Machines C
Exam

University of Chicago Science Olympiad Invitational 2020

Saturday, January 11, 2020

1. Please **do not start** the test until told to!

2. Please **do not write on the test** unless the supervisor says it's okay to (the tests may be reused between blocks).

There are 14 problems in total, with some are multi-part questions. The last few questions are a bit longer and therefore worth more points.

The emphasis of this test is not on significant figures, so three sig figs should be enough for every question (but no penalty for extra).

Some questions may ask for fractions, which you don't need to convert into decimals. Lowest terms would be nice to the graders, though.

Leave answers in terms of π where appropriate.

Quick terminology review

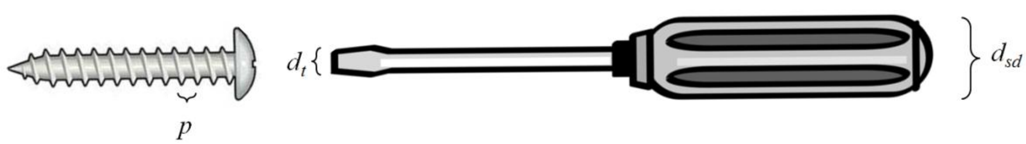
IMA/Ideal Mechanical Advantage: ratio of distance out to distance in.
AMA/Actual Mechanical Advantage: ratio of force in to force out.
Efficiency: AMA/IMA, i.e., fraction of work not lost to friction.
Work: $W=F*\Delta x$, i.e., force exerted along a distance (the component of distance parallel to force).
Torque: $\tau = r * F_{perpendicular}$, or the force exerted at a distance (as leverage).
Statics: All net forces and all net torques are 0 everywhere (a net nonzero force causes motion).

Problem 1 [5 pts]. Bus wheel/Car Wheel

Suppose your car steering wheel is a bit hard to turn, so you decide to tape on a bus steering wheel on top of it to steer with. Say your car's steering wheel has radius $r_c = 18\ cm$ and the bus steering wheel has radius $r_b = 22\ cm$. How much more do you need to move your hands on the bus steering wheel than the car steering wheel (in order to turn the same angle)? Answer as a ratio of the distances. Feel free to write a fraction or decimal.

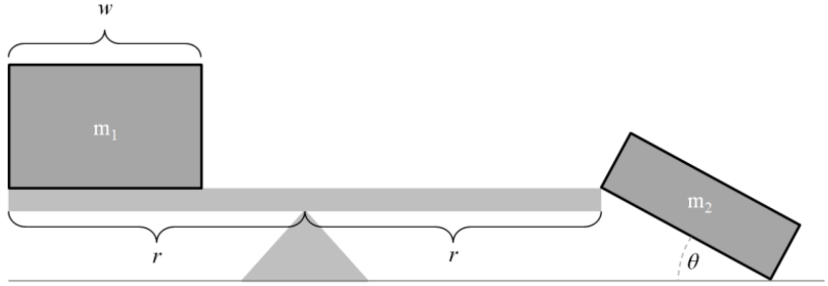
Problem 2 [5 pts]. Screwdriver

Consider a screwdriver with a handle $d_{sd} = 20\ mm$ across and tip $d_t = 7\ mm$ across. If the screw has pitch $p = 1\ mm$, what is the IMA of the system?

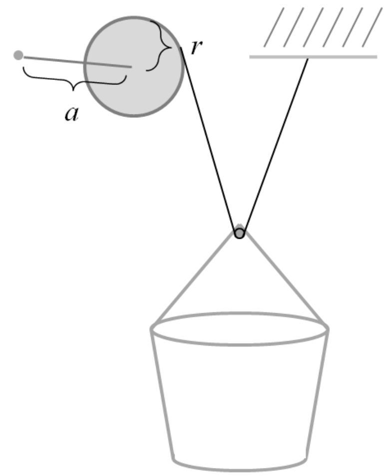


Problem 3 [5 pts]. See-saw

Consider a see-saw with two masses. The first has mass m_1 with width of w, and is placed on the left end of the see-saw. The second mass is propped against the see-saw at an angle θ and has mass m_2 (its length is irrelevant -- assume uniform mass density). Each side of the see-saw has length r. If we know $w/r = \alpha$, find the ratio of masses, m_1/m_2, that balances the see-saw.
Note: you should find an expression in terms of α and θ but not of other variables.



