* Wave Nature of Particles: The PE effect, Compton effect, & the pair production show that light has dual nature - it can behave both like wave & particle.

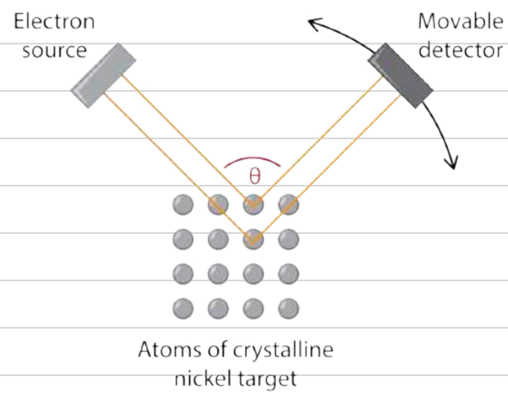
In 1924 Louis de Broglie proposed that wave-particle duality is universal - in other words, according de Broglie, particle must also exhibit wave nature.

A photon has momentum $p = \frac{h\nu}{c} = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p}$. Motivated by this de Broglie postulated that a material particle with momentum p behaves like a wave with wave length

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\gamma m v}} \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

This is called de Broglie wavelength.

* de Broglie's hypothesis was experimentally proven in the Davison-Germer experiment where it was found that electrons also show diffraction like X-Ray



Waves of What? Born Interpretation:

Water wave: what changes periodically is the height of the wave

Sound Wave : pressure changes in periodic manner

Light Wave: the electric & the magnetic field vary

What about de Broglie wave?

Max Born interpreted that the quantity whose variation makes up the "particle wave" is the wave function Ψ. The value of the wave function at any point x, y, z & at time t, is the likelyhood of finding the particle at that location and at that time.

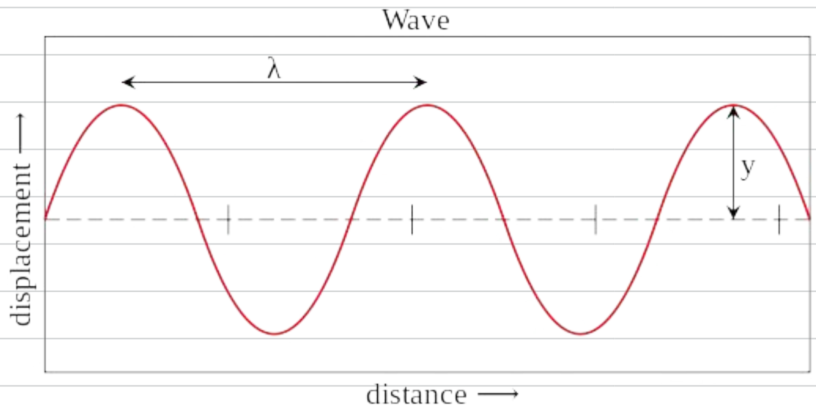
* Wave function ψ of particle wave can be positive, negative, or even be a complex number. Therefore, Ψ has no physical significance - it can not be measured in experiment.

* What is of physical significance is $|\Psi|^2$ - it is called the "probability density" of finding a particle of wave function ψ at a small region around

x, y, z and t. A large $|\psi|^2$ mean large probability of finding the particle.

* Note that even when ψ describes a particle in terms of a wave that is spread out, the particle in itself is not spread out - when experiment is performed to detect the particle, it is always found at a given location at some time t.

* Describing a Wave: Suppose we describe a particle by a wave propagating along the x-direction. If ν is the frequency and λ is the wavelength then the wave can be represented as



λ = wavelength
y = amplitude

$$\psi = A \cos\left[2\pi\left(\nu t - \frac{x}{\lambda}\right)\right]$$

The velocity of the wave is

$$v_p = \nu\lambda \quad \text{where } \lambda = \frac{h}{\gamma m v} \; ; \; \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

where v is the velocity of the particle.

