# Machines C Exam
# State 2021

Saturday, April 10, 2021

Instructions:

1. Be sure to enter all answers into Scilympiad!
2. Many problems are multiple-choice to make grading easier. Not all the options make physical sense, though!! Whenever possible, check that your answers make sense before moving on!
3. Make sure to ask on scilympiad if you have any questions about the test.
4. We hope you enjoy and learn something new from this test! Best of luck!

# 1 Warm-up and general questions

**Question 1** [1 pt]. What is the main cause for mechanical advantages to not match ideal levels?

A) Incorrect configuration

B) Friction

C) Backlash

D) Human error

E) Selecting multiple choice answers without checking if they make sense

**Question 2** [3 pt]. Which of the following is NOT always true of a simple machine? ($e$ is the efficiency)

A) $0 \leq e \leq 1$

B) $AMA \leq IMA$

C) $W_{out} = eW_{in}$

D) $e \leq AMA \cdot IMA$

**Question 3** [1 pt]. Is there a theoretical upper limit to IMA? (i.e., is it impossible for any machine to get infinite IMA?)

A) Yes

B) No

**Question 4** [2 pt]. Jack watches Jill get buckets of water using a single pulley with IMA 1. Jack is lazy and would like to get water with less work but has not studied physics. Jack can raise the IMA of the system to 2 by switching attaching the pulley to the bucket rather than the top of well. Does Jack save any work by doing this? (assume perfect efficiency)

A) Yes

B) No

**Question 5** [3 pt]. If the pulley has an efficiency of 0.4, will Jack end up performing more work with the IMA 2 configuration compared to the original IMA 1 configuration?

A) Yes

B) No

**Question 6** [3 pt]. Two friends want to lift a 100 kg treasure chest out of a hole, but together they are unable to supply the 980 N required (they don't lift, sadly). They do have a 4:1 pulley system lying around but nowhere to attach it to. After they've attached one side to the chest, Friend 1 thinks Friend 2 should hold the other pulleys in place (as an anchor) as she (Friend 1) pulls the rope. How much force would Friend 2 need to provide for this to work? (Friend 1's rope first passes through the pulley on the treasure chest rather than Friend 2's pulley)

A) 980/4 N

B) 980/3 N

C) 980/2 N

D) $980 \cdot 2/3$ N (possible if Friend 2 were twice as strong as Friend 1)

E) $980 \cdot 3/4$ N (too much)

# 2 Simple Machines

## 2.1 Units and dimensional analysis

An important aspect of physics is **dimensional analysis**. In this context, the dimension of a quantity essentially refers to the type of units. For example, the radius of a wheel has dimensions of length, but the actual value may be given in units of meters or feet.

It's very useful to be able to glance at the units or dimensions of a quantity and tell whether your result makes physical sense. For example, knowing that IMA or efficiency is unitless immediately tells us not to put it down if we're looking for a force.

**Question 7** [2 pt]. Which of the following units has dimension of acceleration?

A) m $s^{-2}$

B) m $s^{-1}$

C) kg m $s^{-2}$

D) kg m $s^{-1}$

**Question 8** [2 pt]. Which of the following quantities has the same dimension as **power**?

A) Work per unit time

B) Energy per unit time

C) The product of force and velocity

D) All of the above

E) None of the above

**Question 9** [2 pt]. Which of the following quantities does **not** have the same dimension as **force**?

A) Change in momentum per unit time

B) Change in Kinetic Energy per unit time

C) Gravitational potential per unit length

D) Tension in a string

**Question 10** [2 pt]. What is the dimension of a unit conversion factor, such as $\frac{2.54 \text{ in}}{1 \text{ cm}}$?

A) Inverse Length squared

B) Inverse Length

C) Dimensionless

D) Length

E) Length squared

## 2.2 Computation questions

**Question 11** [2 pt]. If the IMA of a machine is 2, and the efficiency of the machine is 0.65, what is the AMA of the machine?

