

In paper, The design constitutes parallel-biquad digital filter.
This page will represent how to set pole position.

First of all, the simple way to set poles is a pole and conjugate of this pole as below

$$p = R e^{\pm j\omega_0} = e^{\sigma_0 \pm j\omega_0}, \quad p_0 = \sigma_0 + j\omega_0$$

The next step is the peaking bandwidth based on poles. In this case, they suggest $\Delta\theta_k$ concepts,

$$p = e^{\Delta\theta_k \pm j\theta_k}, \quad \begin{cases} k=0, & \Delta\theta_0 = \theta_1 - \theta_0 \\ 0 < k < N, & \Delta\theta_k = (\theta_{k+1} - \theta_{k-1})/2 \\ k=N, & \Delta\theta_N = \theta_N - \theta_{N-1} \end{cases}$$

And they use The target frequency scale is the logarithmic frequency,

$$f_k = 10^{f_0 + k}$$

By refering above equation, it can get Bandwidth, 26% for Quality factor pf)

$$\theta_k = 2\pi \cdot \frac{f_k}{f_s}$$

$$0 < k < N, \quad f_{k+1} - f_{k-1} = 10^{f_0 + nk + n} - 10^{f_0 + nk - n} \\ = 10^{f_0 + nk} (10^n - 10^{-n})$$

In this paper $n=0.1$, for getting 10 band between 20-22050 Hz

$$f_{k+1} - f_{k-1} = 10^{f_0 + nk} (10^{0.1} - 10^{-0.1}) \\ = 0.46 f_k$$

$$k=0, \quad f_1 - f_0 = 10^{f_0 + n} - 10^{f_0} = 10^{f_0} (10^n - 1) \approx 0.25 f_0$$

$$k=N, \quad f_N - f_{N-1} = 10^{f_0 + Nn} - 10^{f_0 + Nn - n} = 10^{f_0 + Nn} (1 - 10^{-n}) \approx 0.20 f_N$$

$$\Delta\theta_k = K_k \theta_k, \quad K_k = \begin{cases} 0.25 & , k=0 \\ 0.23 & , 0 < k < N \\ 0.20 & , k=N \end{cases}$$

$$p, \bar{p} = e^{\Delta\theta_k}, e^{\pm j\theta_k}$$

$$Q_k = \frac{w_k}{26k} = \frac{2\pi f_k}{2 \cdot \alpha_k \theta_k} = \frac{f_s}{2\alpha_k} = f_s/0.5, k=0$$

$$f_s/0.46, 0 < k < N$$

$$f_s/0.4, k=N$$