PH21 Assignment 2

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1. (a)

$$\widetilde{h_k} = \frac{1}{L}\int_0^L h(x)e^{2\pi i f_k x}dx$$

$$h(x) = \sum_{k=-\infty}^{\infty} \widetilde{h_k}e^{-2\pi i f_k x}, f_k = \frac{k}{L}$$

Plugging 2 into 1

$$h(x) = \sum_{k=-\infty}^{\infty}[\frac{1}{L}\int_0^L h(x')e^{2\pi ikx'/L}dx']e^{-2\pi ikx/L}$$

We can swap the sum and the integral, pulling out $h(x')$ to get

$$h(x) = \int_0^L h(x')\delta(x'-x)dx' = h(x), x' \neq xs$$

(b) We use Euler's trig relation, which says $sin(2\pi x/L+\phi) = \frac{e^{-i(2\pi x/L+\phi)}+e^{-i(2\pi x/L+\phi)}}{2i}$. Pulling out the terms that we want, we get that a linear combination of $e^{-2\pi ix/L}$ and $e^{2\pi ix/L}$ with coefficients $A\frac{e^{\pm i\phi}}{2i}$, which are both constant, can represent any function $Asin(2\pi x/L+\phi)$.

Since we know that sines and cosines are offset by some phase, we know that this can also represent any cosine function.

(c) $\widetilde{h}_{-k} = \widetilde{h}_k^*$?
$\widetilde{h}_{-k} = \int_0^L \frac{1}{L}h(x)e^{-2\pi ikx/L}dx$
$\widetilde{h}_k^* = [\int_0^L \frac{1}{L}h(x)e^{2\pi ikx/L}dx]^*$
Since $h(x)$ is real, we can write
$\widetilde{h}_k^* = \int_0^L \frac{1}{L}h(x)[e^{2\pi ikx/L}dx]^*$
Since $[e^{2\pi ikx/L}dx]^* = e^{-2\pi ikx/L}dx$, $\widetilde{h}_k^* = \int_0^L \frac{1}{L}h(x)e^{-2\pi ikx/L}dx = \widetilde{h}_{-k}$

(d) The graphical interpretation of a convolution product is the area that becomes revealed as a window shaped like $h^{(1)}(x)$ is dragged over $h^{(2)}(x)$; the convolution is the integral at every point along this meeting.

If we were to convolve a function H with a smooth function h1 at $k = 0$ with an unit impulse h2 at $k = 50$, we would end up with the value of the original function at $k = 50$ and the shape of h2 overlaid on H at $k = 0$.

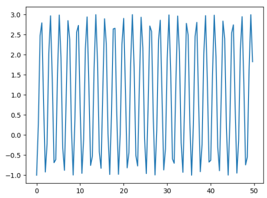

Figure 1: original cosine plot

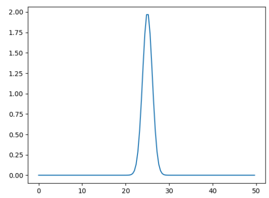

Figure 2: original gaussian plot

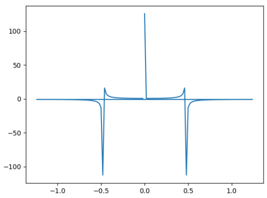

Figure 3: fft cosine plot

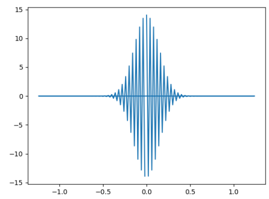

Figure 4: fft gaussian plot

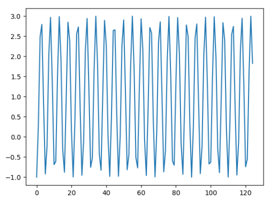

Figure 5: inverse fft cosine plot

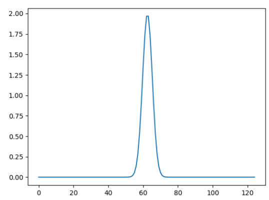

Figure 6: inverse fft gaussian plot

(e) We can see that we recover the same plot at the beginning and at the end. The tighter the original gaussian was, the more rounded the fft peak.

