Machines B - Illinois Regionals 2020

“So... logically, if she weighs the same as a duck, she’s made of wood.”

“And therefore...”

“A witch!”

A widespread medieval usage of pulley scales, depicted in 1975.

Instructions:

Do not open test until instructed!

Write all answers on the **answer sheet**. There are 7 questions. No credit will be given for work on the test if there is no answer on the answer sheet.

Three significant figures should be sufficient for all questions, but more will not be penalized.

Write symbolic answers in **simplest form** so the grader can easily verify your work.

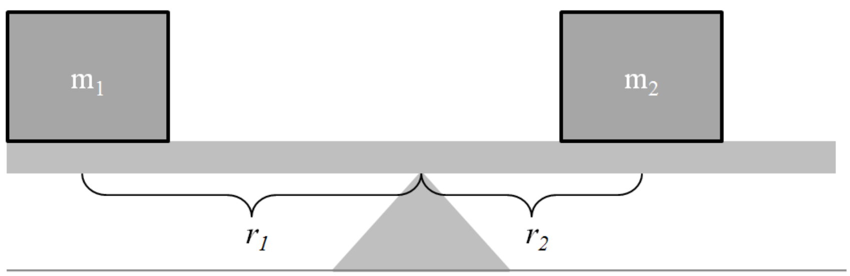
Question 1 [10 pts]. Mass range

Two boxes are placed on a see-saw. Box 1 has mass $\boldsymbol{m_1 = 1\ kg}$ and is $\boldsymbol{r_1 = 3\ m}$ from the fulcrum. Box 2 has mass m_2 and is $\boldsymbol{r_2 = 2\ m}$ from the fulcrum. Suppose the fulcrum is rusty and only transmits **70%** of force (i.e., it has an efficiency of 0.7).

a [5 pts]. What is the smallest possible m_2 that results in a balanced see-saw?

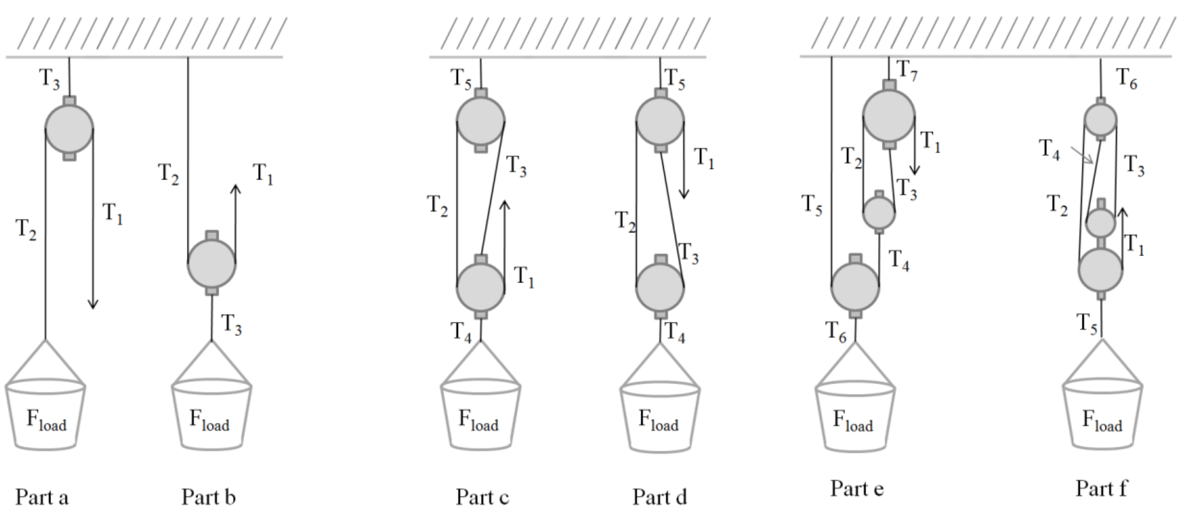
b [5 pts]. What is the largest possible m_2 that results in a balanced see-saw?

Make sure your answer to part b is larger than your answer to part a!



