Machines C - Illinois Regionals 2020

"So... logically, if she weighs the same as a duck, she's made of wood."

"And therefore..."

"A witch!"

A widespread medieval usage of pulley scales, depicted in 1975.

Instructions:

Do not open test until instructed!

Write all answers on the **answer sheet**. There are 7 questions. No credit will be given for work on the test if there is no answer on the answer sheet.

Three significant figures should be sufficient for all questions, but more will not be penalized.

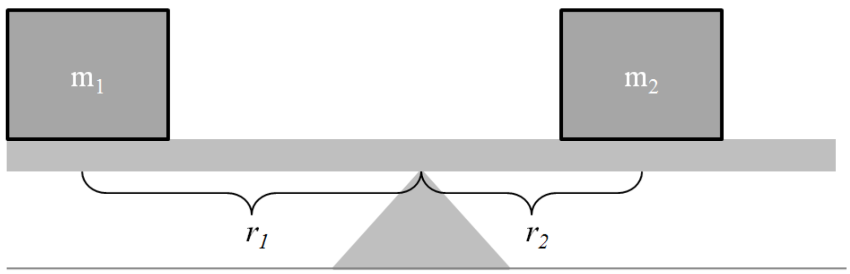
Write symbolic answers in **simplest form** so the grader can easily verify your work.

Question 1 [10 pts]. Mass range

Two boxes are placed on a see-saw. Box 1 has mass m_1 and is r_1 from the fulcrum. Box 2 has mass m_2 and is r_2 from the fulcrum. Suppose the fulcrum is rusty and only transmits 70% of force (i.e., it has an efficiency of 0.7).

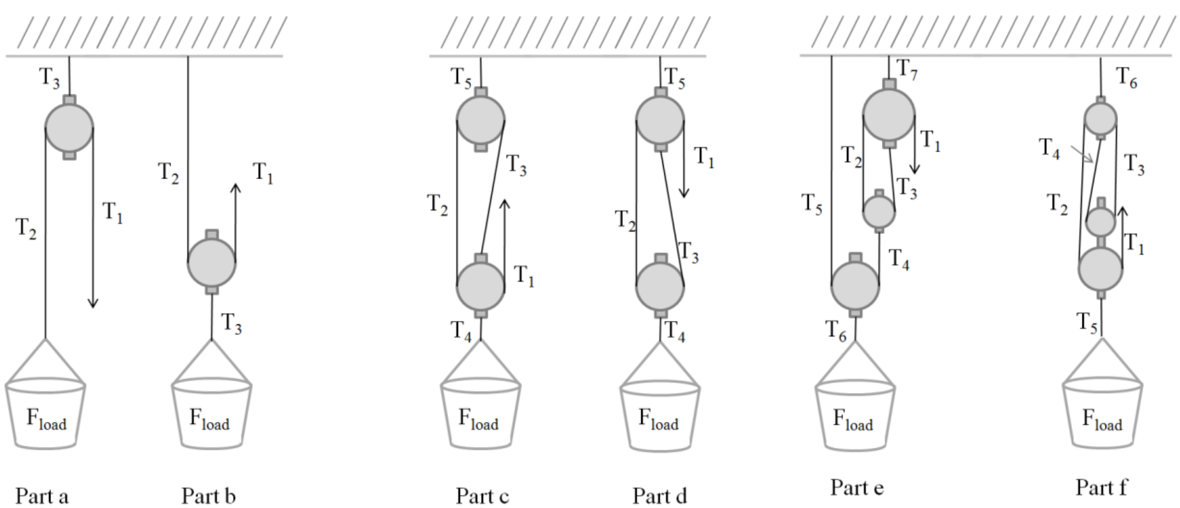
a [5 pts]. What is the minimum m_2 that balances the system? Answer in terms of m_1, r_1, and r_2.

b [5 pts]. What is the maximum m_2 that balances the system? Answer in terms of m_1, r_1, and r_2.



