# SCIENCE-I

## ASSIGNMENT-2

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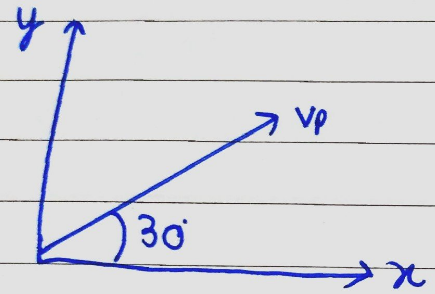
Q1 A particle is moving with speed 0.7c in x-y plane at angle 30° to x axis of an observer O. There is another observer O' who is travelling with speed −0.6c along common x-x' axis. Assuming O, O' are parallel to each other. What is angle that particle make with x' axis.




We are given two inertial frame O and O'. O' relative velocity wrt O : −0.6c [x-axis]

say $v_{O'} = -0.6c\hat{x}$,
In O frame velocity of particle is $V_P \cos 30° [x] + V_P \sin 30° [y]$

hence, $V_P = \frac{0.7\sqrt{3}c}{2}\hat{x} + \frac{0.7c}{2}\hat{y}$

we can now use lorentz transformation, since $V_{O'}$ is only in x direction hence $y' = y$; $z' = z$.

$$x' = \frac{x - v_{O'}t}{\sqrt{1 - \left(\frac{v_{O'}^2}{c^2}\right)}} , \quad t' = \frac{t - v_{O'}x/c^2}{\sqrt{1 - \frac{v_{O'}^2}{c^2}}}$$

$$V_P = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = \left(\frac{0.7\sqrt{3}c}{2}\hat{x} + \frac{0.7c}{2}\hat{y} + 0.\hat{z}\right)$$

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now since $y'=y$, $z'=z$ hence $dy'=dy$ and $dz'=dz$

$x' = \gamma(x - V_0' t)$

$dx' = \gamma(dx - V_0' dt)$

$$\frac{dx'}{dt} = \gamma\left(\frac{dx}{dt} - V_0'\right) = \gamma(V_{Px} - V_0') \quad \text{— (I)}$$

Similarly,

$$t' = \gamma\left(t - \frac{V_0' x}{c^2}\right)$$

$$dt' = (dt)(\gamma)\left(1 - \frac{V_0' V_{Px}}{c^2}\right)$$

$$\frac{dt'}{dt} = \gamma\left(1 - \frac{V_0' \cdot V_{Px}}{c^2}\right) \quad \text{— (II)}$$

(I)/(II) we get: $$\frac{dx'}{dt'} = \frac{\cancel{\gamma}(V_{Px} - V_0')}{\cancel{\gamma}\left(1 - \frac{V_0' \cdot V_{Px}}{c^2}\right)} = \frac{\frac{0.7\sqrt{3}c}{2} - (-0.6c)}{1 + \frac{(0.6)(0.7\sqrt{3}) \cdot \cancel{c^2}}{2 \cdot \cancel{c^2}}}$$

$$\boxed{V_{Px}' = \frac{(0.606 + 0.6)c}{1 + (0.363)} = \frac{(1.206)c}{(1.363)} = 0.884c} \quad \text{— (III)}$$

calculating velocity in y direction wrt O' frame.

$$V_{py'} = \frac{dy'}{dt'} = \frac{dy}{dt'} \quad (\text{since } dy' = dy)$$

$$= \frac{dy}{\gamma \cdot dt\left(1 - \frac{V_{o'}V_{px}}{c^2}\right)} = \frac{(dy/dt)}{\gamma\left(1 - \frac{V_{o'} \cdot V_{px}}{c^2}\right)}$$

$$V_{py'} = \frac{(0.7c/2) \cdot \sqrt{1 - \left(\frac{0.6c}{c}\right)^2}}{(1) \cdot \left(1 - \frac{(-0.6c)(0.7\sqrt{3}c)}{2c^2}\right)}$$

$$V_{py'} = \frac{(0.35c) \cdot \sqrt{1 - 0.36}}{(1)\left(1 + \frac{0.6 \times 0.7\sqrt{3}}{2}\right)} = \frac{(0.35)(\sqrt{0.64})c}{(1 + (0.3)(0.7)(\sqrt{3}))}$$

