#### Problem 1.1

```
 \begin{aligned} x &= 0; & /\!/ O(1) \\ y &= 0; & /\!/ O(1) \\ for & (i=1; i < n; i++) \\ & ... & /\!/ O(n-1) \\ return & (x - y); /\!/ O(1) \end{aligned}
```

In this loop, what's shown is the value i will go from 1 to n. Where n is the size of the array, the value n will increase linearly. Therefore, analyzing the running time complexity will be O(n).

### Problem 1.2

```
 \begin{split} & x = 0; \\ & i = 0; \\ & \textbf{while} \; (i < n) \; \{ \\ & y = 0; \\ & j = 0; \\ & \textbf{while} \; (j < n) \; \{ \\ & k = 0 \\ & \textbf{while} \; (k <= j) \; \{ \\ & y = y + a[k]; \\ & k = k + 1; \\ & \} \\ & j = j + 1; \\ & \} \\ & if \; (b[i] == y) \; \{ \\ & x + +; \\ & \} \; i = i + 1; \\ & \} \\ & return \; x; \\ \end{split}
```

There are three while-loops, and they can be described as a nested loop.

The outer first while loop, the value of i will run from 0 to n-1 so, it will run for n times.

The inner second while loop will also run for n times but for every iteration of the first loop. So, for each value of i, the value of j will move from 0 to n-1.

The inner third while loop will for j times for each iteration of the second loop. For each value of j, the value k moves from 0 to j depending on the value of j.

The third loop will run for 1 time when j=0, 2 times when j=1, 3 times when j=2, ..., n times when j=n. It will run for: (1+2+3+4+5+...+n) times = n\*(n+1)/2 times.

Being that the first loop will run for n iteration, the second and third loop will combinedly run for n\*(n+1)/2. Therefore, the analyzed running time complexity will be n\*n\*(n+1)/2 which is  $O(n^3)$ .

#### Problem 1.3

```
 \begin{split} &\text{if } (i == 0) \; \{ \; /\!/ O(1) \\ &p[0] = a[0]; \\ &p[1] = a[0]; \\ &\} \\ &\text{else } \{ \\ &\text{method3}(a,i\text{-}1,p); \; /\!/ O(n\text{-}1) \\ &\text{if } (a[i] < p[0]]) \; \{ \; /\!/ O(1) \\ &p[0] = a[i]; \\ &\} \\ &\text{if } (a[i] > p[1]]) \; \{ \; /\!/ O(1) \\ &p[1] = a[i]; \\ &\} \\ &\} \\ \end{aligned}
```

The starting value of i is n-1. With the new i value as i-1, the recursive calls are being made. This function is being called recursively n times before reaching the base case so the analyzing running time complexity will be O(n).

### **Problem 1.4**

```
 \begin{split} & \text{if } (x>=y) \; \{ \; /\!/ O(1) \\ & \text{return } a[x]; \\ & \} \\ & \text{else } \{ \\ & z=(x+y) \, / \, 2; \, /\!/ O(1) \\ & u=\text{method4}(a,\,x,\,z); \, /\!/ O(n) \\ & v=\text{method4}(a,\,z+1,\,y); \, /\!/ O(n) \\ & \text{if } (u< v) \; \text{return } u; \, /\!/ O(1) \\ & \text{else return } v; \\ & \} \\ & \} \\ \end{aligned}
```

The analyzed running time complexity will be O(n) because each node will be split into 2 nodes also known as the child nodes. So, it will be equal to the number of the internal nodes (2n-1) as a function will be called for 2n-1 times. Therefore, using the Master theorem,  $T(n)=a.T(n/b)+O(n^d)$  where a=2, b=2, d=0 as  $a>b^d$  there by making the running time complexity  $O(n^{(\log_2 2)})$  which is O(n)

# Problem 2.1

Operation	Return Value	Stack Contents
push(10)	-	(10)
pop()	10	()
push(12)	-	(12)
push(20)	-	(12,20)
size()	2	(12,20)
push(7)	-	(12,20,7)
pop()	7	(12,20)
top()	20	(12,20)
pop()	20	(12)
pop()	12	()
push(35)	-	(35)
isEmpty()	false	(35)

# Problem 2.2

Operation	Return Value	Queue Contents (first $\leftarrow$ Q $\leftarrow$ last)
enqueue(7)	1	(7)
dequeue()	7	()
enqueue(15)	-	(15)
enqueue(3)	-	(15,3)
first()	15	(15,3)
dequeue()	3	(15)
dequeue()	15	()
first()	null	()
enqueue(11)	1	(11)
dequeue()	11	()
isEmpty()	true	()
enqueue(5)	-	(5)