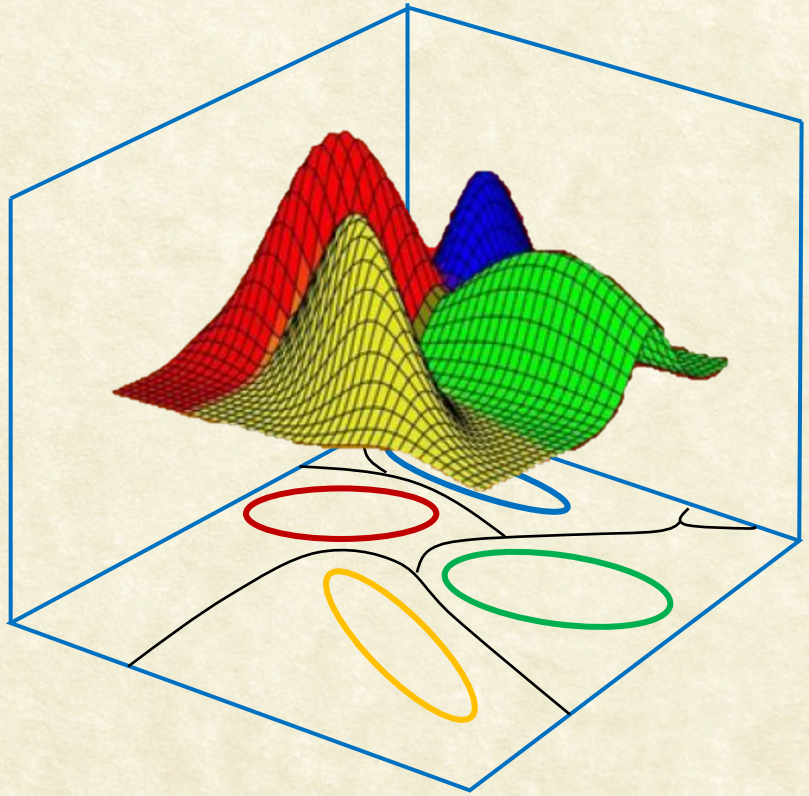
CS7.404: Digital Image Processing

Monsoon 2023: Frequency Domain

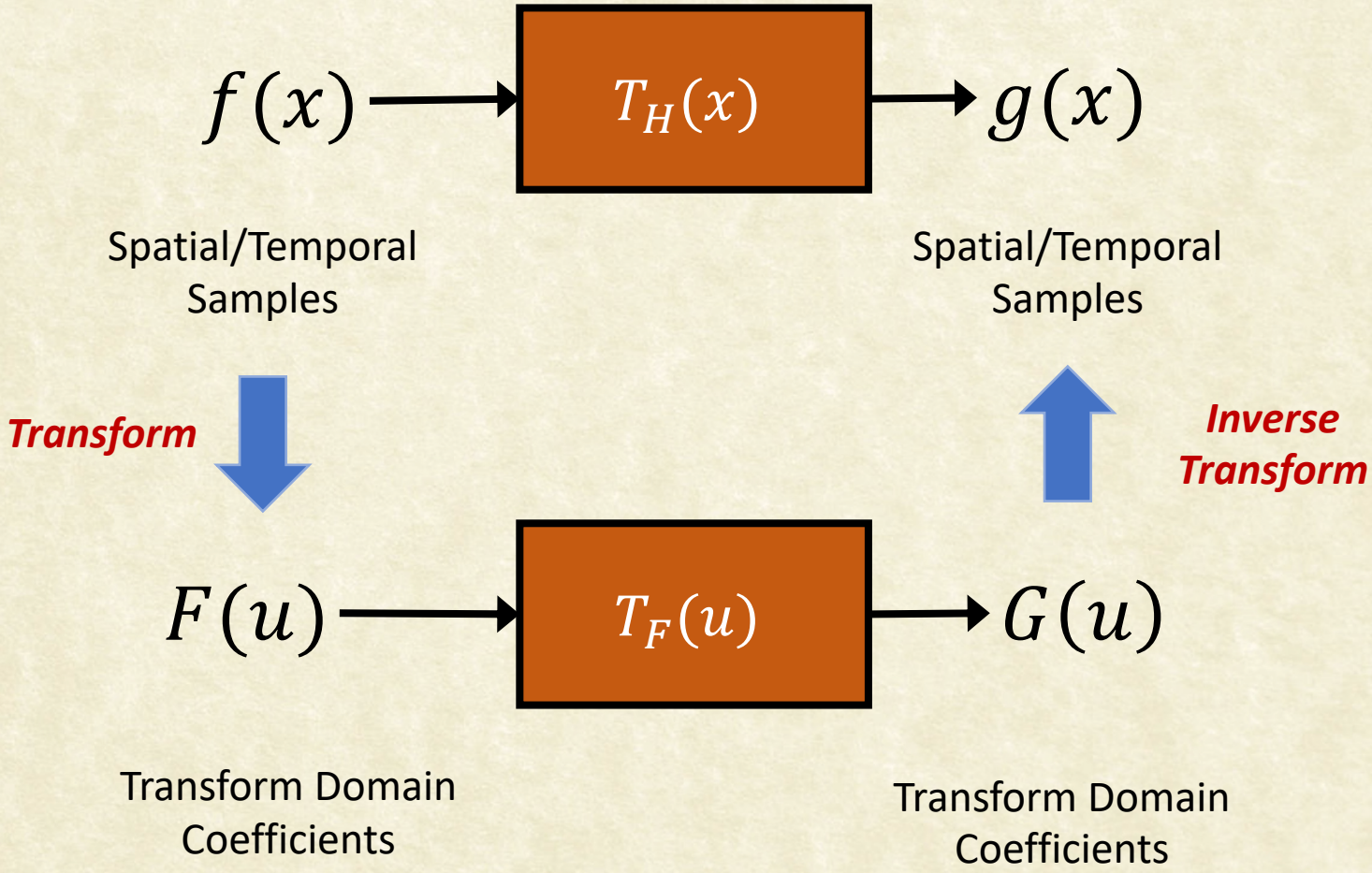


Anoop M. Namboodiri

Biometrics and Secure ID Lab, CVIT,

IIIT Hyderabad

The Systems View



Spatial vs. Transform Domain Processing

Bandhani / Bandhej

Tie Dye

Spatial vs. Transform Domain Processing

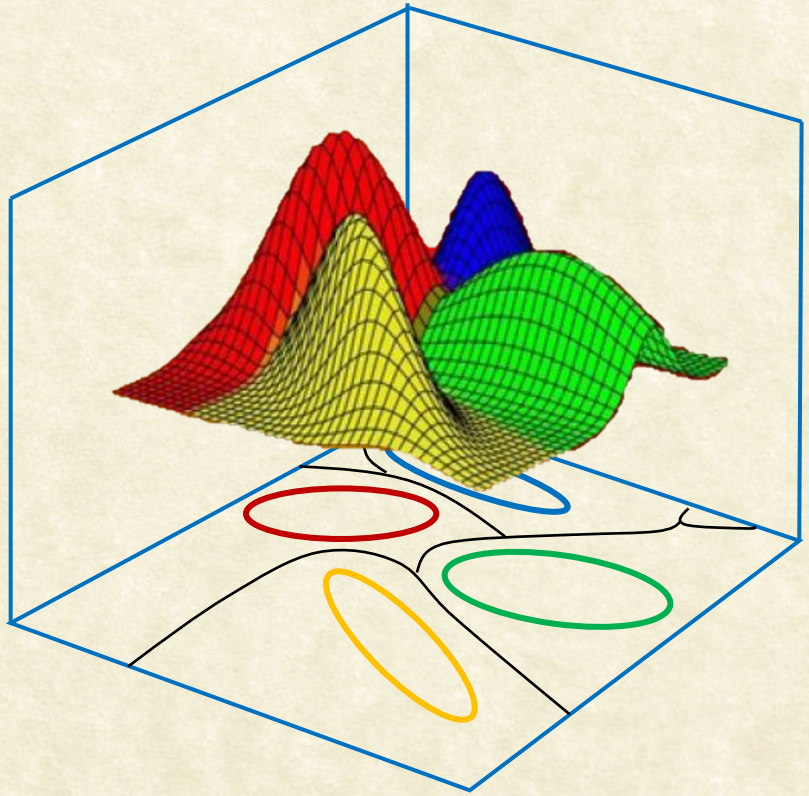
Transform (Tie)