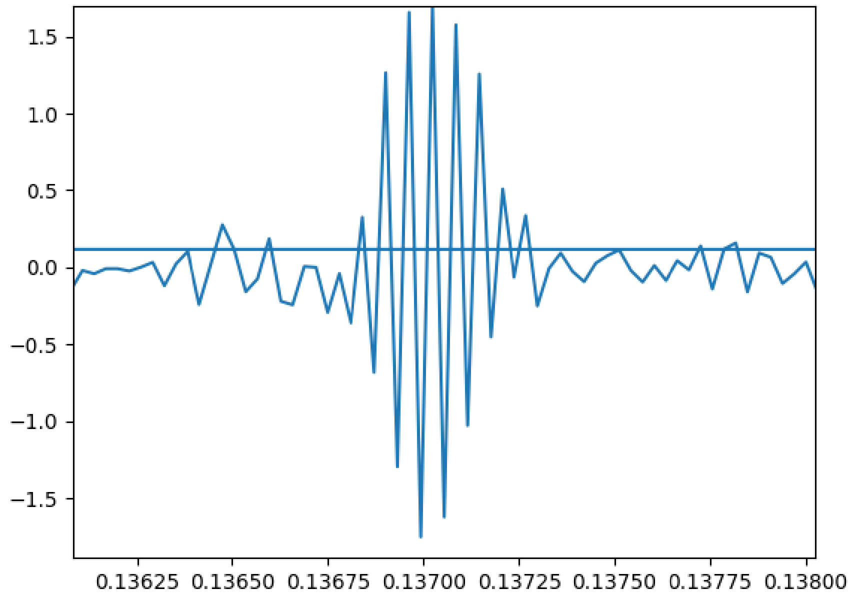

Figure 7: Arescibo peak

2. (a) According to the plot above, the peak frequency is around 0.137 cycles/ms; since the x axis is shifted down 1420 MHz, the peak is at 1420 MHz + 137 kHz. The signal is pretty round, suggesting that the original data was pointed.

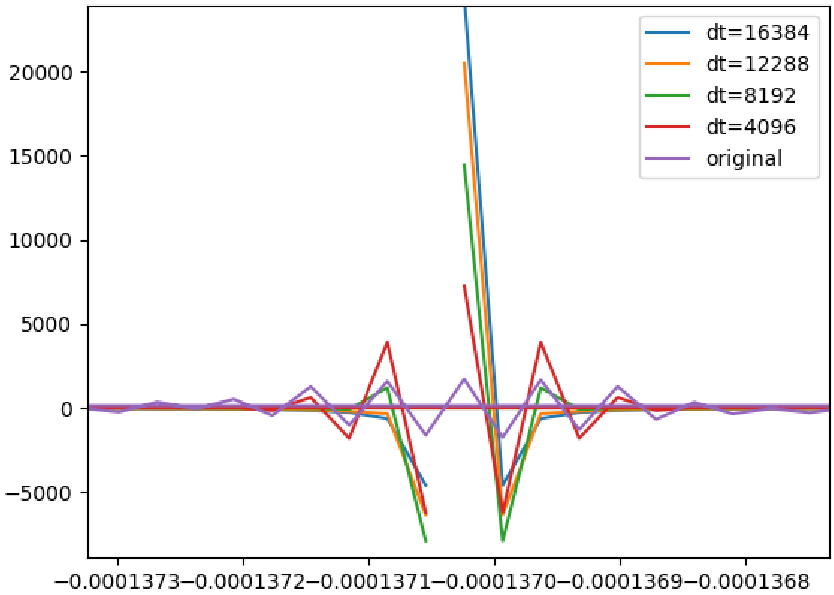

Figure 8: Arescibo peak

(b) Looking at the plot above, the best estimate for the time constant Δt is 4096 ms.

3. (a) (nothing to do)

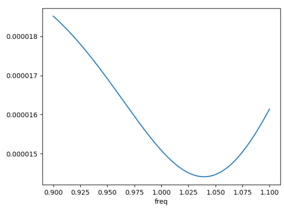

Figure 9: lombscargle for evenly spaced gaussian

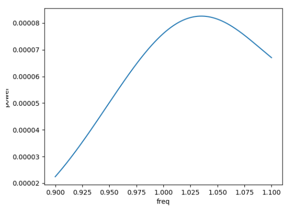

Figure 10: lombscargle for evenly spaced arescibo data

(b) These both give much less information than fft.

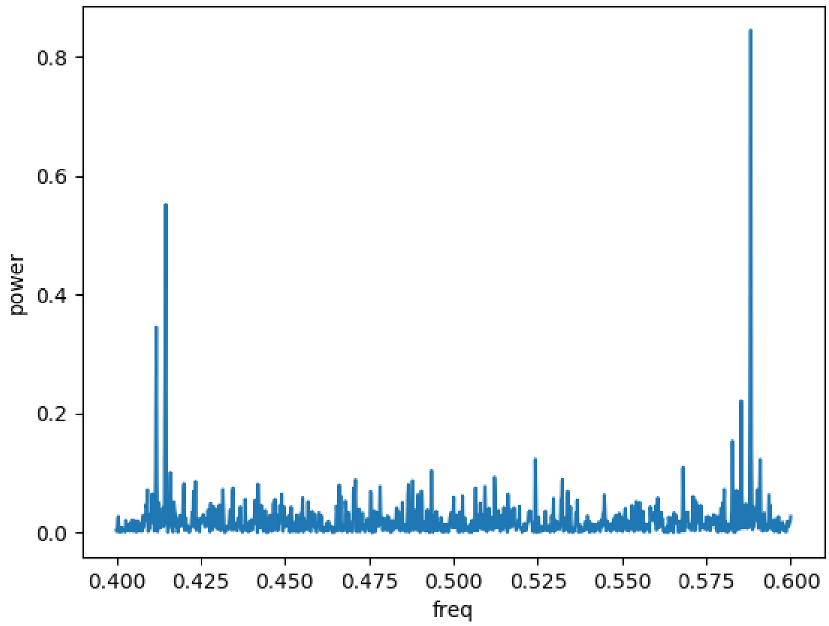

Figure 11: Uneven HER-X1 data

(c) We can see the peak at $1/1.7 = 0.5882$; we can also see a secondary peak near 0.418, suggesting some other significant period of around 2.5 days.

Other significant frequencies could come from background noise in the observatory (radios, cellphones?).