$$V_{py'} = \frac{0.28c}{(1 + (0.363))} = \frac{0.28}{1.363}c = 0.20c$$

$$\boxed{V_{py'} = 0.20c}$$

calculating velocity in z' direction

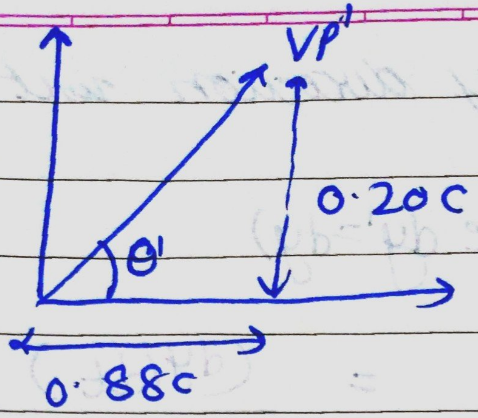
$$V_{pz'} = \frac{dz'}{dt'} = \frac{dz}{dt'} = \frac{dz}{\gamma \cdot dt\left(1 - \frac{V_{o'}V_{px}}{c^2}\right)} = \frac{dz/dt}{\gamma\left(1 - \frac{V_{o'} \cdot V_{px}}{c^2}\right)} = 0$$

hence,

$$V_{p'} = 0.88c\hat{i} + 0.20c\hat{j} + 0\hat{z}$$

$$V_{p'} = \sqrt{(0.88)^2 + (0.20)^2}\, c = \sqrt{(0.7744) + 0.04}\, c$$

$$\boxed{= 0.90c}$$




$$\tan\theta' = \frac{0.20c}{0.88c} = 0.227$$

$$\theta' = \tan^{-1}(0.227) = \boxed{12.7 \text{ degree}}$$

hence we get velocity observed by $\hat{O}$ as $\boxed{0.9c}$ which is $\boxed{0.88c\,\hat{x} + 0.20c\,\hat{y} + 0\,\hat{z}}$ making angle of 12.7 degree with x axis.

Q3) calculate momentum of Electron that has total Energy of 1MeV. Hint: MeV: unit of Energy, $1\,MeV = 1.603 \times 10^{-13}\,J$. Electron rest mass Energy $m_0 = 0.511\,MeV/c^2$.

using relativistic mechanics we know that momentum of particle $p = mv = m_0 \gamma \cdot v$

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ $\quad m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ → rest mass

Total Energy $= mc^2 = m_0 \gamma c^2 = \frac{m_0 \cdot c^2}{\sqrt{1 - v^2/c^2}}$

using relation that; $E^2 = (pc)^2 + (m_0 c^2)^2$

$$(1\,MeV)^2 = (pc)^2 + (0.511\,MeV)^2$$

$$\frac{1 - (0.511)^2}{c^2} = p^2$$

$$p = \sqrt{\frac{1 - (0.511)^2}{c^2}} = \frac{\sqrt{1 - (0.511)^2}}{c}$$

$$= \frac{\sqrt{0.738879}}{c} = \frac{0.8595}{c}\,MeV$$

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$$p = \frac{0.8595 \times 1.603 \times 10^{-13}}{3 \times 10^{8}} = 0.459 \times 10^{-21} \text{ kgm/sec}$$