A) 2.65

B) 2

C) 1.35

D) 1.3

**Question 12** [2 pt]. Consider a type 3 lever, with load of 60N applied at 0.8 m from the pivot. In order to maintain equilibrium, where (as in distance from the pivot) should effort of 90 N be exerted?

A) 1.33 m

B) 0.83 m

C) 0.53 m

D) 0.13 m

**Question 13** [3 pt]. A double-start screw has 1/16 inch between adjacent threads. If I'm using a screwdriver whose handle has a radius of 2 inches, what's the IMA of the screw-screwdriver combination?

A) $\pi/2$

B) $\pi/4$

C) $\pi/8$

D) $8\pi$

E) $16\pi$

F) $32\pi$

**Question 14** [3 pt]. If a gear system with 5 gears (3 between the driver and driven) has an IMA of 1000, but each of the 3 intermediate gears transmits only 0.9 of the force, what is the AMA of the system?

A) 590

B) 656

C) 700

D) 729

E) 810

F) 900

**Question 15** [5 pt]. An elevator starts at the top of a 100 m shaft at rest, but its descent is slowed due to friction on the cable. By the time the elevator reaches the ground, its speed is 30 m/s. What fraction of the energy was dissipated by friction?

A) 0.322
B) 0.459
C) 0.541
D) 0.678
E) Need to know the mass of the elevator

**Question 16** [2 pt]. College is tiring and lowers students' efficiency by 15% every year. If a freshman moving into the dorms can bring their 20 kg luggage up two flights of stairs in 80 seconds, how long would it take them to bring up the same luggage at the beginning of senior year (after three full years)?

A) 44 s
B) 80 s
C) 130 s
D) 145 s
E) 160 s

## 2.3 Gear Ratio

I find it easy to get tripped up by the IMA of a gear, also known as the gear ratio. Hopefully this section will help you to avoid confusing it with its reciprocal.

Assume there are two gears with $N_{in}$ and $N_{out}$ teeth, respectively. We turn the "in" gear (which is known as the "driver" within the gear system even though it is itself driven), which drives the "out gear" (also called the "driven" gear).

**Question 17** [2 pt]. Suppose $d_{in}$ is the distance that the edge of the "in" gear travels, and $d_{out}$ is the distance that the edge of the "out" gear travels. What is the value of $d_{in}/d_{out}$? Hint: this is typically NOT called the IMA, despite appearing to match the formula (we usually don't care about the distance itself).

A) 1/2
B) 1
C) 2
D) $N_{out} + N_{in}$
E) $N_{out} - N_{in}$
F) $N_{in} - N_{out}$

**Question 18** [2 pt]. The value above is not very interesting or useful. Instead, let's think about the mechanical advantage as the ratio of torques (angular analog of forces), so that $AMA = \tau_{out}/\tau_{in}$. Assume conservation of energy so that $IMA = AMA$ for now. What is

the IMA in terms of the angles each gear turns? (Let $\theta_{in}$ be the angle the input gear turns, while $\theta_{out}$ is the angle that the output gear turns)
Remember that work is given by $W = \theta\tau$, where $\tau$ is the torque applied to turn the gear by an angle of $\theta$, so with conservation of energy, we know $W_{in} = \theta_{in}\tau_{in} = W_{out} = \theta_{out}\tau_{out}$.

A) $\theta_{in}/\theta_{out}$
B) $\theta_{out}/\theta_{in}$
C) $\theta_{in} - \theta_{out}$
D) $\theta_{out} - \theta_{in}$

**Question 19** [3 pt]. Let's put this in terms of $N_{in}$ and $N_{out}$ now. Remember that $d_{in} = N_{in}\theta_{in} = N_{out}\theta_{out} = d_{out}$ is the distance each gear travels at its edge (hint hint for a previous question). What's the IMA of the gears in terms of the number of teeth?

The IMA is also called the Gear Ratio. You can think of it as the number of revolutions the input gear needs to turn in order for the output gear to make one full revolution.