Process (Dye)

Inverse Transform (Untie)

CS7.404: Digital Image Processing

Monsoon 2023: Frequency Domain Basics



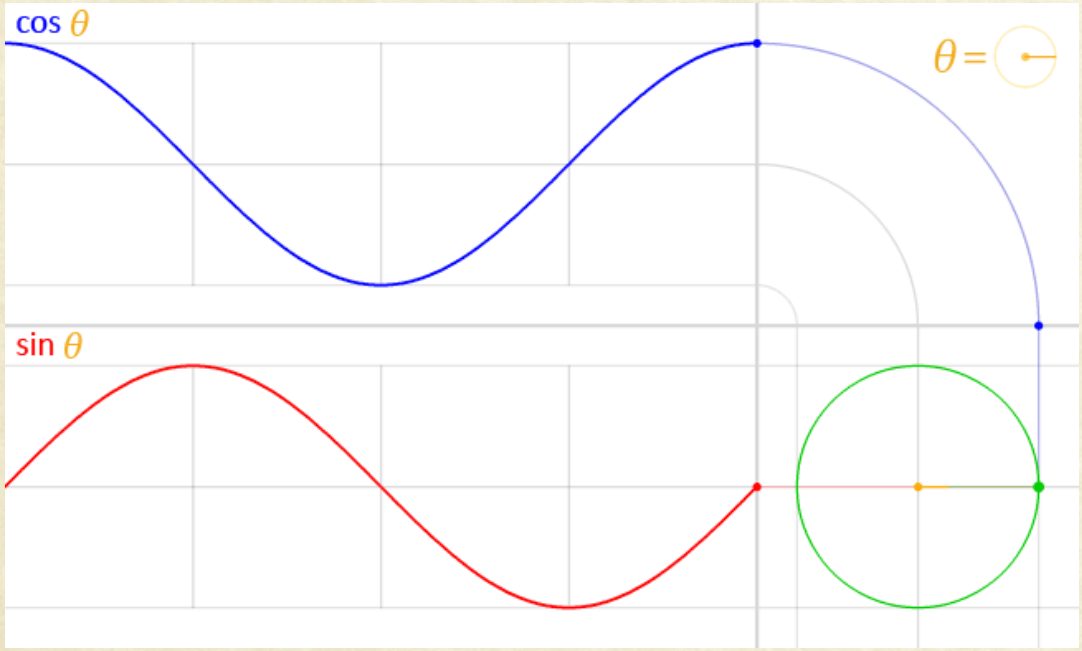
Anoop M. Namboodiri

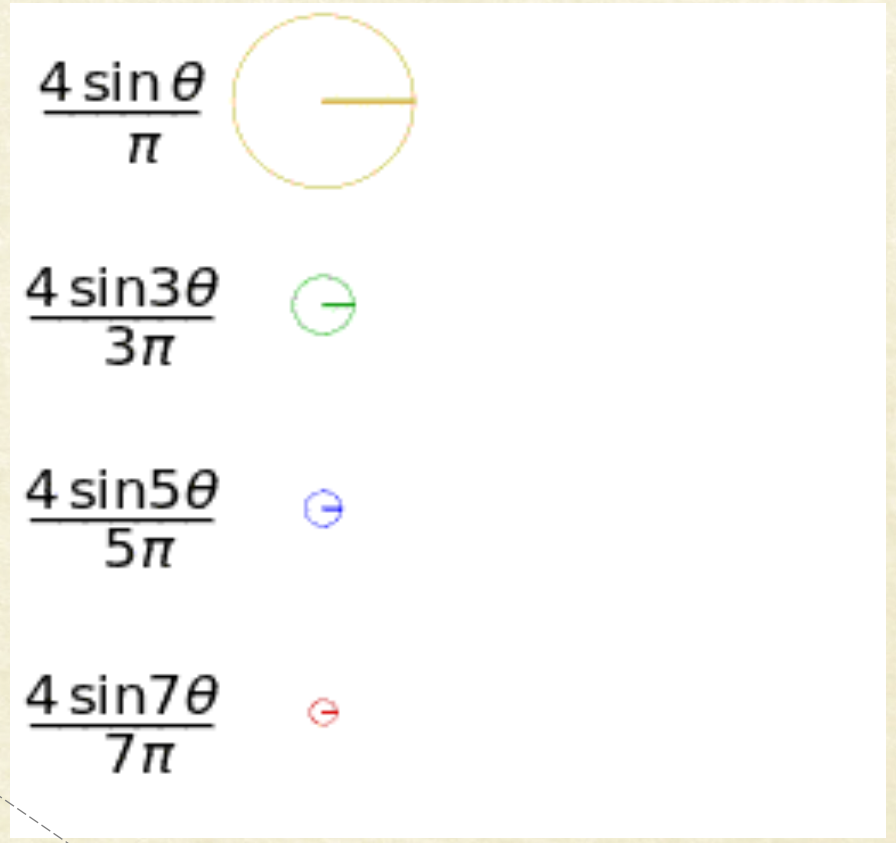
Biometrics and Secure ID Lab, CVIT,

IIIT Hyderabad

Periodic Signals

- Periodic → **Frequency** of occurrence
  - Repetitions/<Unit> (cycles/sec = Hz)





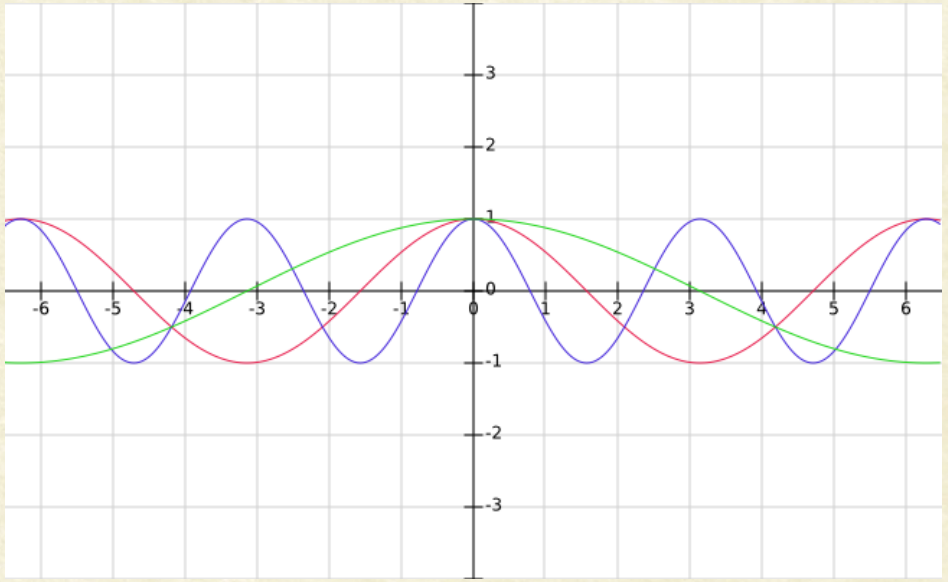
$$x(t) = A\cos(\omega t) = A\cos(2\pi f t) = A\cos\left(\frac{2\pi}{T}t\right)$$