$$= \boxed{4.6 \times 10^{-22} \text{ kgm/sec}}$$

Q2 Consider a particle of rest mass mo — — — — — — —

— — — — — — — — — — — — — — with correct units

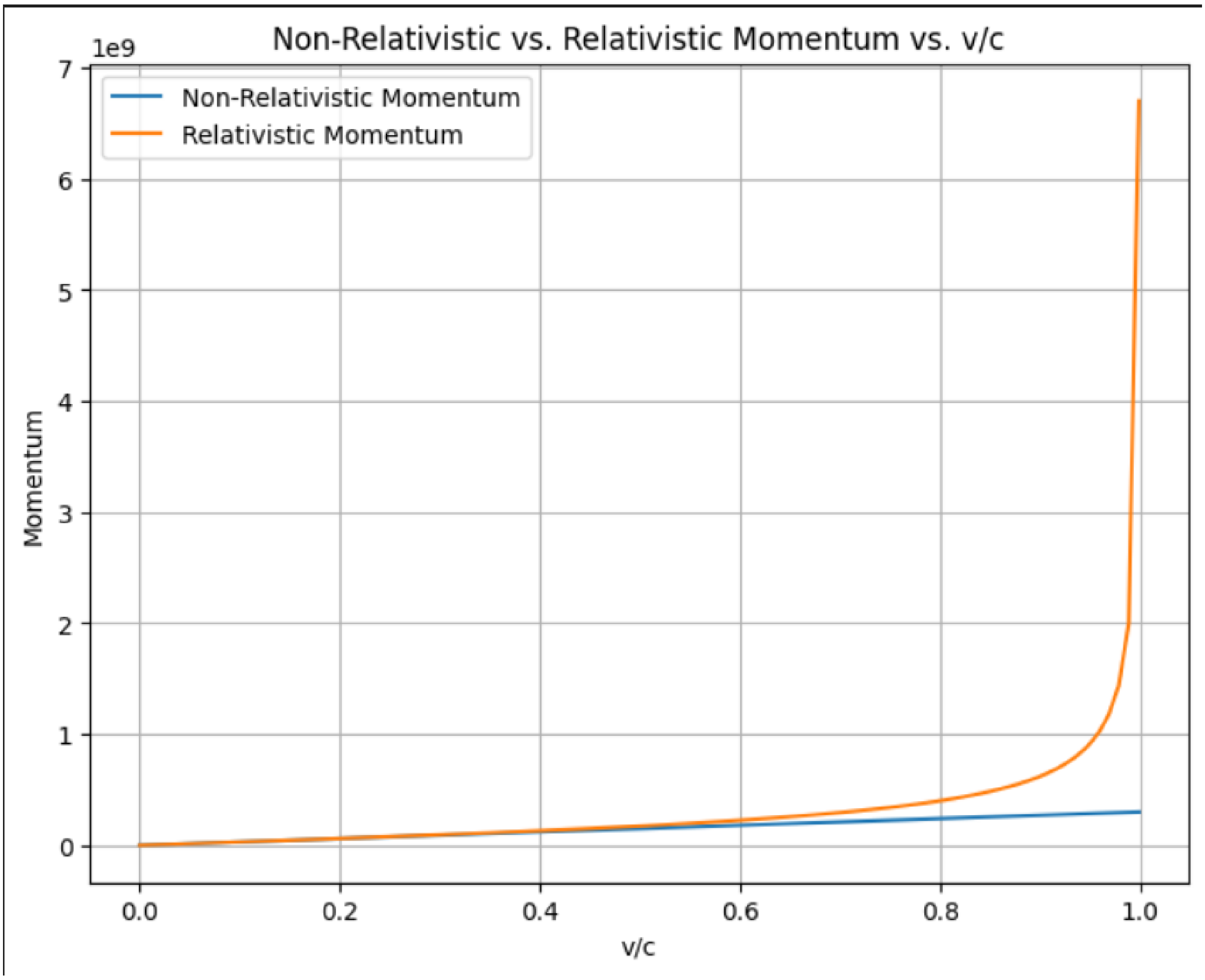
→ Difficulty in travelling with speed of light:

Momentum: from plot we can observe that as an object velocity increases to speed of light i.e (v/c) approaches to 1. Its relativistic momentum approaches infinity. This means as object approaches speed of light it becomes increasing difficult to accelarate it further hence momentum required become astronomically large.

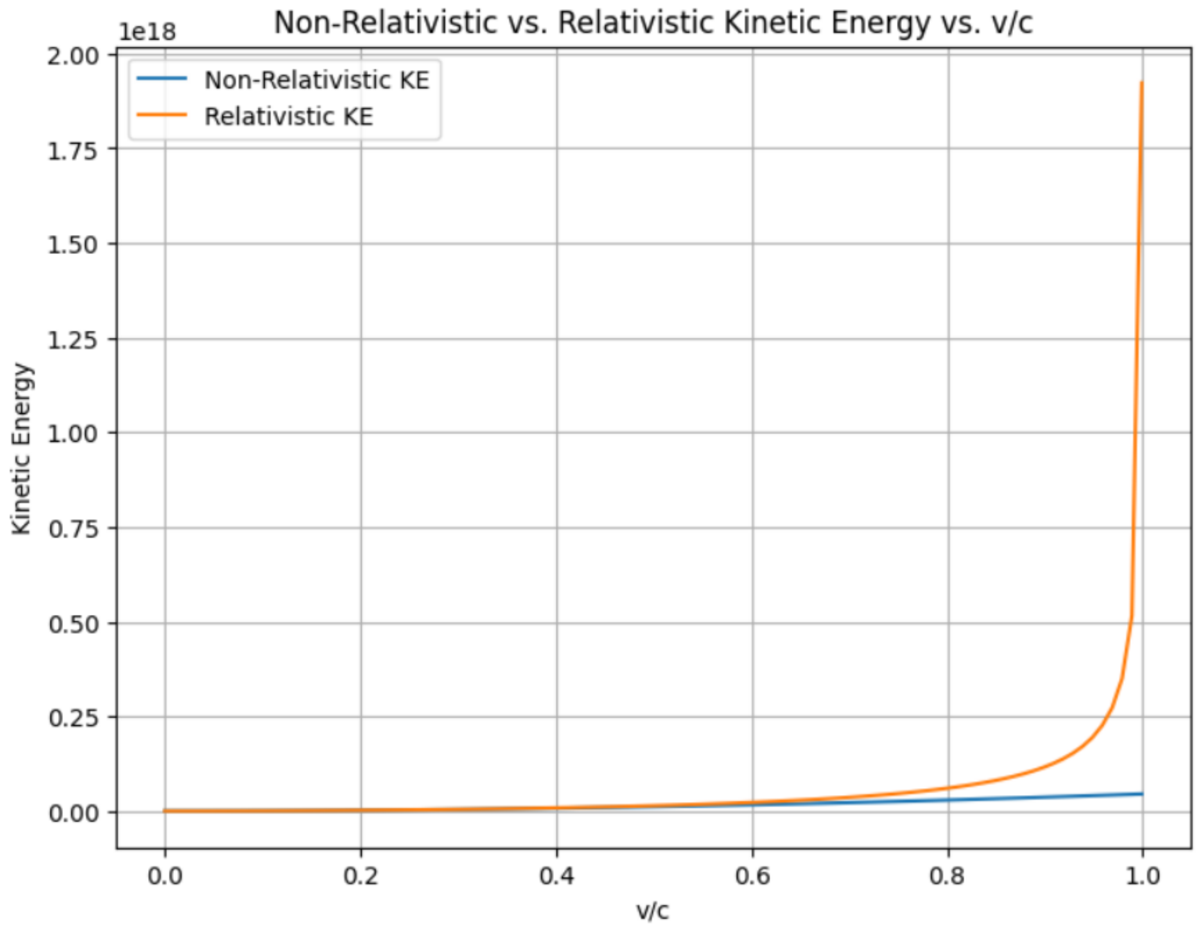
Kinetic Energy: Plot of kinetic Energy vs v/c demonstrate that relativistic kinetic Energy increase significantly as v/c approaches 1. hence as object reaches speed of light it gains enormous amount of kinetic Energy leading to massive increase in effective mass

