UNIVERSITY OF CALIFORNIA, BERKELEY

EE-120: Homework 4

Oscar Ortega

November 3, 2019

PROBLEM 1

If we consider the fact the fan is actually rotating clockwise, the slowest the fan can go is $\frac{3}{4}$ of a revolution every $\frac{1}{24}$ of a second. Given our samples, this is equivalent to performing $\frac{7}{4}$ of a turn or $\frac{11}{4}$ of a turn. In other words, the number of possible revolutions per every $\frac{1}{24}$ of a second is as follows:

$$\frac{\text{revolutions}}{\frac{1}{24}\text{sec.}} = \frac{3+4k}{4} k \in \mathbb{Z}^+ \tag{0.1}$$

We multiply by 24 to get the number of possible revolutions per seconds

$$24 * \frac{3+4k}{4} k = 18 + 24k \in \mathbb{Z}^+ \tag{0.2}$$

PROBLEM 2

- a:
  We can sample below the nyquist rate at 19khz by making use of the free real-estate from (-10,-6) and (6,10) and having the "copies" of $X(j\omega)$ intersect the regions of zero-frequency content within the power spectrum of $x(t)$
- b:
  Work shown below:

PROBLEM 3

- a:
  let x_1, x_2 be two signals being sampled by the impulse train $p(t)$ and define $\hat{x}(t) = \alpha x_1 + b x_2(t)$:

$$
\begin{aligned}
\hat{x}_p(t) &= \hat{x}(t)p(t) && (0.3)\\
&= \hat{x}(t)\sum_{n=-\infty}^{\infty}\delta(t-kT) && (0.4)\\
&= (\alpha x_1 + b x_2)(t)\sum_{n=-\infty}^{\infty}\delta(t-kT) && (0.5)\\
&= \alpha x_1(t)\sum_{n=-\infty}^{\infty}\delta(t-kT) + \beta x_2(t)\sum_{n=-\infty}^{\infty}\delta(t-kT) && (0.6)\\
&= \alpha x_{1p}(t) + \beta x_{2p}(t) && (0.7)
\end{aligned}
$$

- b:

$$
\begin{aligned}
x(t) &= \cos(2\pi t) && (0.8)\\
x_p(t) &= \cos(2\pi t)\sum_{n=-\infty}^{\infty}\delta(t-kT) && (0.9)\\
&= \sum_{k=-\infty}^{\infty}\delta(t-kT) && (0.10)\\
X_p(j\omega) &= \sum_{k=-\infty}^{\infty}\delta(\omega-k\omega s) && (0.11)\\
X_r(j\omega) &= X_p(j\omega)H_{lp}(j\omega) && (0.12)\\
&= \delta(t) && (0.13)\\
x_r(t) &= 1 && (0.14)
\end{aligned}
$$

- c:

$$
\begin{aligned}
\hat{x}(t) &= \cos(2\pi(t-\frac{1}{2})) && (0.15)\\
x_p(t) &= \cos(2\pi(t-\frac{1}{2}))\sum_{n=-\infty}^{\infty}\delta(t-kT) && (0.16)\\
&= -\sum_{n=-\infty}^{\infty}\delta(t-kT) && (0.17)\\
X_p(j\omega) &= -\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega s) && (0.18)\\
X_r(j\omega) &= X_p(j\omega)H_{lp}(j\omega) && (0.19)\\
&= -\delta(t) && (0.20)\\
\hat{x}_r(t) &= -1 && (0.21)\\
& && (0.22)
\end{aligned}
$$

Therefore, the system is not time invariant, as the two signals are not equal.

Problem 4

- a:

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega \tag{0.23}$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} j\omega e^{j\omega n}d\omega \tag{0.24}$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} j\omega(\cos(\omega n)) + j\sin(\omega n))d\omega \tag{0.25}$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} j\omega(\cos(\omega n))d\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi} -\omega(\sin(\omega n))d\omega \tag{0.26}$$
$$= \frac{1}{\pi}\int_{0}^{\pi} -\omega(\sin(\omega n))d\omega \tag{0.27}$$
$$= \begin{cases} \frac{\cos(\pi n)}{\pi n} - \frac{\sin(\pi n)}{\pi n^2} : n \neq 0 \\ 0 : n = 0 \end{cases} \tag{0.28}$$

- b:

$$\hat{H}_d(e^{j\Omega})|_{\Omega=0} = \sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n}|_{\Omega=0} \tag{0.29}$$
$$= \sum_{n=-\infty}^{\infty} h[n] \tag{0.30}$$
$$= h[-1] + h[0] + h[1] \tag{0.31}$$
$$= 0 \tag{0.32}$$

$$\hat{H}_d(e^{j\Omega}) \approx \hat{H}_d(e^{j\Omega})|_{\Omega=0} + \frac{d\hat{H}_d(e^{j\Omega})}{d\Omega}(e^{j\Omega}) \tag{0.33}$$
$$\approx 0 + \frac{d\hat{H}_d(e^{j\Omega})}{d\Omega}|_{\Omega=0}(e^{j\Omega}) \tag{0.34}$$
$$\approx \frac{je^{j\Omega}}{T} \tag{0.35}$$

If we set $\alpha = \Omega e^{-j\Omega}$ Then we can recover the original frequency response with the first order approximation. Plot below.