Question 2 [17.5 pts]. Deriving the IMA of simple pulley systems

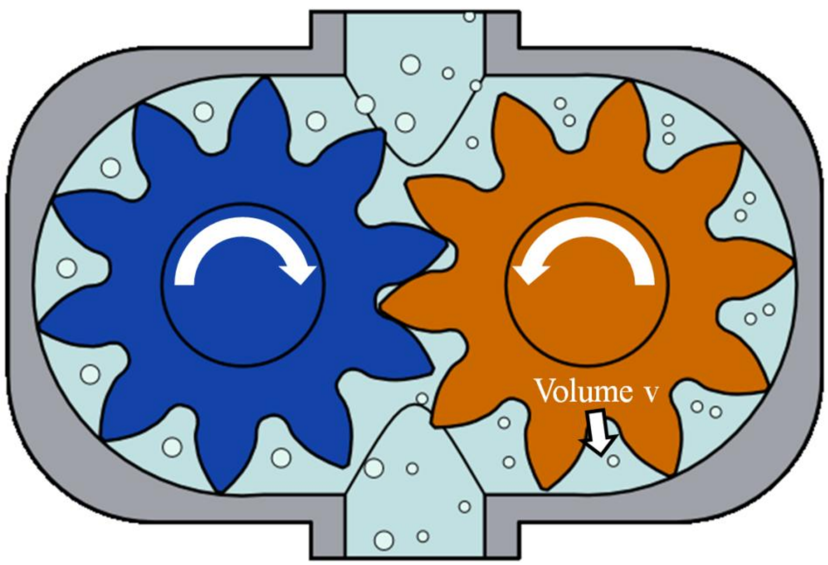
a-f [0.5 pt per entry]. For each system below, write the tension of each section of string, given T_1 = 1 N. Then find the maximum load that can be supported (F_{load}) given T_1 as the input. Then find the IMA of the system (assume massless, frictionless pulleys so IMA=AMA). Hint: it is normal to have many repeated answers between different portions of string, but not every piece of string has the same tension. Make sure to write down your answers in the correct field!



Note: why bother track each tension for these simple setups? As demonstrated in a/b and c/d, you need to be careful in applying any mnemonic you may have learned (e.g. "draw a line and count the number of times string passes through it"). This method of tracking tensions is generally more robust and hopefully will help you to never get tricked by a pulley question again!

Question 3 [12.5 pts]. Kepler pump

Around 1600, Johannes Kepler designed a simple gear system that pumps water in one direction as the gears are driven. Due to the gear teeth shape, a minimal amount of fluid is able to travel in the reverse direction in the middle.



a [2.5 pts]. Does the water flow up or down? The left gear spins clockwise as the picture indicates.

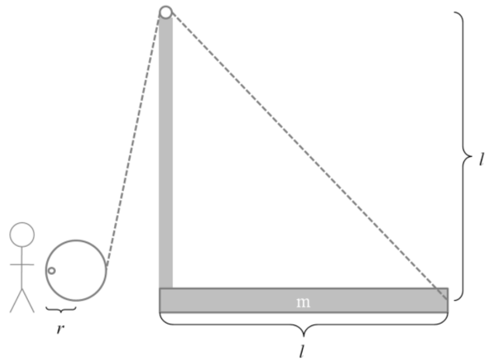
b [3 pts]. When the left gear is turned by 360º, how much liquid does it move (by volume)? Each gear has 10 teeth and 10 corresponding pockets, each with volume $v = 1$ mL.

c [2 pts]. When the left gear is turned by 360º, how much total liquid does the system move?

d [5 pts]. How fast should the left gear be driven to result in a flow of 6 liters per minute? Answer in revolutions per minute.

Question 4 [5 points]. Drawbridge

A drawbridge of length *l* has a mass *m* and can be lifted or lowered by means of a chain connected to a hand-cranked windlass with radius *r*.



How much work is required to pull up the drawbridge? Answer in terms of *m, l,* and gravitational acceleration *g*. (Hint: the usage of the windlass does not change the work)

