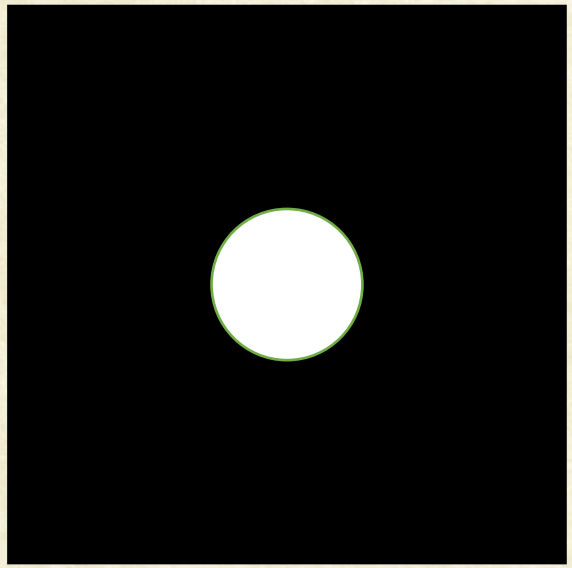
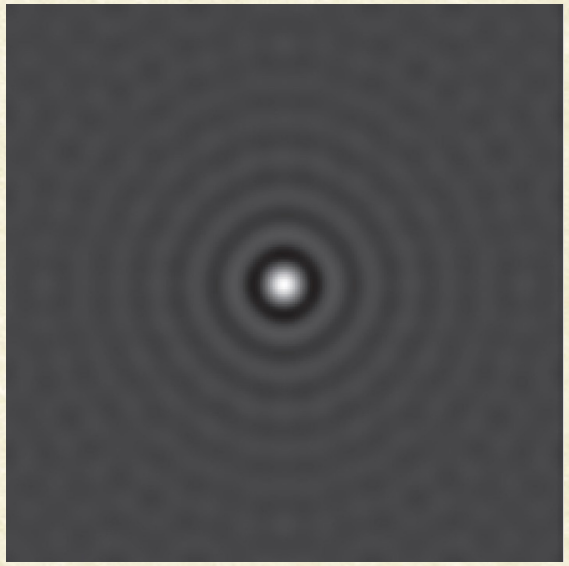
CS7.404: Digital Image Processing

Monsoon 2023: Lowpass Filtering in Fourier Domain





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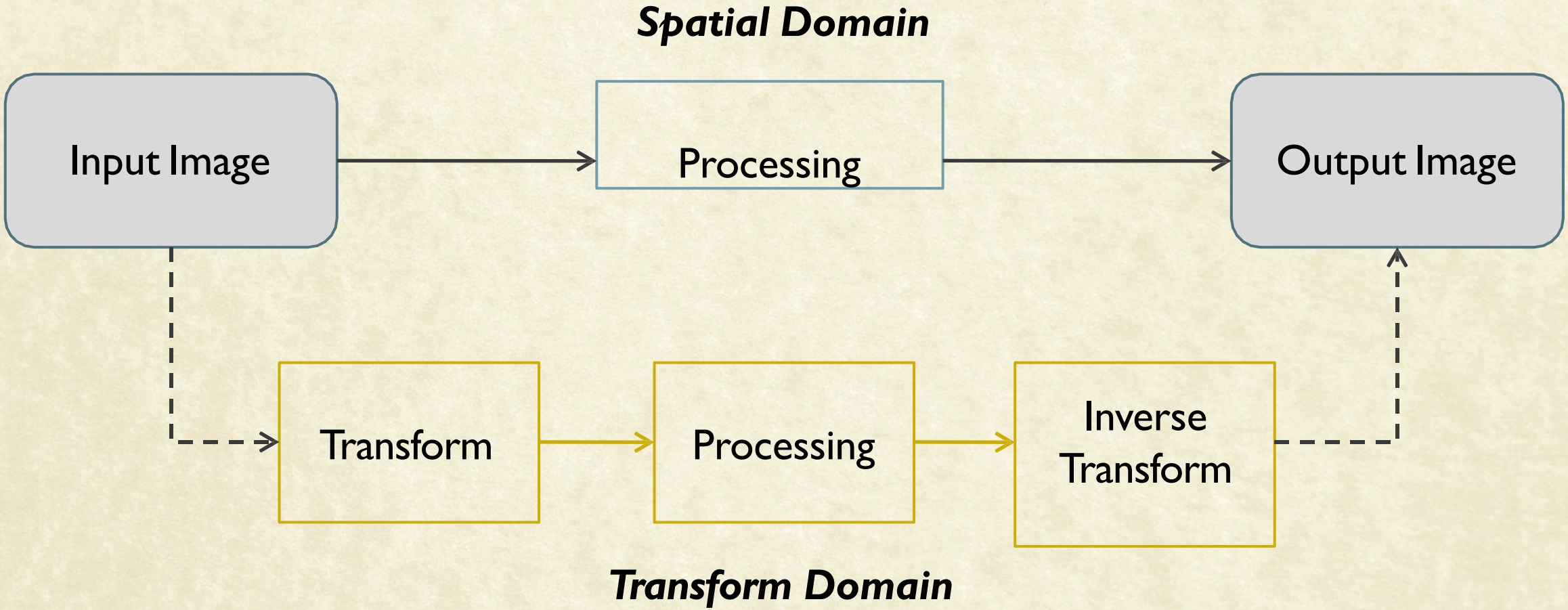
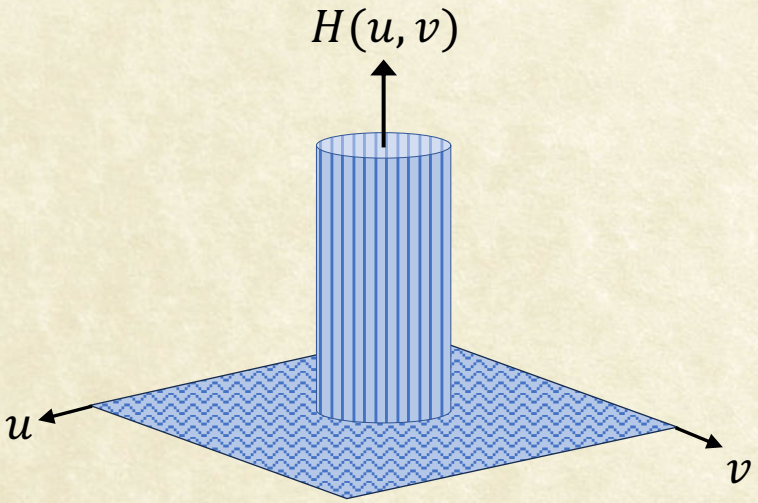
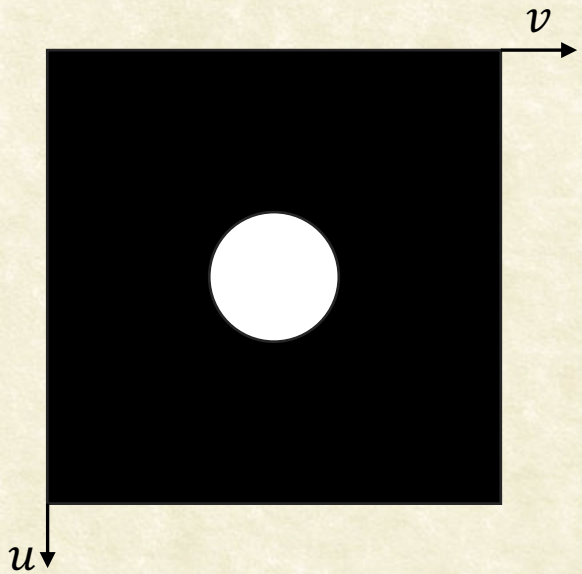
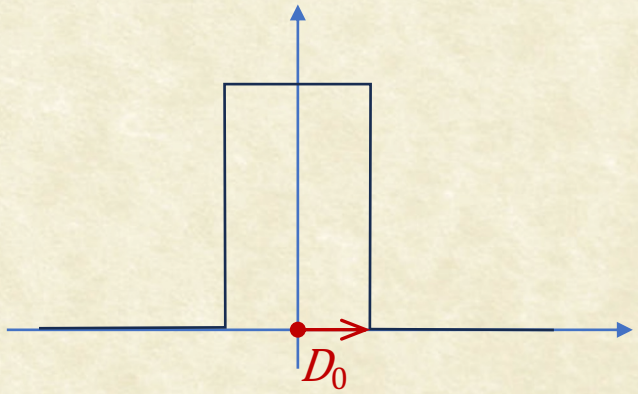
Spatial vs. Transform Domain Processing