A) $N_{in} - N_{out}$
B) $N_{out} - N_{in}$
C) $N_{in}/N_{out}$
D) $N_{out}/N_{in}$

**Question 20** [2 pt]. If an input gear with 10 teeth is driving an output gear with 1000 teeth, what is the gear ratio or IMA? (sometimes visualizing extreme cases makes it easier to remember which way things should go)

A) 1/100
B) 1
C) 100

## 2.4 Miscellaneous Gear Questions

**Question 21** [3 pt]. Three gears are connected to form a gear system. They each have $N_1$, $N_2$, and $N_3$ teeth, respectively, where gear 1 is the driver of the system and gear 3 is the driven gear. The first gear gets damaged so that it only has $2N_1/3$ teeth intact. How many revolutions should gear 1 make in order for gear 3 to make one full revolution? (basically, what is the gear ratio over the course of a full cycle)

A) $\frac{2N_3}{3N_1}$
B) $\frac{2N_1}{3N_3}$
C) $\frac{N_3}{N_1}$
D) $\frac{N_1}{N_3}$
E) $\frac{3N_3}{2N_1}$
F) $\frac{3N_1}{2N_3}$

**Question 22** [2 pt]. Which of the following could be used to give irrational gear ratios?

A) Worm gears
B) Planetary gears
C) Harmonic gearing
D) Differential gear
E) Continuous variable transmission
F) Helical bevel gear

**Question 23** [1 pt]. What is the purpose of a rack-and-pinion gear system?

A) Spread onions on a rack of ribs
B) Stretch pines across a rack
C) Decrease friction with the gear's axle
D) Change the direction of gears' rotations
E) Translate between rotational and linear motion

**Question 24** [1 pt]. You've probably noticed gear tooth shapes tend to have a rounded (involute) shape. What is NOT an advantage of this shape over the more obvious triangular tooth?

A) Constant radius of contact point
B) Reduced backlash
C) More easily paired with gears of different sizes
D) Continuous contact
E) More robust to manufacturing errors

## 2.5 Pulley Party!

To replace all the pool parties all us scioly peeps would've gone to...

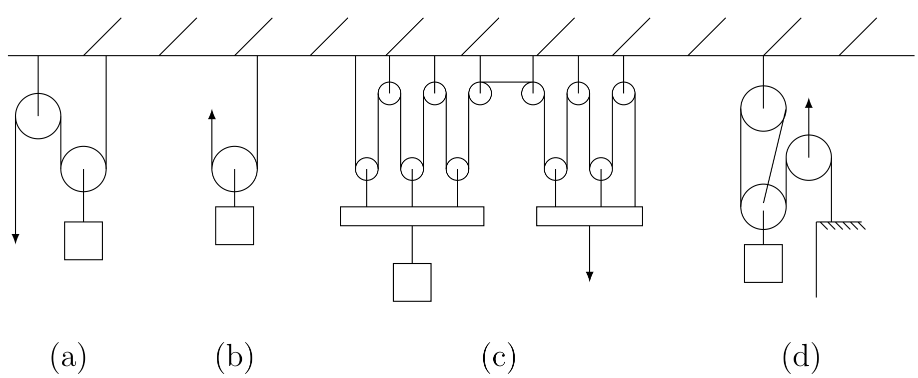


**Figure 1:** Pulleys gallore!

**Question 25** [1 pt]. What is the IMA of the pulley system in Figure 1 part (a)?

A) 1/2
B) 1
C) 3/2
D) 2
E) 4

**Question 26** [1 pt]. What is the IMA of the pulley system in Figure 1 part (b)?

A) 1/2
B) 1
C) 3/2
D) 2
E) 4

**Question 27** [4 pt]. What is the IMA of the pulley system in Figure 1 part (c)?

A) 2/3
B) 5/6
C) 1
D) 6/5
E) 4/3
F) 3/2

**Question 28** [5 pt]. What is the IMA of the pulley system in Figure 1 part (d)?

A) 1/2
B) 1
C) 3/2
D) 2
E) 5/2
F) 4

## 2.6 Efficiency of the Wedge

Let's compute the efficiency of a wedge if we know the coefficient of friction $\mu$. Consider a wedge that is $L$ long and $t$ wide being driven by a force $f$ into a piece of wood. The wedge's tip has an angle of $2\theta$, where $\tan\theta = \frac{t}{2L}$.