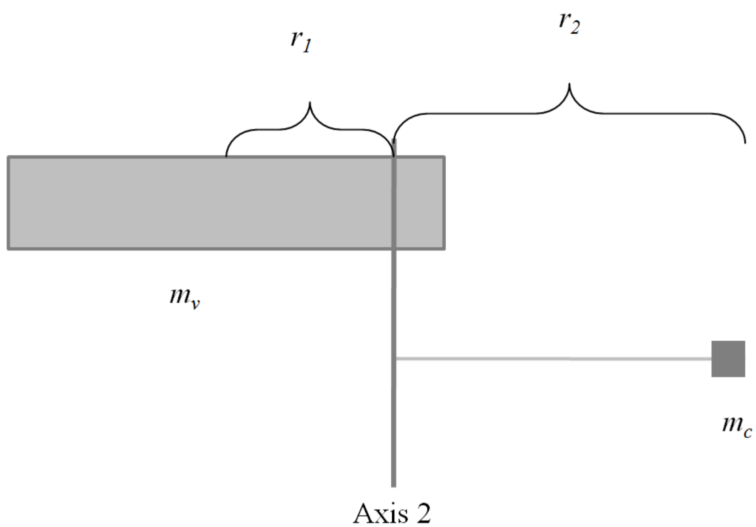
Question 5 [15 pts]. Equatorial mounted telescope

Here is a picture of an equatorial mounted telescope. To adjust for the Earth's rotation throughout a night, this telescope is able to rotate about an axis (right-ascension) that is parallel to the Earth's axis of rotation (polar axis). In order for this to be easily adjusted, the telescope itself is balanced by a counterweight.

a [5 pts]. Suppose the main telescope tube has mass 5 kg and is centered 30 cm from Axis 1. If the counterweight is located 50 cm from Axis 1, what is its mass?

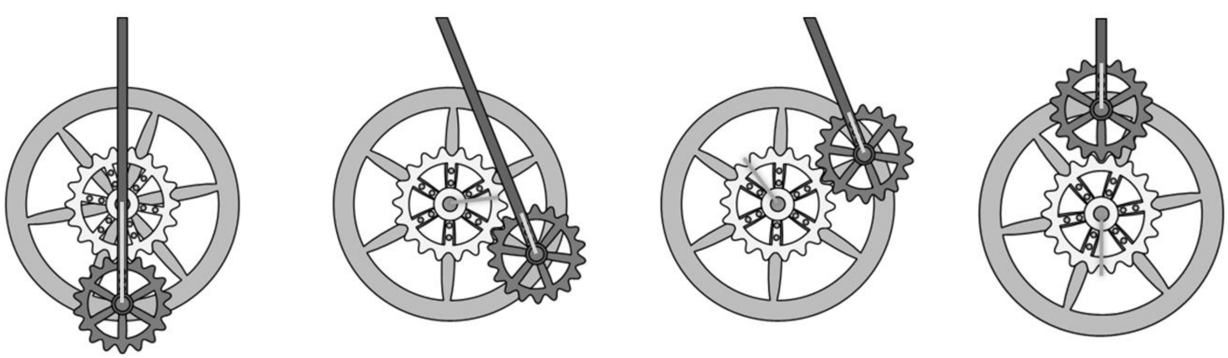
b-c [10 pts]. Suppose the main telescope is balanced around axis 2, so that the smaller weight (to the right) counterbalances the viewfinder (top tube). We can easily measure the positions of the centers of these objects: the viewfinder is centered r_1 = 10 cm to the left of axis 2, while the counterweight is positioned r_2 = 40 cm.

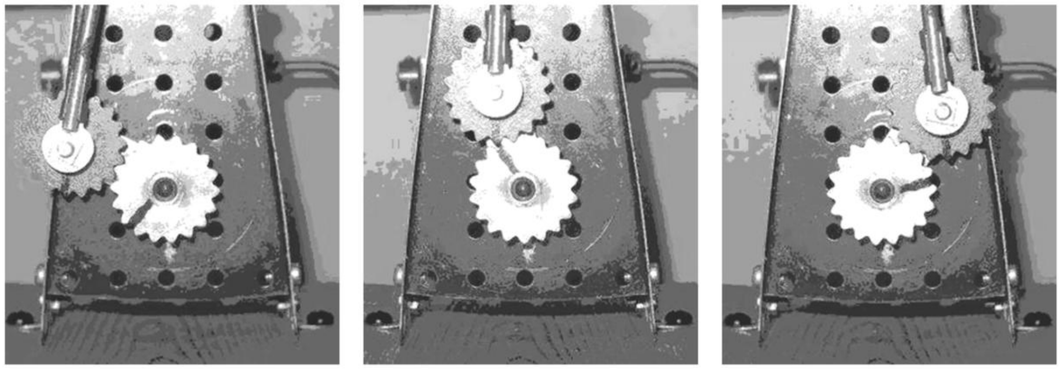
Based on your knowledge of class 1 levers, you should be able to figure out the masses of the counterweight and viewfinder (assume a uniform mass distribution). If we know m_v+m_c = 2 kg (supposing the main telescope tube has mass 3 kg), then what are m_c and m_v? Make sure that these add up to 2 kg and also correspond with the correct object!



