Machines C Answer Key
Regionals 2021

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Instructions:

1. Be sure to enter all answers into Scilympiad!
2. Many problems are multiple-choice to make grading easier. Not all the options make physical sense, though!! Whenever possible, check that your answers make sense before moving on!
3. Make sure to ask on scilympiad if you have any questions about the test.
4. We hope you enjoy and learn something new from this test! Best of luck!

1 Obtaining the test document

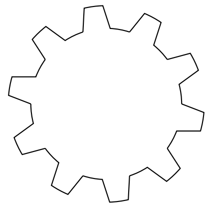

Figure 1: Simple pic

Question 1 [1 pt]. What is pictured in Figure 1 of the pdf? (if you're reading this you're set :))

A) Class 2 lever
B) Gear
C) Pulley
D) Antikythera mechanism
E) Pyramid

Answer: B) Gear

2 General Machines Properties

Question 2 [1 pt]. Which has higher IMA?

A) Class 2 levers
B) Class 3 levers

Answer: A) Class 2 levers

Question 3 [1 pt]. Which machine has an IMA in the form $\frac{2\pi r}{p}$?

A) Wheel-and-axle
B) Screw
C) Helical Gear
D) Class 1 lever

Answer: B) Screw

Question 4 [2 pt]. Which of the following is a valid expression for the AMA of a machine?

A) $\frac{d_{in}}{d_{out}}$
B) $\frac{d_{out}}{d_{in}}$
C) $\frac{F_{in}}{F_{out}}$

D) $\frac{F_{out}}{F_{in}}$

Answer: D) $\frac{F_{out}}{F_{in}}$

Question 5 [4 pt]. Consider two simple machines with IMAs IMA_1 and IMA_2 and efficiencies e_1 and e_2, respectively. If the two machines are connected so the output force of machine 1 provides the input force to machine 2, what is the IMA of the compound system?

A) IMA_1

B) IMA_2

C) $IMA_1 + IMA_2$

D) $IMA_1 \cdot IMA_2$

E) max $\{IMA_1, IMA_2\}$ (i.e., whichever is greater)

Answer: D) $IMA_1 \cdot IMA_2$

Question 6 [5 pt]. What is the efficiency of the resulting system? Hint: the AMA behaves like the IMA.

A) e_1

B) e_2

C) $e_1 + e_2$

D) $e_1 \cdot e_2$

E) $\frac{e_1 IMA_1 + e_2 IMA_2}{IMA_1 + IMA_2}$

Answer: D) $e_1 \cdot e_2$

Question 7 [3 pt]. Let W_1 be the work required to push a block up a ramp inclined at angle θ to a height h, and let W_2 be the work required to push the same block up height h. What is the ratio W_1/W_2?

A) $\cos\theta$

B) $\sin\theta$

C) 1

D) $1/\sin\theta$

E) $1/\cos\theta$

Answer: C) 1

Question 8 [4 pt]. This time, suppose pushing the block up a ramp has efficiency e_1 and takes work W_1', and pushing the block straight up has efficiency e_2 and takes work W_2'. What is W_1'/W_2'?

A) $\frac{e_1 W_1}{e_2 W_2}$

B) $\frac{e_2 W_1}{e_1 W_2}$

C) $\frac{W_1}{W_2}$

D) $\frac{e_1 W_2}{e_2 W_1}$

E) $\frac{e_2 W_2}{e_1 W_1}$

Answer: B) $\frac{e_2 W_1}{e_1 W_2}$

3 Units and Dimensions

Question 9 [1 pt]. What is the SI unit for work?

A) Watt (W)

B) Joule (J)

C) Newton (N)

D) Kilogram-meter per second (kg m s^{-1})

Answer: B) Joule (J)

Question 10 [2 pt]. What is the most typical way to denote the SI units for torque?

A) Kilogram-meter squared per second squared (kg m^2 s^{-2})

B) Joule (J)

C) Newton-meter (N·m)

D) Watt-second (W·s)

Answer: C) Newton-meter (N·m)

Question 11 [2 pt]. What are the dimensions of IMA? For reference, "the dimension of IMA" basically means "what kind of units does IMA have?" without choosing between SI, Imperial, or some other unit system.