Ideal Lowpass Filters





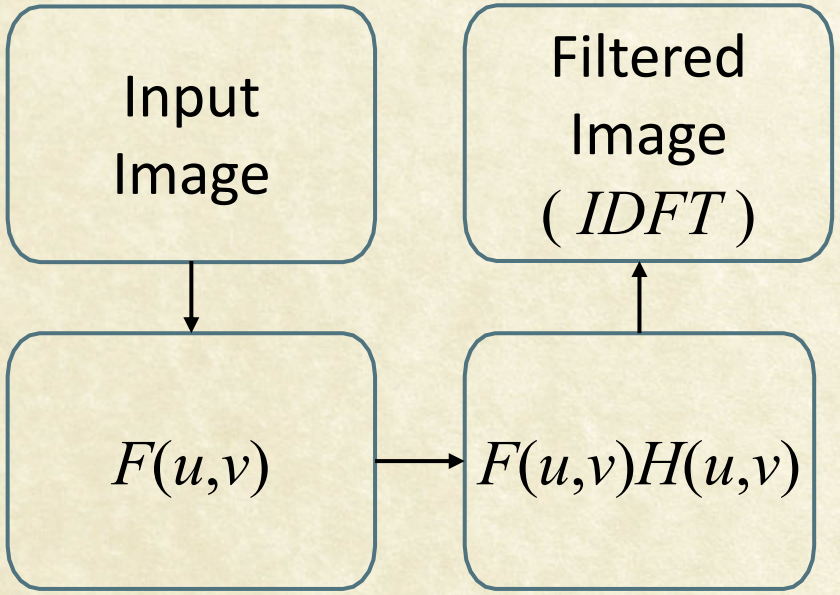


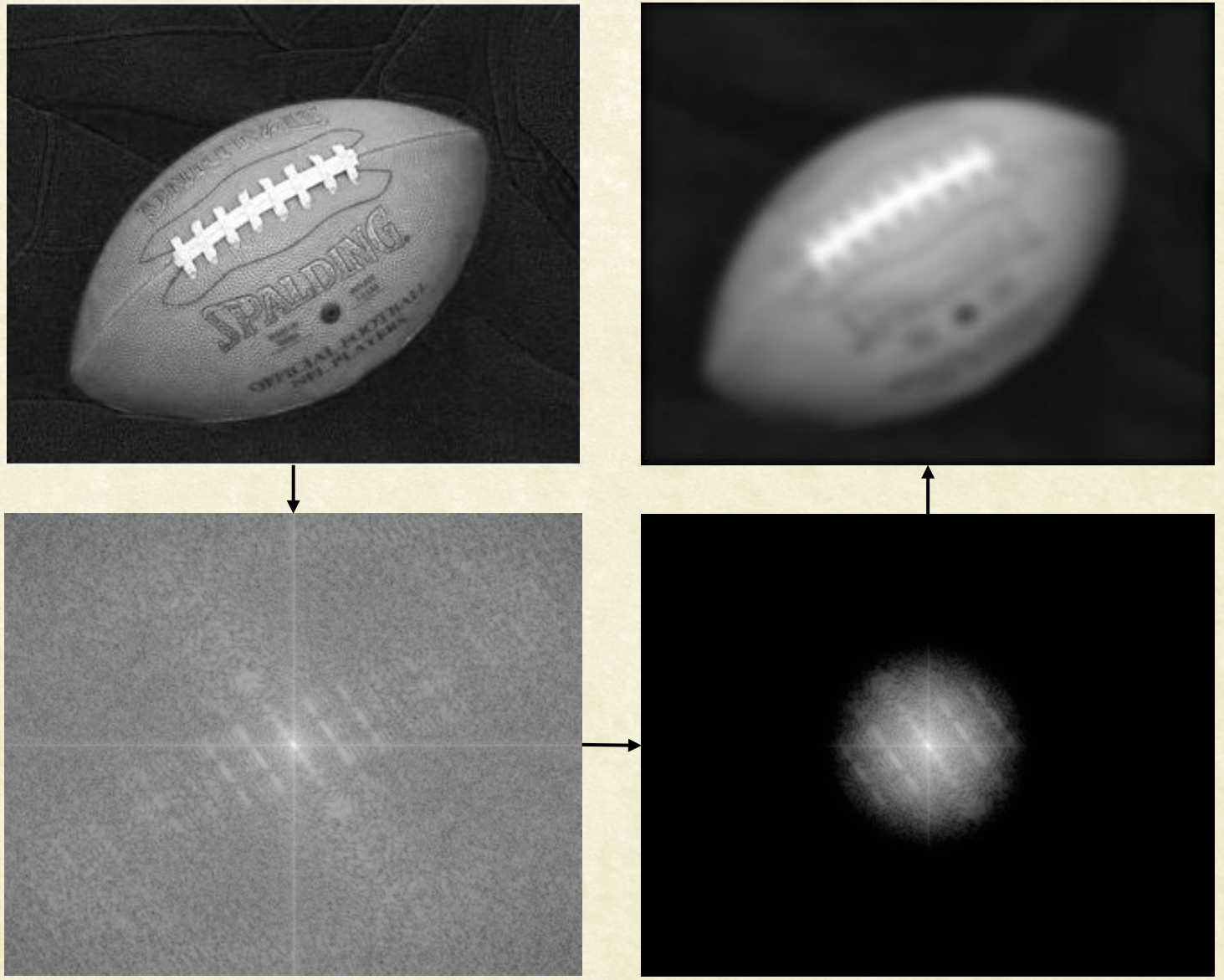
$$H(u,v) = \begin{cases} 1 \text{ if } D(u,v) \le D_0 \\ 0 \text{ if } D(u,v) > D_0 \end{cases}$$

where $D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$

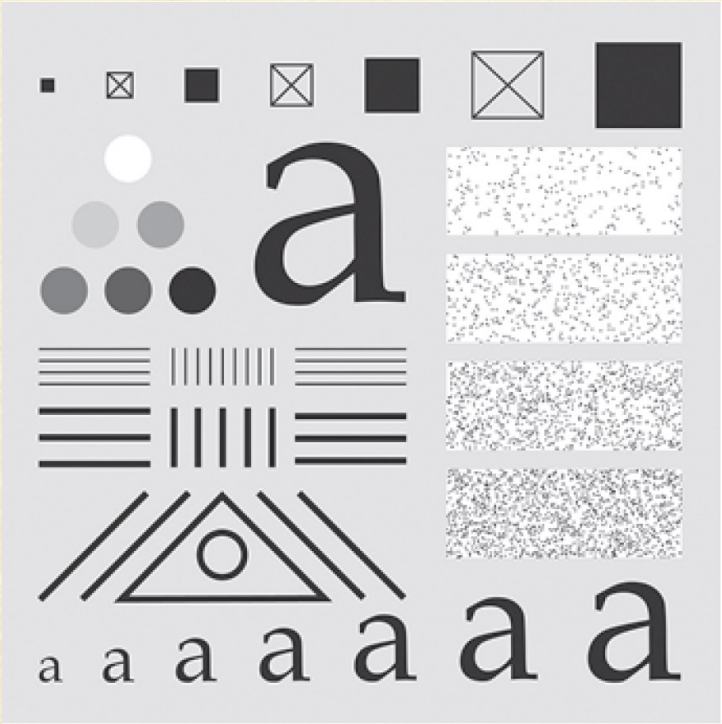
D_0 → cut off frequency

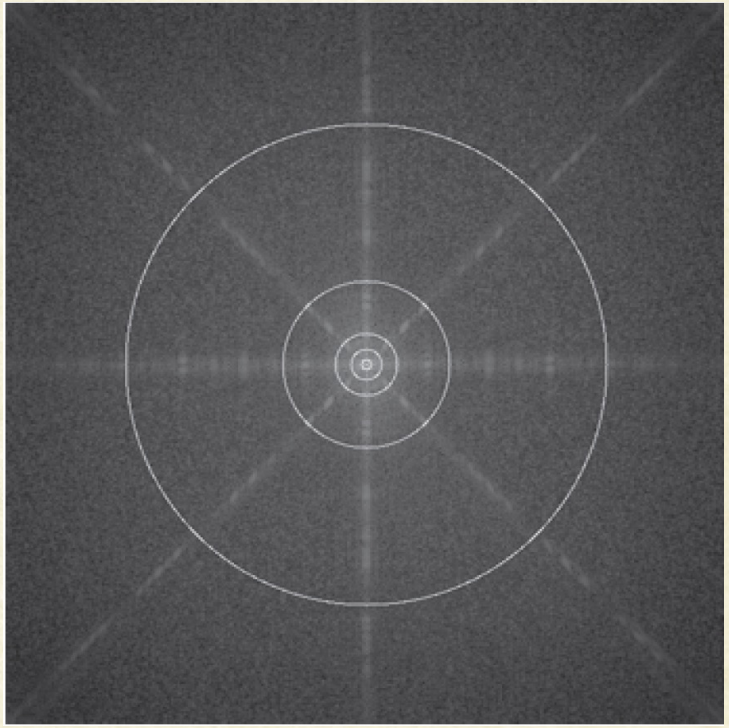
Ideal Lowpass Filters





ILPF: Filter Size





Radii 10,30,60,160 and 460 → power 87, 93.1, 95.7, 97.8 and 99.2

$$\alpha = \frac{100}{P_T} \sum_u \sum_v P(u,v)$$

Ideal Lowpass Filters

Filter Radius:

Orig	10	30
60	160	460

Power Removed: 13.1, 7.2, 4.9, 2.4, and 0.6%

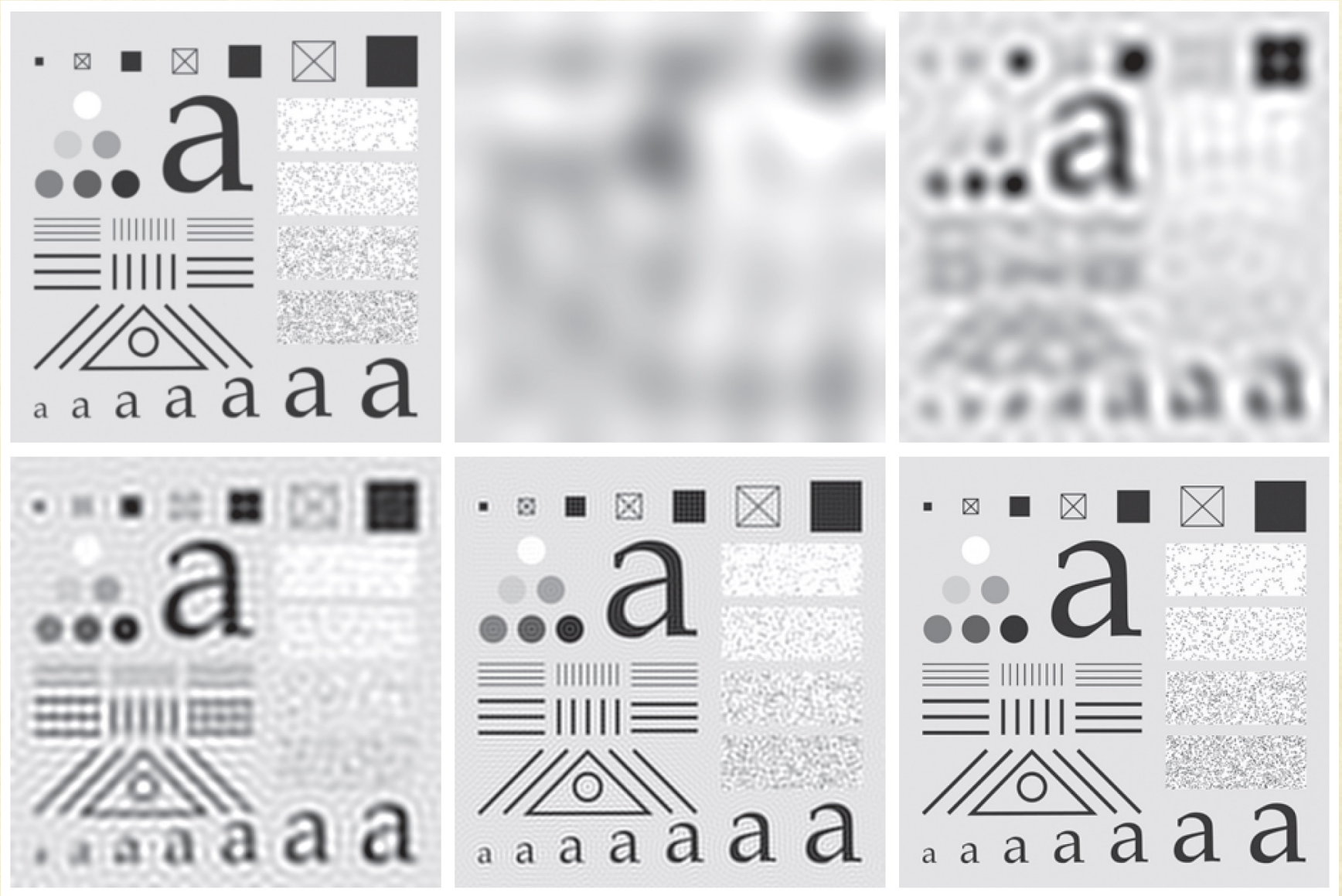
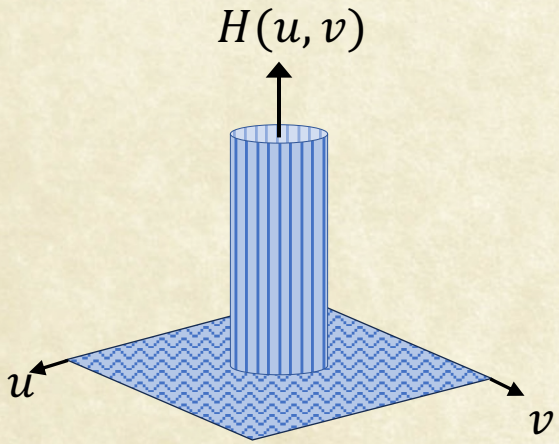
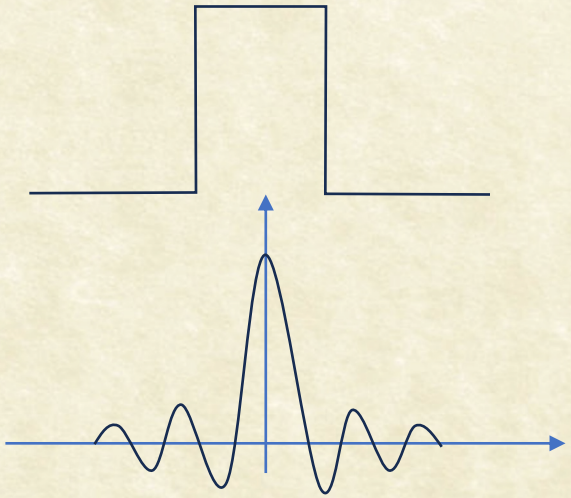


Image courtesy: Gonzalez and Woods

Ideal Lowpass Filters: Ringing Artifacts



ILPF radius 60



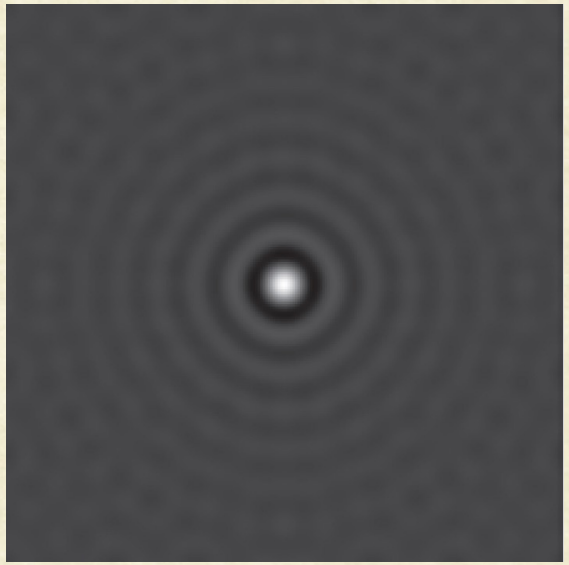
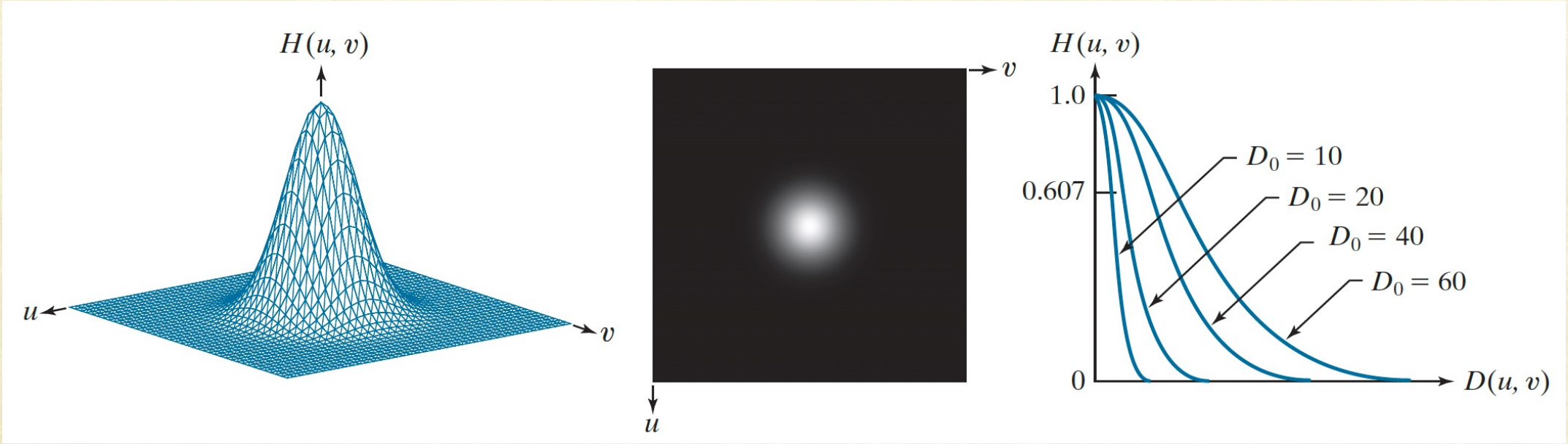


Image courtesy:
Gonzalez and Woods

Gaussian Lowpass Filters



$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

D_0 is the cut-off frequency (0.607 of its max value 1.0)

Image courtesy: Gonzalez and Woods

Gaussian Lowpass Filters (GLPF)

Filter Radius:

Orig	10	30
60	160	460

Power Removed: 13.1, 7.2, 4.9, 2.4, and 0.6%

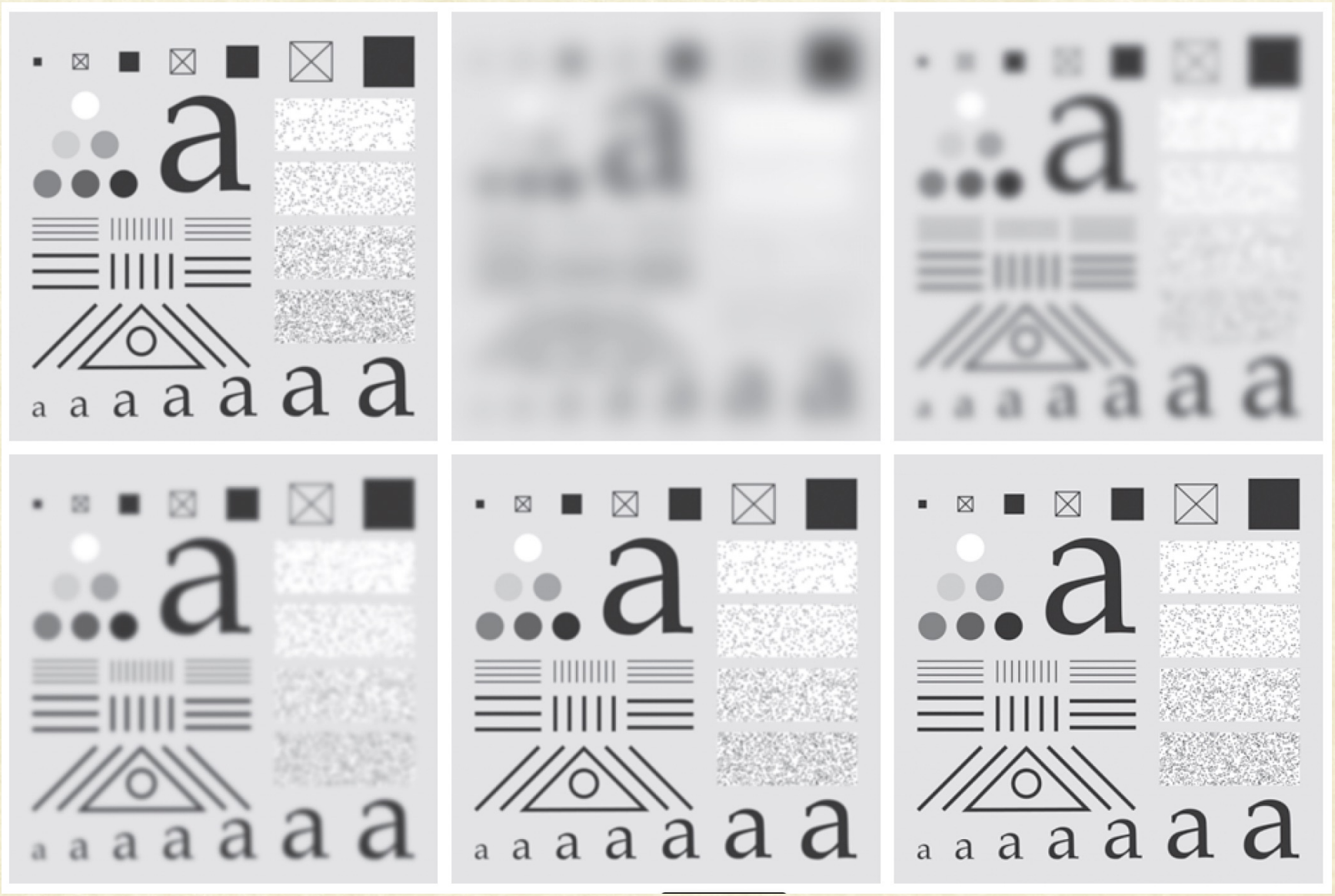
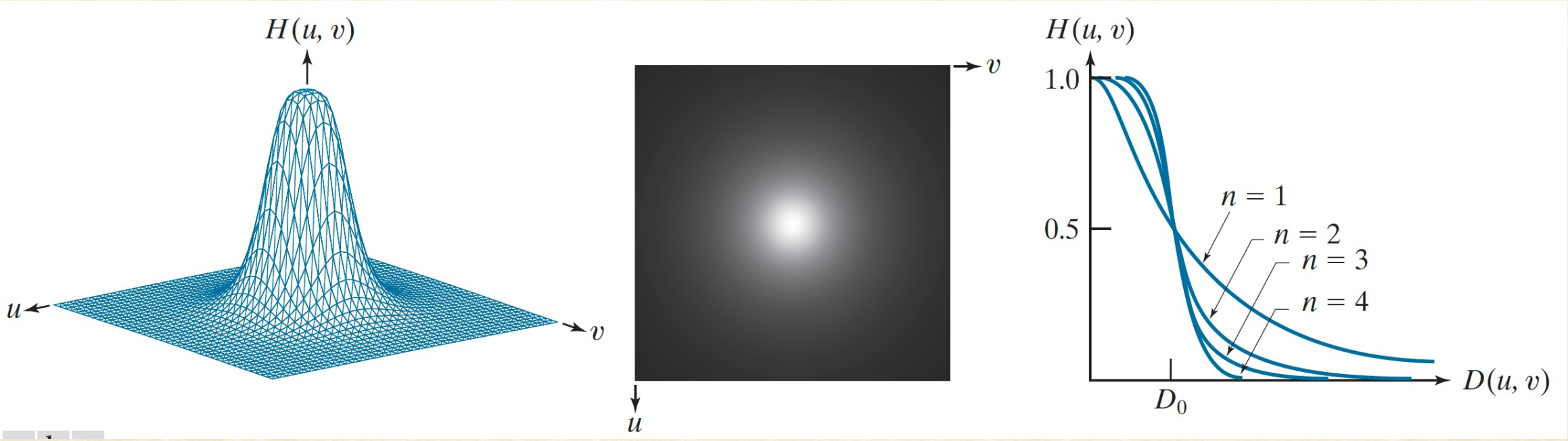


Image courtesy: Gonzalez and Woods

Butterworth Lowpass Filter (BLPF)