Angular frequency

Fundamental Period

Simple periodic signals

- $x(t) = A\cos(t)$
- $x(t) = A\cos(2t)$
- $x(t) = A\cos(t/2)$

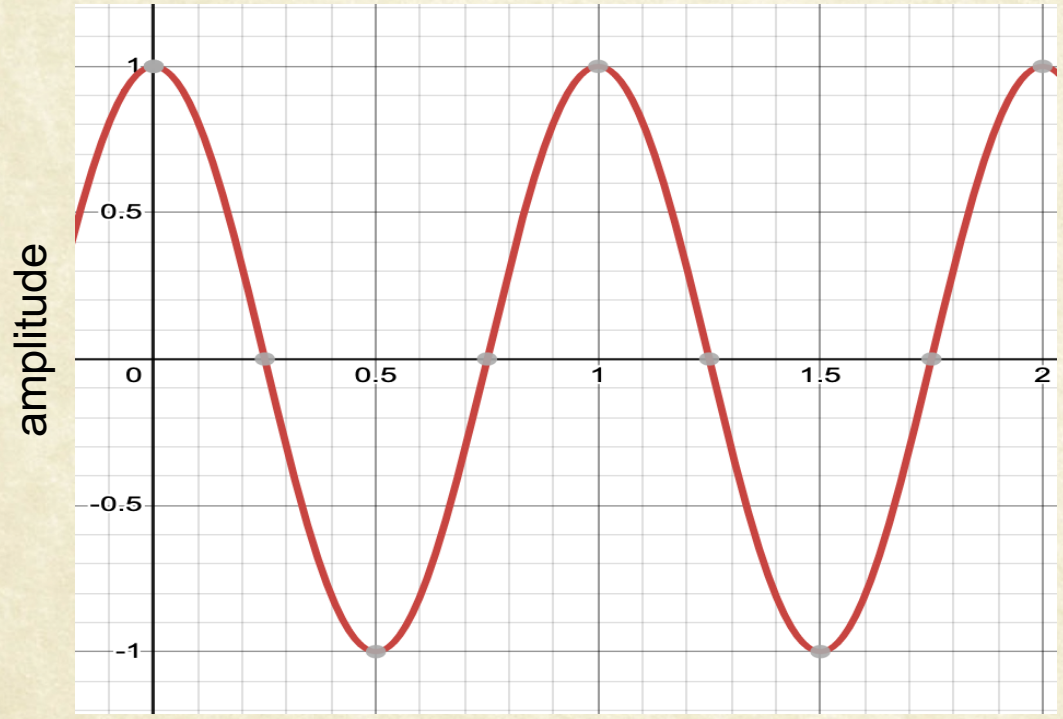


- $x(t) = A\cos(\omega t) = A\,cos(2\pi f t) = A\,cos(\frac{2\pi}{T}t)$

Angular frequency

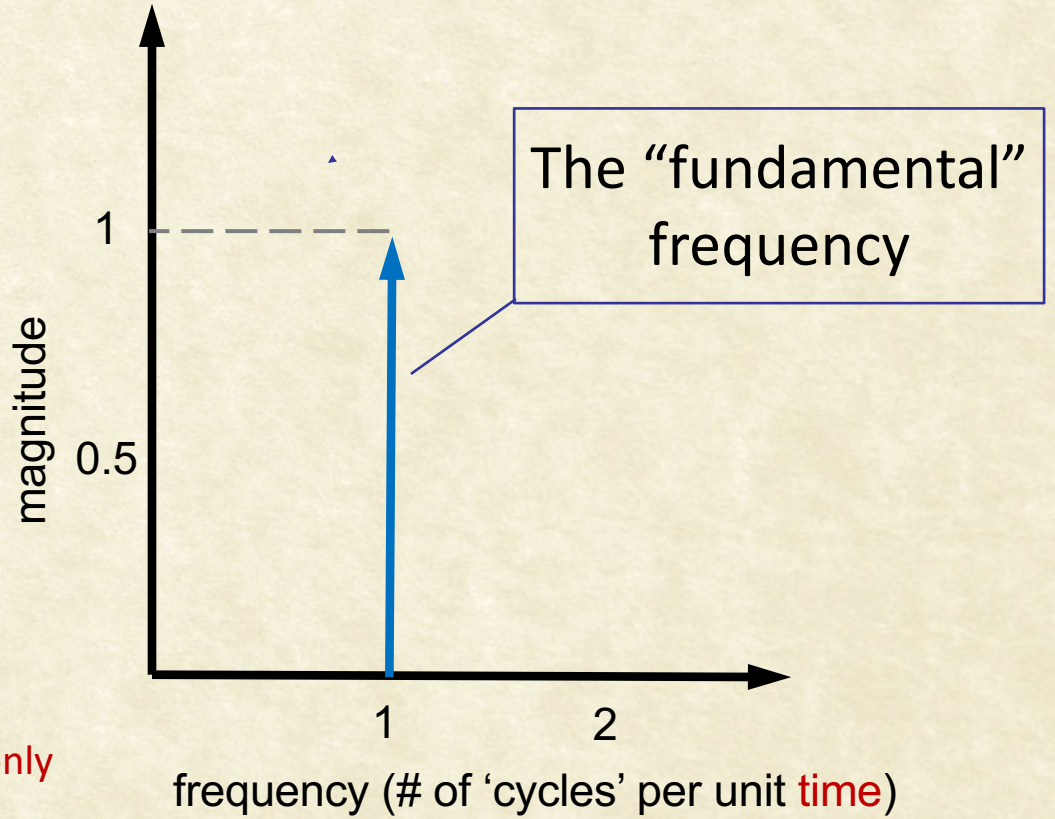
Signal and Frequency Domains

$$y = A\,cos(2\pi f t) = cos(2\pi t)$$



NOTE: t=0 is time from where signal is considered

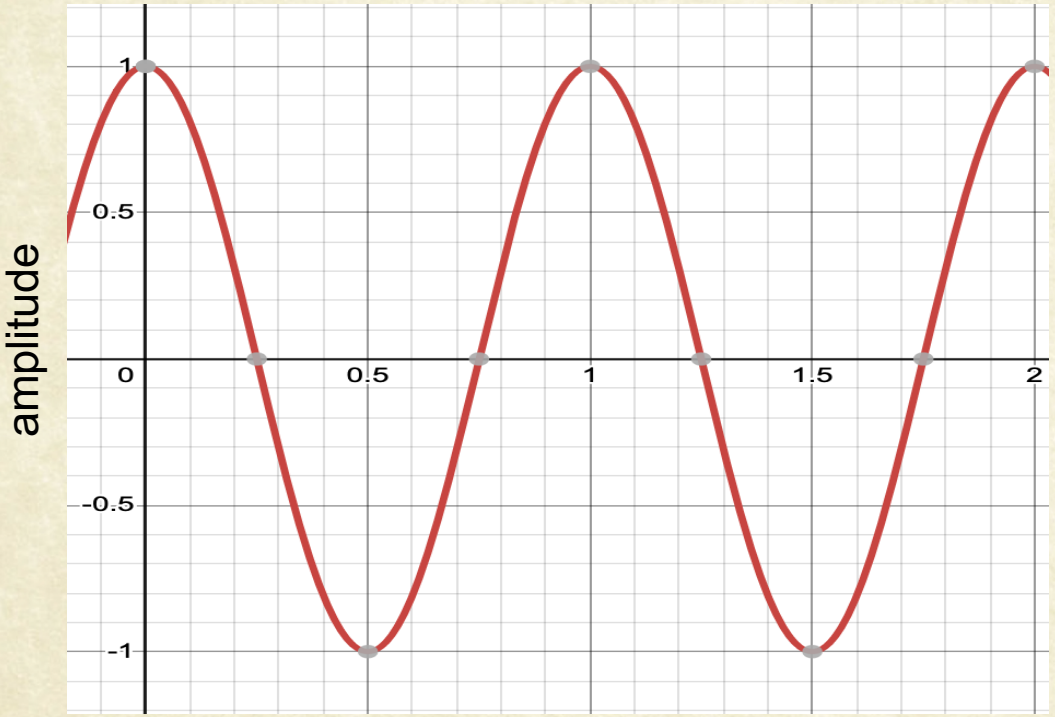
NOTE: t is bounded only for physical signals