Question 6 [10 pts]. Sun-and-planet gear

Here's a sun-and-planet gear system. A planet gear is driven around the center sun gear by a rod moving up and down (e.g. pushed by a piston). While the sun gear and attached frame are free to rotate, the planet's teeth are fixed with respect to the rod. Here are a couple pictures from the Wikipedia page (not to be confused^{TM} with planetary gears):



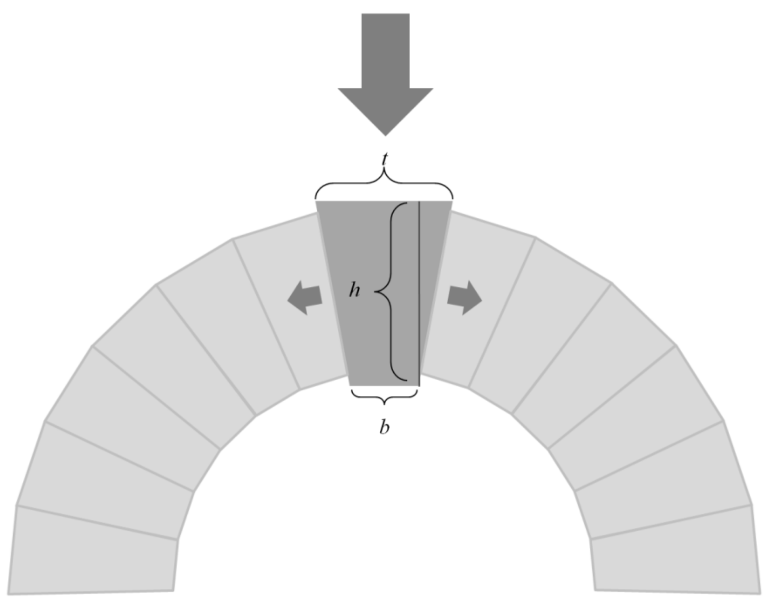


a [5 pts] As in the pictures, consider a system where the sun gear and planet gear have the same number of teeth. How many rotations does the sun turn while the planet gear makes one full revolution?

b [5 pts]. Now consider a setup where the sun gear has **twice** as many teeth as the planet gear (and therefore also has twice the radius). Now how many rotations does the sun turn while the planet gear makes one full revolution? (note this is fractional and is between 1 and 2)

Question 7 [15 pts]. Keystone of an arch

Here's a keystone in an arch. It redirects force to the left and right so that no support is needed directly under it.



a [2 pts]. What simple machine does the keystone most behave like?

b [5 pts]. The keystone measures as follows: the top width is t = 30 cm, the bottom width b = 15 cm, and the height is h = 45 cm. If the input force is the weight from above and the output is pushing neighboring bricks to the left and right, what is the IMA of this keystone as a simple machine?

c [5 pts]. If the keystone supports 100 kN in load, how much **horizontal** force does it impart on each brick next to it? You can find this using the keystone as the simple machine identified in part (a).

d [3 pts]. What is the total force on each brick? Hint: the vertical force has to be 50 kN since the 100 kN load is split across two sides. You can use Pythagoras' theorem.

Machines B -- Answer Sheet

Test score: _____/85

Team name: ______________________

Page 1 score: ____/40

Team number: _________

Page 2 score: ____/45

Question 1 [10 pts].

a [5 pts]. ___________________ kg

b [5 pts]. ___________________ kg

Question 2 [17.5 pts; 0.5 pt per blank]

a [2]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

F_{load}=____ N

IMA=____

b [2]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

F_{load}=____ N

IMA=____

c [3]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

T_4 = ____ N

T_5 = ____ N

F_{load}=____ N

IMA=____

d [3]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

T_4 = ____ N

T_5 = ____ N

F_{load}=____ N

IMA=____

e [4]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

T_4 = ____ N

T_5 = ____ N

T_6 = ____ N

T_7 = ____N

F_{load}=____ N

IMA=____

f [3.5]. T_1 = 1 N

T_2 = ____ N

T_3 = ____ N

T_4 = ____ N

T_5 = ____ N

T_6 = ____ N

F_{load}=____ N

IMA=____

Question 3 [12.5 pts].