$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

n: order of the filter

BLP filter can give better control over cutoff with minimal ringing artifacts

GLPF vs BLPF vs ILPF

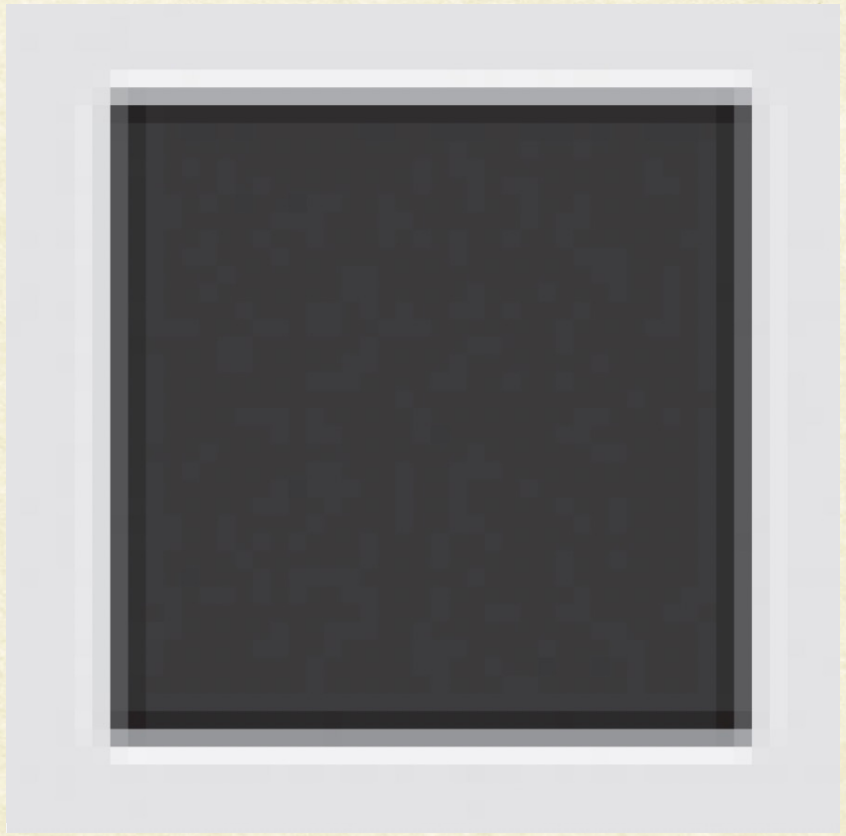
GLPF

BLPF

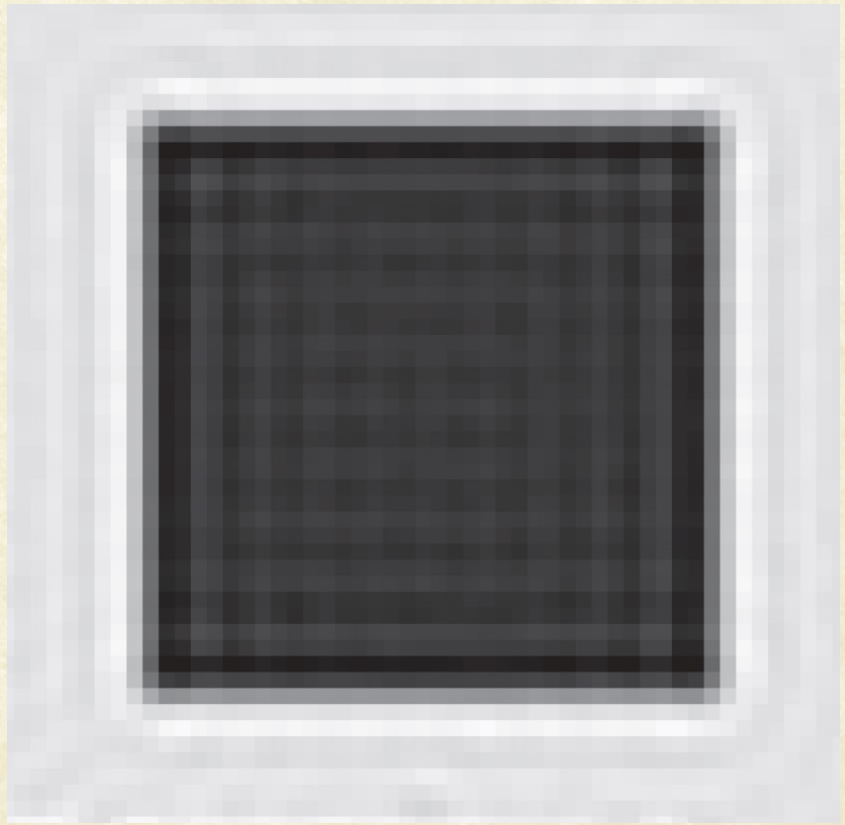
ILPF

GLPF vs BLPF vs ILPF

GLPF



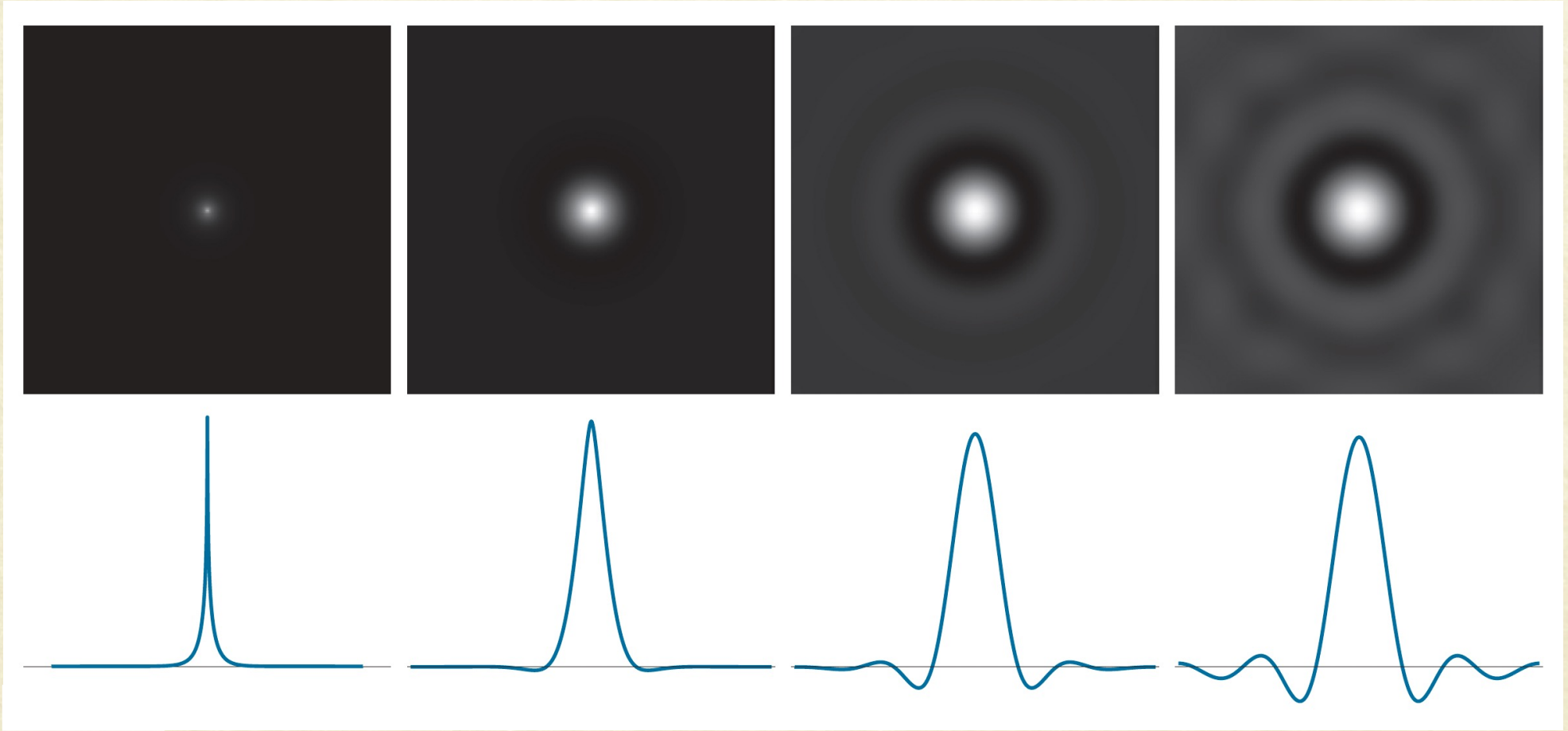
BLPF



ILPF

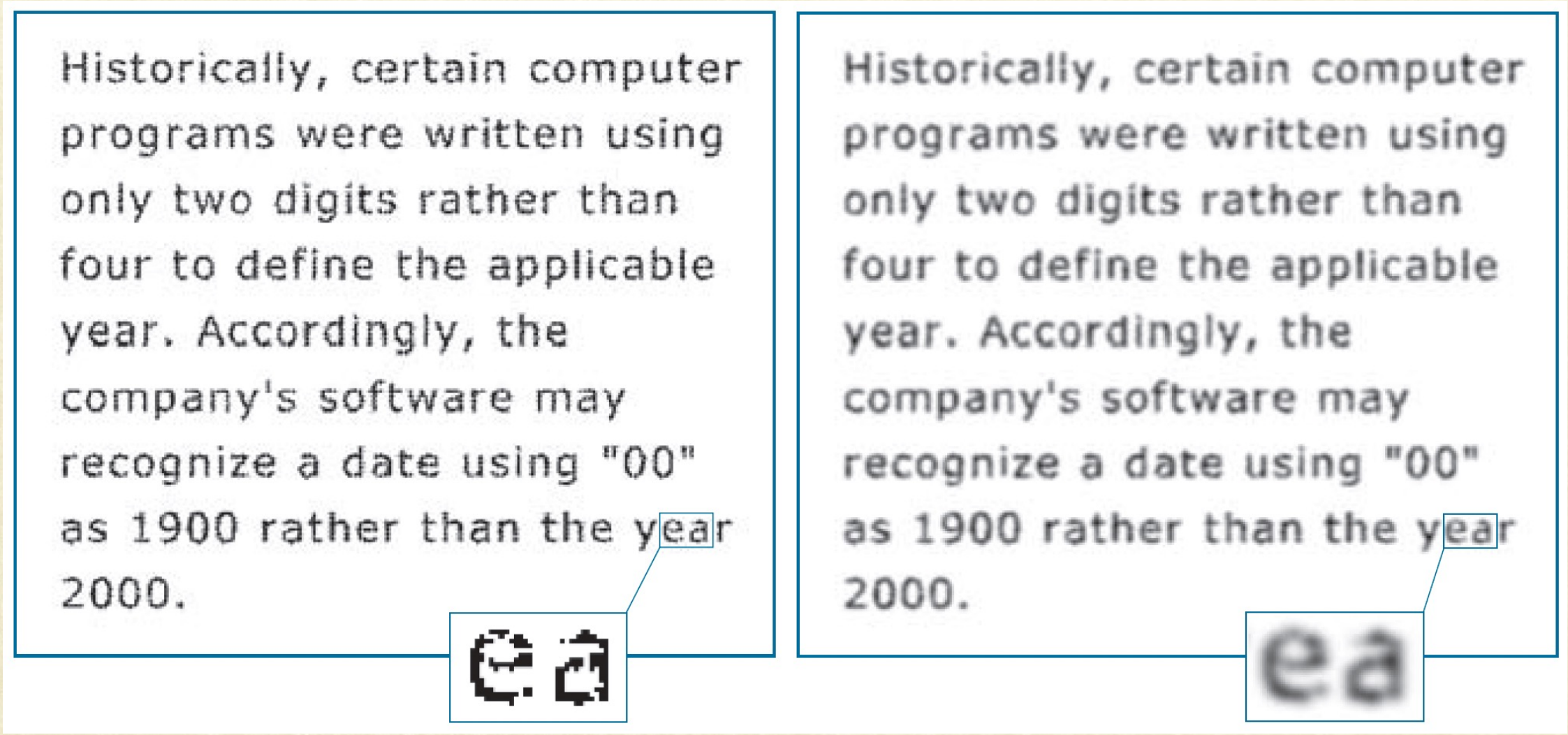
BLPF in Spatial Domain

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$



Order (n): 1 2 5 20

LPF Application: Character Recognition



GLPF with $D_0 = 120$

Image courtesy: Gonzalez and Woods

GLPF to Increase Resolution!!

LPF Application: Photography

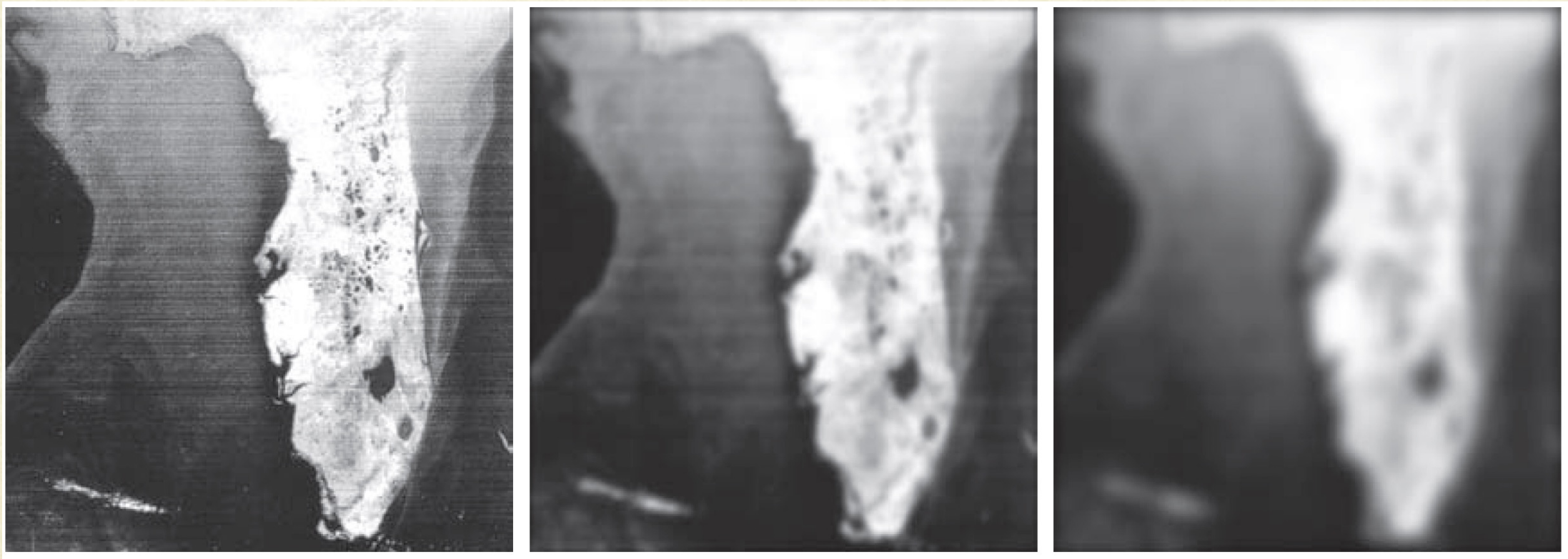
Wrinkle lines are reduced by GLPF

$D_0 = 150$

$D_0 = 130$

LPF Application: Photography

Scan lines reduced by GLPF from satellite images



$D_0 = 50$ $D_0 = 20$

Questions?

