International Institute of Information Technology Hyderabad

Modern Complexity Theory (CS1.405)

Assignment 1
Deadline: August 26, 2023 (Saturday), 17:00 PM
Venue for Submission: CSTAR, A3-110, Vindhya Block, IIIT Hyderabad
Total Marks: 100

NOTE: It is strongly recommended that no student is allowed to copy from others.
No assignment will be taken after deadline.
Write the following while submitting ONLY HARDCOPY:

Modern Complexity Theory (CS1.405)
Assignment 1
Name:
Roll No.:

1. The following are the state diagrams of two DFAs, say M_1 and M_2 as shown in Figure 1. Answer the following:

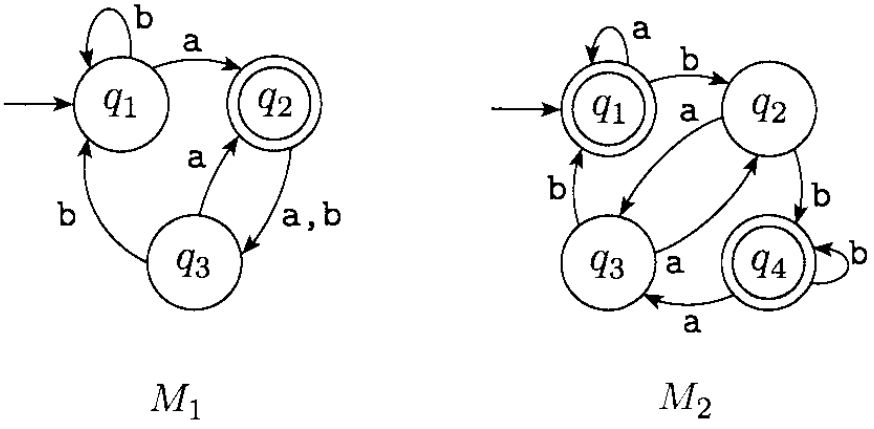

Figure 1: State diagrams of two DFAs M_1 and M_2

(a) What are the set of accept states for M_1 and M_2?

(b) What sequence of states do the machines M_1 and M_2 go through on an input `aabba`?

(c) Give the formal descriptions of the machines M_1 and M_2.

[2 + (2+2) + (2 + 2) = 10]

2. Give state diagrams of DFAs recognizing the following languages.

   (a) $\{w|w\}$ contains the substring 0101, that is, $w = x0101y$ for some x and $y\}$.

   (b) All strings except the empty string.

[5 + 5 = 10]

3. The symmetric difference between two sets A and B is defined as

   $$(A - B) \cup (B - A).$$

   If L_1 and L_2 be two regular languages (over some alphabet Σ), then show that their symmetric difference $(L_1 - L_2) \cup (L_2 - L_1)$ is also regular.

[10]

4. Let $\Sigma = \{a, b\}$ be an alphabet. Consider a language as follows:

   $L = \{\alpha \in \Sigma^* | \alpha$ starts with $aa,$ but it does not end with $aa\}$.

   Design a Deterministic Finite Automaton (DFA) whose language will be L.

[10]

5. Suppose that a MTM has four tapes and the tapes contents are shown below:
   tape 1: $0 \quad 1^{\downarrow} \quad 0 \quad 1 \quad 0 \quad \bigsqcup \ldots$
   tape 2: $a \quad a \quad a^{\downarrow} \quad \bigsqcup \ldots$
   tape 3: $b^{\downarrow} \quad a \quad \bigsqcup \ldots$
   tape 4: $x \quad y \quad z^{\downarrow} \quad \bigsqcup \ldots$
   where $\downarrow$ is the current tape position for a tape.

   Design a single-tape Turing machine, S for the above MTM.

[10]

6. Consider the state diagrams of two finite state transducers (FST), say T_1 and T_2 as shown in Figure 2. Given the sequence of states entered and the output produced in each of the following parts:

   (i) T_1 on input 020211

   (ii) T_2 on input `bbbbbbab`

[5 + 5 = 10]

7. We call a *write-once Turing machine* as a single-tape TM that can alter each tape square at most once (including the input portion of the tape). Show that this type of variant Turing machine model is equivalent to the ordinary Turing machine model.

[10]

8. Show that the collection of decidable languages is closed under complementation.

[10]

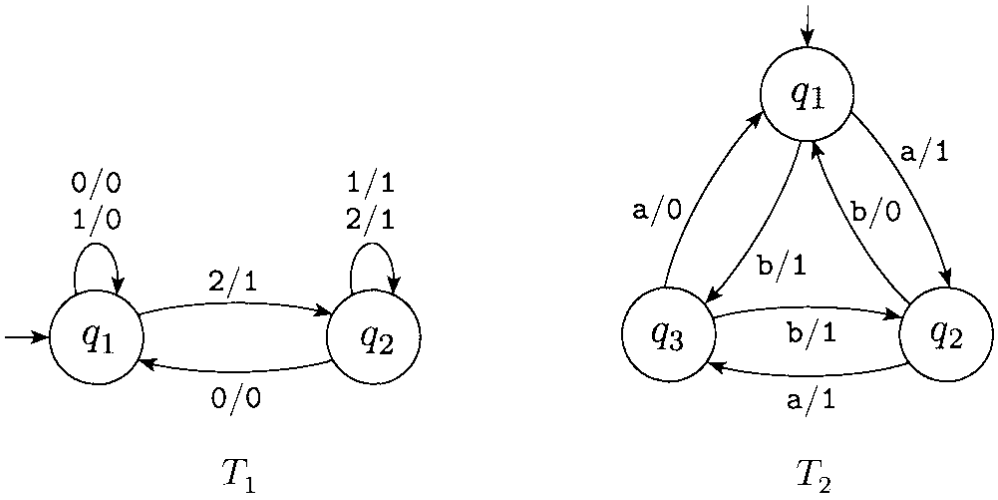

Figure 2: State diagrams of two DFAs M_1 and M_2

9. Show that the collection of Turing-recognizable languages is closed under star.

[10]

10. Prove that a language is Turing-recognizable if and only if some nondeterministic Turing machine recognizes it.

[10]

All the best!!!