The wedge experiences normal forces from both sides (resistance from the wood), and we can call the output force $f_{out} = N\cos\theta$. The reasoning is that we only want the vertical component of force imparted by the wedge onto the wood (the total force is the normal force $N$). (it's a little sketchy but if I say $2N\cos\theta$ then the AMA is off by a factor of 2...)

**Question 29** [3 pt]. Which equation describes the horizontal forces acting on the wedge?

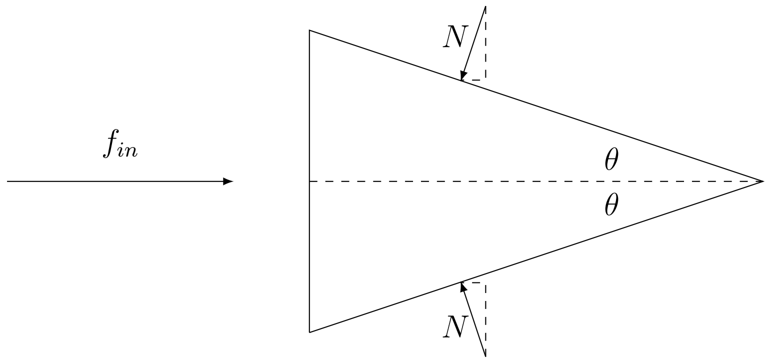


**Figure 2:** Forces on a wedge, without friction

A) $F_x = 0 = f_{in} - 2N \sin\theta$
B) $F_x = 0 = f_{in} + 2N \sin\theta$
C) $F_x = 0 = f_{in} - 2N \cos\theta$
D) $F_x = 0 = f_{in} + 2N \cos\theta$

**Question 30** [2 pt]. So what is the AMA $f_{out}/f_{in}$? Since there is not yet friction, this should match the equation for IMA if you rewrite it in terms of $L$ and $t$.

A) $\frac{1}{2}$
B) $\frac{1}{2\sin\theta}$
C) $\frac{1}{2\cos\theta}$
D) $\frac{1}{2\tan\theta}$

Now let's add in friction, which points roughly to the left but parallel to the wedge's surface.

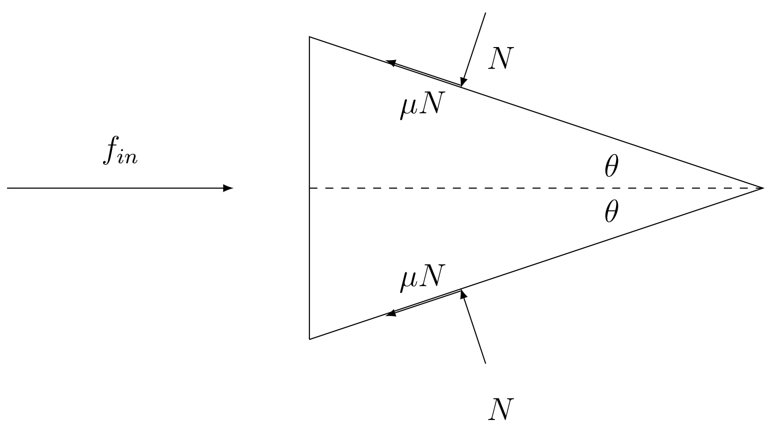


**Figure 3:** Forces on a wedge friction

**Question 31** [6 pt]. Now which equation describes the horizontal forces acting on the wedge?

A) $F_x = 0 = f_{in} - 2N \sin\theta - 2\mu N \cos\theta$
B) $F_x = 0 = f_{in} + 2N \sin\theta + 2\mu N \cos\theta$

C) $F_x = 0 = f_{in} - 2N\cos\theta - 2\mu N\sin\theta$

D) $F_x = 0 = f_{in} + 2N\cos\theta + 2\mu N\sin\theta$

**Question 32** [5 pt]. So, what is the efficiency of our wedge? (Hint: compute the AMA again and divide by the IMA from before)

