Inverse Lorentz transformation from S' to S frame: Class 2

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \quad ; \quad t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - v^2/c^2}}$$

$$y = y' \quad ; \quad z = z'$$

* Spacetime & Event : Space & time transform as component of a single entity called the "spacetime". Any point in the spacetime is called an 'event' (x, y, z, t).

* For $v \ll c$ Lorentz transformation $\rightarrow$ Galelian transformation: DIY

* Relativistic Velocity Addition: Refer to the previous S & S' frames. Consider an moving object along the $+x$ (or $+x'$) direction. Velocity measured in S frame $V_x = \frac{dx}{dt}$, $V_y = 0$, $V_z = 0$

In S' frame

$$V_x' = \frac{dx'}{dt'}, \; V_y' = 0, \; V_z' = 0$$

From the LT relations

$$dx = \frac{dx' + v\,dt}{\sqrt{1 - v^2/c^2}} \quad ; \quad dt = \frac{dt' + \frac{v}{c^2}\,dx}{\sqrt{1 - v^2/c^2}}$$

Then

$$V_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v}{c^2}\,dx'}$$

$$= \frac{dx'/dt' + v}{1 + \frac{v}{c}\,dx'/dt'} = \frac{V_x' + v}{1 + \frac{v}{c^2}\,V_x'}$$

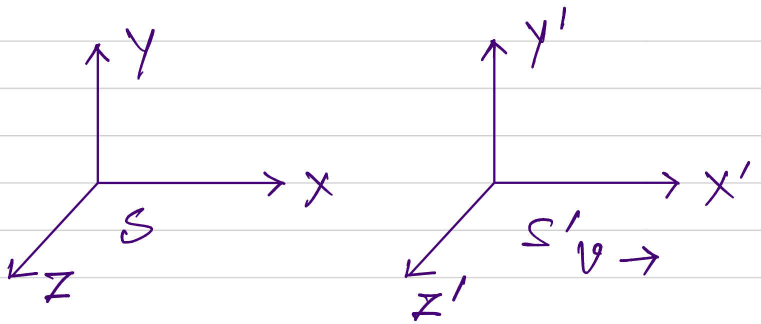
If we assume that in the S' frame $V_x' = c$, the speed of light then in the S frame

$$V_x = \frac{c + v}{1 + \frac{v}{c^2}\,c} = \frac{c + v}{1 + v/c} = c$$

Hence the speed of light is same in both S & S' frame according to the postulate of special theory of relativity.

* Nothing travels faster than light in vacuum. In some medium particles can travel faster than light - but it does not break relativity. Example: Cherenkov Radiation.

* Time Dilation: Moving clocks runs slow with respect to an observer at rest. In the following figure there are identical clocks in S & S' frames. From the POV of the S, the S' is traveling with v along the x axis of S.



The Lorentz transformation of time (going from S' to the S frame)

$$t = \frac{t' + \frac{v}{c^2} x'}{\sqrt{1 - v^2/c^2}}$$

Assume that in the S' the clock is placed at x'. Let say that two consecutive ticks happen at t_1' & t_2'. The observer on S who is looking at the clock rest on the S' frame would say that the ticks happen at t_1 & t_2 according to his/her clock.

Then $$t_1 = \frac{t_1' + \frac{v}{c^2} x'}{\sqrt{1 - v^2/c^2}} , \quad t_2 = \frac{t_2' + \frac{v}{c^2} x'}{\sqrt{1 - v^2/c^2}}$$

The observer on the S frame will conclude that the interval between two ticks of the S' clock is

