

# International Institute of Information Technology Hyderabad

## Modern Complexity Theory (CS1.405)

### Assignment 3

**Deadline: November 4, 2023 (Saturday), 17:00 PM**

**Venue for Submission: CSTAR, A3-110, Vindhya Block, IIIT Hyderabad**

**Total Marks: 100**

**NOTE:** It is strongly recommended that no student is allowed to copy from others.

No assignment will be taken after deadline.

Write the following while submitting ONLY HARDCOPY:

Modern Complexity Theory (CS1.405)

Assignment 3

Name:

Roll No.:

1. Show that PSPACE is closed under the operations complementation and star.  
[10 + 10 = 20]
2. Show that any SPACE-hard language is also NP-hard.  
[10]
3. Show that TQBF restricted to formulas where the part following the quantifiers is in CNF (conjunctive normal form) is still PSPACE-complete.  
[20]
4. Define  $UCYCLE = \{\langle G \rangle \mid G \text{ is an undirected graph that contains a simple cycle}\}$ . Show that  $UCYCLE \in L$ , where  $L$  is the class of languages that are decided in logarithmic space on a deterministic Turing machine (DTM), that is,  $L = SPACE(\log n)$ .  
[10]
5. A *linear bounded automaton (LBA)* is a one-tape one-head nondeterministic TM with the tape finite and just big enough to hold the entire input. If an LBA tries to move its head outside its tape at either end, some mechanism prevents it from doing so, and the head remains in the old position. Deduce that the language

$$A_{LBA} := \{\langle M, \alpha \rangle \mid \text{LBA } M \text{ accepts } \alpha\}$$

is PSPACE-complete.

[20]

6. The generalized tic-tac-toe game is played on an  $n \times n$  board with player  $X$  starting and with players  $X$  and  $O$  making alternate moves. The player who first places his marker in five consecutive cells in a row, column or diagonal wins. The game ends in a draw if no such sequence is present when all of the  $n^2$  cells are marked. Show that the language

$GT := \{\langle c \rangle \mid c \text{ is an intermediate configuration of the board with next move by } X \text{ and with a winning strategy for } X\}$

is in PSPACE. (Hint: Use recursion.)

[20]

**All the best!!!**