A) $1 - \mu$

B) $\frac{1}{1+\mu/\tan\theta}$

C) $\frac{1}{1-\mu/\tan\theta}$

D) $1 - \mu/\sin\theta$

E) $1 - \mu/\tan\theta$

**Question 33** [1 pt]. How does the efficiency scale as a function of $\theta$? (for $0 < \theta < \pi/2$)

A) Increasing

B) Decreasing

C) Neither

# 3 Compound Machines

## 3.1 Lever at nontrivial angles (with a wheel)

Levers are nice simple machines when they stay still (fine for statics). However, when levers turn a lot, their motion becomes more obviously circular rather than straight and linear. In this section we'll be looking at levers' behavior in the circular regime.

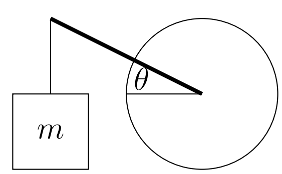


**Figure 4:** Mass suspended from a rod attached to a wheel

Figure 4 shows a rod attached to a wheel, and a mass $m$ is suspended from the tip of the rod with string. The rod is held at an angle $\theta$ from the ground. The wheel has radius $r$, and the rod has length $\ell$.

**Question 34** [4 pt]. How much torque (in the clockwise direction) must be provided to the wheel to keep the mass in place?

A) $mg\ell$

B) $mg\ell\sin\theta$

C) $mg\ell\cos\theta$

D) $mg\ell\tan\theta$

E) $mg\ell/\sin\theta$

F) $mg\ell/\cos\theta$

**Question 35** [1 pt]. If force is provided at the edge of the wheel (i.e., at radius $r$), how much force is needed to keep the mass suspended?

A) $mgr/\ell$

B) $mg\ell/r$

C) $mg\ell\sin\theta/r$

D) $mg\ell\cos\theta/r$

E) $mg\ell\tan\theta/r$

**Question 36** [3 pt]. If we regard the load on the system to be $mg$ rather than some fractional component of $mg$, then what is the AMA of our system (consider the ratio of forces and assume perfect efficiency)?

Note that this is not constant in $\theta$ and is therefore not considered a simple/compound machine (although if you apply force vertically to the lever directly rather than through a wheel, it will be subject to the same sin/cos/tan term and still act like a simple machine).

A) $\ell\sin\theta/r$

B) $\ell\cos\theta/r$

C) $\ell\tan\theta/r$

D) $r\sin\theta/\ell$

E) $r\cos\theta/\ell$

F) $r\tan\theta/\ell$

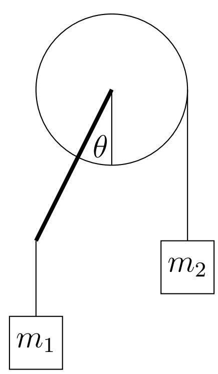


**Figure 5:** Mass suspended from a rod attached to a wheel, counterbalanced by another mass suspended from the wheel

**Question 37** [5 pt]. Consider a modification of the previous setup, shown in Figure 5. Here, mass $m_1$ hangs from the bottom, and a counterweight with mass $m_2$ is suspended from the wheel. Note that the definition of $\theta$ has changed, so you may need to switch the trigonometric function you were using previously.

At what angle $\theta$ is the system at equilibrium? Like last time, the rod is length $\ell$, and the wheel has radius $r$.

A) $\sin^{-1}\left(\frac{m_1\ell}{m_2 r}\right)$

B) $\cos^{-1}\left(\frac{m_1\ell}{m_2 r}\right)$

C) $\tan^{-1}\left(\frac{m_1\ell}{m_2 r}\right)$

D) $\sin^{-1}\left(\frac{m_2 r}{m_1\ell}\right)$

E) $\cos^{-1}\left(\frac{m_2 r}{m_1\ell}\right)$

F) $\tan^{-1}\left(\frac{m_2 r}{m_1\ell}\right)$

**Question 38** [2 pt]. Is this a stable equilibrium? That is, if I $\theta$ up or down a little, will the masses restore the angle to the equilibrium value?