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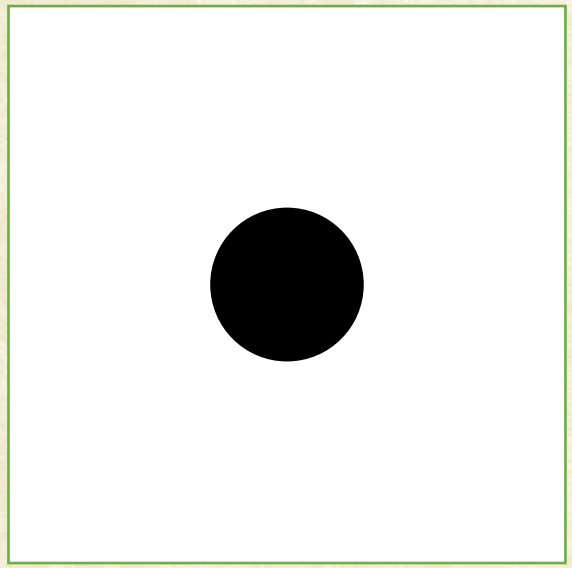
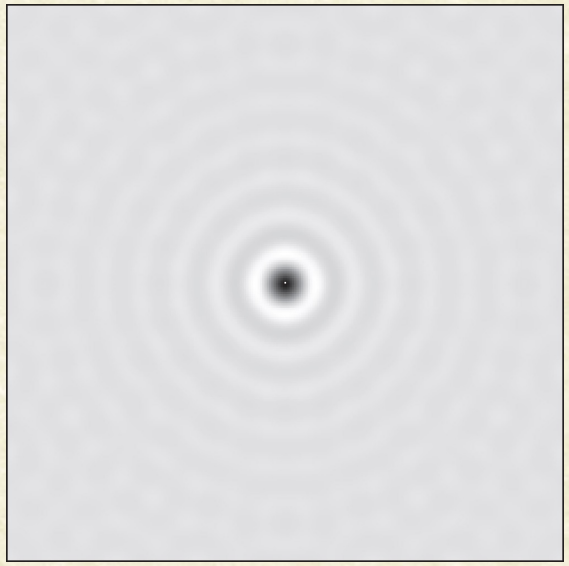
Monsoon 2023: Highpass Filtering in Fourier Domain





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Highpass Filter

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

Ideal HPF:

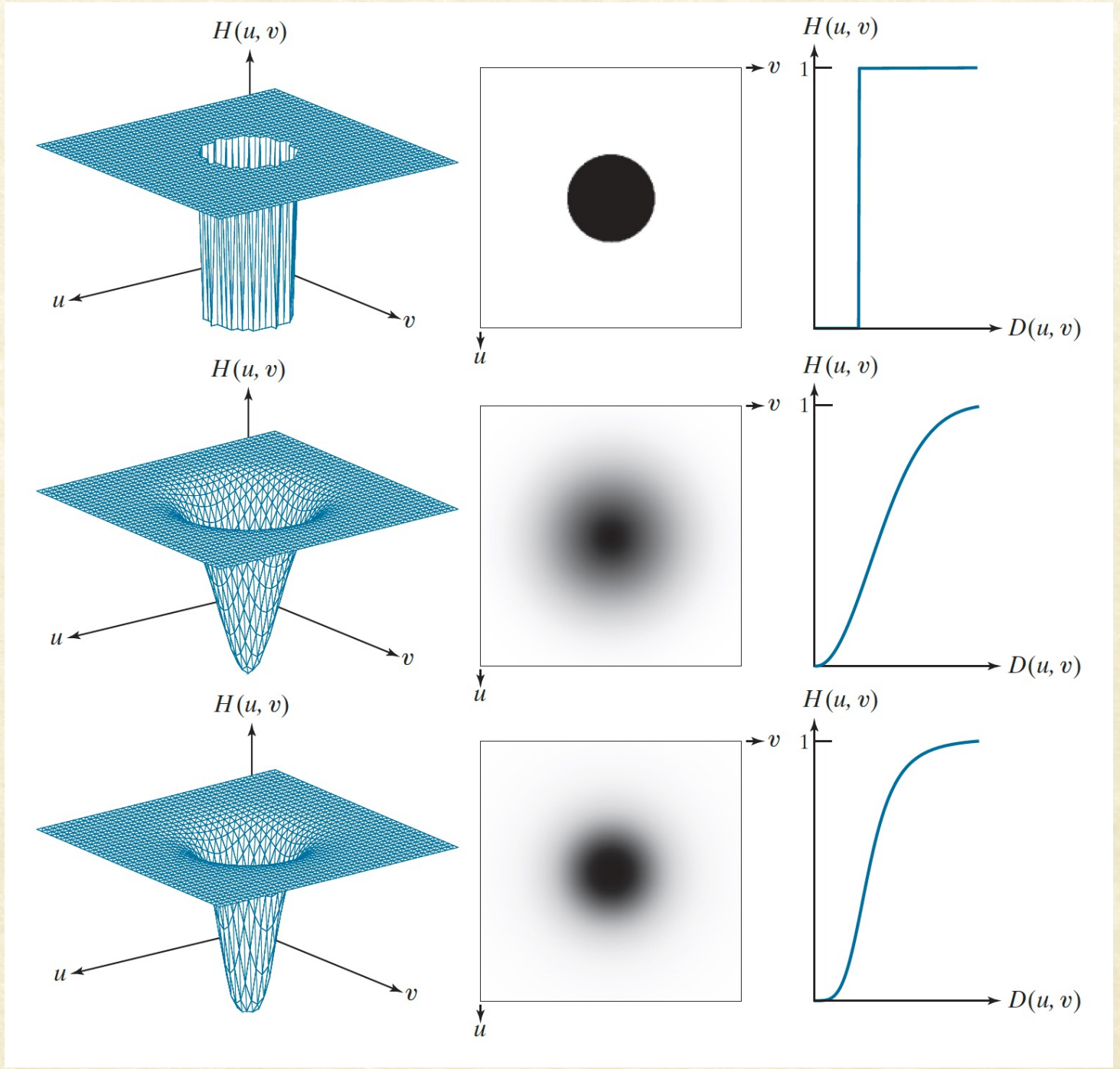
$$H(u,v) = \begin{cases} 0 \text{ if } D(u,v) \le D_0 \\ 1 \text{ if } D(u,v) > D_0 \end{cases}$$

GLPF:

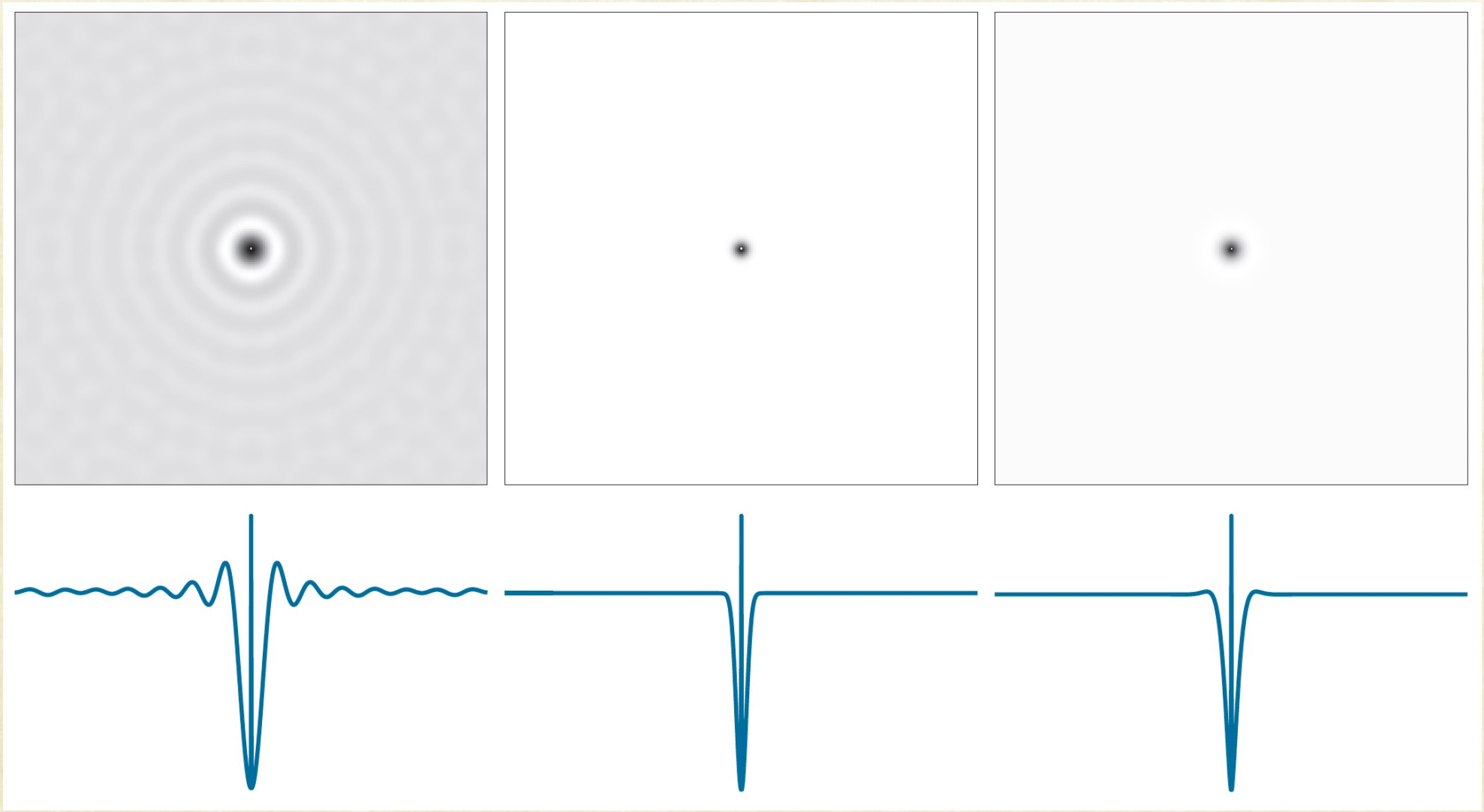
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