Time domain ⟷ **Frequency domain**

Signal and Frequency Domains

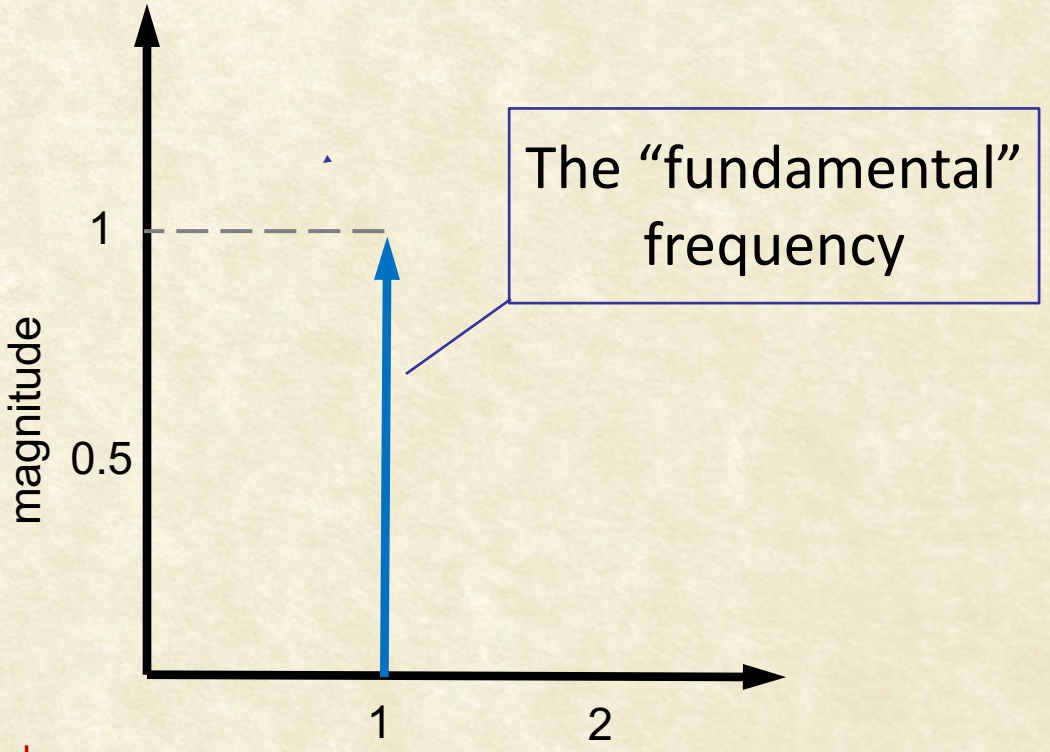
$$y = A\,cos(2\pi f s) = cos(2\pi s)$$



NOTE: s=0 is time from where signal is considered

Spatial distance (s)

NOTE: s is bounded only for physical signals



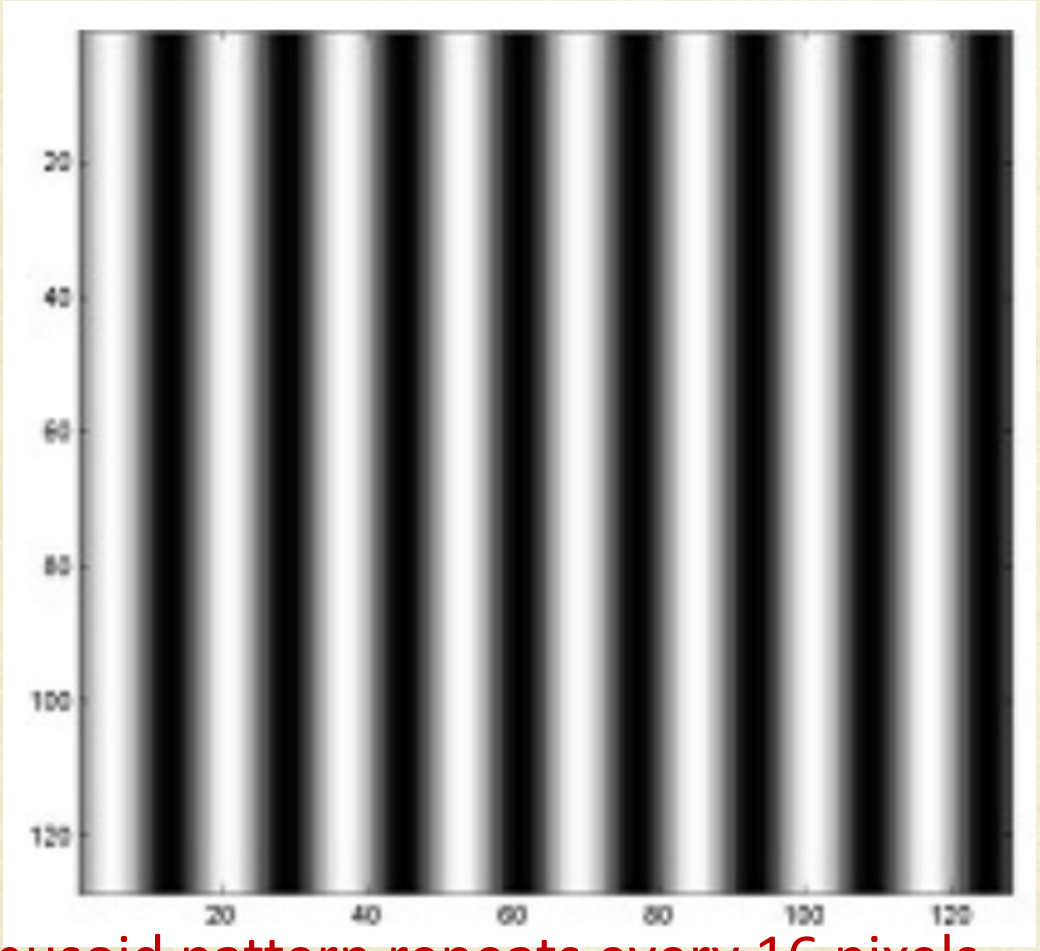
frequency (# of ‘cycles’ per unit distance)