A) Yes

B) No

## 3.2 Planetary Gears

In this section, we'll look at the motion of gears within a planetary gear as it might be used in a wind turbine, with the annulus fixed and the sun as the driver. Instead of trying too hard to visualize everything in motion, we'll use a table to help track all the angles.

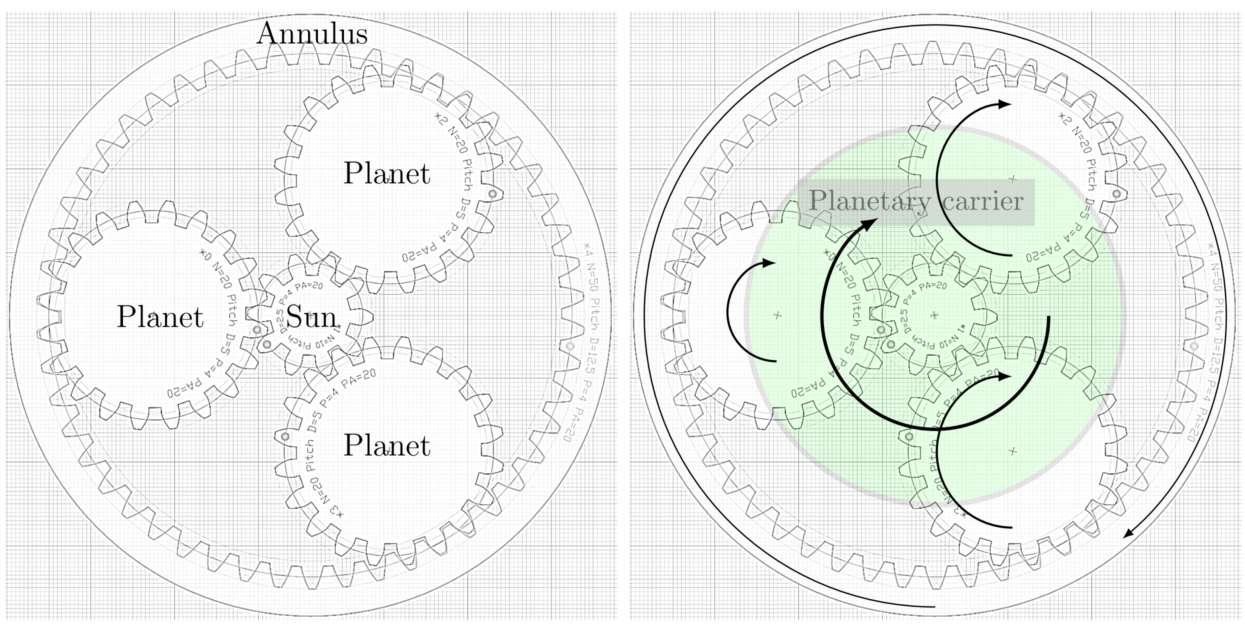


**Figure 6:** Planetary Gear, created with `geargenerator.com`. Each planet, the annulus, and the planetary carrier (in green) all rotate clockwise. The sun does not turn here.

Terminology:

- Sun: center gear - radius $s$

- Planets: outer circular gears (connected with the sun) - radius $r$
- Planetary carrier: structure holding the planets together - radius $r + s$
- Annulus: outer ring - radius $\ell$

Here's a table that shows how we can track the relative motions of all the different moving pieces. The scenario is that we hold the planetary carrier fixed while we turn the annulus by one full revolution.

The first column says the distance that each gear moves at its edge, with positive values indicating clockwise rotations and negative values indicating counterclockwise rotations. The second column indicates the number of revolutions each gear makes when the planetary carrier is fixed. Since we're interested in fixing the annulus in place, we can rotate the entire frame by just subtracting one off of every value in the second column to get the values in the third column.

| Gear | Path length | # revs (fixed carrier) | # revs (fixed annulus) |
|---|---|---|---|
| Sun ($s$) | $-2\pi\ell$ | $-\ell/s$ | $-\ell/s - 1$ |
| Planet ($r$) | $2\pi\ell$ | $\ell/r$ | $\ell/r - 1$ |
| Planetary Carrier ($r + s$) | 0 | 0 | $-1$ |
| Annulus ($\ell$) | $2\pi\ell$ | 1 | 0 |

**Figure 7:** Table to help compute the gear ratios

**Question 39** [7 pt]. When we use the sun as the input and the planetary carrier as the output, what is the gear ratio? You might want to double-check against your answer for question 18 to avoid getting the reciprocal of the correct answer!

