UNIVERSITY OF CALIFORNIA, BERKELEY

# EE-120: Lecture 1

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## 1 DEFINITIONS

- Signal: Function of one or more variables. In this class we usually deal with only one dimensional signal.
- Note that we will typically denote continuous time signals as functions of time $t$ in the following way: $x(t)$. We will reserve discrete time signals as functions of $n$ in the following way: $x[n]$

## 2 TRANSFORMATIONS OF SIGNALS

- Drag: if we consider the signal $y(t) \rightarrow x(t-T) : T \in \mathbb{F}$, we can intuitively think of the following variable transformation as moving the new signal $y$ forward by $T$ time-steps. This is because the signal at $y(t=t') = x(t=t'-T)$
- Flip: if we consider the signal $y(t) = x(-t)$, we can intuitively think of the following variable transformation as flipping the new signal.
  In other words, the signal at $y(t=t') = x(t=-t')$

## 3 MORE COMPLEX VARIABLE TRANSFORMATIONS

We can also consider the following composition of variable transformations and the effect they have on signals. Also take note that the following operations are not commutative.

- Drag, then Flip:
  1. Consider an intermediary signal $y(t) = x(t - T)$. As you can see this is a dragging of the original time steps by $T$ units.
  2. So if we consider the new signal $z(t) = y(-t)$. This is just a flipping of the new signal.
     In other words: $z(t) = y(-t) = x(-(t - T)) = x(-t + T)$
- Flip, then drag
  1. Consider an intermediary signal $y(t) = x(-t)$. As you can see this is a flip of the original signal.
  2. So if we consider the new signal $z(t) = y(t-T)$. This is then just a dragging forward of the intermediary signal by $T$ time-steps.
     In other words $z(t) = y(t - T) = x(-t - T)$

# 4 Famous Signals

- Unit Impulse:

$$\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases} \tag{4.1}$$

- Unit Step:

$$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases} \tag{4.2}$$

- Take note of the following:

$$\delta[n] = u[n] - u[n-1] \tag{4.3}$$

$$u[n] = \sum_{i=0}^{\infty} \delta[n-i] \tag{4.4}$$

# 5 More Definitions

- Systems: a process by which "input" signals are transformed to "output" signals.
- Eg: the *moving average filter* which is defined as follows is an example of a system:

$$y[n] = \frac{1}{3}(x[n-1] + x[n] + x[n+1]) : \forall n \in \mathcal{N} \tag{5.1}$$

- This is also what is known as a *low-pass filter* as it reduces high-frequency signals.

# 6 PROPERTIES OF SYSTEMS

- Memorylessness: If the output of a given signal is not dependent on the past, then the signal is said to be *memoryless.*
  As an example, the *moving average filter* is not memoryless as the signal $y(t)$ depends on $x(t-1)$.

- Causal: A system is said to be causal iff the output only depends on present or past time.
  As an example, the moving average filter is also not causal as the output signal $y(t)$ is dependent on future input $x(t+1)$

- Stability: A system is said to be *stable* if bounded inputs generate bounded outputs.

- Unstable: more formally we can say a system is *unstable* if the following is true:

$$\exists t \in \{\mathbb{F}/\{-\infty,\infty\} \text{ s.t } y(t) = \infty \vee y(t) = -\infty \tag{6.1}$$

- examples of unstable systems include integrator - accumulator circuits defined as follows.

$$y(n) = \int_{-\infty}^{n} x(n)dn \tag{6.2}$$

$$y[n] = \sum_{-\infty}^{n} x[n] \tag{6.3}$$

- Time Invariance: A system is said to be time-invariant if a time shift in the input results in an identical time shift in the output for any input-output pair and any shift.
  In other words a signal is said to be *time-invariant* if the following holds:

$$y(t) = f(x(t)) \tag{6.4}$$

$$f(x(t-T)) = y(t-T) : \forall T \in \mathbb{F} \tag{6.5}$$

- Linearity: A system is said to be linear if two following properties hold for all input-output pairs:

$$cx(t) \rightarrow cy(t) \tag{6.6}$$

$$x_1(t) \rightarrow y_1(t) \text{ and } x_2(t) \rightarrow y_2(t) \text{ then } x_1 + x_2(t) \rightarrow y_1 + y_2(t) \tag{6.7}$$

# 7 LINEAR TIME-INVARIANT (LTI) SYSTEMS

A system that is both linear and time-invariant is referred to a *Linear Time-Invariant System* or LTI System.
Property: Knowing the response to a unit impulse of an LTI system is enough to predict the response to any other input.

Proof: Let $h[n]$ denote the *impulse response* of an LTI system to the unit impulse $\delta[n]$

$$x[n] = \sum_{i=-\infty}^{\infty} x[i]\delta[n-i] \tag{7.1}$$

Since $\delta[n] \to h[n]$ and because $\delta[n-i] \to h[n-i] : \forall i$ by the time-invariance of the system.

$$y[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]. \tag{7.2}$$

The operation defined above is known as the *convolution* of the two signals $x, h$ and is denoted as $(x * h)$. In other words what we have proved is that for any discrete LTI system: $y[n] = (x * h)[n]$.