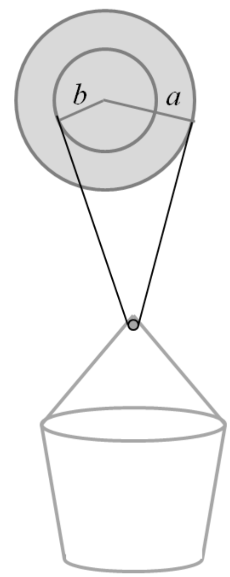
Problem 4 [5 pts]. Windlass with a ceiling



This windlass is a cylinder with rope coiled about it at a radius *r* and has a handle of length *a* that can be used as a crank. This pulls up a bucket hanging at the bottom of the rope (without friction of course!). The rope comes back up and is attached to the ceiling. *Sidenote: the standard windlass does not necessarily need the rope to come back up.*

Find the IMA of this system as an expression of *a* and *r*.

Problem 5 [5+5 pts]. Differential Windlass

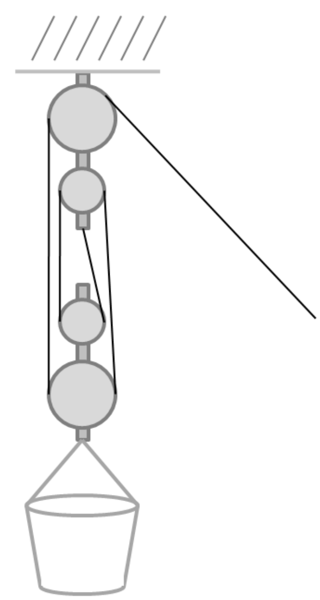


The differential windlass has two cylinders: one with radius *a*, and the other with radius *b*. This time, the rope comes down from the inner cylinder and loops back to onto the outer cylinder. For simplicity, suppose that we just apply force at radius *a*.

i. Check the bounds: what happens when $a=b$? What about when $b=0$? (describe with just a few words)

ii. Find the IMA of the differential windlass in terms of *a* and *b*. *Hint: it's called differential, so you should expect to find* (a-b) *in your expression.*

Problem 6 [5+5+5 pts]. Simple Pulley (aka Double Tackle)



Here's a relatively simple pulley system that may be found in a sailboat. (although in a boat they'd probably have something other than a bucket as the load)

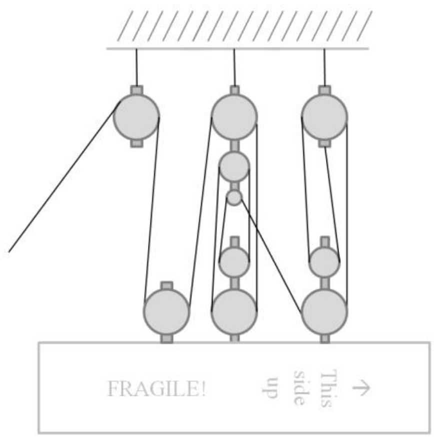
i. Find the IMA of this system.

ii. Suppose Clumsy Joe spilled tar on the pulley system, and now each individual pulley has an efficiency of 80%. What's the AMA of the overall system?

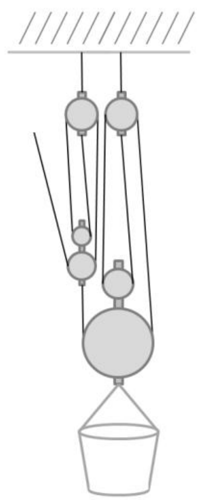
iii. What's the IMA of the system if we flipped it up-side down, so that the load is where the ceiling currently is, and the bucket is fixed in place?

Problem 7 [5 pts]. Pulley System

Find the IMA of the following system used to lift a mysterious box.



