UNIVERSITY OF CALIFORNIA, BERKELEY

EE-120: Lecture 7

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July 16, 2021

1 More Continuous Time Fourier Transforms

Again, consider the following time- frequency signal pairs $x(t), X(\omega)$ where x is a signal and X is the corresponding Fourier transform of x.

- Derivative properties:
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  $$\frac{x(t)}{dt} \leftrightarrow j\omega X(\omega) \tag{1.1}$$
  -
  $$-jtx(t) \leftrightarrow \frac{dX(\omega)}{d\omega} \tag{1.2}$$

  Note the duality that takes place between the time and frequency domains.

- Frequency shifting:
  $$e^{j\omega_0 t}x(t) \leftrightarrow X(\omega - \omega_0) \tag{1.3}$$

  Again, take note to how this is dual to shifting in the time domain.

- Multiplication property:
  $$s(t)p(t) \leftrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\theta)P(\omega-\theta)d\theta = \frac{1}{2\pi}(S * P)(\omega) \tag{1.4}$$

  Take note how this is dual to the convolution property we stated earlier.

2 LTI Systems

Recall that we denote the impulse response of a signal y as h where $h(t)$ is the response of signal y when $x(t) = \delta(t)$. Also recall that we defined the frequency response of a signal y as equal to the following:

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t} dt \tag{2.1}$$

This is simply the fourier transform of the impulse response h!

3 Frequency Response of Continuous Time LTI Systems

We know bring our attention to LTI systems of the following form:

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \tag{3.1}$$

With our newfound knowledge of FT properties, we can find the Frequency response of the following signal quite easily!
By our derivative properties we know the Fourier Transform is as follows:

$$\sum_{k=0}^{N} a_k (j\omega)^k Y(j\omega) = \sum_{k=0}^{M} b_k (j\omega)^k \tag{3.2}$$

We also know that $(x * h)(t) = y(t) \to Y(\omega) = X(\omega)H(\omega)$ by our convolution property.

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{N} a_k (j\omega)^k}{\sum_{k=0}^{M} b_k (j\omega)^k} \tag{3.3}$$

Which $H(j\omega)$ is a simple form for the frequency response of the signal when dealing with nth order differential systems.