- c:

$$\frac{je^{j\Omega}}{T} = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} \tag{0.36}$$
$$y[n] = Tx[n - \Omega] \tag{0.37}$$

PROBLEM 5

- a:
  Generalizing from the 1D case we arrive at the following:

$$X_p(e^{j\omega_1}, e^{j\omega_2}) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} X(e^{j(\omega - k_1 \omega_{s_1})}, e^{j(\omega - k_2 \omega_{s_2})}) : \omega_{si} = \frac{2\pi}{N_i} \tag{0.38}$$

- b:
  To avoid aliasing, we need to ensure that the Nyquist Conditions hold in both ω_{s1} and ω_{s2} in other words, the following needs to hold:

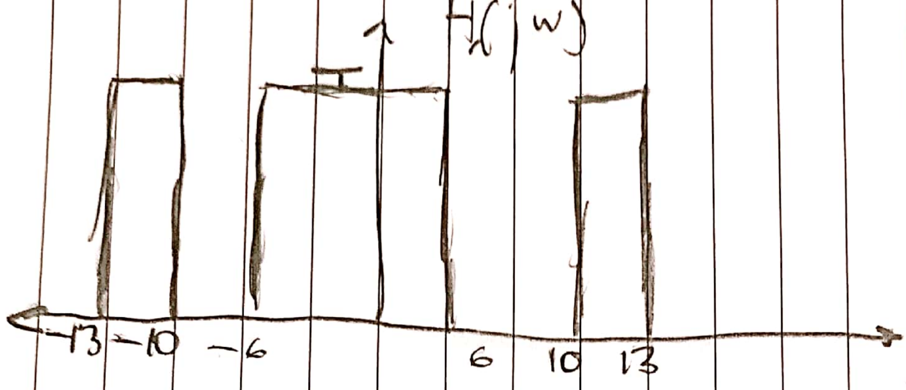
$$\omega_{s1} > 2\omega_{M1} \tag{0.39}$$

$$\omega_{s2} > 2\omega_{M2} \tag{0.40}$$

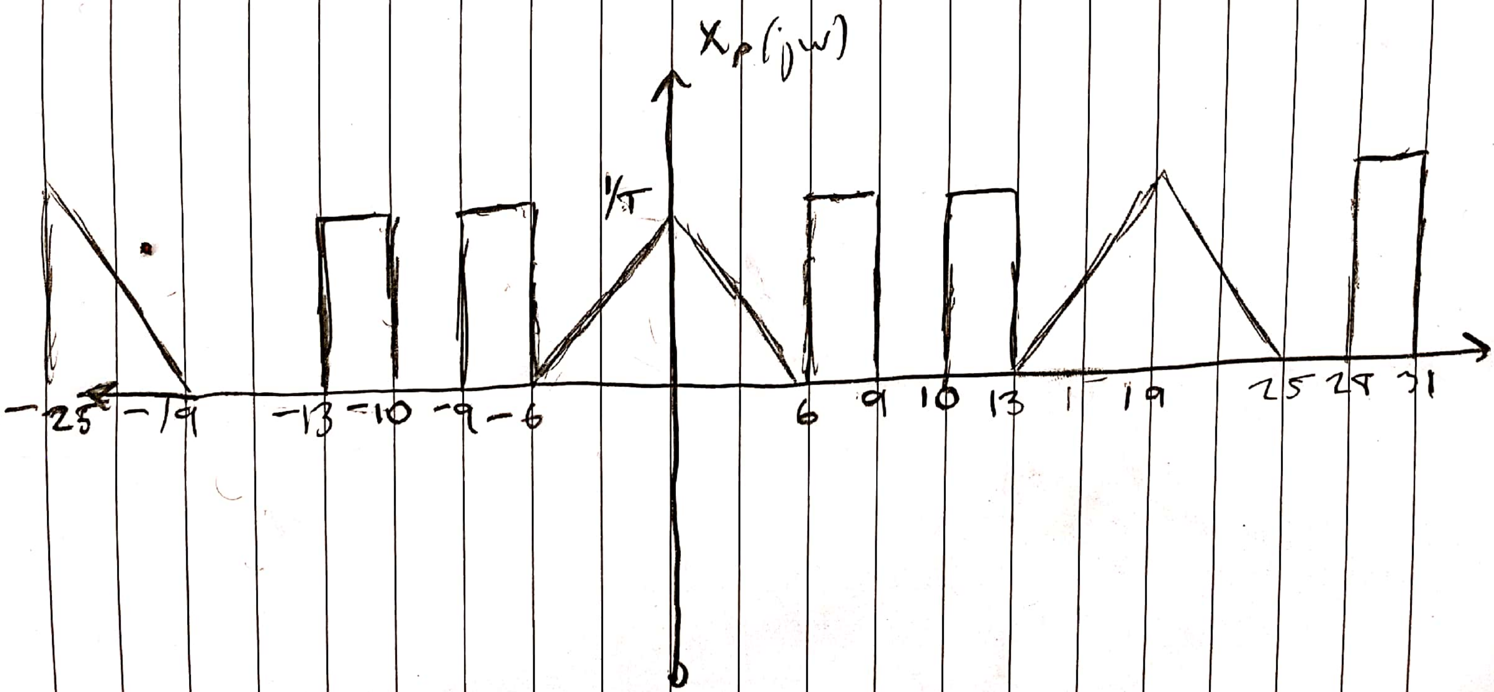
  Where the ω_{M1} is the bandwidth of the $x[n_1, n_2]$ in the n_1 direction etc.s