Problem 8 [5+5 pts]. Another pulley system

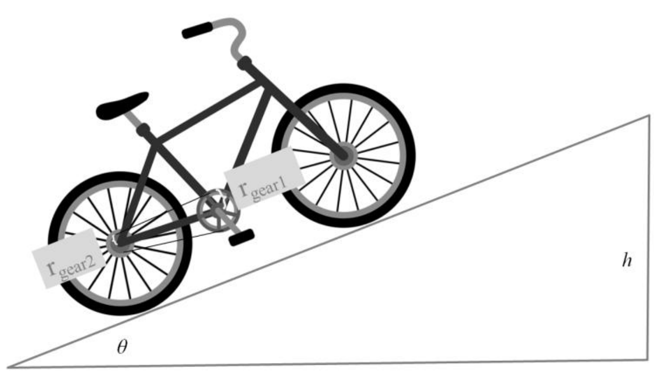


i. What's the IMA of this system?

ii. If the rope can withstand a maximum of 10 N, what's the heaviest load that can be lifted before the rope breaks? (answer in Newtons)

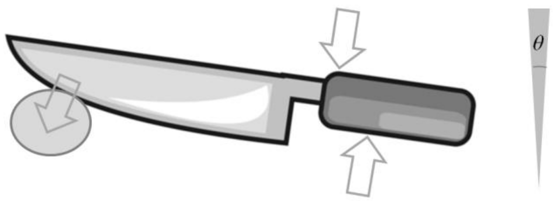
Problem 9 [5 pts]. Bicycle on a Ramp

The pedal is at a radius of r_p = 20 cm, and the front gear has a radius of r_{gear1} = 14 cm. The back gear has radius r_{gear2} = 10 cm, and the rear wheel has radius r_w = 60 cm. The ramp has an incline of θ = 17º. What's the IMA of the system, supposing our goal is to go up by *h* (and don't care about horizontal motion)?



Problem 10 [2+3+5 pts]. Knife as a compound machine

You could consider the knife as a lever if you apply force with your thumb on the top of a knife while the rest of your hand acts as a fulcrum. Suppose that the thumb above is located 1 inch from the index finger below, and that the thing you're cutting is 4 inches from your thumb. The knife's blade is also a wedge, making the knife a compound machine. Say $\theta = 0.573^{\circ} = 0.01$ (radians).



i. What class lever is the knife being used as?

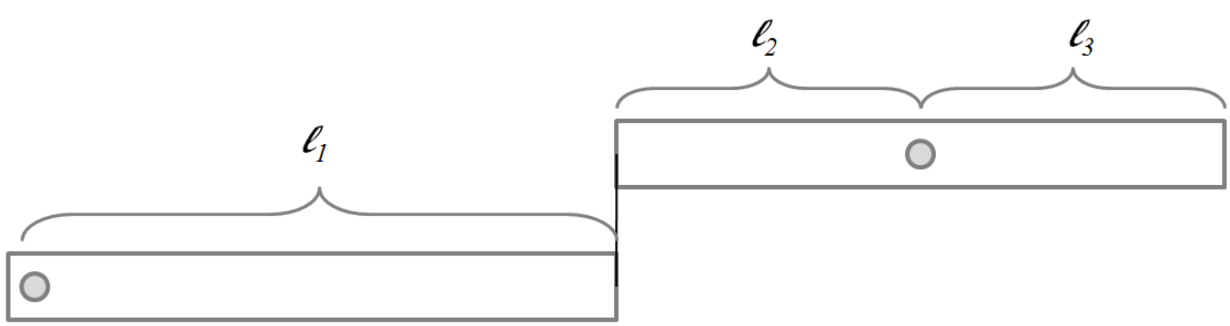
ii. What's the mechanical advantage of the wedge? Since θ is small and given in radians, you can easily use the small angle approximation: $\tan\theta = \sin\theta = \theta$.

iii. What's the IMA of the whole system, assuming all the force is applied by the thumb?

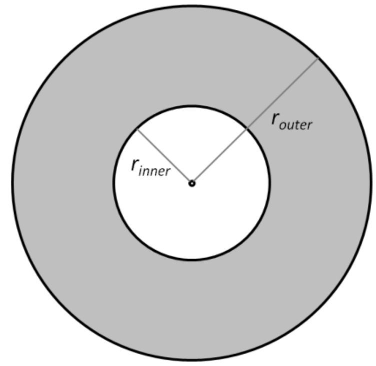
Problem 11 [10 pts]. Balancing the build portion of this event

Consider the build portion of this event. There are two levers of equal length, ℓ_1, connected in the middle by string. The right lever is a class 1 lever with a fulcrum splitting its length into ℓ_2 and ℓ_3. Find the ratio ℓ_2/ℓ_3 that balances the two levers (i.e. such that no counterweight is needed).

