International Institute of Information Technology Hyderabad

Modern Complexity Theory (CS1.405)

Assignment 4
Deadline: November 18, 2023 (Saturday), 17:00 PM
Venue for Submission: CSTAR, A3-110, Vindhya Block, IIIT Hyderabad
Total Marks: 100

NOTE: It is strongly recommended that no student is allowed to copy from others.
No assignment will be taken after deadline.
Write the following while submitting ONLY HARDCOPY:

Modern Complexity Theory (CS1.405)
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Name:
Roll No.:

1. Show that $Pr_R[\mathbf{P^R} = \mathbf{NP^R}] = 0$, where the probability $Pr(\cdot)$ is taken over a random oracle $\mathbf{R} \in \{0,1\}^*$.

   [10]

2. Show that a language $\mathbf{L} \in \mathbf{ZPP}$ if and only if $\mathbf{L}$ has an average polynomial time algorithm that always gives the right answer.

   [15]

3. Show that $\mathbf{RP} \subseteq \mathbf{BPP}$.

   [10]

4. If $\mathbf{coNP} = \mathbf{NP}$, then for every $i \geq 2$, $\Sigma_i = \mathbf{NP}$.

   [10]

5. Let $\mathbf{L} = \{(\langle M \rangle, w) : M$ rejects w in less than $t(|\langle M \rangle|, w)$, time steps, $|\langle M \rangle| \leq \sqrt{\log\ t(|\langle M \rangle, w|)}$ and M's tape alphabet has size $4\}$.

   (a) $\mathbf{L} \notin \mathbf{TIME}(o(t(n)))$.
   (b) $\mathbf{L} \in \mathbf{TIME}(O(t(n) \cdot \log\ t(n)))$.

   [10 + 10 = 20]

6. Show that $\mathbf{ZPP} \subseteq \mathbf{RP} \cap \mathbf{coRP}$.

   [15]

7. Show that $\mathbf{RP}$ is in the first level of the polynomial hierarchy.

   [10]

8. Suppose we have $\mathbf{PP} = \{\mathbf{L} : x \in \mathbf{L}$ is decided by a probabilistic polynomial time algorithm$\}$. By decidable, we mean that the probability of the algorithm being wrong is $\leq 1/2$. Why is $\mathbf{PP}$ less useful than $\mathbf{BPP}$? Prove that $\mathbf{BQP} \subseteq \mathbf{PP}$.

[2 + 8 = 10]