\vec{r}$ where $d\vec{r}$ is the displacement.

If there is no other force acts then this work is converted to Kinetic energy

$$d(KE) = \vec{F}\cdot d\vec{r}$$

Total Kinetic Energy to move the object from 0 to distance r is

$$KE = \int_0^r \vec{F}\cdot d\vec{r}$$

The relativistic version of this KE is

$$KE = \int_0^r \frac{d}{dt}\left(\frac{m_0\vec{v}}{\sqrt{1-v^2/c^2}}\right)\cdot d\vec{r} = \int_0^r d\left(\frac{m_0\vec{v}}{\sqrt{1-v^2/c^2}}\right)\cdot\frac{d\vec{r}}{dt} = \int_0^u d\left(\frac{m_0\vec{v}}{\sqrt{1-v^2/c^2}}\right)\cdot\vec{v}$$

$$= \int_0^u v\, d\left(\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right)$$ { Since this is scalar we remove vector sign.

We Know $\int_a^b x\,dy = xy\Big|_a^b - \int_a^b y\,dx$; take $x = v$

$$y = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$$

Hence $KE = \dfrac{m_0 u^2}{\sqrt{1-u^2/c^2}} - m\displaystyle\int_0^u \dfrac{v\,dv}{\sqrt{1-v^2/c^2}}$

$$= \frac{m_0 u^2}{\sqrt{1-u^2/c^2}} + \left[m c^2 \sqrt{1-v^2/c^2}\right]_0^u$$

$$= \frac{m_0 u^2}{\sqrt{1-u^2/c^2}} + m_0 c^2 \sqrt{1-u^2/c^2} - m_0 c^2$$

$$= \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} - m_0 c^2 =$$

$$= m c^2 - m_0 c^2$$

$\boxed{E = mc^2 = KE + m_0 c^2}$ this is the total energy of the object

$\Rightarrow$ Total Energy E = Kinetic Energy + Rest mass energy

If the particle is at rest then $KE = 0$, so

$$\boxed{E = m_0 c^2}$$

For $u \ll c$ one can perform series expansion of $\frac{1}{\sqrt{1-u^2/c^2}}$. We get

$$KE = m_0 c^2 \left[1 + \frac{1}{2} u^2/c^2 + \cdots - 1 \right]$$

$$\approx \frac{1}{2} m_0 u^2$$

* Energy & Momentum: Total Energy $E = mc^2 = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}}$

Total momentum $p = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$

With some algebra it can be shown that

$$E^2 - p^2c^2 = (m_0 c^2)^2$$

$$\Rightarrow E^2 = (m_0 c^2)^2 + p^2 c^2$$