Note: the figure below is definitely not to scale.



Problem 12 [2.5 x 4 = 10 pts]. Vinyl Record



A vinyl record player turns the record at a constant 33 1/3 rpm. Its outer radius is r_{outer} = *6 inches*, and the inner radius where the grooves end is at r_{inner} = *2 inches*. Suppose grooves are *0.005 inches* apart.

i. How much music can you fit on each side of this record?

ii. What is the length of groove that corresponds to one second on the outside of the record? (in.)

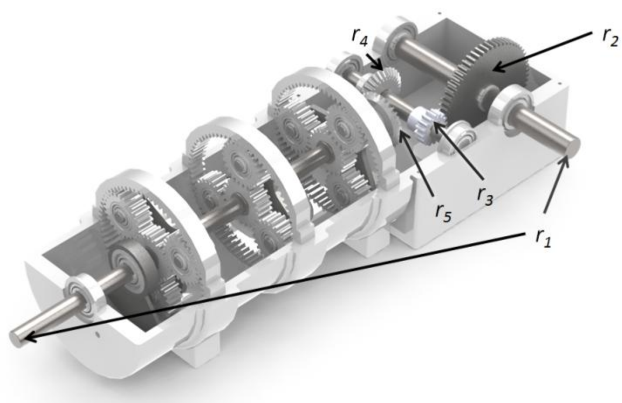
iii. What is the length of groove that corresponds to one second on the inside of the record? (in.)

iv. If you want your song to be the highest quality, should you put it at the beginning or end of the track? (fellow kids - the needle starts on the outside and goes inward)

Problem 13 [5+10 pts]. Gear system

Given r_1 = *5 mm*, r_2 = *24 mm*, r_3 = *8 mm*, r_4 = *10 mm*, and r_5 = *15 mm*, suppose the three planetary systems have sun gears (in the center) with the same number of teeth as each of the planets.

Note that the radii given for the bevel gears, r_4 and r_5, are slightly ambiguous as the two gears' teeth come into contact at a range of radii. Consider r_4 and r_5 the outermost radii where the teeth meet (we could choose any point as long as we're consistent).



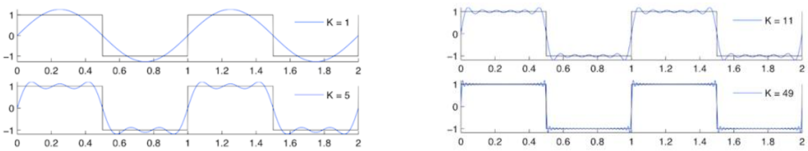
Picture: http://www.trapeziumengineering.com/wp-content/uploads/2014/01/Gearbox-Render-2.png

i. What's the IMA of each of the planetary gears? For clarity: the annulus is fixed, while the planetary carrier is driven, and the sun is used as the output.

ii. What's the IMA of the entire system?

Problem 14 [10+10 pts.] Michelson's Harmonic Analyzer

Albert Michelson (founder of the physics department at UChicago; famous for measuring the speed of light with Edward Morley) designed a machine to mechanically evaluate/plot Fourier Series, which is very useful in physics and digital signal processing. The main idea is that any function can be rewritten as a sum of sines and cosines (with varying periods and amplitudes). For example, here's an approximation of a square wave using increasingly many sines and cosines (exact in the infinite limit):

