Modeling Temporary Market Impact

Problem Setup

We are given high-frequency limit order book data for three tickers: `CRWV`, `SOUN`, and `FROG`. Our objective is not to optimize a specific trading strategy, but to **characterize and model temporary price impact** (slippage) as a function of trade size and market conditions.

We define the **temporary slippage function** for a trade of size x executed at time t as:

$g_t(x) :=$ average slippage (in dollars or %) incurred when trading x shares at time t.

Our goal is to:

- Understand how $g_t(x)$ behaves across different tickers and times,
- Evaluate how well different models fit the observed behavior,
- Discuss implications for volume allocation over time.

Data Normalization

To ensure comparability across tickers, we normalize slippage by the **midprice** at each time t. The normalized slippage becomes:

$$\tilde{g}_t(x) := \frac{g_t(x)}{P_{\text{mid},t}}$$

This expresses slippage as a percentage and helps account for differences in stock price magnitude across tickers.

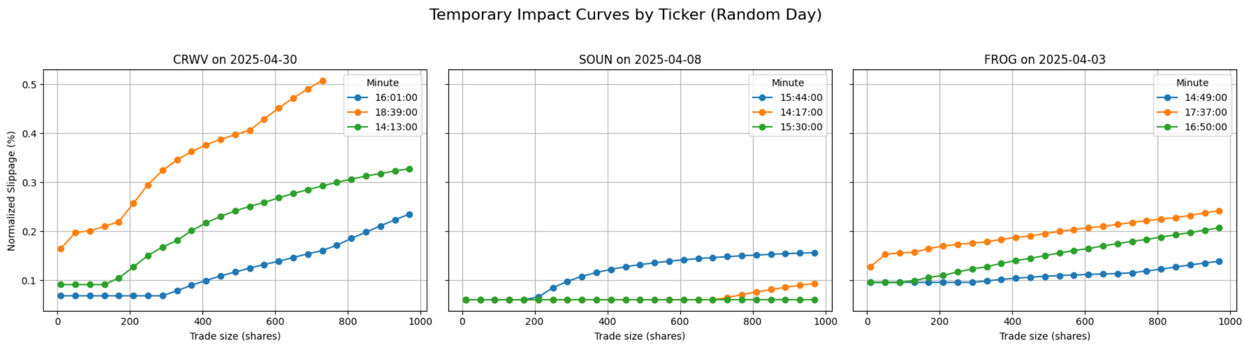
Empirical Observations

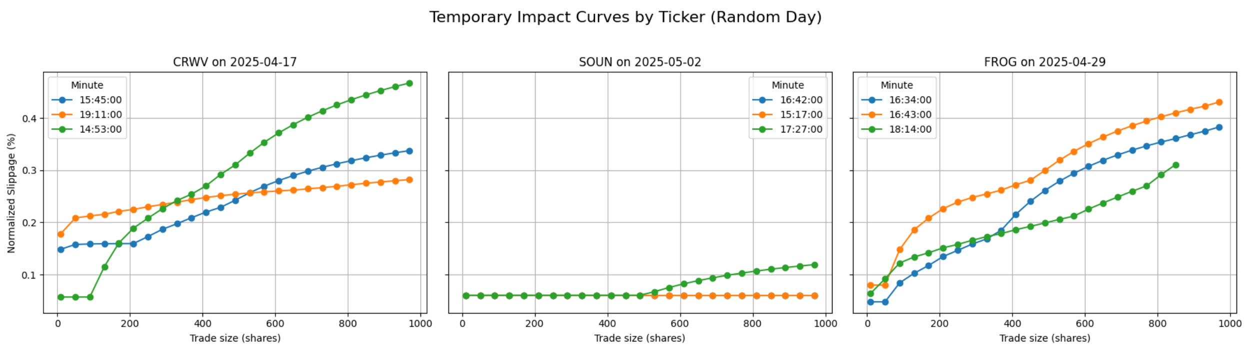
We visualize $\tilde{g}_t(x)$ across random 1-minute intervals for each ticker and day. Key findings include:

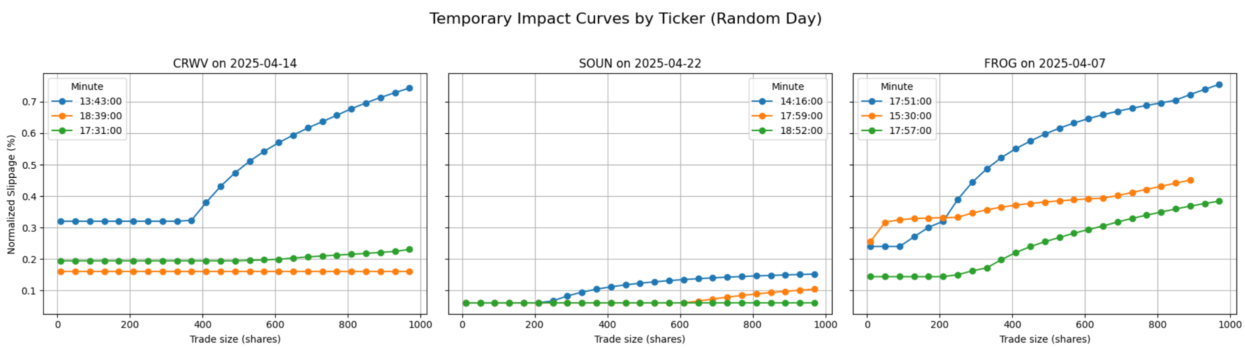
- Slippage increases nearly monotonically with trade size, as expected.

- Some minutes exhibit near-flat curves (indicating deep liquidity), while others show rapid slippage growth.
- There is significant intra-day and inter-day variability, even after normalization.

Sample plots of normalized slippage can be included as:







Modeling Approach

We propose a **ReLU-style piecewise-linear model** to describe the normalized temporary slippage function:

$$\tilde{g}_t(x) = \begin{cases} 0, & x \leq x_0 \\ \eta_{t,s} \cdot (x - x_0), & x > x_0 \end{cases}$$

where:

- x_0 is a *liquidity threshold*, the trade size that can be absorbed without significant impact,

- $\eta_{t,s}$ is a *slope parameter* encoding the market impact per unit of excess volume. It varies with time t and ticker s.

Motivation

This model reflects two key stylized facts:

1. **Flat region for small trades:** Small orders exhibit near-zero slippage.
2. **Linear growth afterward:** Slippage increases approximately linearly beyond a threshold.

Advantages

- **Interpretability:** Parameters x_0 and η_t have clear economic meaning.
- **Robustness:** Avoids overfitting and generalizes across tickers.
- **Microstructure Alignment:** Matches book structure—liquidity deep near midprice, sparse further away.

Limitations

- **Non-smooth transition:** Sharp kink at x_0 may not reflect real-world transitions.
- **Estimation challenge:** Identifying x_0 per minute is non-trivial.
- **No concavity:** Lacks diminishing marginal impact that power laws exhibit.

Possible Approaches

- **Empirical Learning:** Fit $g_{t,s}(x)$ to historical data. Use features like spread, depth, and volatility to estimate x_0 and $\eta_{t,s}$.
- **Execution Optimization:** Formulate as:

$$\min_{\{x_t\}} \sum_t g_t(x_t), \quad \text{subject to } \sum_t x_t = X_{\text{total}}$$

- **Robustness Evaluation:** Assess stability across days, symbols, and market regimes.
- **Simulation Testing:** Compare naive vs. optimized execution paths using historical data.

Summary

Normalized impact curves reveal consistent patterns: a flat-slippage region up to a threshold, followed by linear or concave growth. Our ReLU-style model captures this with interpretable parameters.

This foundation enables:

- **Empirical prediction** of impact parameters,
- **Optimization-based scheduling** of execution to minimize slippage.