A) $\frac{s}{\ell-s}$
B) $\frac{s}{\ell}$
C) $\frac{s}{\ell+s}$
D) $\frac{\ell}{s} - 1$
E) $\frac{\ell}{s}$
F) $\frac{\ell}{s} + 1$

**Question 40** [3 pt]. What happens to this gear ratio at the limit with very small planets, so that $s \approx \ell$? (check that your answer makes sense – if it doesn't, revisit your previous answer)

A) 0
B) 1/2
C) 1
D) 2
E) $\infty$ (in the limit that $s \to \ell$)

# 4 Energy

**Question 41** [3 pt]. A pendulum of length $\ell$ is pushed to an angle of $5°$. How much potential energy $E$ has it gained if the bottom bob has mass $m$?

A) $mg\ell \cos 5°$
B) $mg\ell \sin 5°$
C) $mg\ell(1 - \cos 5°)$
D) $mg\ell(1 - \sin 5°)$

**Question 42** [3 pt]. If the pendulum is released, what is the peak speed the bob experiences?

A) $\sqrt{2E/m}$
B) $\sqrt{E/m}$
C) $\sqrt{2E/m} \cos 5°$
D) $\sqrt{2E/m} \sin 5°$
E) $\sqrt{E/m} \cos 5°$
F) $\sqrt{E/m} \sin 5°$

**Question 43** [2 pt]. Before becoming a physics professor at the University of Chicago, Arthur Compton discovered that X-ray photons scattered by free electrons could be modeled by inelastic collisions between particles. He later earned the Nobel Prize in 1927 for this work (the Compton Effect or Compton scattering).

When an incoming X-ray photon bounces off a free electron, which of the following quantities is not conserved in the inelastic collision?

A) Energy of the system
B) Momentum of the system
C) Energy of the photon (and therefore wavelength)
D) Mass of the electron

# 5 Tie-breaker

**Question 44** [1 pt]. Open-ended tie-breaker: simplified trebuchet Here's a simplified trebuchet that is basically a counterweight-powered catapult (or a good-old class 1 lever!). The length of the arm between the fulcrum and projectile is $\ell$, and the length from the fulcrum to the weight is $r$. I've written $\theta$ as the angle that of the arm from the vertical. You can assume $m \ll M$.

What can you tell me about the trebuchet? For example: potential energy in the beginning, velocity of the projectile, mechanical advantage, etc.

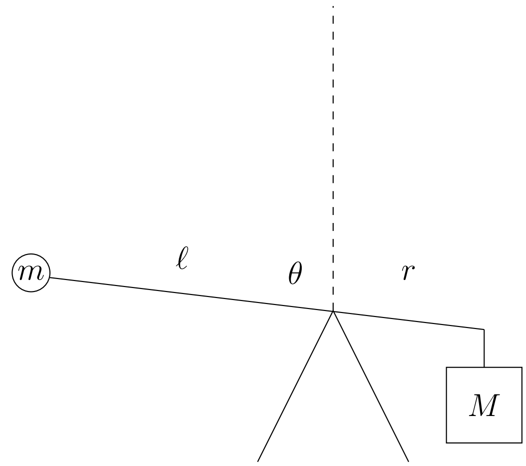


**Figure 8:** Trebuchet

Feel free to make (and state/briefly explain) any assumptions that seem reasonable (e.g. maybe you just want to see the velocity if the projectile is released at the top). If you prefer to write with different variable names, just state them clearly.

You could also use this space to come up with a machines/trebuchet meme :) (although note that you'll be more likely to win a tie if you can say something about the trebuchet)

Point total: 116