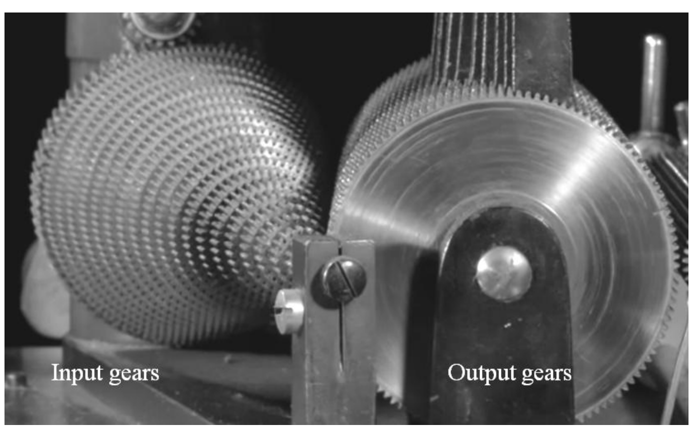


The Fourier Series looks like this:

$$F(x) = \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

We will just consider the sine term, since the cosine term effectively behaves the same. In this problem we'll see how: (i) the *sin(nx)* terms are produced, (ii) how we adjust the amplitude b_n, and (iii) (no question asked) how the sines are summed together. Obviously the machine must consider a finite number of these terms. We'll consider a machine with the first 20 sine terms.

i. The sines with different frequencies are produced by a series of gears, with gear ratios varying between 1:1 to 20:1. All the input gears are driven at a single frequency, so all the output gears amplify that frequency by the gear ratio.

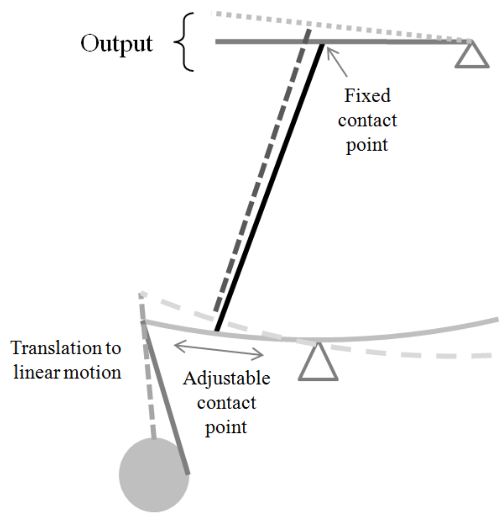


Source: youtube, engineerguy (https://www.youtube.com/watch?v=8KmVDxkia_w)

Suppose I drive the input gear shaft at a rate of 30 rpm. How fast does the output gear connected with the smallest input gear spin (in rpm)? How about the output gear connected to the *n*th smallest input gear? (include *n* in your response)

ii. The output gears from part (i) are used to drive a lever up and down, translating rotational into linear motion. This is where the sine function comes from.

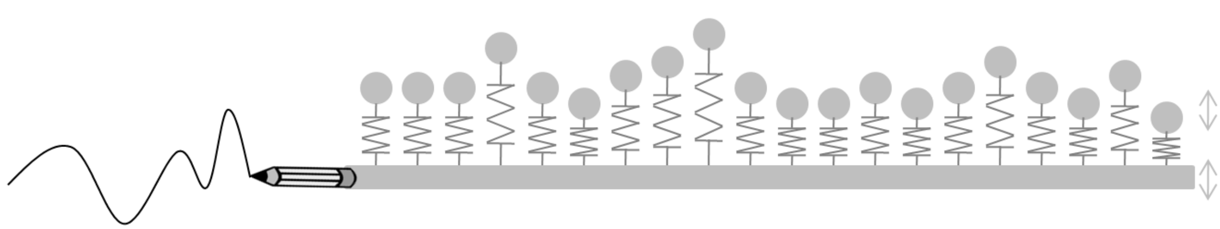
The dotted lines in the figure below show how the system changes as the bottom (output) gear turns: the bottom lever rocks up and down, which causes the long arm to drive the top lever up and down. We change the amplitude of this output by adjusting the contact point between the long arm and bottom lever.



Suppose the long arm connecting the two levers is positioned x cm to the *left* of the bottom fulcrum, and suppose that the range of motion of the top lever is 2 cm. What is the range of motion of the top lever if we move the long arm to $3x/2$ cm to the *right* of the bottom fulcrum?

iii. (no question) At this point, we have 20 arms moving sinusoidally but want a single output. The great insight of this machine is to connect all the arms with springs to a single bar, which effectively averages out all the displacements and is like a scaled summation of sines!

You can attach a pen to the very end of this device and draw the desired function. Below, the gray circles represent the ends of the top lever arms from part (ii) that move up and down.



Thanks for taking this test! I hope you learned something and found the problems fun!