Question 2 [17.5 pts]. Deriving the IMA of simple pulley systems

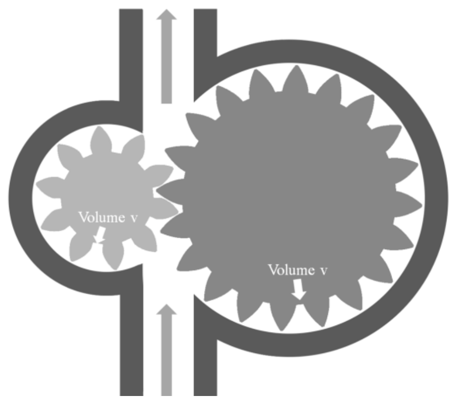
a-f [0.5 pt per entry]. For each system below, write the tension of each section of string if $T_1 = 1$ N. Then find the maximum load that can be supported by 1 N of force inputted into the system (F_{load}). Use this information to find the IMA of the system (we're assuming massless, frictionless pulleys so IMA=AMA). Hint: there should be lots of repeated answers, but not every tension within each section should be the same. Be careful with the numbering when you write your answers down!



Note: why bother track each tension for these simple setups? As demonstrated in a/b and c/d, you need to be careful in applying any mnemonic you may have learned (e.g. "draw a line and count the number of times string passes through it"). This method of tracking tensions is generally more robust and hopefully will help you to never get tricked by a pulley question again!

Question 3 [12.5 pts]. Kepler pump variant

Around 1600, Johannes Kepler designed a simple gear system that pumps water in one direction as the gears are driven. Due to the gear teeth shape, a minimal amount of fluid is able to travel in the reverse direction in the middle. Here the right gear has twice radius and number of teeth as the left.



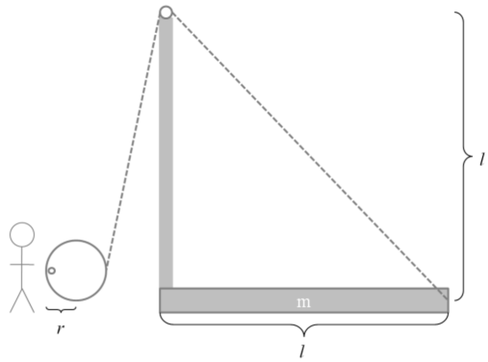
a [1.5 pts]. Which gear pushes water across faster?

b [1 pts]. Should the left gear be driven clockwise or counterclockwise to force water upward?

c [10 pts]. Suppose each gap between two teeth and the pump wall has volume v = 1 mL (for both gears). The left gear has 10 teeth, while the right gear has 20 teeth. How fast should the left gear be driven to result in a flow of 6 liters per minute (=1.57 gallons per minute)? Answer in revolutions per minute.

Question 4 [10 points]. Drawbridge

A drawbridge of length l has a mass m and can be lifted or lowered by means of a chain connected to a hand-cranked windlass with radius r.



a [5 pts]. How much work is required to pull up the drawbridge? Answer in terms of *m, l,* and gravitational acceleration g.

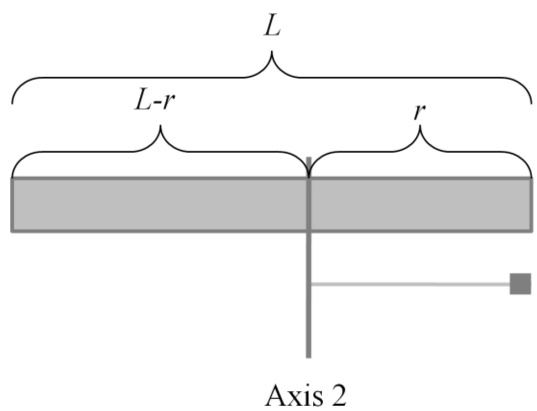
b [5 pts]. How many turns of the windlass are required to lift the drawbridge completely up? Write your answer in the simplest exact form in terms of l and r (don't convert irrational numbers like π or square roots to decimal, but make sure you've simplified your expression).

Question 5 [20 pts]. Equatorial mounted telescope

Here is a picture of an equatorial mounted telescope. To adjust for the Earth's rotation throughout a night, this telescope is able to rotate about an axis (right-ascension) that is parallel to the Earth's axis of rotation (polar axis). In order for this to be easily adjusted, the telescope itself is balanced by a counterbalance.