A) Force

B) Force squared

C) Distance

D) Distance squared

E) Dimensionless

Answer: E) Dimensionless

Question 12 [2 pt]. Which of the following is not dimensionless?

A) Efficiency

B) Gear ratio

C) Angle of inclination

D) Angular velocity

E) Mass ratio (as in m_a/m_b or m_b/m_c in the build portion)

Answer: D) Angular velocity. Note that for C) Angle of inclination, degrees and radians are not proper units but rather scalings of a unitless ratio!

4 Specific Machine Questions

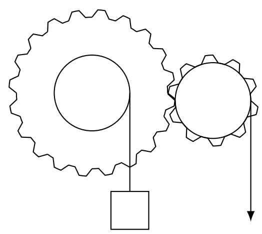

Figure 2: The gear version of an Atwood machine

Question 13 [4 pt]. Here (Figure 2) is a gear system inspired by the Atwood machine. The left gear has a weight attached at radius 1 and has an outer radius 2. The right gear has radius 1 and also has a rope attached at radius 1. What is the IMA of this system? (the weight is the load here)

A) 1/4

B) 1/2

C) 1

D) 2

E) 4

Answer: C) 1

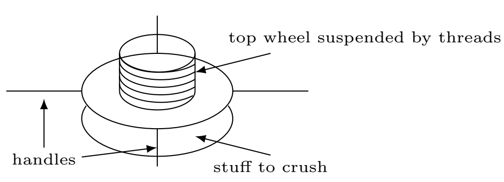

Figure 3: Imagine there is a structure from above holding the large screw by its threads. As the top wheel is turned, it slowly moves down and is able to crush things placed on the bottom "wheel."

Question 14 [7 pt]. Consider the large screw in Figure 3 with handles sticking off the sides. The screw has radius of 1 m with a pitch of 2 cm. Four people push on the handles at a radius of 2 m. Suppose these people placed pistachios on the bottom wheel and need 20000

N of force to crack them (imagine there are a lot of pistachios and they're the kind that you can't pry open). How much force would each person need to supply?

A) $\frac{25}{4\pi}$ N ≈ 1.98 N
B) $\frac{25}{2\pi}$ N ≈ 3.97 N
C) $\frac{25}{\pi}$ N ≈ 7.96 N
D) $\frac{50}{\pi}$ N ≈ 15.9 N
E) $\frac{100}{\pi}$ N ≈ 31.8 N

Answer: C) $\frac{25}{\pi}$ N ≈ 7.96 N
IMA: $\frac{2\pi 2 \text{ m}}{2 \text{ cm}} = 200\pi$ so $F = \frac{20000\text{N}}{4 \cdot 200\pi} = \frac{25}{\pi}$ N ≈ 7.96 N

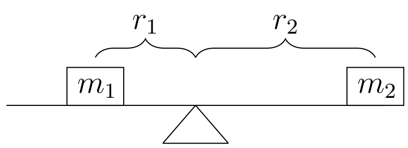

Figure 4: Simple see-saw / lever setup

Question 15 [4 pt]. Consider the see-saw in Figure 4. Which of the following equations best represents the fact that there is 0 torque about the fulcrum?

A) $r_1 m_1 g - r_2 m_2 g = 0$
B) $r_1 m_1 g + r_2 m_2 g = 0$
C) $r_1 m_2 g - r_2 m_1 g = 0$
D) $r_1 m_2 g + r_2 m_1 g = 0$

Answer: A) $r_1 m_1 g - r_2 m_2 g = 0$

Pulley fun

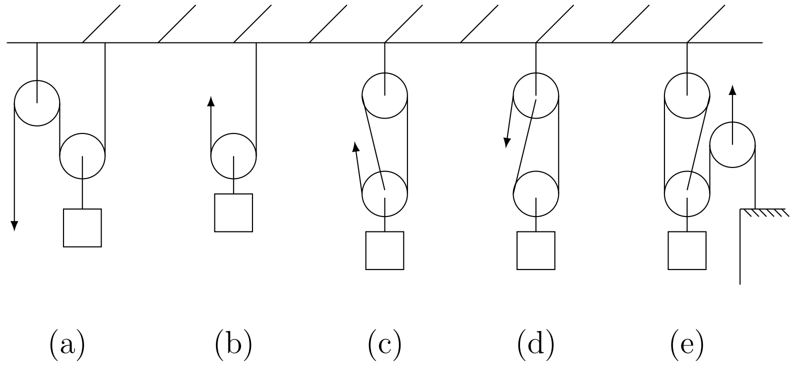

Figure 5: Pulleys gallore!

