Homework 6 writeups

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Section: AMATH 301 B

Problem 1

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy
```

Part a - Create a meshgrid

In [ ]:
```
# Define the theta values
theta_span = np.linspace(-3 * np.pi, 3 * np.pi, 25)

# Define the v values
v_span = np.linspace(-3, 3, 25)

# Create the mesh
theta, v = np.meshgrid(theta_span, v_span)
```

Part b - Create a quiver plot

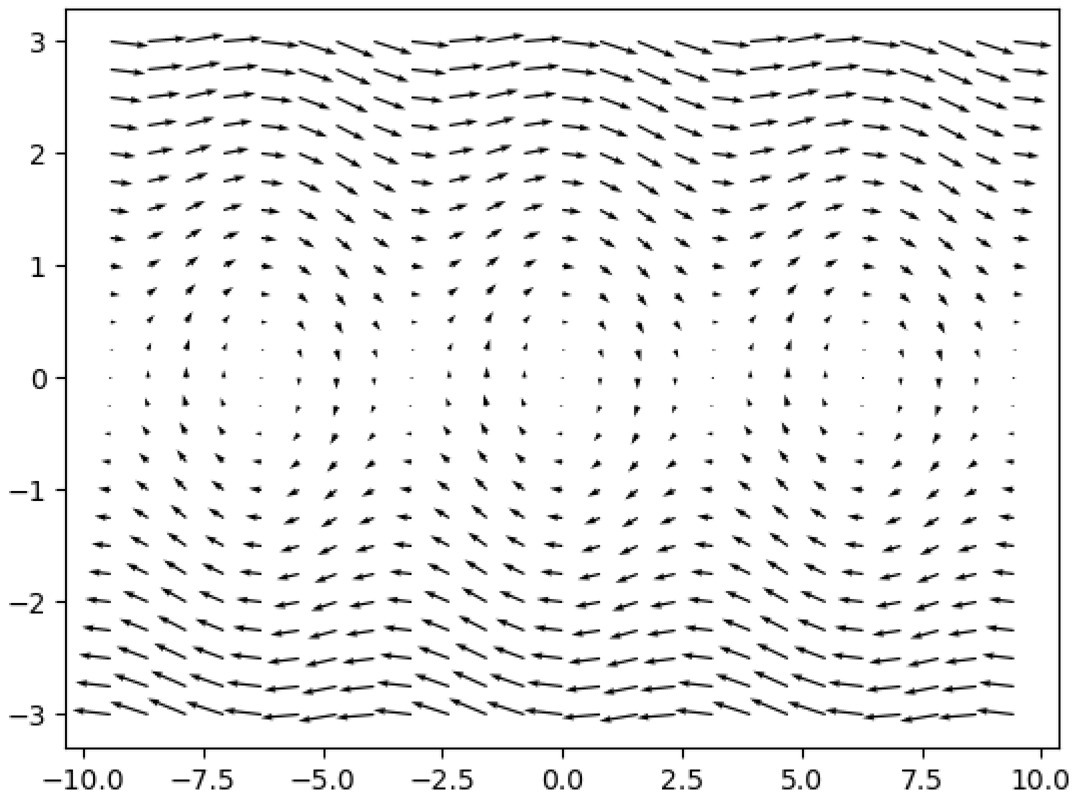
In [ ]:
```
fig, ax = plt.subplots()

g = 9.8
L = 11
sigma = 0.12

theta_prime = lambda theta, v: v
v_prime = lambda theta, v: -g/L*np.sin(theta) - sigma*v

ax.quiver(theta, v, theta_prime(theta, v), v_prime(theta, v))
```

Out[ ]:
```
<matplotlib.quiver.Quiver at 0x7fd558a5eeb0>
```



Part c - Label the axes. Here you should have the complete quiver plot.

In [ ]:
```
ax.set_xlabel(r'$\theta$')
ax.set_ylabel(r'$v$')
ax.set_title('Phase Portrait')
```

Out[ ]:
```
Text(0.5, 1.0, 'Phase Portrait')
```

Part d - Include trajectories.

In [ ]:
```
odefun = lambda t, p: np.array([theta_prime(p[0], p[1]), v_prime(p[0], p[1])])
tspan = np.linspace(0, 50, 5000)

sol1 = scipy.integrate.solve_ivp(odefun, np.array([tspan[0], tspan[-1]]), np.ar
sol2 = scipy.integrate.solve_ivp(odefun, np.array([tspan[0], tspan[-1]]), np.ar
sol3 = scipy.integrate.solve_ivp(odefun, np.array([tspan[0], tspan[-1]]), np.ar
sol4 = scipy.integrate.solve_ivp(odefun, np.array([tspan[0], tspan[-1]]), np.ar