To find the frequency we use

$$h\nu = \gamma m c^2$$

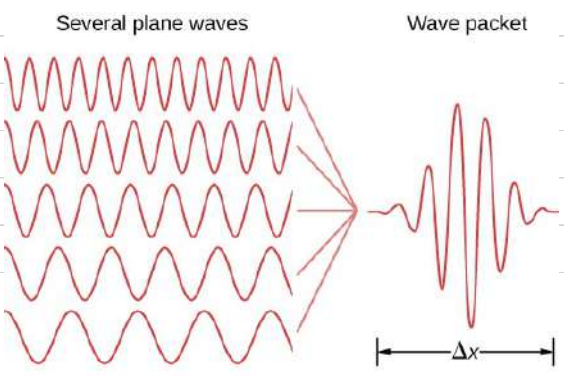
$$\Rightarrow \nu = \frac{\gamma m c^2}{h}$$

Hence the velocity of the de Broglie wave is

$$v_p = \nu\lambda = \frac{\gamma m c^2}{h} \cdot \frac{h}{\gamma m v} = \frac{c^2}{v}$$

As $v < c$ we have $v_p > c$ $\Rightarrow$ this is unacceptable.

* Particle wave or de Broglie wave can not be described by a simple wave-like equation $\psi = A \cos 2\pi(\nu t - x/\lambda)$. What we need to do is to superimpose a number of waves with slightly different wavelength/frequency - the superposition form a wave packet or group wave as shown below



In the wavepacket, the wave may travel at $v_p > c$ but the packet itself travels with $v_g = v$ { v_p and v_g are called phase and group velocity, respectively)

* The Uncertainty Principle: Describing a particle by a wavepacket ψ limits the precision in the position & momentum measurements

$|\psi|^2 \rightarrow$ probability density of finding the particle - this is maximum at the middle of the wavepacket - but not equal to 0 elsewhere.

- One can increase the accuracy by making the packet very narrow - but it would lead to increase the uncertainty in the measurement of λ, and hence in the momentum measurement.

* The reverse of the above is also true - if the wavepacket is wider then the momentum measurement is more precise - but the position measurement will be less precise

The impossibility to precisely measure the position & the momentum at same time is called Heisenberg's uncertainty principle. . Quantitatively it is given as follows; if Δx is the uncertainty in the position measurement & Δp is the uncertainty in the momentum measurement then

$$\Delta x \Delta p \geq \frac{h}{4\pi} \left(\text{or } \frac{\hbar}{2}, \ \hbar = h/2\pi = 1.054 \times 10^{-34} \, J.s \right)$$

* Another form of the uncertainty principle is in the form of energy E & time t. If E is the emitted energy of a particle in the form of electromagnetic wave and the time interval of emission is Δt then

$$\Delta E \Delta t \gtrsim \frac{h}{4\pi}$$

* As the reduced Planck constant $\hbar = 1.054 \times 10^{-34}$ J.s is very small the uncertainty principle is significant at the atomic scales

* Example 3.7 of Beiser : Atomic nucleus radius $\sim 5\times 10^{-15}$ m.

$$\Rightarrow \Delta x = 5\times 10^{-15}\ \text{m} \quad \Rightarrow \Delta p \geq \frac{\hbar}{2\Delta x} \geq \frac{1.054\times 10^{-34}\ \text{J.s}}{2\times 5\times 10^{-15}\ \text{m}}$$

$$\geq 1.1\times 10^{-20}\ \text{Kg m/s}$$

The momentum p of the electron must be at least of this order

Kinetic energy of electron $= pc \geq (1.1\times 10^{-20}\ \text{kg m/s})(3\times 10^{8}\ \text{m/s})$

$$\geq 3.3\times 10^{-12}\ \text{J}$$

$$\approx 20\ \text{MeV}$$

In radioactive decay of nucleus the energy of emitted electrons are not more than a few MeV. Hence the electrons are not coming from the nucleus.