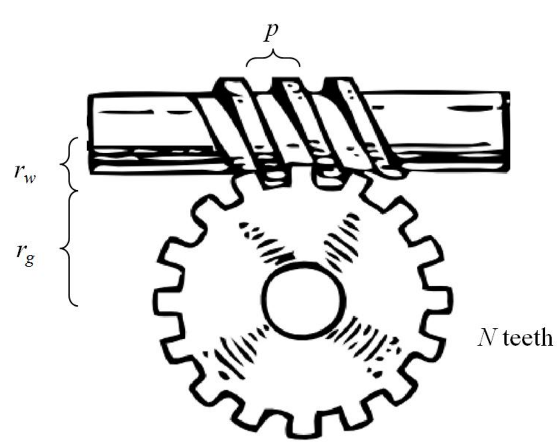
a [5 pts]. Suppose the main telescope tube has mass 5kg and is centered 30 cm from Axis 1. If the counterweight is located 50 cm from Axis 1, what is its mass?

b [15 pts]. Now, consider the counterweighting about Axis 2. Suppose the telescope's mass is evenly distributed and totals $m_T = 4.5\ kg$, while this counterweight is $m_c = 500\ g$. The counterweight rests at a distance r from axis 2, and so does the right side (eyepiece) of the telescope. Find r/L assuming the system is balanced. Hint: if you can find the center of mass of this system, that point should be located on axis 2.



Question 6 [10 pts]. Worm gear

Here is a worm gear, where the top threaded shaft is referred to as the worm. Suppose the worm has pitch p and radius r_w. The gear has N teeth at a radius r_g. What is the gear ratio of this system when the worm is driven? Answer in terms of p, r_w, N, and r_g. Hint: you may not need all these quantities.



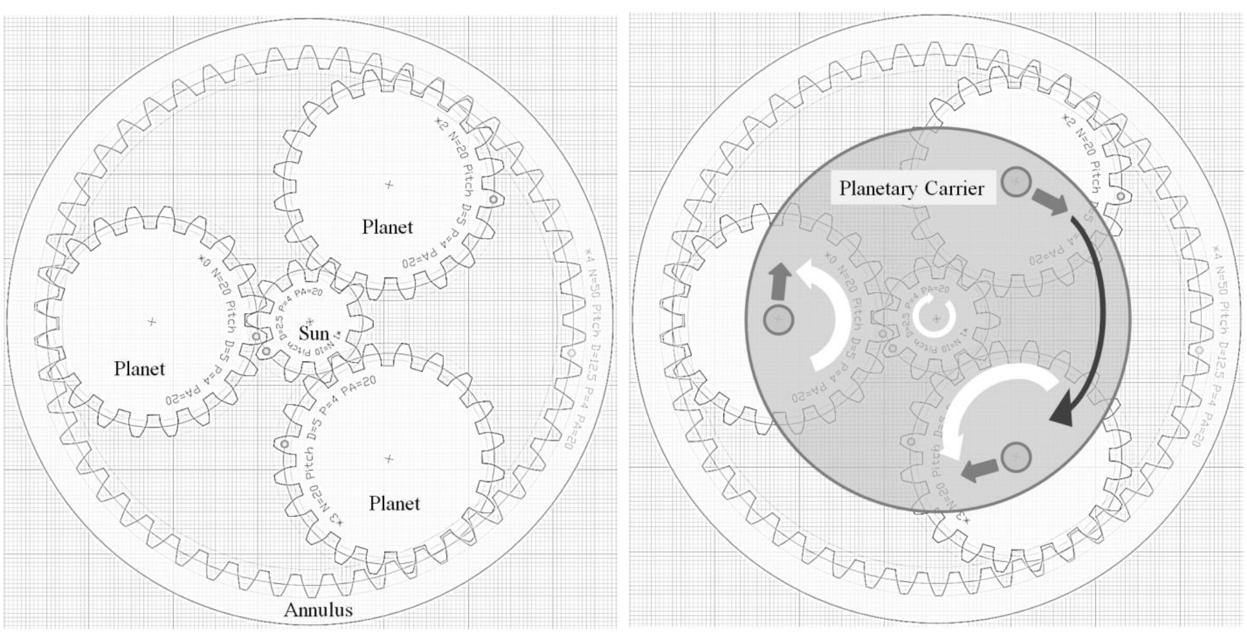
Question 7 [25 pts]. Walkthrough for a planetary gear system

For the last question, we'll walk through how to find the IMA of a planetary gear system in one particular setting. A planetary gear system involves gears whose centers move while the teeth spin, so they can be a bit tricky to visualize. Planetary gears are pretty efficient ways to achieve high IMAs and are used in many places, including wind turbines (low blade angular speed but with high torque gets converted to a higher speed for electricity production).