- c:
  Generalizing from the 1D case and assuming $h[n_1, n_2] = h[n_1]h[n_2]$ The impulse response of the ideal reconstruction filter is equal to the following:

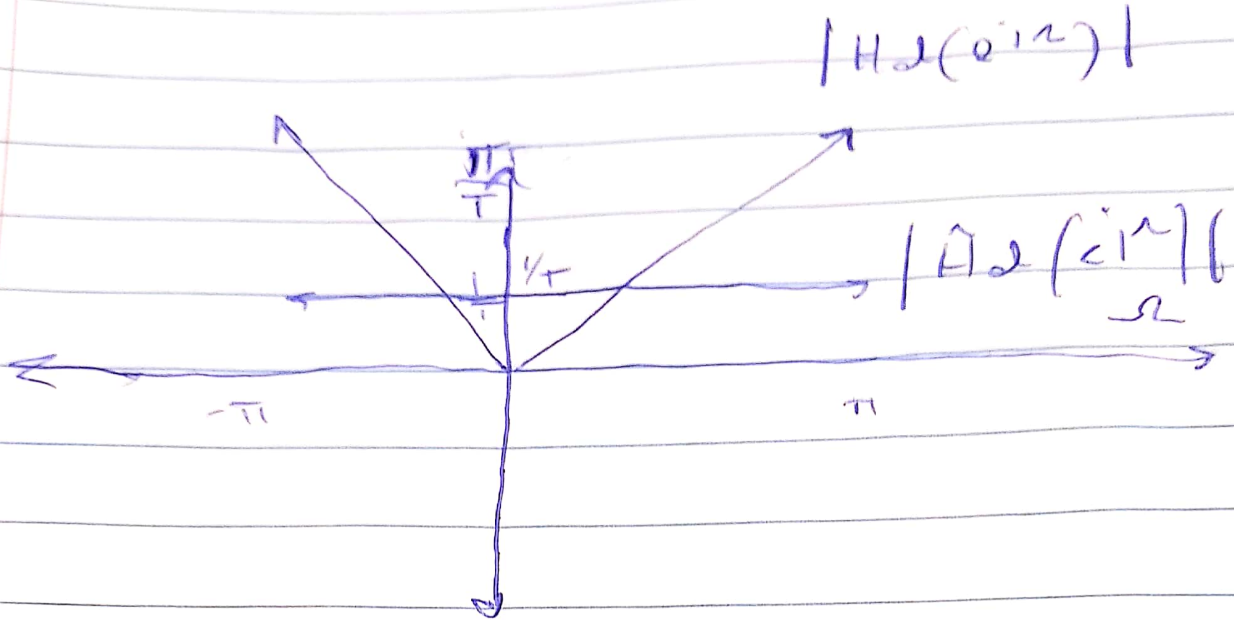
$$h[n_1, n_2] = \text{sinc}\Big(\frac{n_1}{N_1}\Big)\text{sinc}\Big(\frac{n_2}{N_2}\Big) \tag{0.41}$$



H(jw)
T
-13
-10
-6
6
10
13



Xp(jw)
1/T
-25
-19
-13
-10
-9
-6
6
9
10
13
19
25
29
31



$|H_d(e^{j\Omega})|$
$\frac{\pi}{T}$
$1/T$
$\frac{1}{T}$
$|A_d(e^{j\Omega})|$
Ω
$-\pi$
π