
EE-120: Lecture 14

Oscar Ortega

July 16, 2021

1 REVIEW OF THE SAMPLING THEOREM

Setup:

We take samples of a continuous-time signal x every 2 seconds:

$$\omega_s = \frac{2\pi}{T} \quad (1.1)$$

We can reconstruct $x(t)$ from the samples $\{x(nT)\}_{n \in \mathbb{Z}}$ if it is the case that for $\omega \notin (-\omega_s, \omega_s)$: $X(\omega) = 0$ and that the sampling frequency $\omega_s > 2\omega_M$ then x_r defined as follows is equal to x

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT) \text{sinc}\left(\frac{t-nT}{T}\right) \quad (1.2)$$

2 FOURIER TRANSFORM OF THE SAMPLED SIGNAL

We can define $x_d[n] = x(nT)$ which as we can see is a discrete-valued time signal. We can relate the DTFT to the CTFT as follows:

$$X_d(e^{j\Omega})|_{\Omega=\omega T} = X_p(\omega) \quad (2.1)$$

Note: because X_p and X_d are defined as follows

$$X_p(\omega) = \sum_{n=-\infty}^{\infty} x_d[n] e^{-j\omega T n} \quad (2.2)$$

$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_d[n] e^{-j\Omega n} \quad (2.3)$$

We can see that X_d is equal to the following:

$$X_d(e^{j\Omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega T - 2\pi k) \quad (2.4)$$