Use the following options for the next five pulley questions:

A) 1
B) 3/2
C) 2
D) 5/2
E) 3
F) 7/2

Question 16 [3 pt]. What is the IMA of the pulley system in Figure 5 section (a)?

Answer: C) 2

Question 17 [3 pt]. What is the IMA of the pulley system in Figure 5 section (b)?

Answer: C) 2

Question 18 [4 pt]. What is the IMA of the pulley system in Figure 5 section (c)?

Answer: E) 3

Question 19 [4 pt]. What is the IMA of the pulley system in Figure 5 section (d)?

Answer: C) 2

Question 20 [5 pt]. What is the IMA of the pulley system in Figure 5 section (e)?

Answer: B) 3/2

Question 21 [4 pt]. In Figure 6, there are two disks, each with an inner wheel of radius 0.3 m and outer wheel of radius 1 m. What's the IMA of this system if force is applied at the arrow, and the square is the load?

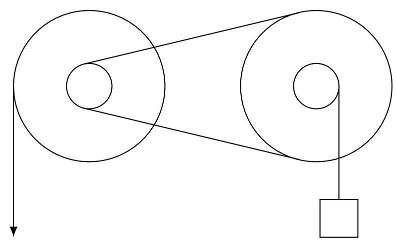

Figure 6: Two disks connected by a chain.

A) 9/100
B) 3/10
C) 1
D) 10/3
E) 100/9

Answer: E) 100/9 since $1/0.3 \cdot 1/0.3 = 100/9$

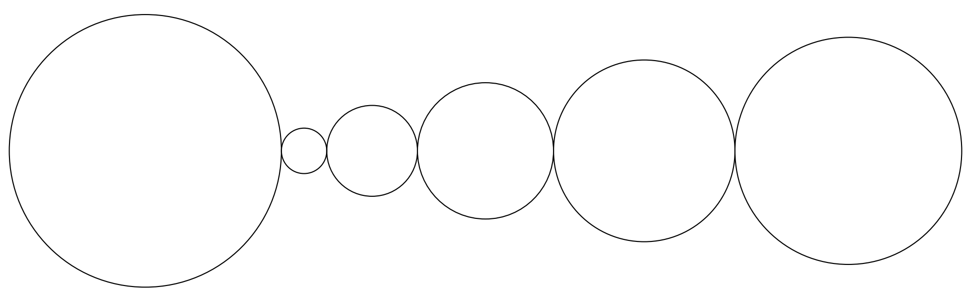

Figure 7: A chain of (lazily-drawn) gears with radii $6, 1, 2, 3, 4$, and 5, respectively.

Question 22 [3 pt]. Consider the chain of gears in Figure 7. If the leftmost gear, with radius 6, is the input, and the rightmost gear, with radius 5, is the output, what is the gear ratio? Hint: remember that the sign of the gear ratio indicates whether both gears rotate in the same or opposite directions.

A) $-6/5$
B) -1
C) $-5/6$
D) $5/6$
E) 1
F) $6/5$

Answer: C) $-5/6$ because they spin in opposite directions and the output gear spins 6/5 revolutions per input revolution

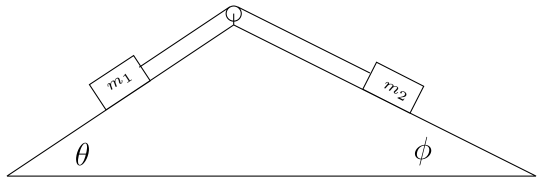

Figure 8: Two masses on a hill (or: doubly inclined atwood machine)

Question 23 [6 pt]. In Figure 8 there are two masses counterbalancing each other. On the left is mass m_1, which lies on a slope inclined at θ, and on the right is mass m_2, which lies on a slope inclined at θ. If these masses are balancing each other, what is the ratio m_1/m_2?

