UNIVERSITY OF CALIFORNIA, BERKELEY

EE-120: Lecture 9

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1 MORE PROPERTIES OF THE DTFT

Again note that x, X are corresponding pairs of signals and their respective Fourier Transform.

- Conjugation and Conjugate Symmetry Property:
  $x^*[n] \leftrightarrow X^*(e^{-j\omega})$ Again, note that if we have a signal x that is even symmetric, then this implies that the Fourier transform $X(e^{j\omega})$ is real valued.

- Time Expansion: If we define the time expended signal $x_{(M)}[n]$ as follows:

$$x_{(M)}[n] = \begin{cases} x[n/M] : n = 0, \pm M, \pm 2m, \ldots \\ 0 : \text{else} \end{cases} \tag{1.1}$$

  Then, $x_{(M)}[n] \leftrightarrow X(e^{j\omega M})$

- Differentiation In Frequency: $nx[n] \leftrightarrow j\frac{dX(e^{j\omega})}{d\omega}$ Note that there is no counterpart to differentiation in time domain, but the following would be the closest counterpart.
  $x[n] - x[n-1] \leftrightarrow (1 - e^{-j\omega})X(e^{j\omega})$

- Parseval's relation:
  $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$

- Multiplication Property:
  $x_1[n]x_2[n] \leftrightarrow \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$ where the integral is known as the periodic convolution of the signal x.

- Convolution Property: $(x_1 * x_2)[n] \leftrightarrow X_1(e^{j\omega})X_2(e^{j\omega})$

1.1 Finding the Frequency Response from a Difference Equation

Consider an LTI system of the following form:

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \tag{1.2}$$

We can find the frequency response of this signal quite easily!
If we recall that the frequency response of a signal is the Fourier Transform of the impulse response, let $x[n] = \delta[n]$ and let $y[n] = h[n]$, then the equation becomes the following:

$$\sum_{k=0}^{N} a_k h[n-k] = \sum_{k=0}^{M} b_k \delta[n-k] \tag{1.3}$$

Note that each $a_k h[n-k]$ term has $a_k e^{-j\omega k} H(e^{j\omega})$ as its corresponding Fourier transform an recall that $b_k \delta[n-k]$ has $b_k e^{-j\omega k} \cdot 1$ as its corresponding Fourier Transform pair. This leads to the following expression if we take the Fourier Transforms of [1.3]:

$$\sum_{k=0}^{N} a_k e^{-j\omega k} H(e^{j\omega}) = \sum_{k=0}^{M} b_k e^{-j\omega k} \tag{1.4}$$

This leads to the following expression for the frequency response:

$$H(e^{j\omega}) = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{k=0}^{N} a_k e^{-j\omega k}} \tag{1.5}$$