UNIVERSITY OF CALIFORNIA, BERKELEY

EE-120: Lecture 10

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1 FIR FILTER DESIGN BY WINDOWING

Lets define the ideal low-pass filter $h[n]$ as follows:

$$h[n] = \frac{w_c}{\pi}\text{sinc}(\frac{w_c}{\pi}n) \tag{1.1}$$

Which has the periodic box fourier transform of height 1, width $2w_c$ and is of period 2π.
Let $w[n]$ be defined as follows:

$$w[n] = \begin{cases} 1 : |n| \leq N_1 \\ 0 : \text{otherwise} \end{cases} \tag{1.2}$$

We can consider to w to be a sort of window function on the impulse response of our signal. In other words our resultant signal $\hat{h}$ will be defined as follows.
Let $\hat{h}[n] = h[n]w[n]s$, a truncated version of $h[n]$. This means our frequency response $\hat{H}(e^{j\omega})$ is as follows:

$$\hat{H}(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} H(e^{j\theta})W(e^{j(\omega-\theta)})d\theta \tag{1.3}$$

(Recall the multiplication property of Fourier Transforms). As we increase the length of the window, the closer we get to achieving the ideal LPF after windowing.
In summary: to obtain a FIR filter we can truncate the ideal filter's impulse response with these window functions. However, to make it causal we shift to the right by the length of the window. Recall the following:

$$\hat{h}[n - N_1] = e^{-j\omega N_1}\hat{H}(e^{j\omega}) \tag{1.4}$$

To make the dc gain $\hat{H}(e^{j0}) = 1$ This is equivalent to scaling the impulse response such that $\sum_n \hat{h}[n] = \sum_n \hat{h}[n]e^{-j\omega n}|_{n=0} = 1$
This means we can implement this as a FIR filter of the following form:

$$y[n] = \sum_{i=1}^{M} \hat{h}[i - N_1]x[n-i] \tag{1.5}$$

2 Discrete Fourier Transform

The discrete fourier transform of a sequence $x[n]$ is a sequence $X[k]$ of the same length in the frequency domain where $X[k]$ is defined as the following.

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} \tag{2.1}$$

3 Connection to Discrete Fourier Series

If we define a period-N sequence $\hat{x}[n]$ by sticking $x[n]$ end to end, then $\hat{x}[n] = x[n \mod N]$ and $X[k] = Na_k : k \in \{0, 1, ..., N-1\}$
We define the synthesis equation for the DFT as follows:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn} : n \in \{0, 1, ..., N-1\} \tag{3.1}$$

4 Conjugate Symmetry in the DFT

If $x[n]$ is real valued then

$$X^*[N-k] = X[k] : k = \{0, 1, ..., N-1\} \tag{4.1}$$