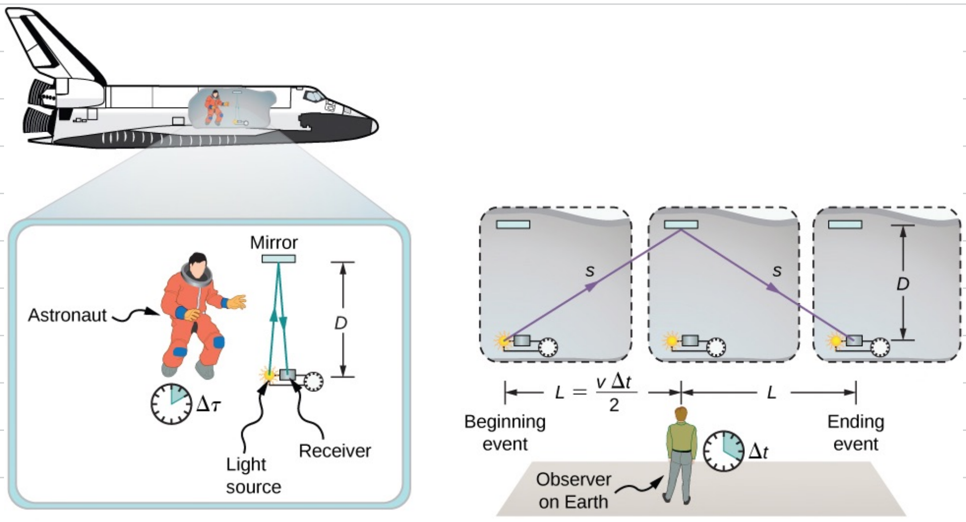
$$t_2 - t_1 = \frac{t_2' + {}^{v}/_{c^2}\, x' - t_1 - \frac{v}{c^2} x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{t_2' - t_1'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

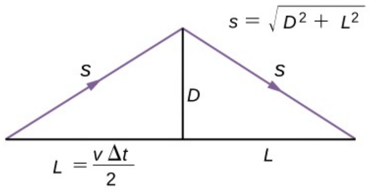
Since $\frac{1}{\sqrt{1 - v^2/c^2}} < 1$ the observer on the S frame concludes that the S' clocks is ticking slower by a factor $\frac{1}{\sqrt{1 - v^2/c^2}}$ which means that the S' clock is dilated. The observer of the S' frame will also make the same conclusion - ie the clock on the S will apper to run slower.

* Proper time: Time measured by an observer with a clock that is rest in the observer's reference frame is called proper time.

Time Dilation Experiment:



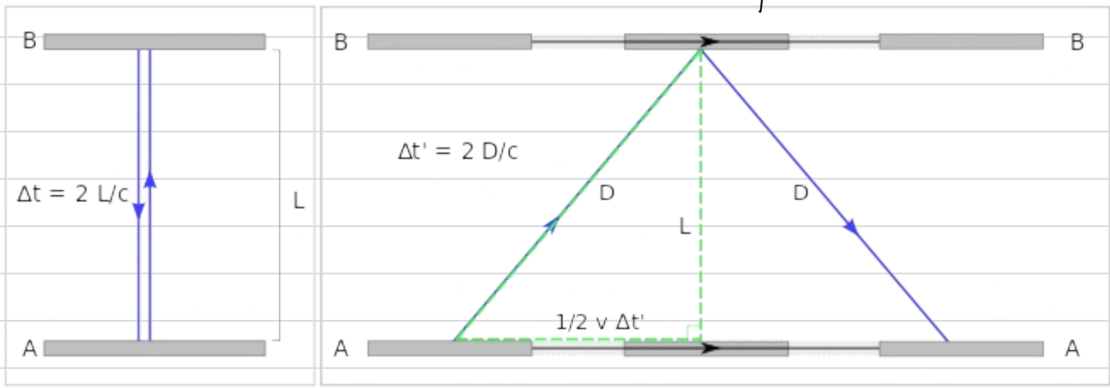
(a) (b)



(c)

Setup Kept at earth

Setup Kept at a space station but viewed from earth.



Two parallel mirrors are Kept at a distance of L. One is Kept at earth and the other one at a Space Station travelling at speed v.

In both the setups, a unit of time is the time it takes for a light ray reflected from the bottom mirror to travel to the top mirror and reflected back from it and reach the bottom mirror.

Earth Setup + observer at earth: A unit of time $\Delta t = 2\,L/c$

Space station setup + observer at earth: $\Delta t' = 2\, D/c$

$$D^2 = L^2 + \left(v \frac{\Delta t'}{2}\right)^2$$

$$\Rightarrow \left(c \frac{\Delta t'}{2}\right)^2 = L^2 + \left(v \frac{\Delta t'}{2}\right)^2$$

$$\Rightarrow (\Delta t')^2 \left[\frac{c^2}{4} - \frac{v^2}{4}\right] = L^2$$

$$\Rightarrow \Delta t' = \frac{L}{\sqrt{\frac{c^2}{4} - \frac{v^2}{4}}} = \frac{2L/c}{\sqrt{1 - v^2/c^2}} = \frac{\Delta t}{\sqrt{1 - v^2/c^2}}$$

Hence for the observer at earth, the clock of the spacestation runs slower. Of course the astronaut will say that the clock at the earth is running slow.