a [2.5 pts]. up / down (circle one)

b [3 pts]. ___________________ mL

c [2 pts]. ___________________ mL

d [5 pts]. ___________________ rpm

Question 4 [5 pts]. ____________________ (symbolic)

Question 5 [15 pts].

a [5 pts]. ____________________ kg

b [5 pts]. m_c = ____________________ kg

c [5 pts]. m_v = ____________________ kg

Question 6 [10 pts].

a [5 pts]. ____________________ revolutions

b [5 pts]. ____________________ revolutions

Question 7 [15 pts].

a [2 pts]. ____________________ (machine type)

b [5 pts]. ____________________ (unitless)

c [5 pts]. ____________________ kN

d [3 pts]. ____________________ kN

Machines B -- Answer KEY (Point total: 85=40+45)

Question 1 [10 pts].

a [5 pts]. **1.05 kg = 21/20 kg** = 7/10 * 1*3/2 = 7/10 * m_1r_1/r_2

b [5 pts]. **2.143 kg = 15/7 kg** = 10/7 * 1*3/2 =10/7 m_1r_1/r_2

Question 2 [17.5 pts, 0.5 pt each -- but don't score T_1 as it is given]

a [2]. T_1 = 1 N

T_2 = **1** N

T_3 = **2** N

F_{load} = **1** N

IMA = **1**

b [2]. T_1 = 1 N

T_2 = **1** N

T_3 = **2** N

F_{load} = **2** N

IMA = **2**

c [3]. T_1 = 1 N

T_2 = **1** N

T_3 = **1** N

T_4 = **3** N

T_5 = **2** N

F_{load}= **3** N

IMA = **3**

d [3]. T_1 = 1 N

T_2 = **1** N

T_3 = **1** N

T_4 = **2** N

T_5 = **3** N

F_{load} = **2** N

IMA = **2**

e [4]. T_1 = 1 N

T_2 = **1** N

T_3 = **1** N

T_4 = **2** N

T_5 = **2** N

T_6 = **4** N

T_7 = **3** N

F_{load} = **4** N

IMA = **4**

f [3.5]. T_1 = 1 N

T_2 = **1** N

T_3 = **1** N

T_4 = **1** N

T_5 = **4** N

T_6 = **3** N

F_{load} = **4** N

IMA = **4**

Grading note: it is incorrect if a team writes "N" after the IMA, which should be unitless. I'd recommend taking off at least 0.5-1 points if a team does this for every entry of IMA as it shows they don't understand that IMA is a ratio.

Question 3 [12.5 pts].

a [1.5 pts]. **up** - the upward traveling happens on the left/right rather than the middle

b [3 pts]. **9 mL**

c [2 pts]. **18 mL**

d [5 pts]. **300 rpm** = (6000 mL/min) / (20 mL (between both gears) per revolution)

Question 4 [5 pts].

$\frac{1}{2}mlg$ (from lifting the center of mass by half the height of the door)

Question 5 [15 pts].

a [5 pts]. **3 kg**

b [5 pts]. $\boldsymbol{m_c}$ **= 2/5 kg = 0.4 kg**

c [5 pts]. $\boldsymbol{m_v}$ **= 8/5 kg = 1.6 kg**

The viewfinder effectively acts as a point mass at its center of mass. m_v and m_c are in a ratio of r1:r2 = 1:4 and add up to 2.

Question 6 [10 pts].

a [5 pts]. **2** revolutions (= 4π = 720º) (accept any form; no units required unless in degrees)

b [5 pts]. **3/2=1.5** revolutions (=3π = 540º) (accept any form)

Question 7 [15 pts].

a [2 pts]. **wedge**

b [5 pts]. **6** = 90 cm / 15 cm = 2 * 45 cm / (30 cm - 15 cm) = 2h / (t-b)

c [5 pts]. **600 kN** = 6 * 100 kN

d [3 pts]. **602 kN** = sqrt(600^2 + 50^2)