Assignment 04: Can Cosmic Rays Unlock Ancient Egyptian Secrets?

Instructions: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

1. **Integrity**: Collaboration among students is encouraged, but each student must submit their own individual work. If working in a group, the names of all collaborators should be included in the submission. Sharing or copying code is strictly prohibited and external code can only be used with permission.
2. **Report**: **The write-up carries 50% of the credits** and should consist of three parts: answers to theory questions, resulting images of each step, and discussions of experiments. Handwritten scans will not be accepted, all answers to theory questions and discussions for experiments should be typed electronically.
3. **Submission**: Start early! The assignment is due on **November 20, 2023, 2359 IST**. Students will maintain private student repositories on GitHub Classroom and submit all code and files via GitHub. The datasets, starter codes, if any, and the layout of the final submission are available on the repositories.

The idea of there being precious items hidden in secret chambers has the potential to spark one's imagination. During the 1960s, Godfrey Hounsfield, a British engineer, considered the possibility of detecting hidden areas in Egyptian pyramids by capturing cosmic rays that traversed through unseen voids.

This notion of *'looking inside a box without opening it'* persisted with Hounsfield over the years, leading to his eventual discovery of a method to use high-energy rays to uncover that which is invisible to the naked eye. As a result, he developed a technique to peer inside the solid skull and obtain an image of the soft brain contained within. The first computed tomography image — a CT scan — of the human brain was made 51 years ago, on October 1, 1971. Hounsfield's stupendous work on CAT scan virtually changed the entire face of medical sciences, both in the domain of diagnosis and therapeutic interventions. He shared the Nobel Prize for Physiology or Medicine in 1979 with Allan MacLeod Cormack.

In the last 35 years, there has been a revolution in image reconstruction techniques in fields from astrophysics to electron microscopy and, most notably, in medical imaging. In each of these fields, one would like to have a precise picture of a 2 or 3-dimensional object which cannot be obtained directly. The data which is accessible is typically some collection of averages. The problem of image reconstruction is to build an object out of the averaged data and then estimate how close the reconstruction is to the actual object.

1 Image Projections and the Radon Transform

Implement radon transform **from scratch** and report the observations on the change in the sinogram as you vary the step size of θ and the number of detectors (range of r).

2 Parallel Beam Tomographic Reconstruction

1. Implement inverse radon transform **from scratch** for tomographic reconstruction by:
   (a) Simple backprojection
   (b) Direct Fourier reconstruction
   (c) Filtered backprojection (FBP)

   We will start with simulated parallel beam data of the Shepp-Logan phantom. The data is in *sl_phantom.mat*, and has the variables *slp* which is an image of the phantom, and *pd* which is the projection data. The projections are the columns of the matrix. There are 256 samples per projection and 402 projections over 180 degrees. Using suitable metrics such as SNR and MSE, check the performance of these algorithms.
2. In filtered backprojection, experiment with the different parameters given below, observe the effects and write down your observations.
   (a) Filters (convolutional kernels)
   (b) Number of views (orientations) and detectors

3 Parallel Beam Reconstruction of the Fan Beam Data

3.1 Display the Fan Beam Sinogram

Next, we will look at the fan beam data provided in the matlab file named *fan_beam_data.mat*. The data contains 888 samples (detectors) that cover an arc of approximately $55°$, and a total of 984 projections were taken during a full 2π rotation. The variable d represents the data, while the variable *fan_angle* contains the fan angle in radians. The data is quite pretty. Display it - an image of the sinogram should be generated. You may need to adjust the window (contrast processing) to enhance its visibility.

3.2 Parallel Beam Reconstruction

Before we get to the fan beam algorithm, it is interesting to try the FBP parallel beam reconstruction directly. Filter the sinogram and backproject the filtered data. Since we have 2π of projection angles, choose any interval of π, and perform the backprojection. The data will support a reconstruction of 512 samples. Display the result. You should be able to tell there is something in the data, but it won't be pretty.

3.3 Rebinning

The next step is to perform the rebinning operation, which involves computing a new parallel projection dataset for each projection angle θ, using the fan-beam sinogram we have with approximately six γ samples for each angle. The rebinning operation essentially resamples the fan-beam sinogram along diagonal lines. Reconstruct parallel-beam data from a fan-beam sinogram. After performing rebinning operation, show the rebinned sinogram. The primary difference is that the trace of particular bright objects now resembles sinusoids, whereas before, it was distorted depending on their location in the field of view.

Use your parallel beam FBP backprojection algorithm to reconstruct the rebinned data. Again, use any set of π projections. Show the reconstruction. It should yield a visually appealing result. Finally, we should correct the rebinned parallel beam data for the non-uniform beam spacing but that's beyond the scope of this assignment.

4 Fundamentals: Fun with Frequencies

Hybrid images are static images that change in interpretation as a function of the viewing distance. The basic idea is that high-frequency tends to dominate perception when it is available but, at a distance, only the low-frequency part of the signal can be seen. By blending the high-frequency portion of one image with the low-frequency portion of another, we get a hybrid image that leads to different interpretations at different distances. This exercise is based on the paper "Hybrid Images", SIGGRAPH 2006 and is intended to familiarize you with image filtering and frequency representations.

1. Get a few pairs of images that you want to make into hybrid images. You can use the sample images for debugging, but you should use your own images in your results. Then, you will need to write code to low-pass filter one image, high-pass filter the second image, and add (or average) the two images. For a low-pass filter, it is suggested to use a standard 2D Gaussian filter. For a high-pass filter, using the impulse filter minus the Gaussian filter (which can be computed by subtracting the Gaussian-filtered image from the original) is suggested. The cutoff-frequency of each filter should be chosen with some experimentation. Try creating a variety of types of hybrid images (change of expression, morph between different objects, change over time, etc.). The project site has several examples that may inspire.

   Show at least three results, including one that doesn't work so well (failure example). Briefly explain how you got the good results (e.g., chosen cut-off frequencies, alignment tricks, other techniques), as well as any difficulties and the possible reasons for the bad results.

2. Demonstrate the process of generating hybrid images using frequency analysis for one of your favourite results. Display the log magnitude of the Fourier transform of both input images, the filtered images, and the resulting hybrid image. You may use any signal processing libraries for computing Fourier transform.

3. Try using colour to enhance the effect of hybrid images. Does it work better to use color for the high-frequency component, the low-frequency component, or both?