Terminology:

> Annulus: outer ring - N_a teeth, radius r_a
>
> Sun: center gear - N_s teeth, radius r_s
>
> Planets: outer circular gears (connected with the sun) - N_p teeth, radius r_p
>
> Planetary carrier: structure holding the planets together - radius (r_p+r_s)

All the teeth are the same size, so $N_a/r_a = N_p/r_p = N_s/r_s$. Moreover, for this problem we will consider the case where r_s:r_p:r_a is 1:2:5. We intend to drive this system by the planetary carrier and use the sun's rotation as the output shaft. The annulus will be fixed in place.



Left: gear parts labeled; Right: Planetary Carrier shown with rotation directions
Picture rendered on geargenerator.com (lets you see gears turn)

However, for the derivation **suppose for now that the planetary carrier is fixed** and that we **let the annulus turn** instead.

a [3 pts]. After the annulus makes one full revolution, how many revolutions does each planet rotate? Again, the planets are fixed for now, so consider only the rotation of each planet's teeth about its center.

b [3 pts]. When each planet makes one revolution, how many revolutions does the sun gear turn?

c [4 pts]. Therefore, for each full revolution of the annulus, how many revolutions does the sun make?

d [5 pts]. What is the gear ratio of this system when the annulus is driven and the sun is used as the output? Be sure to write the correct sign to indicate whether the annulus and sun rotate in the same or different directions.

Now it's time to **fix the annulus in place** and **drive the planetary carrier** as I said we'd do at the beginning. It may be helpful to imagine viewing the same system but from a frame rotating at the same rate as the annulus.

e [5 pts]. In the same time that it would have taken the annulus to make a full revolution in parts a-d, how many revolutions does the **planetary carrier** turn?

f [5 pts]. In the same time that it takes the **planetary carrier** to make a full revolution, how many times does the sun gear rotate? Remember to adjust for the change in annulus rotation.

g [5 pts]. What is the gear ratio of the overall system, with the planetary carrier as the input and sun gear as the output and fixed annulus?

Machines C -- Answer Sheet Test score: _____/110

Team name: ______________________ Page 1 score: ____/40

Team number: _________ Page 2 score: ____/70

Question 1 [10 pts].

a [5 pts]. ___________________ (symbolic)

b [5 pts]. ___________________ (symbolic)

Question 2 [17.5 pts; 0.5 pt per blank]

a [2]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

F_{load}=____ N

IMA=____

b [2]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

F_{load}=____ N

IMA=____

c [3]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

T_4 = ____ N

T_5 = ____ N

F_{load}=____ N

IMA=____

d [3]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

T_4 = ____ N

T_5 = ____ N

F_{load}=____ N

IMA=____

e [4]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

T_4 = ____ N

T_5 = ____ N

T_6 = ____ N

T_7 = ____N

F_{load}=____ N

IMA=____

f [3.5]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

T_4 = ____ N

T_5 = ____ N

T_6 = ____ N

F_{load}=____ N

IMA=____

Question 3 [12.5 pts].

a [1.5 pts]. left / same / right (circle one)

b [1 pts]. clockwise / counterclockwise (circle one)

c [10 pts]. ___________________ rpm

Question 4 [10 pts].

a [5 pts]. ____________________ (symbolic)

b [5 pts]. ____________________ (symbolic)

Question 5 [20 pts].

a [5 pts]. ____________________ kg

b [15 pts]. ____________________ (unitless)

Question 6 [10 pts]. ____________________ (symbolic)

Question 7 [30 pts].