ax.plot(sol1.y[0, :], sol1.y[1, :], 'r')
ax.plot(sol2.y[0, :], sol2.y[1, :], 'g')
ax.plot(sol3.y[0, :], sol3.y[1, :], 'b')
ax.plot(sol4.y[0, :], sol4.y[1, :], 'y')
```

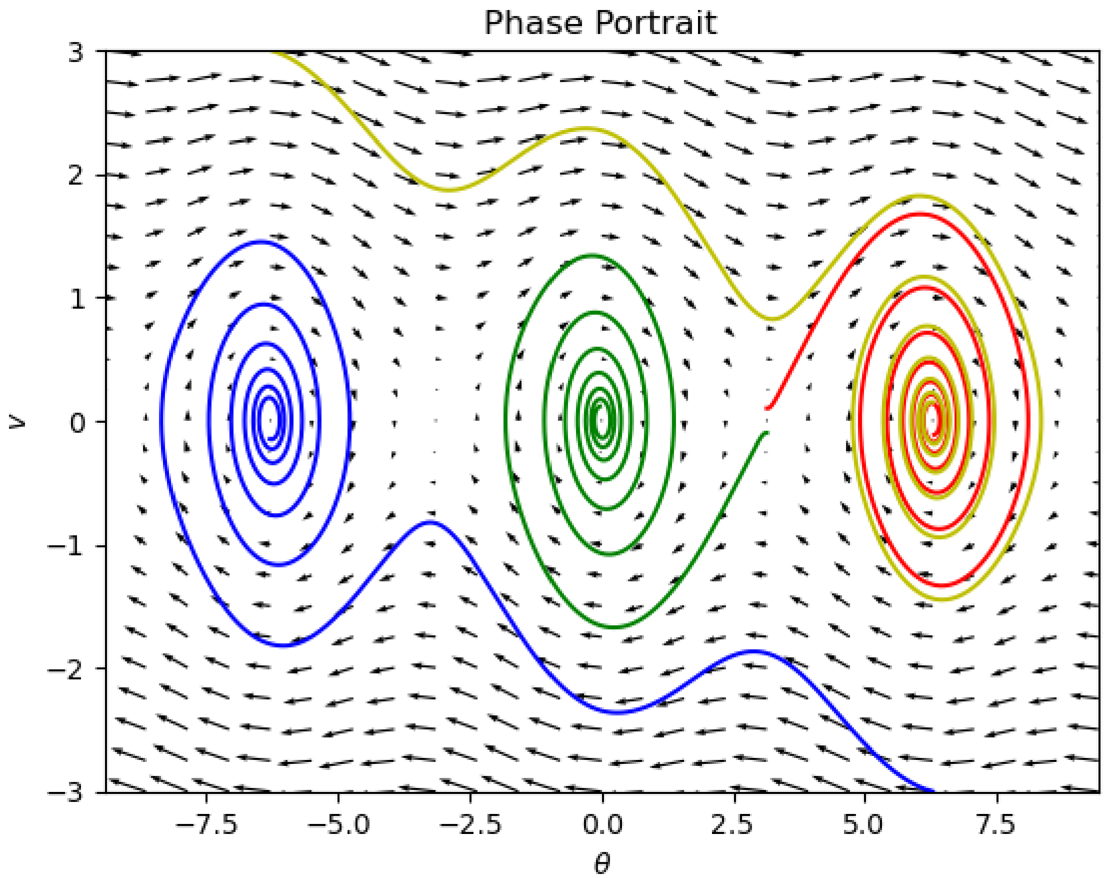
Out[ ]:
```
[<matplotlib.lines.Line2D at 0x7fd579d40c40>]
```

Part e - axis display

In [ ]:
```
ax.set_xlim([-3*np.pi, 3*np.pi])
ax.set_ylim([-3, 3])

fig
```

Out[ ]:



Part f - Discussion

Part (i) - long-term behavior

As time approaches infinity, the velocity of the pendulum will approach 0 for any given θ_0 due to the damping effect from friction, air resistance, or other forces.

Part (ii) - Comparing two solutions with $\theta_0 = \pi$.

In both the solutions, $\theta_0 = \pi$, which implies that the pendulum starts held exactly top of the hinge. As velocity is positive 0.1 in the first case, the pendulum will start swinging anticlockwise because θ increases as t increases, resulting in positive velocity. On the other hand, in the second case, we have negative 0.1 as the velocity, which means that the pendulum starts swinging clockwise because θ decreases as t increases, resulting in negative velocity. Essentially, the pendulum starts at the same initial position and swings with the same speed (magnitude of velocity is equal) in both cases but in the opposite directions.

Part (iii) - Comparing two solutions with equal and opposite θ_0 and v_0.

In the two solutions here, $(\theta, v) = (2\pi, -3)$ and $(-2\pi, 3)$, the pendulum starts held exactly below the hinge because $\theta = 2\pi, -2\pi$ have the same position as $\theta = 0$ as they represent the θ value after completing one full circle around the hinge anticlockwise and clockwise respectively. In the first case, we have initial velocity negative 3, which means that the pendulum will swing to the left (in clockwise direction) because decreasing θ with increasing t will result in negative velocity. On the other hand, in the second case, we have initial velocity positive 3, which means that the pendulum will again swing to the left (in clockwise direction) because decreasing negative θ with increasing t will result in positive velocity. Therefore, same as part (ii), the pendulum starts at the same initial position and swing at the same speed (magnitude of velocity is equal) in both cases, but in the same direction too here.