A) $\frac{\sin\phi}{\sin\theta}$
B) $\frac{\sin\theta}{\sin\phi}$
C) $\frac{\cos\phi}{\cos\theta}$
D) $\frac{\cos\theta}{\cos\phi}$
E) 1

Answer: A) $\frac{\sin\phi}{\sin\theta}$

5 Kinematics and the Static case

5.1 Nested Atwood Machine

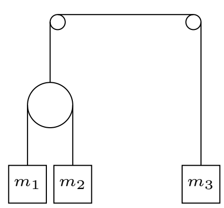

Figure 9: Nested Atwood Machine

For the following problems, consider the upward direction positive. For example, a object in freefall would experience acceleration $-g$. The top two pulleys are fixed, but the large lower left pulley can move up or down. Assume all string and pulleys are massless.

Question 24 [3 pt]. Which of the following equations best represents the forces acting on m_3?

A) $m_3a_3 = (m_1 + m_2 - m_3)g$
B) $m_3a_3 = (m_1 + m_2 + m_3)g$
C) $m_3a_3 = (m_1 - m_2 - m_3)g$
D) $m_3a_3 = (m_1 - m_2 + m_3)g$

Answer: A) $m_3 a_3 = (m_1 + m_2 - m_3)g$

Question 25 [5 pt]. What is the acceleration a_1 of mass m_1?

A) $\frac{m_1 - m_2 + m_3}{m_1} g$
B) $\frac{m_1 - m_2 - m_3}{m_1} g$
C) $\frac{m_1 + m_2 - m_3}{m_1} g$
D) $\frac{-m_1 - m_2 + m_3}{m_1} g$
E) $\frac{-m_1 + m_2 + m_3}{m_1} g$

Answer: E) $\frac{-m_1 + m_2 + m_3}{m_1} g$

Question 26 [3 pt]. Which ratio of masses $m_1 : m_2 : m_3$ will result in no acceleration for any mass?

A) 1:1:1
B) 1:1:2
C) 1:2:1
D) 2:1:1
E) 1:1:3

Answer: B) 1:1:2

5.2 Pulling masses

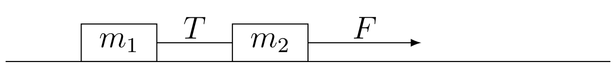

Figure 10: Two masses being pulled by rope

Question 27 [7 pt]. In Figure 10, mass m_2 is pulled by a force F and pulls along a mass m_1 through a string with tension T. What are the equations of motion for m_1 and m_2?

A) $m_1 a_1 = m_2 T$
$m_2 a_2 = F - m_1 T$
B) $m_1 a_1 = F + T$
$m_2 a_2 = F$
C) $m_1 a_1 = T$
$m_2 a_2 = F - T$
D) $m_1 a_1 = m_2 a_2$
$m_2 a_2 = F$
E) $m_1 a_1 = T/(T + F)$
$m_2 a_2 = F/(T + F)$

F) $m_1 a_1 = T/(T+F)$
$m_2 a_2 = F$

Answer: C) $m_1 a_1 = T$
$m_2 a_2 = F - T$

Question 28 [4 pt]. If the two masses stay a constant distance apart, i.e. $a_1 = a_2$, then what is the tension T as a function of F?

A) F

B) $\frac{m_1}{m_1+m_2}F$

C) $\frac{m_2}{m_1+m_2}F$

D) $\frac{m_1-m_2}{m_2}F$

E) $\frac{m_2-m_1}{m_2}F$

Answer: B) $\frac{m_1}{m_1+m_2}F$

Question 29 [2 pt]. If there is friction on just m_1, would T/F increase or decrease? (still assume $a_1 = a_2$)

A) Increase

B) Decrease

Answer: A) Increase because m_1 would need relatively more force supplied to maintain the same acceleration

Question 30 [2 pt]. If there is friction on just m_2, would T/F increase or decrease? (still assume $a_1 = a_2$)

A) Increase

B) Decrease

Answer: B) Decrease because more of F is lost to friction

5.3 Pulley with Weighted Rope

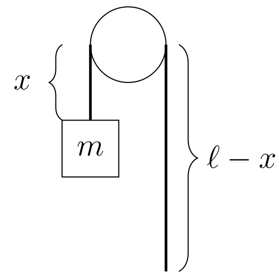

Figure 11: A pulley with a weighted rope

The pulley in Figure 11 has a weighted rope with density λ so that the left piece of rope has mass λx and the right has mass $\lambda(\ell - x)$.