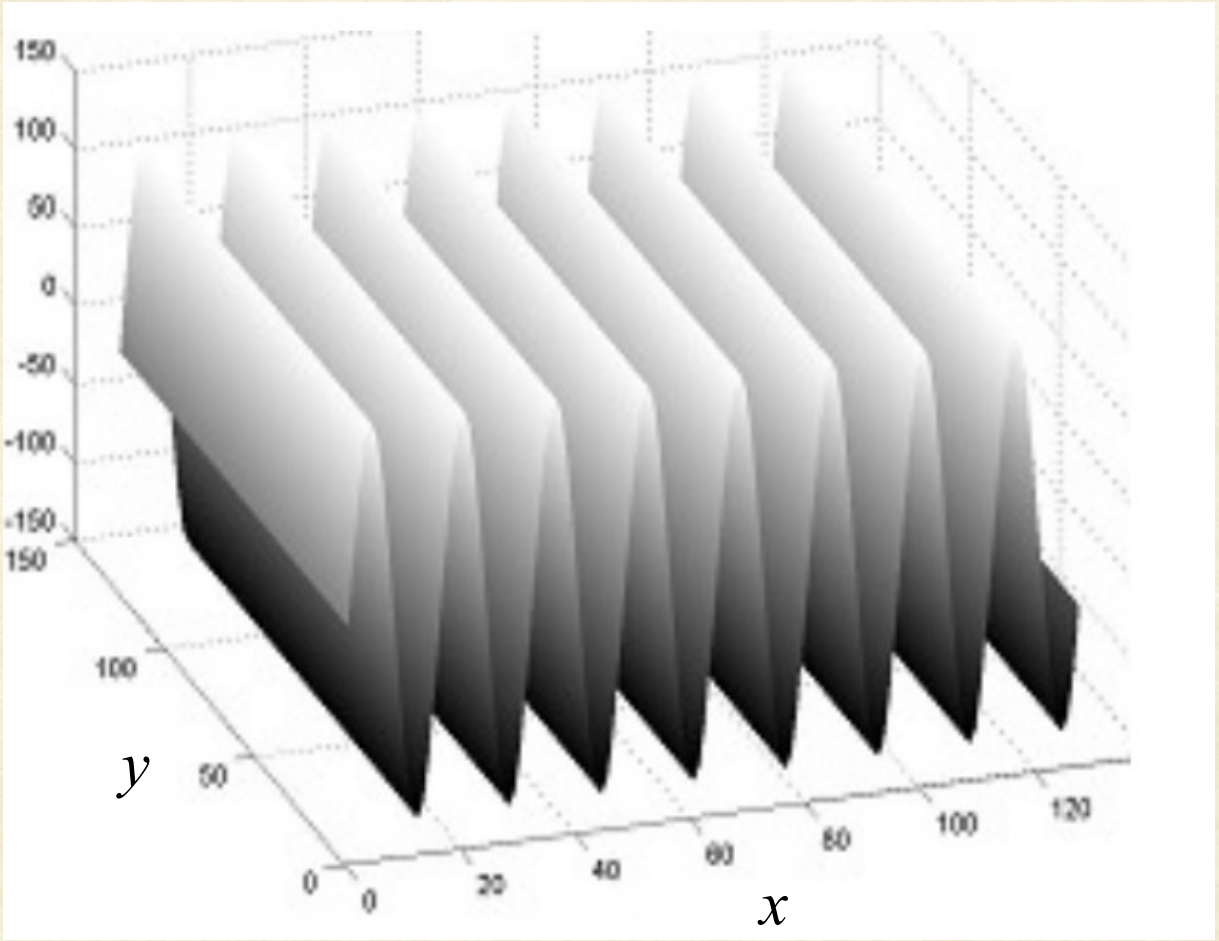
Spatial domain ⟷ **Frequency domain**

Periodic Signals in Images

$$I(x, y) = 128\ sin(2\pi x/16)$$

128 x 128 grayscale image



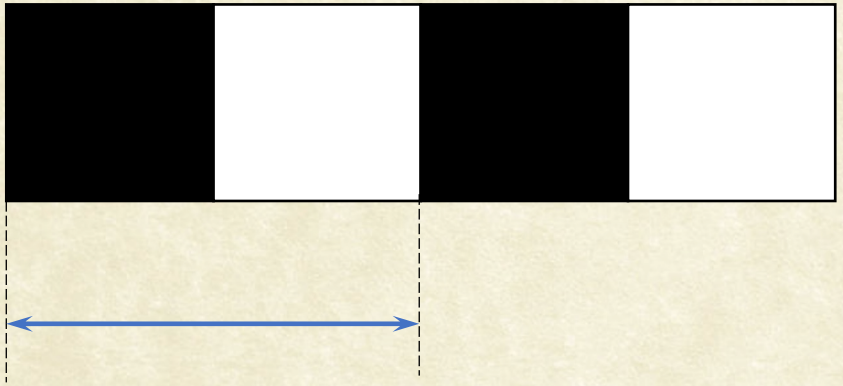


Sinusoid pattern repeats every 16 pixels

f = 1/16 cycles/pixel

Periodic Signals in Images

- Spatial period = Minimal # of pixels between two identical patterns in a “periodic” image

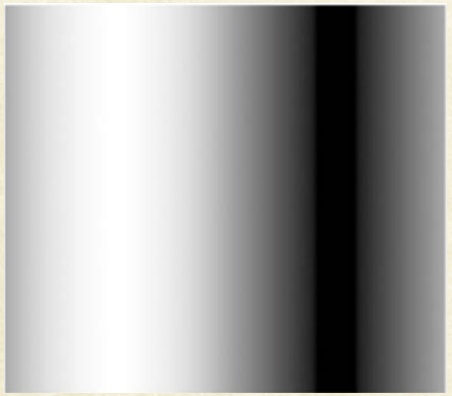


Minimal Period:
2 pixels

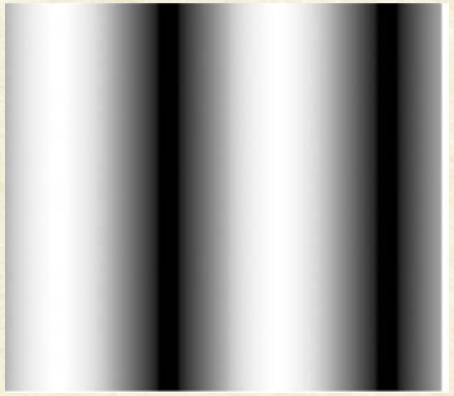
$$\nu_{max} = \frac{1}{minimal\ period} = \frac{1}{2}$$