BHPF:

$$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$$



Highpass Filter Kernels in Spatial Domain

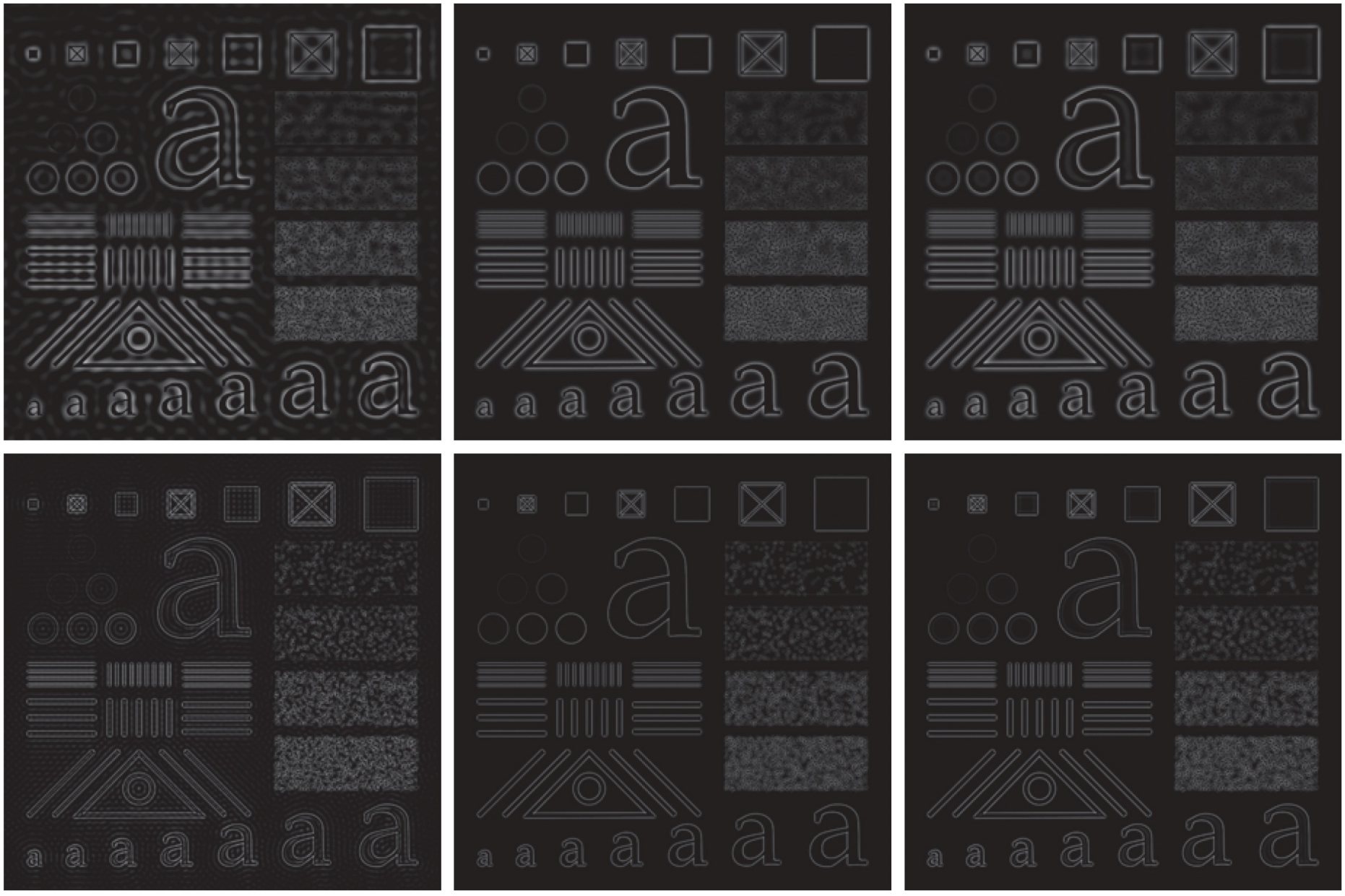


Ideal HPF Gaussian HPF Butterworth HPF

HPF Results: IHPF, GHPF, and BHPF

D_0=60
(2^{nd} order)

D_0=160



HPF Applications: Fingerprint Enhancement

Smudged fingerprint, Highpass filtered image and Thresholded image

Laplacian Filter in Frequency Domain

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y)$$

$$H(u,v) = -4\pi^2 D^2(u,v)$$

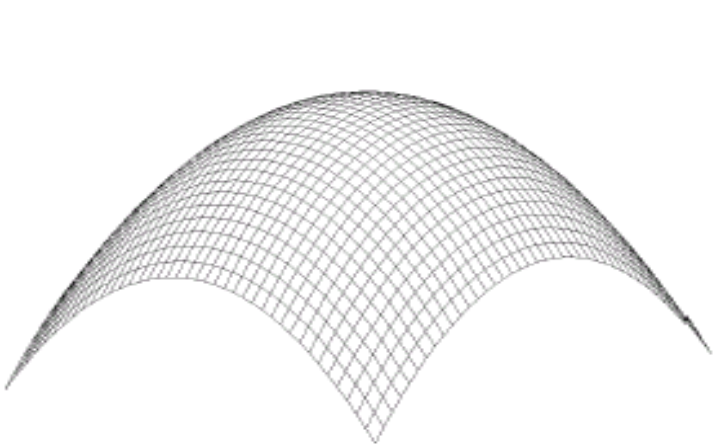
$$\Im[\frac{d^n f(x)}{dx^n}] = (ju)^n F(u)$$

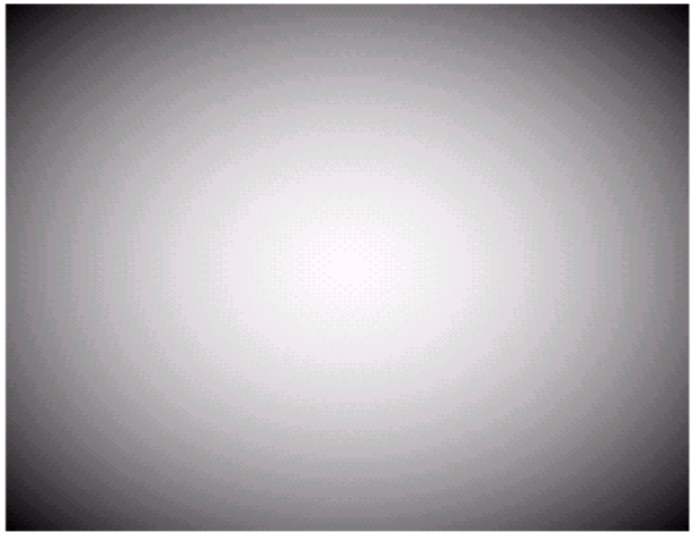
$$\Im[\frac{\partial^2(f(x,y))}{\partial x^2} + \frac{\partial^2(f(x,y))}{\partial y^2}] = (ju)^2 F(u,v) + (jv)^2 F(u,v)$$

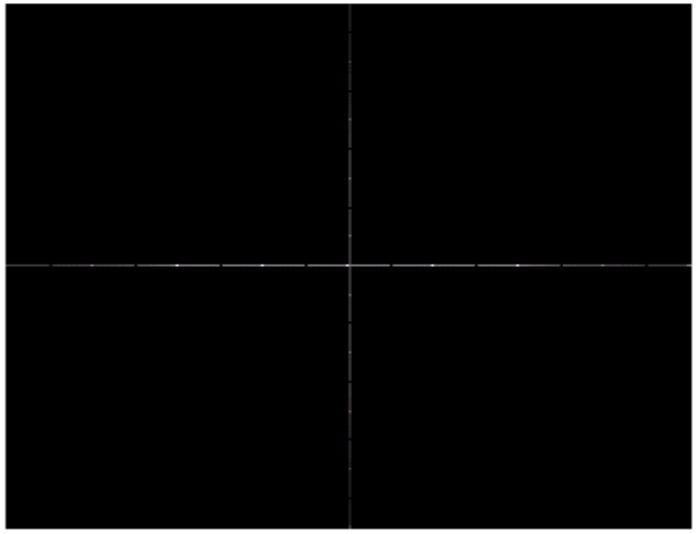
$$= -(u^2 + v^2) F(u,v)$$

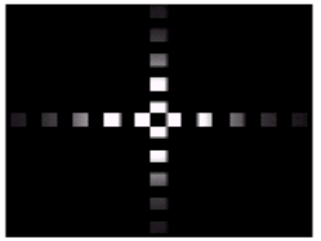
Laplacian in Frequency Domain

$$
\begin{aligned}
g(x,y) &= \Im^{-1}\left\{F(u,v) - H(u,v)F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 - H(u,v)\right]F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 + 4\pi^2 D^2(u,v)\right]F(u,v)\right\}
\end{aligned}
$$









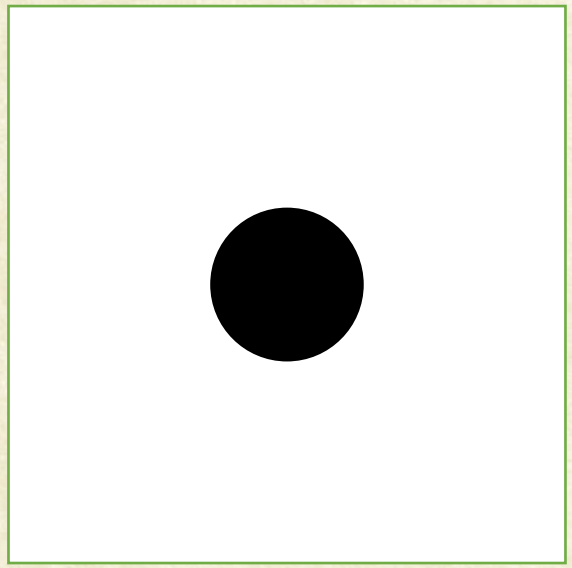
0	1	0
1	−4	1
0	1	0

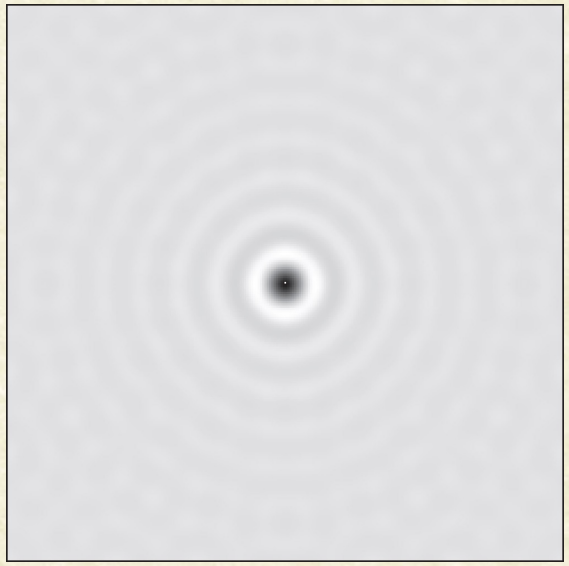
Image courtesy: Gonzalez and Woods

Questions?

