UNIVERSITY OF CALIFORNIA, BERKELEY

EE-120: Lecture 14

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1 QUICK SUMMARY

Recall:
If we have a signal that is not LTI, but is linear and the sampling from the analogue to digital conversion does not cause aliasing, then we can in fact reconstruct $Y(\omega) = H_d(e^{j\omega T})X(\omega)$

2 EXAMPLES

Ideally: We chose $y(t) = \frac{d}{dt}x(t)$
We also know that if we are sampling a signal at intervals of T, we can approximate the derivative at time T with the slope of the secant line between the two points. In other words, we have the following signal:

$$y_d[n] = \frac{x_d[n+1] - x_d[n]}{T} \tag{2.1}$$

What is the frequency response of this signal?
Well, what is the impulse response?

$$h_d[n] = \frac{\delta[n+1] - \delta[n]}{T} \tag{2.2}$$

The fourier response is just the DTFT of the impulse response!

$$\delta[n] \leftrightarrow 1 : \delta[n+1] \leftrightarrow e^{j\Omega} \tag{2.3}$$

$$H_d(e^{j\Omega}) = \frac{1}{T}(e^{j\Omega} - 1) \tag{2.4}$$
$$= \frac{1}{T}e^{j\frac{\Omega}{2}}(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}) \tag{2.5}$$
$$= 2j\sin(\frac{\Omega}{2}) \tag{2.6}$$

Recall from the hw, that the ideal differentiator has frequency response $H(j\omega) = j\omega$, higher frequencies are attenuated a bit, but it seems okay if we our signals don't change too quickly.

3 Delaying Input Signal

We want $y(t) = x(t - \triangle)$ eg: $\delta = T$ this will correspond to a discrete time $y_d[n] = x_d[n-1]$ But what if $\frac{\triangle}{}$ is not an integer? Well, an equivalent formulation is to try and recreate $H(\omega) = e^{-j\omega\triangle}$ Our goal becomes to try and recreate $H_d(e^{j\Omega}) = H(\omega)|_{w=\frac{\Omega}{T}} = e^{-j\frac{\triangle}{T}\Omega}$ Now we can invert the Fourier transform to find the impulse response.

$$h_d[n] = \text{sinc}(n - \frac{\triangle}{T}) \tag{3.1}$$

In general, when dealing with non-integer shifts, we know the following:

$$y_d[n] = (h_d * x_d)[n] \tag{3.2}$$
$$= \sum_k x_d[k]h_d[n-k] \tag{3.3}$$
$$= \sum_k x_d[k]\text{sinc}(n - \frac{\triangle}{T} - k) \tag{3.4}$$
$$\tag{3.5}$$

4 Sampling of Discrete-Time Signals

We obtain samples of a discrete-time signal x[n] by multiplying it with a discrete-time impulse train with period N:
eg: with period equal to 2, we only keep the odd-valued lolipops. This practice is known as "down-sampling if you don't store the zero-valued signals in the middle of the impulses that are kept. Otherwise, this is known as *impulse train sampling*. The same reconstruction sufficiency holds in the discrete case. To *upsample* a signal, it means to stretch the signal by a factor of N and pad zeroes in between the non-zero values of the signal. What are some applications of down-sampling and upsampling? Well, zooming in or sooming out of a camera is an example of down/up-sampling of a 2d signal.

5 2D sampling

Given $x(t_1, t_2)$ and sampling periods T_1, T_2

$$x_d[n_1, n_2] = x(n_1T_1, n_2T_2) \tag{5.1}$$

We define $x_p = x(t_1, t_2)p(t_1, t_2)$ where p is the 2d impulse train defined as follows:

$$p(t_1, t_2) = \sum_{n_1}\sum_{n_2}\delta(t_1 - n_1 T_1, t_2 - n_2 T_2) \tag{5.2}$$

This gives us the following 2d CTFT:

$$X_p(\omega_1, \omega_2) = \frac{1}{T_1 T_2}\sum_{K_1}\sum_{K_2} X(\omega_1 - k\omega_{s_1}, \omega_2 - k_2\omega_{s_2}) \tag{5.3}$$

The perfect sinc interpolation will be expressed as follows:

$$x_r(t_1, t_2) = \sum_{k=-\infty}^{\infty} x(n_1 T_1, n_2 T_2)\text{sinc}(\frac{t_1 - n_1 T_1}{T_1})\text{sinc}(\frac{t_2 - n_2 T_2}{T_2}) \tag{5.4}$$

As usual, $x_r = x$ does not perform aliasing which will be dependent wether the Shannon Nyquist conditions hold in both dimensions of the current signal.