Extending sinusoids to 2D

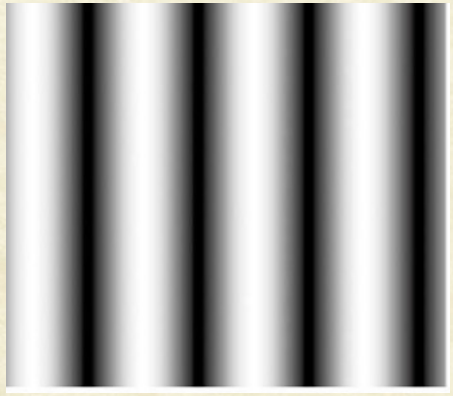
$$s(x, y) = \sin[2\pi(u_0 x + v_0 y)]$$



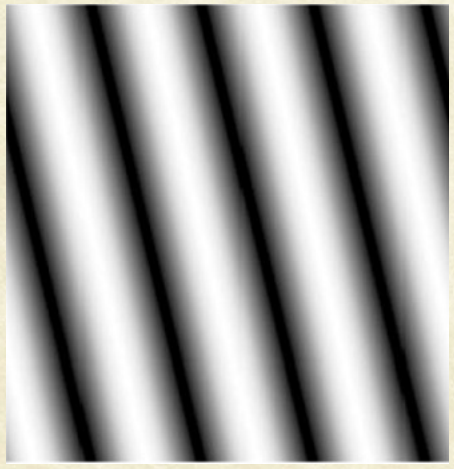
$u_0 = 1,\ v_0 = 0$



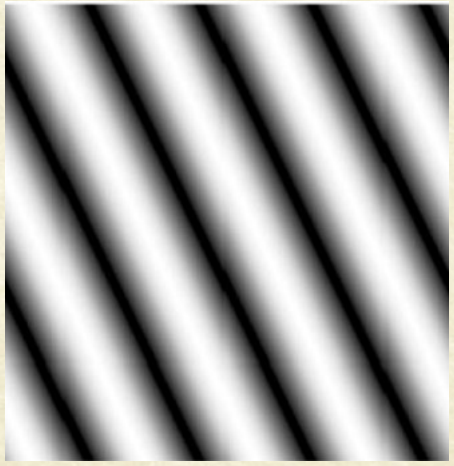
$u_0 = 2,\ v_0 = 0$



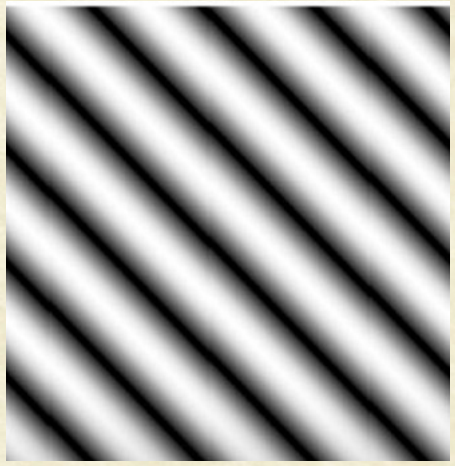
$u_0 = 4,\ v_0 = 0$



$u_0 = 4,\ v_0 = 1$

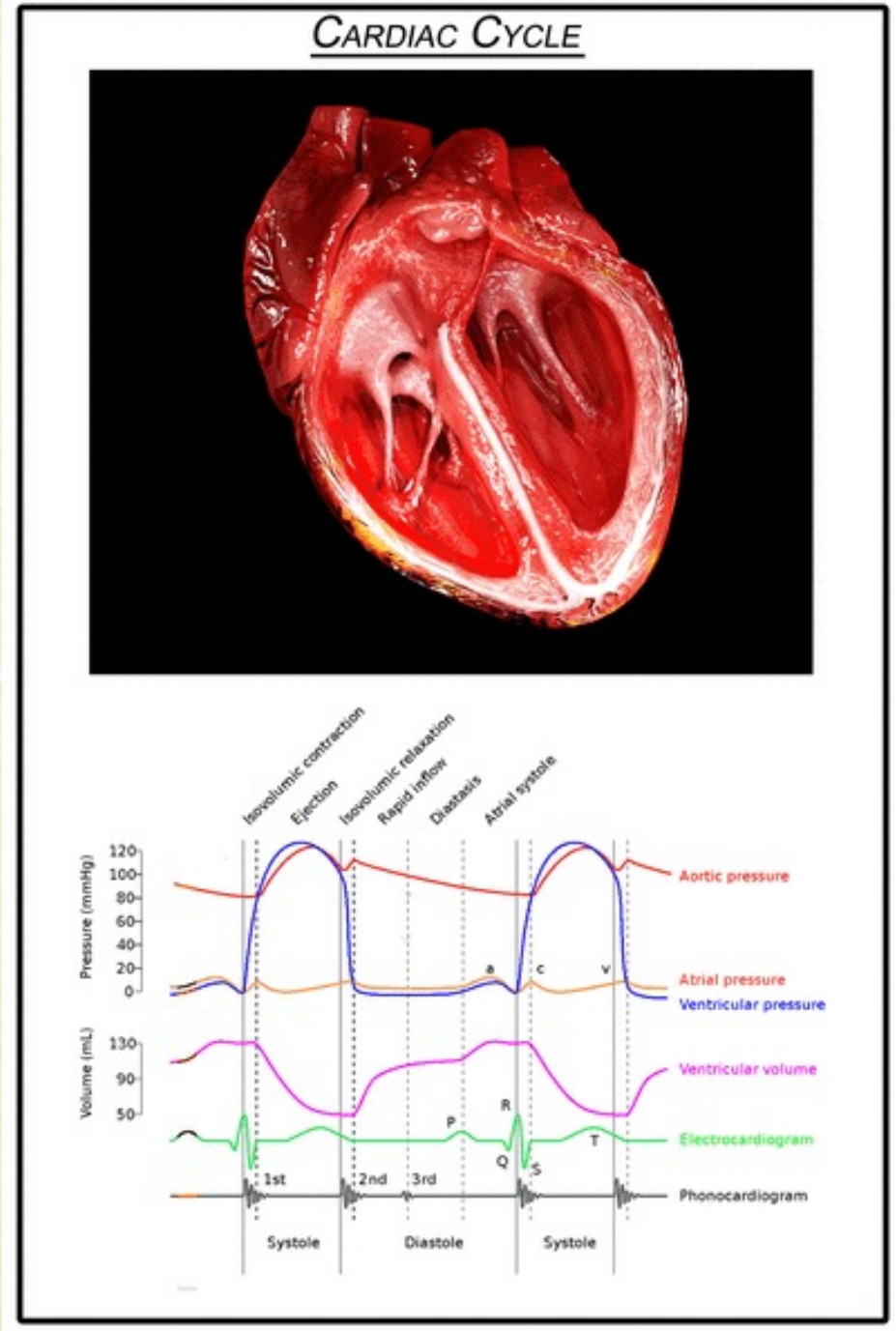


$u_0 = 4,\ v_0 = 2$



$u_0 = 4,\ v_0 = 4$

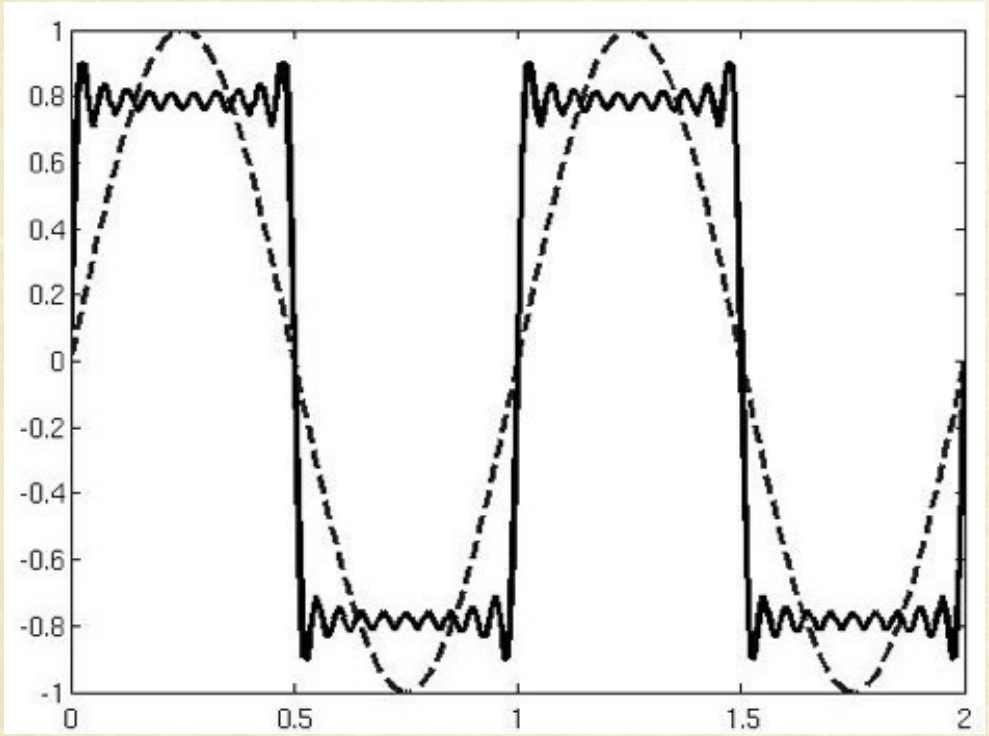
Many natural phenomena (signals) are periodic
but not necessarily sinusoidal

