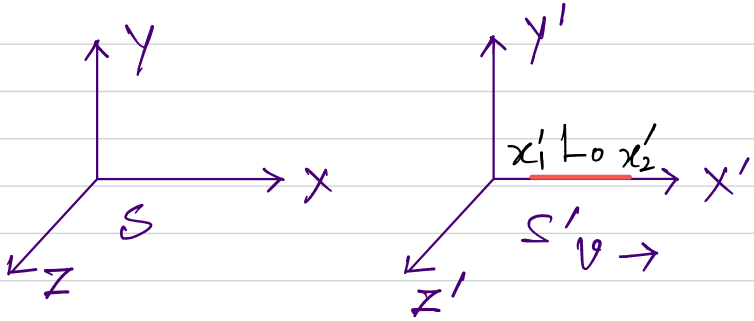
* **Length Contraction**: The length of an object in motion appear shorter along the direction of motion.



In the S' frame there is a stick of length L. The S' moves with v along X direction of S frame.

In the S' frame : $L_0 = x_2' - x_1'$

In the S' frame : $L = x_2 - x_1$

Now $L_0 = x_2' - x_1' = \dfrac{x_2 - vt_2 - x_1 + vt_1}{\sqrt{1 - v^2/c^2}}$

As both the ends of the stick is measured at the same time $t_2 = t_1$

$$\Rightarrow L_0 = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}} = \frac{L}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow L = L_0 \sqrt{1 - v^2/c^2} < L_0$$

Hence to the S frame the length L is shorter than the actual length. L_0

Class 3

Doppler Effect in Light: In case of sound the Doppler effect is

$$f = f_0 \left(\frac{1 + v/v_s}{1 - V/v_s} \right)$$

f_0 = original frequency, f - observed frequency, v_s → speed of sound,
v → speed of observer (+ for to and − for away from source)
V → " " source (+ " " " " " observer)

Since sound waves need a medium to propagate, the velocity of both the observer and the source is important. This is different in light.

* Observer moves perpendicular to him and light source:

f_0 → frequency of light, then proper time in the frame of the source for one clock tick is $t_0 = 1/f_0$. The same for the observer is $t = t_0/\sqrt{1 - v^2/c^2}$. Hence the observed frequency

$$f = 1/t = \frac{\sqrt{1 - v^2/c^2}}{t_0} = f_0 \sqrt{1 - v^2/c^2} \Rightarrow f < f_0$$

Observer moving away from source: In this case the observer moves away a distance of vt between two consequtive ticks. So the light waves take vt/c longer to reach the observer. Hence the total time between two consequtive waves is $T = t + vt/c$

$$= t_0 \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} = t_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$$

v is negative as the observer recedes.

Hence the observed freqnency is given by

$$f = \frac{1}{T} = f_0 \sqrt{\frac{1 - v/c}{1 + v/c}} < f_0$$

Observer is approaching source: The same calculation as above, but write

$$T = t - vt/c$$

One then gets
$$f = f_0 \sqrt{\frac{1 + v/c}{1 - v/c}} > f_0$$

v is positive for approach

* Lights coming from far away stars/galaxies exhibit Doppler effect that indicate that they are receeding — this hints to a expanding universe.

Simultaneity in Relativity: Due to relative nature of time & space, events that are simultaneous in one frame are not simultaneous in another.

At earth, at time t_0, there are two explosions at x_1 & x_2. With respect to the earth there is a spaceship moving at speed v. In the space ship the two events occur at

$$t_1' = \frac{t_0 - vx_1/c^2}{\sqrt{1 - v^2/c^2}} \qquad t_2' = \frac{t_0 - vx_2/c^2}{\sqrt{1 - v^2/c^2}}$$

Hence the time difference is

$$\Delta t = t_2' - t_1' = \frac{v(x_2 - x_1)/c^2}{\sqrt{1 - v^2/c^2}}$$

So in the spaceship the events are not simultaneous. Observations of both stationary observer on earth & the moving observer on the spaceship are correct.

Spacetime: It is clear by now that time and distance measurements are not independent of each other. Due to relativity we have talk about events in spacetime. An event is a set of four coordinates (t, x, y, z) in spacetime. An 'interval' between two events in spacetime is

$$\tau^2 = (ct)^2 - x^2 - y^2 - z^2$$

Spacetime intervals are invariant under Lorentz transformation - this means the following. In S frame two events are (x_1, y_1, z_1, t_1) & (x_2, y_2, z_2, t_2). In S' frame the events are (x_1', y_1', z_1', t_1') & (x_2', y_2', z_2', t_2'). Then

$$(ct_2 - ct_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2$$

$$= (ct_2' - ct_1')^2 - (x_2' - x_1')^2 - (y_2' - y_1')^2 - (z_2' - z_1')^2$$

In analogy with vector in cartesian coordinate the set of four spacetime coordinate (x, y, z, t) is called "four vector".

Defining $\Delta t = t_2 - t_1$, $\Delta x = x_2 - x_1$ ... etc, the space time interval is

$$\Delta\tau^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \quad \text{- Invariant under LT}$$

$$\equiv (c\Delta t)^2 - (\Delta x)^2 \quad \left\{ \text{for simplicity } (\Delta x)^2 \equiv (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \right.$$

* Timelike Interval: For timelike interval $(\Delta\tau)^2 > 0$

$$\Rightarrow (c\Delta t)^2 > (\Delta x)^2$$

Time like intervals are causally related - that means one event can affect the other. Example - two explosions happening at the same space location (x, y, z) but at two different times.

* Spacelike Interval: Here $(\Delta\tau)^2 < 0 \Rightarrow (c\Delta t)^2 < (\Delta x)^2$

Spacelike intervals are causally disconnected. For example two explosions occur in a reference frame at two different locations but at the same time t.

Light like Interval: For $(\Delta\tau)^2 = 0 \Rightarrow (c\Delta t)^2 = (\Delta x)^2$

Lightlike events are causally connected. Consider a frame in which two explosion occurs at a distance 3×10^8 m apart in space and 1s apart in time. In this case the light ray starting journey from the first explosion can reach the 2nd explosion just in time.

* World line: Trajectories of ordinary particles or light in the 4D spacetime are called worldlines.