Example 3.8 of Beiser: Hydrogen atom radius is 5.3×10^{-11} m. This can be used to calculate the minimum energy of an electron in H-atom:

Ans: $\Delta x = 5.3 \times 10^{-11}$ m

$$\Delta p \geq \frac{\hbar}{2\Delta x} \geq 9.9 \times 10^{-25} \text{ kg m/s}$$

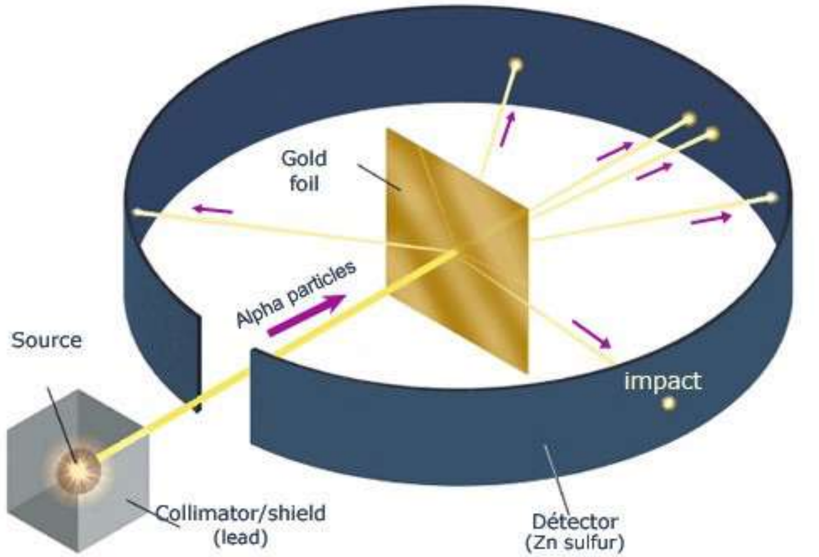
↳ this momentum is non-relativistic. Hence the kinetic energy of an electron in an H atom is

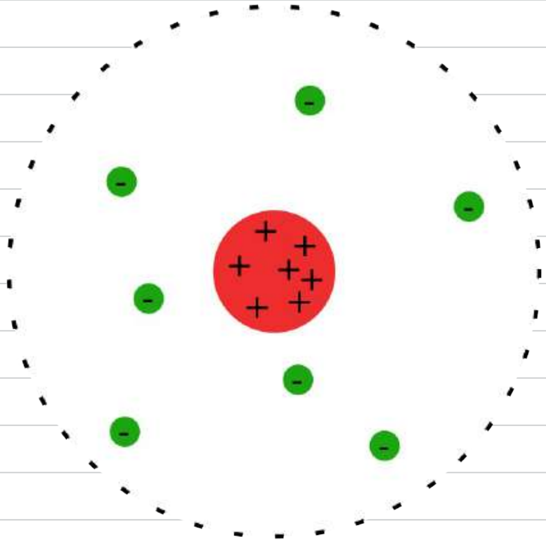
$$KE = p^2/2m \geq \frac{(9.9 \times 10^{-25} \text{ kg m/s})^2}{2 \times (9.1 \times 10^{-31} \text{ kg})} \geq 5.4 \times 10^{-19} \text{ J}$$

$$\geq 3.4 \text{ eV}.$$

In H-atom the lowest energy of H atom is 13.6 eV.

* Rutherford-Geiger-Marsden Expt:





Rutherford's Model of Atom

↓

the electrons revolve the nucleus in circular orbits just like the planets around the sun.

* Failure of Rutherford atom model

1. Fails to explain stability of atoms
2. Fails to explain atomic spectra.

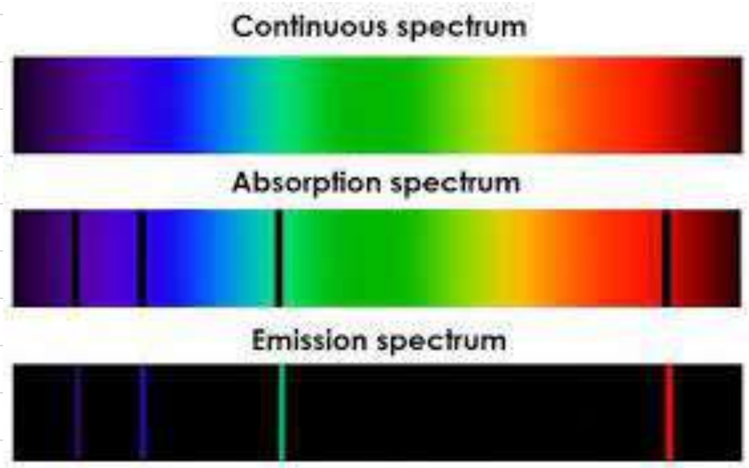
Atomic Spectra: Atoms and molecules in rarefied gases when suitably excited, emit radiation of certain wavelengths that are characteristics to the atoms and molecules

* Emission Spectra: line spectra when excited

* Absorbtion Spectra: White light passing through a cold & dilute gas found to absorb only certain frequency of light - it gives some black lines in a bright background.

