UNIVERSITY OF CALIFORNIA, BERKELEY

# EE-120: Lecture 4

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## 1 Fourier Series

Given the following definitions of periodicity and of $\omega_0$, we can define the fourier series as the the following weighted sum of sinusodials $e^{jk\omega_0 t} : k = 0 \pm 1, \dots \pm \infty$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \tag{1.1}$$

This equation is known as the **synthesis equation** because it synthesizes a periodic signal from sinusoidal components.

- The $k = 0$ term is referred as the **DC** component
- the $k = \pm i$'th terms are referred to the ith harmonics of the Fourier series.

### 1.1 Conjugate Symmetry Property

If $x(t)$ has Fourier series coefficients $a_k$ then $x^*(t)$ has Fourier series coefficients $b_k = a^*_{-k}$
Corollary:
If x is real-valued, then $a_k = a^*_{-k}$

### 1.2 How to we even find $a_k$ though?

$$\int_0^T x(t)e^{-jn\omega_0 t} dt = \int_0^T \left( \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t} dt \right) \tag{1.2}$$

$$= \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\omega_0 t} dt \tag{1.3}$$

$$\int_0^T e^{j(k-n)w_0 t} dt = \begin{cases} T : k = n \\ 0 : k \neq n \end{cases} \tag{1.4}$$

$$= \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)w_0 t} dt = T a_n \tag{1.5}$$

Which motivates the following equation.

$$a_n = \frac{1}{T} \int_0^T x(t) e^{-jnw_0 t} dt \tag{1.6}$$

This is known as the **analysis equation**. Take note that the DC component is simply the average of $x(t)$ over one period.

## 1.3 Convergence of Fourier Series

Let $x_m(t)$ be defined as the following:

$$x_m(t) = \sum_{k=-m}^{m} a_k e^{jkw_0 t} \tag{1.7}$$

Up until this point we have taken our synthesis equation for granted. However is it true that $\lim_{m\to\infty} x_m(\tau) = x(\tau)$?

Theorem: Suppose x is piecewise continuous with piecewise continuous derivative, and periodic with fundamental period T and frequency $\omega_0$. Then if x is continuous at $t = \tau$ then:

$$\lim_{m\to\infty} x_m(\tau) = x(\tau) \tag{1.8}$$

If x is discontinuous at $t = \tau$, then:

$$\lim_{m\to\infty} x_m(\tau) = \frac{1}{2} x(\tau^-) + x(\tau^+) \tag{1.9}$$

Where $x(\tau^-)$ and $x(\tau^+)$ are the left and right hand limits of the function respectively. Note that because the pointwise limit defined is simply the average of the left and right hand limits, our convergence result does not reach any contradictions when dealing with oscillatory phenomena such as **Gibbs Phenomenom**

## 1.4 Properties of the Fourier Series

- Linearity: If two signals x,y with identical periods have FS coefficients $a_k, b_k$, the Ax + By has FS coefficients $Aa_k + Bb_k$
- Time shift: If x has FS coefficients $a_k$, then $\hat{x}(t) = x(t - t_0)$ has FS coefficients $a_k e^{-jk\omega_0 t_0}$
- Time Reversal: If $x$ has FS coefficients $a_k$, then $\hat{x}(t) = x(-t)$ has FS coefficients $a_{-k}$, So if x is even symmetric and x is real valued, then we can say $a_k = a_{-k} = a_k^*$. This means the coefficients are real valued.