# CODE:

```
m0 = 1
c = 3e8

v_over_c = np.linspace(0, 0.999, 100)

velocity = v_over_c * c
non_relativistic_momentum = m0 * velocity
relativistic_momentum = m0 * velocity/ np.sqrt(1 - v_over_c**2)


plt.figure(figsize=(8, 6))
plt.plot(v_over_c, non_relativistic_momentum, label='Non-Relativistic Momentum')
plt.plot(v_over_c, relativistic_momentum, label='Relativistic Momentum')
plt.xlabel('v/c')
plt.ylabel('Momentum')
plt.title('Non-Relativistic vs. Relativistic Momentum vs. v/c')
plt.legend()
plt.grid(True)

plt.show()
```




```
non_relativistic_kinetic_energy = 0.5 * m0 * ((velocity)**2)
gamma = 1 / np.sqrt(1 - v_over_c**2)
relativistic_kinetic_energy = (gamma - 1) * m0 * (c**2)

plt.figure(figsize=(8, 6))
plt.plot(v_over_c, non_relativistic_kinetic_energy, label='Non-Relativistic KE')
plt.plot(v_over_c, relativistic_kinetic_energy, label='Relativistic KE')
plt.xlabel('v/c')
plt.ylabel('Kinetic Energy')
plt.title('Non-Relativistic vs. Relativistic Kinetic Energy vs. v/c')
plt.legend()
plt.grid(True)

plt.show()
```

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**<u>Non-relativistic (Linear) Momentum vs. v/c and Relativistic Momentum vs. v/c:</u>**
These plots show how the momentum of a particle changes as its velocity increases relative to the speed of light (v/c). The non-relativistic momentum, given by `p = m * v`, increases linearly with velocity. However, the relativistic momentum, which considers the effects of special relativity, follows a different trend. It starts to deviate from linearity as v/c approaches 1. This means that as a particle gets closer to the speed of light, the increase in momentum becomes less significant for a given increase in velocity. In practical terms, it becomes increasingly difficult to accelerate the particle further as it gets closer to the speed of light.

**<u>Non-relativistic Kinetic Energy vs. v/c and Relativistic Kinetic Energy vs. v/c:</u>**

These plots illustrate how the kinetic energy of a particle changes with its velocity relative to the speed of light. The non-relativistic kinetic energy, `KE = 0.5 * m * v^2`, increases quadratically with velocity. In contrast, the relativistic kinetic energy, `KE = (γ - 1) * m * c^2`, shows a sharp increase as v/c approaches 1. This indicates that as a particle's velocity nears the speed of light, its kinetic energy grows substantially. This poses a fundamental challenge because it implies that an infinite amount of energy would be required to accelerate a massive object to the speed of light, which is clearly unattainable with finite resources.

In conclusion, these plots visually demonstrate the key challenges of approaching the speed of light:

- **Increasing Momentum:** As a particle's velocity approaches the speed of light, the amount of momentum needed to continue accelerating grows significantly, making it impractical to provide the necessary force to maintain this acceleration.
- **Increasing Energy:** The relativistic kinetic energy rises without limit as v/c approaches 1. To reach or surpass the speed of light, an object would require an infinite amount of energy, which is impossible to achieve in practice due to resource constraints.