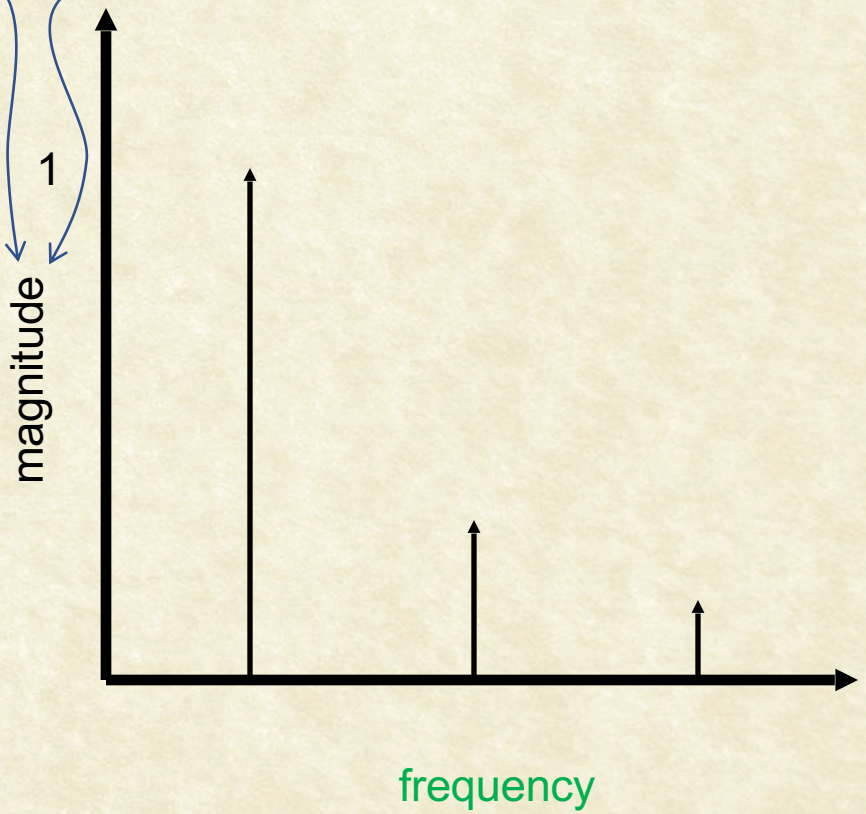


https://commons.wikimedia.org/wiki/File:Cardiac-Cycle-Animated.gif

Fourier Series

$$y(t) = \sum_n a_n \sin(nf \times 2\pi t) + \sum_n b_n \cos(nf \times 2\pi t)$$

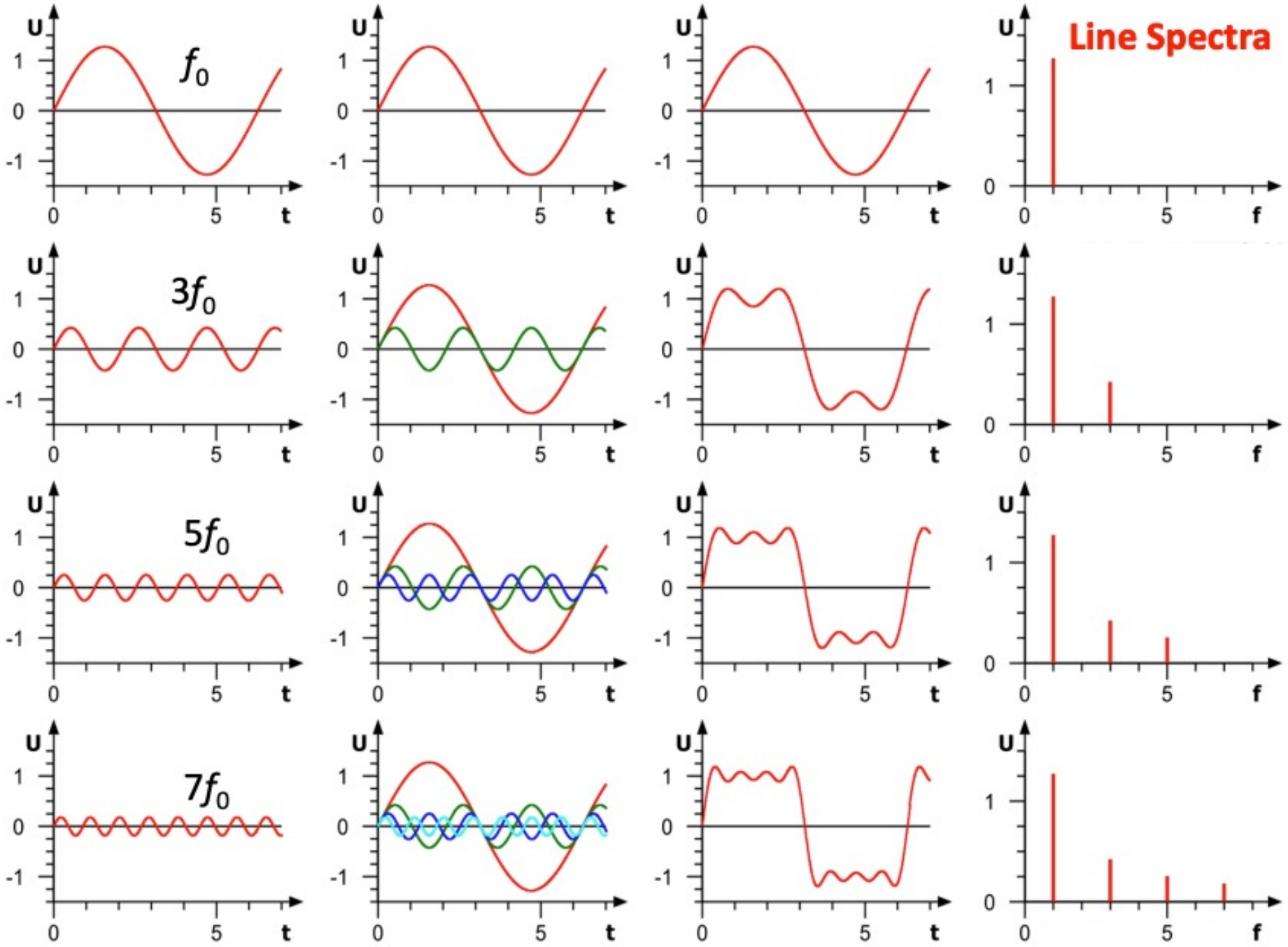




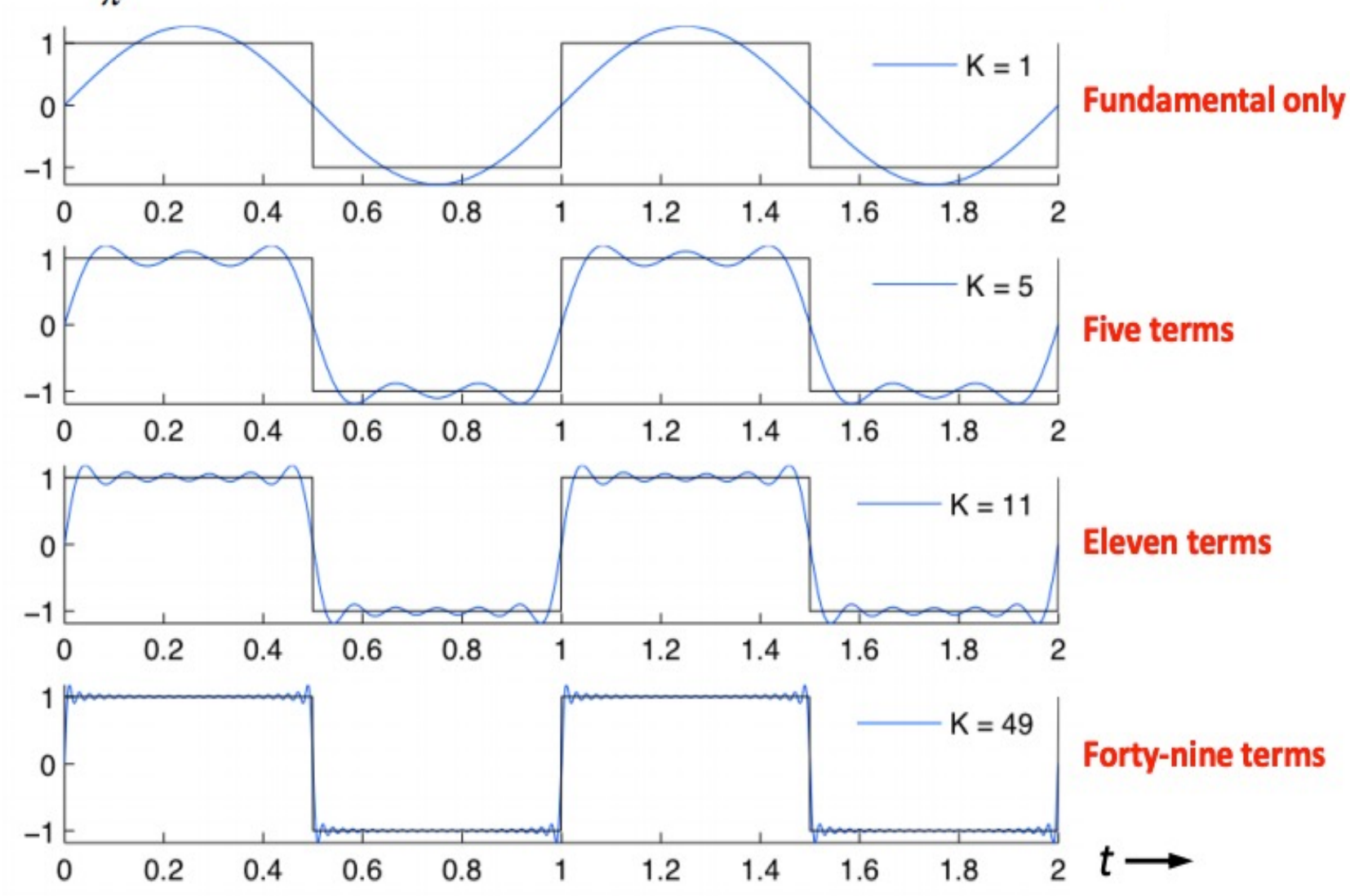
Signal domain ⟺ **Frequency Spectrum**

Fourier Series, visually

Example: Periodic Square Wave as Sum of Sinusoids



$$f(t) = \frac{4}{\pi}\left[\sin(\pi t) + \frac{1}{3}\sin(3\pi t) + \frac{1}{5}\sin(5\pi t) + \frac{1}{7}\sin(7\pi t) + \cdots\right]$$