Question 31 [6 pt]. What is the equation of motion for the suspended mass?

A) $ma = (\lambda\ell - m)g$
B) $ma = (\lambda\ell + m)g$
C) $ma = (\lambda\ell - \lambda x - m)g$
D) $ma = (\lambda\ell - \lambda x + m)g$
E) $ma = (\lambda\ell - 2\lambda x - m)g$
F) $ma = (\lambda\ell - 2\lambda x + m)g$

Answer: E) $ma = (\lambda\ell - 2\lambda x - m)g$

Question 32 [5 pt]. What value of x results in a static system?

A) $\frac{\ell}{2} - \frac{m}{\lambda}$
B) $\frac{\ell}{2} - \frac{m}{2\lambda}$
C) $\frac{\ell}{2}$
D) $\frac{\ell}{2} + \frac{m}{2\lambda}$
E) $\frac{\ell}{2} + \frac{m}{\lambda}$

Answer: B) $\frac{\ell}{2} - \frac{m}{2\lambda}$

Question 33 [2 pt]. If the mass is placed above this value of x (i.e. its value of x is too small), will the mass move up, down, or neither?

A) Up
B) Down
C) Neither

Answer: A) Up

6 Miscellaneous

6.1 Determining an object's center of mass using a lever

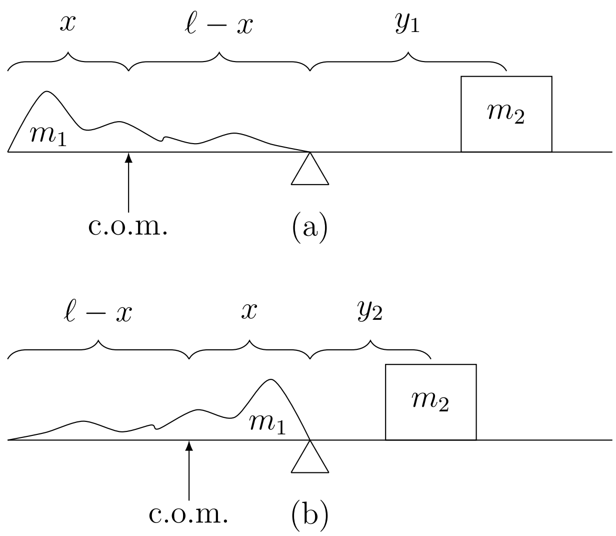

Figure 12: Mystery object on a see-saw

In Figure 12 is a mystery object with mass m_1 being counterbalanced by a mass m_2. In (a), the center of mass of m_1 is located x from the left edge. In (b), we flip the mystery object, readjust the counterbalance, and measure the new position y_2.

Question 34 [6 pt]. Which of the following equations are true when both systems (a) and (b) are balanced?

A) $m_2 y_1 = m_1 x$
$m_2 y_2 = m_1(\ell - x)$

B) $m_2 y_1 = m_1(\ell - x)$
$m_2 y_2 = m_1 x$

C) $m_1 y_1 = m_2(\ell - x)$
$m_1 y_2 = m_2 x$

D) $m_1 y_1 = m_2 x$
$m_1 y_2 = m_2(\ell - x)$

Answer: B) $m_2 y_1 = m_1(\ell - x)$
$m_2 y_2 = m_1 x$

Question 35 [4 pt]. By dividing one equation by the other, we can find the center of mass of our mystery object. What is x/ℓ?