* Studying a spectra can tell what kind of element is present in the gas -

* Astronomers studying spectrum of stars determine the elements present on the star & also the atmosphere - like the pressure & density



Spectral Series: 100 years before QM, JJ Balmer found out that in hydrogen wavelengths of spectral lines can be written in terms of a series.

Balmer Series $\quad \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

$n = 3 \Rightarrow H_\alpha$ line of $\lambda = 656.3\,nm$

$n = 4 \Rightarrow H_\beta$ line of $\lambda = 486.3\,nm$

and so on

the contineous spectra for low wavelengths correspond to $n \to \infty$

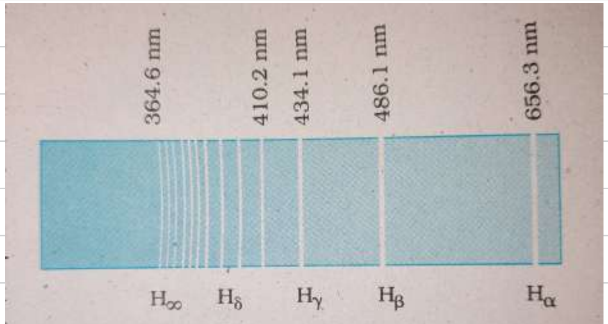


FIGURE 12.6 Balmer series in the emission spectrum of hydrogen.

R is called the Rydberg constant and Balmer determined that $R = 1.097 \times 10^7\ m^{-1} = 0.01097\ nm^{-1}$

* Balmer series is in the visible spetra. Later many other such spectral series were found

Lyman Series (UV): $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$; $n = 2, 3, 4$

Paschen (IR): $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4, 5, 6$

In all these series the constant R was the same - it means that spectral lines are due to some underlying structure of the atoms.

* Bohr Atom Model: Electrons move in circular orbits with velocity v

Centripetal force $F_c = \frac{mv^2}{r}$. The electron is hold by electric force

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

these two are equal $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$

$$\Rightarrow v = \frac{e}{\sqrt{4\pi\epsilon_0 r}}$$

this is a non-relativistic velocity, so $v^2/c^2 \ll 1$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \approx 1$$

The de Broglie wavelength $\lambda = \frac{h}{mv}$

$$= \frac{h}{e}\sqrt{\frac{4\pi\epsilon_0 r}{m}}$$

this is called the orbital wavelength.

Total energy of electron $E = KE + PE$

$$= \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

$$= \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{4\pi\epsilon_0 r}$$

Expt: indicates that $E = 13.6\,eV$ is required to remove an electron in H atom from its orbit. Now $13.6\,eV = 2.2 \times 10^{-18}\,J$

$$\Rightarrow r = -\frac{e^2}{8\pi\epsilon_0 E} = -\frac{(1.6 \times 10^{-19}\,C)^2}{8\pi\,(8.85 \times 10^{-12}\,F/m)\,(-2.2 \times 10^{-18}\,J)}$$

$$= 5.3 \times 10^{-11}\,m$$

$$\boxed{\Rightarrow \text{this also leads to } v = 2.2 \times 10^6\,m/s \ll c}$$

Hence the atomic circumference $= 2\pi r = 33 \times 10^{-11}\,m$

Using the value of r

$$\lambda = \frac{h}{e}\sqrt{\frac{4\pi\epsilon_0 r}{m}}$$

$$= \frac{6.63\times10^{-34}\,J.S}{1.6\times10^{-19}\,C}\sqrt{\frac{(4\pi)(8.85\times10^{-12}\,C^2/N.m^2)(5.3\times10^{-11}\,m)}{9.1\times10^{-31}\,kg}}$$

$$= 33\times10^{-11}\,m$$

The circumference of an atomic orbit (in H atom) is such that the de Broglie wave of an electron is joined end to end. Otherwise the wave will create destructive interference.