CS7.404: Digital Image Processing

Monsoon 2023: Other Filters





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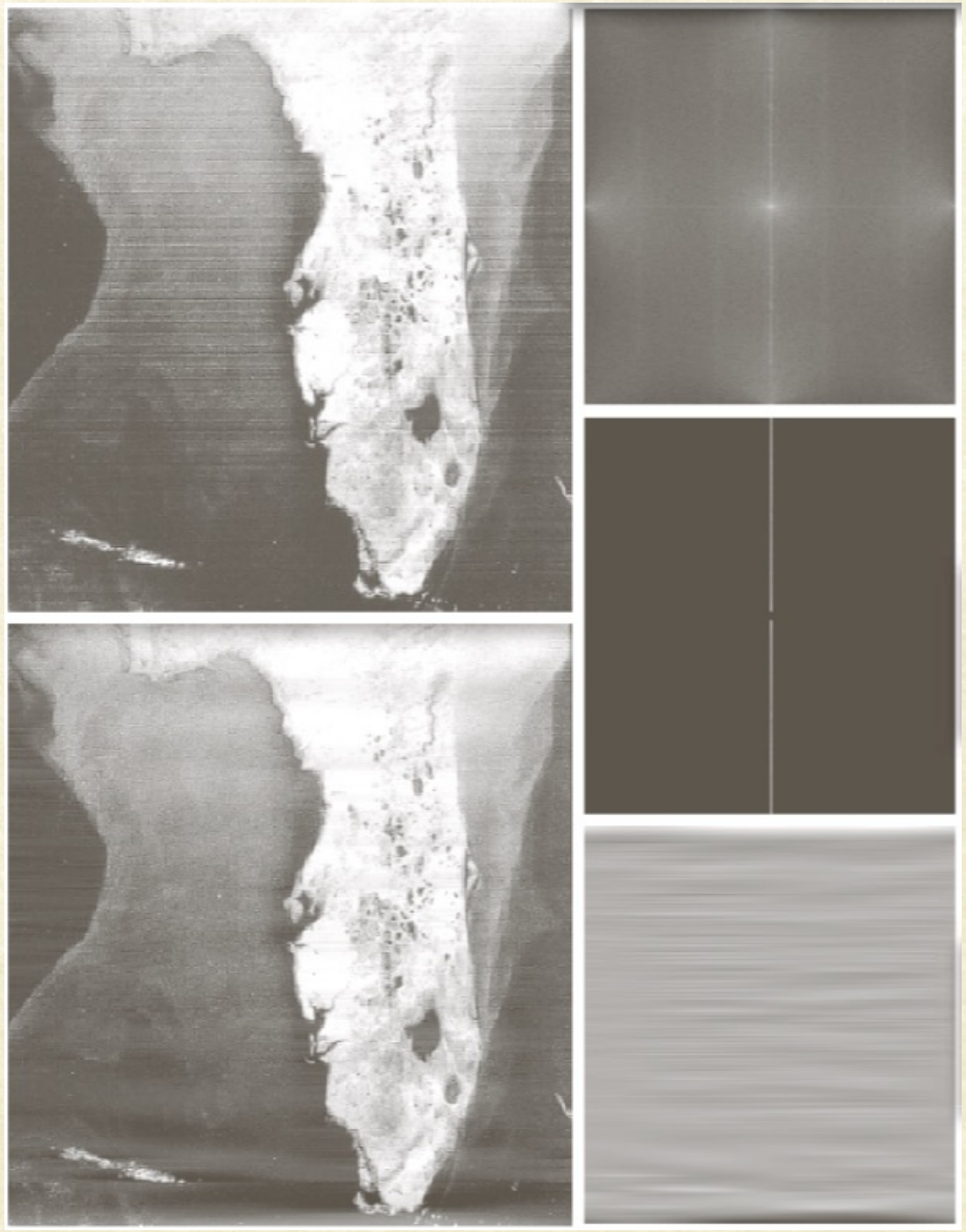
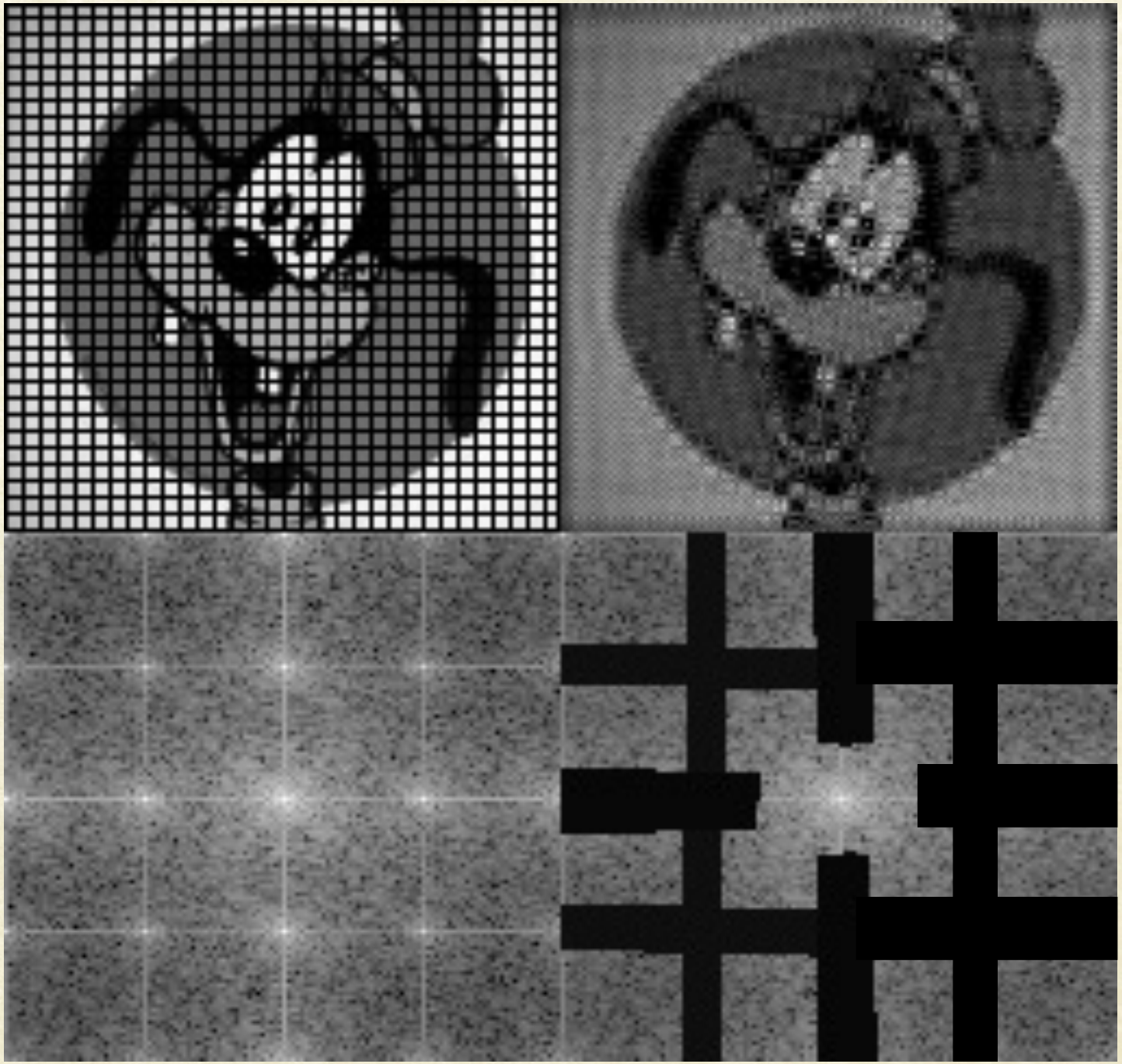
Notch Reject filter (Notch pass filter)

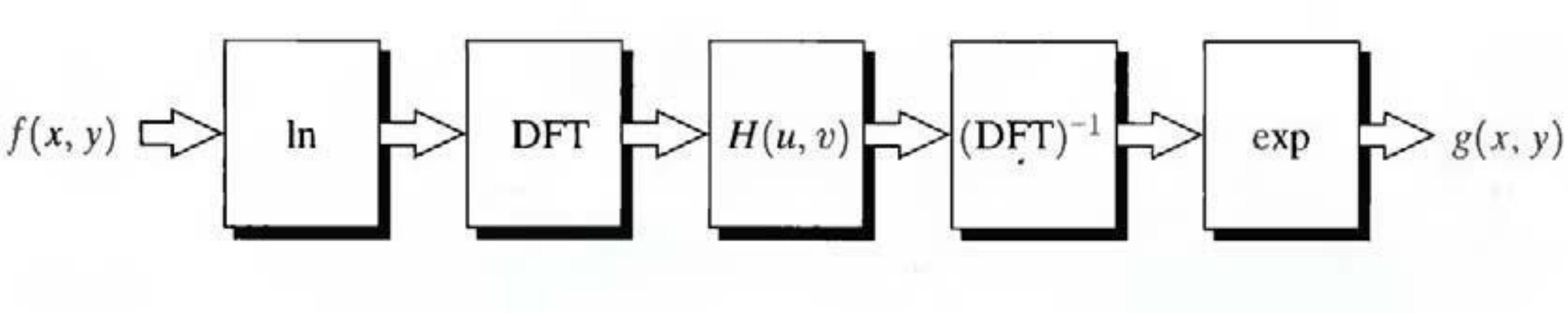


Artifact Removal



Filtering in frequency domain

- Band reject (Band pass filters)
- Unsharp Masking and High boost filtering
- Homomorphic filtering



Questions?