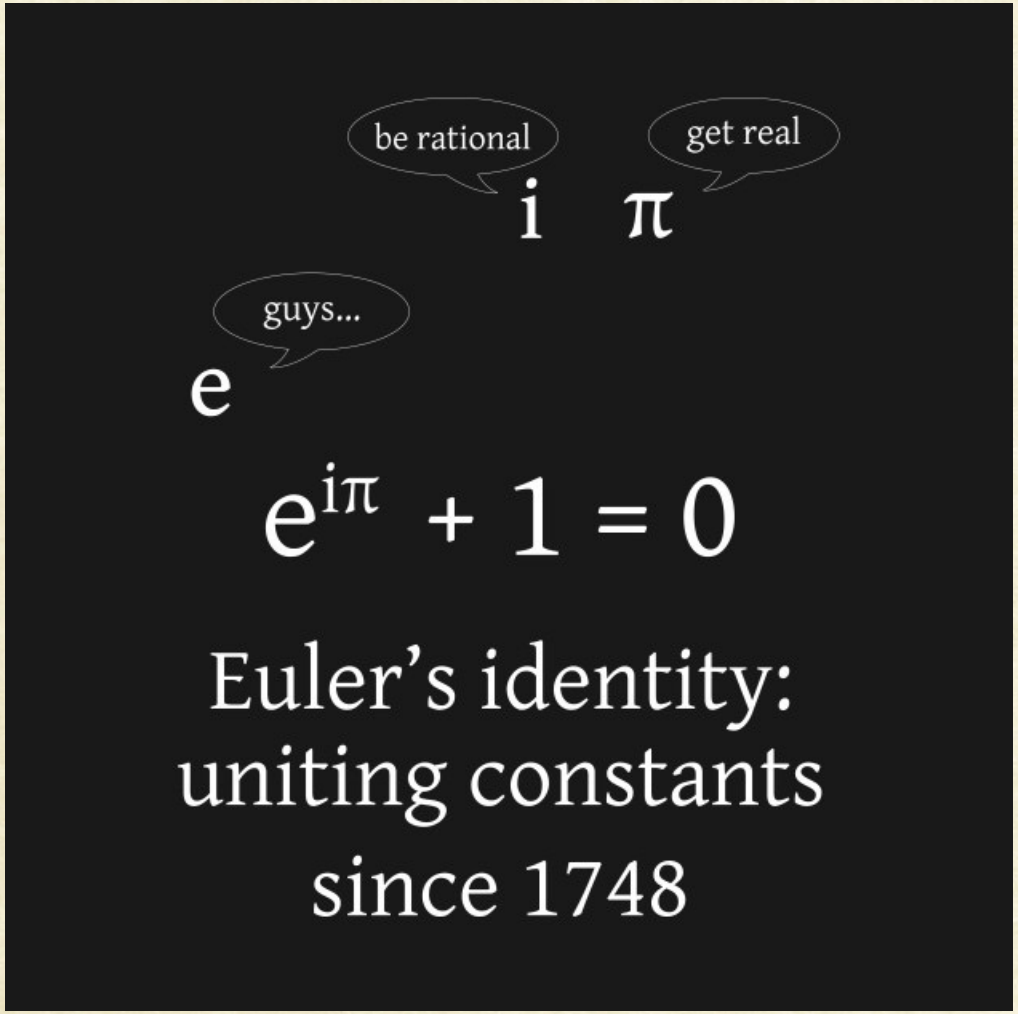
Fundamental only

Five terms

Eleven terms

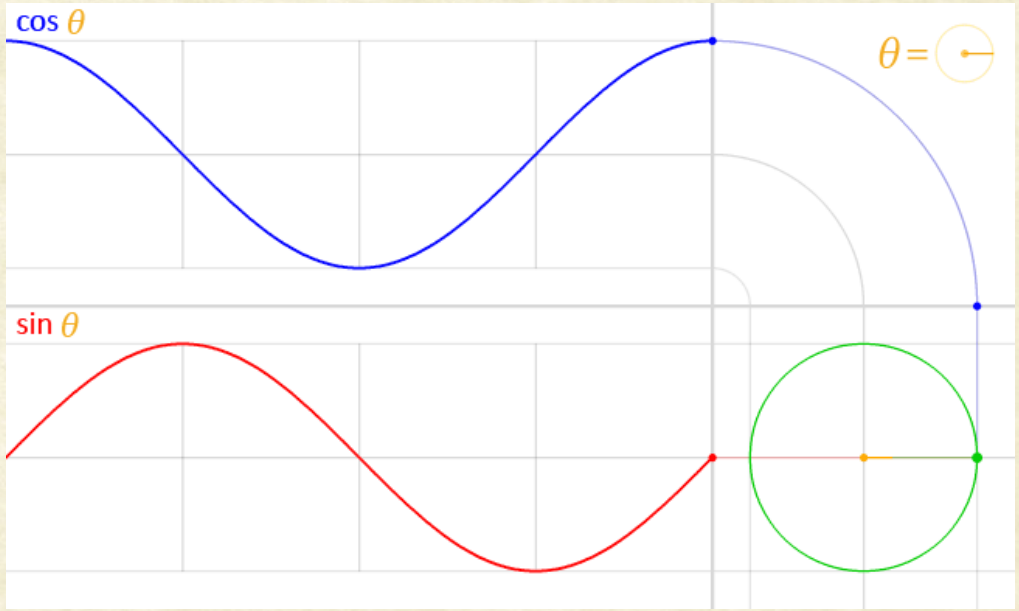
Forty-nine terms

http://ceng.gazi.edu.tr/dsp/fourier_series/description.aspx



$$e^{it} = \cos t + i \sin t$$

$$i = \sqrt{-1}$$

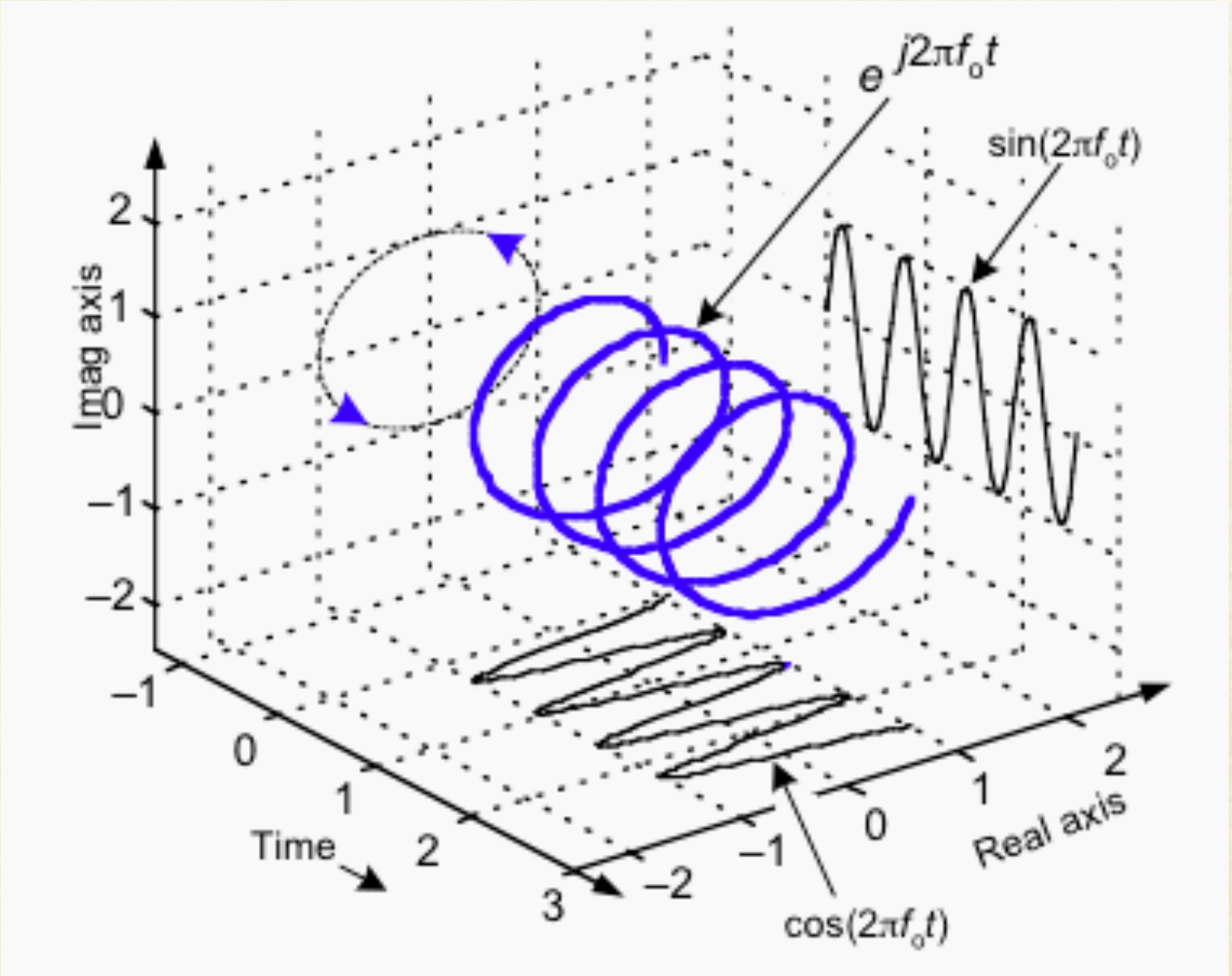


https://res.cloudinary.com/teepublic/image/private/s--s4GX0hG3--/t_Preview/b_rgb:191919,c_limit,f_jpg,h_630,q_90,w_630/v1486157785/production/designs/1177382_1.jpg

Complex sinusoid

$$e^{it} = \cos t + i \sin t$$

$$i = \sqrt{-1}$$



Questions?