A) $\frac{y_1}{y_2}$

B) $\frac{y_2}{y_1}$

C) $\frac{y_1}{y_1+y_2}$

D) $\frac{y_2}{y_1+y_2}$

E) $\frac{m_1 y_1}{m_1 y_1+m_2 y_2}$

F) $\frac{m_2 y_1}{m_2 y_2}$

Answer: D) $\frac{y_2}{y_1+y_2}$

6.2 Inclined Plane with friction

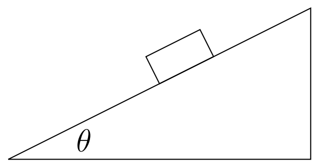

Figure 13

Consider pushing a mass up a ramp with friction, as in Figure 13. The coefficient of friction is μ, and the ramp is inclined at an angle θ.

Question 36 [3 pt]. What is the IMA of this system? (Find the expression that matches the familiar L/h form)

A) $\sin\theta$

B) $\tan\theta$

C) $\cos\theta$

D) $\frac{1}{\cos\theta}$

E) $\frac{1}{\tan\theta}$

F) $\frac{1}{\sin\theta}$

Answer: F) $\frac{1}{\sin\theta}$

Question 37 [7 pt]. What is the AMA of this system?

A) $\mu\tan\theta$

B) $\sin\theta+\mu\cos\theta$

C) $\cos\theta+\mu\sin\theta$

D) $\frac{\mu}{\tan\theta}$

E) $\frac{1}{\sin\theta+\mu\cos\theta}$

F) $\frac{1}{\cos\theta+\mu\sin\theta}$

Answer: E) $\frac{1}{\sin\theta+\mu\cos\theta}$

Question 38 [3 pt]. What is the efficiency of the system?

A) $1-\mu$

B) $(1-\mu)\tan\theta$

C) $\frac{\cos\theta}{\cos\theta+\mu\sin\theta}$

D) $\frac{\mu\cos\theta}{\cos\theta+\mu\sin\theta}$

E) $\frac{\sin\theta}{\sin\theta+\mu\cos\theta}$

F) $\frac{\mu\cos\theta}{\sin\theta+\mu\cos\theta}$

Answer: E) $\frac{\sin\theta}{\sin\theta+\mu\cos\theta}$

Question 39 [3 pt]. What angle θ would maximize the system's efficiency?

A) $0°$

B) $45°$

C) $90°$

D) $\arctan(\mu)$

E) $\arctan(1/\mu)$

Answer: C) $90°$

6.3 Tiebreaker inspired by the build portion (rip)

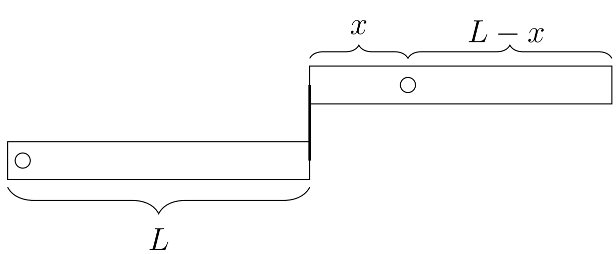

Figure 14: Compound lever

Question 40 [0 pt]. In Figure 14 is a sketch of the compound lever from this event's build event. Assume that each lever has mass m, length L, and uniform mass density of $\lambda = \frac{m}{L}$. Write down the equation representing the fact that there is zero torque about the right fulcrum (you can do this in terms of moments).

If you have time, try to show that the system is balanced when $x/L = 1/3$.

This is a tiebreaker question (marked as 0.9 points on Scilympiad), so don't sweat it too much. Grading will mainly be for clarity and demonstration that you understand how to write down the relevant equations for this problem.

Answer: Sample equation:

$$\begin{aligned}
(\lambda(L-x))\left(\frac{1}{2}(L-x)\right) &= \left(\frac{1}{2}\lambda L\right)x + (\lambda x)\left(\frac{1}{2}x\right) \\
\frac{1}{2}\lambda(L-x)^2 &= \frac{1}{2}\lambda Lx + \frac{1}{2}\lambda x^2 \\
L^2 - 2Lx + x^2 &= Lx + x^2 \\
L^2 - 3Lx &= 0 \\
L(L-3x) &= 0
\end{aligned}$$

So $x/L = 1/3$.

Point total: 140