a [3 pts]. ____________________ (unitless)

b [3 pts]. ____________________ (unitless)

c [4 pts]. ____________________ (unitless)

d [5 pts]. ____________________ (unitless)

e [5 pts]. ____________________ (unitless)

f [5 pts]. ____________________ (unitless)

g[5 pts]. ____________________ (unitless)

Machines C -- Answer KEY (Point total: 110=40+70)

Question 1 [10 pts].

a [5 pts]. 7/10 m_1r_1/r_2 (= 7/10 * m_1r_1/r_2)

b [5 pts]. 10/7 m_1r_1/r_2 (= 1.43 * m_1r_1/r_2)

Question 2 [17.5 pts, 0.5 pt each -- but don't score T_1 as it is given]

a [2]. T_1 = 1 N

T_2 = **1** N

T_3 = **2** N

F_{load} = **1** N

IMA = **1**

b [2]. T_1 = 1 N

T_2 = **1** N

T_3 = **2** N

F_{load} = **2** N

IMA = **2**

c [3]. T_1 = 1 N

T_2 = **1** N

T_3 = **1** N

T_4 = **3** N

T_5 = **2** N

F_{load}= **3** N

IMA = **3**

d [3]. T_1 = 1 N

T_2 = **1** N

T_3 = **1** N

T_4 = **2** N

T_5 = **3** N

F_{load} = **2** N

IMA = **2**

e [4]. T_1 = 1 N

T_2 = **1** N

T_3 = **1** N

T_4 = **2** N

T_5 = **2** N

T_6 = **4** N

T_7 = **3** N

F_{load} = **4** N

IMA = **4**

f [3.5]. T_1 = 1 N

T_2 = **1** N

T_3 = **1** N

T_4 = **1** N

T_5 = **4** N

T_6 = **3** N

F_{load} = **4** N

IMA = **4**

Grading note: it is incorrect if a team writes "N" after the IMA, which should be unitless. I'd recommend taking off at least 0.5-1 points if a team does this for every entry of IMA as it shows they don't understand that IMA is a ratio.

Question 3 [12.5 pts].

a [1.5 pts]. **same** (the left side spins twice as fast but has half as many teeth as the right)

b [1 pts]. **clockwise**

c [10 pts]. **300 rpm** = (6000 mL/min) / (20 mL (between both gears) per revolution of the left gear); the 20 mL per revolution is because left gear has 10 teeth gaps (10 mL), and the right gear sends the same amount as the left (as in part a) (10 mL more).

Question 4 [10 pts].

a [5 pts]. $\frac{1}{2}mlg$ (from lifting the center of mass by half the height of the door)

b [5 pts]. $\frac{l\sqrt{2}}{2\pi r} = 0.2251\frac{l}{r}$ (note that the question requests exact form, but this is mainly for grading convenience. I leave it to your discretion whether to deduct points for converting from exact form).

Question 5 [20 pts].

a [5 pts]. **3 kg**

b [15 pts]. **9/20 = 0.45** (note this must be unitless as a ratio of lengths)

Comes from: $\frac{r}{L} = \frac{1}{2}\frac{m_T}{m_c+m_T} = \frac{1}{2} \cdot \frac{4.5\ kg}{5\ kg} = \frac{9}{20} = 0.45$, where the second expression shows how much to the left the telescope skews the center of mass from the counterweight. If $m_c=0$ then we have the center of mass in the middle; if $m_T=0$ then the center of mass is all the way on the right.

Question 6 [10 pts]. ***N*** - definitely a trick question with no dependence on p, r_w, or r_g.

Question 7 [30 pts].

a [3 pts]. **5/2 = 2.5** (either form is acceptable) (don't accept -5/2)

b [3 pts]. **2/1 = 2** (accept -2)

c [4 pts]. 5/2 * 2/1 = **5** (accept -5)

d [5 pts]. **-5** (don't accept +5 -- that's the same direction) (optional: give partial credit for +5 or for the negative version of the answer given for c)

e [5 pts]. **1** (accept -1)

f [5 pts]. **+6** (optional: 2-3 pts for 4 -- they subtracted 1 instead of adding 1 from part c)

g[5 pts]. **+6** (or give full marks for matching part f -- here the goal is to understand it's positive and not negative (output spins in the same direction as the input))