PH21 Assignment 4 Report updated last section

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1 Coin

true probability H = 0.4:

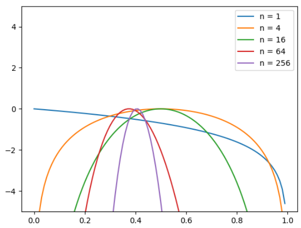

Figure 1: gauss, within 1 sigma

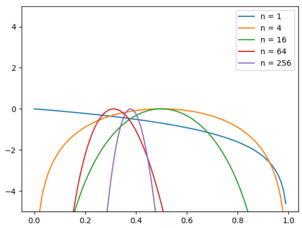

Figure 2: gauss, within 3 sigma

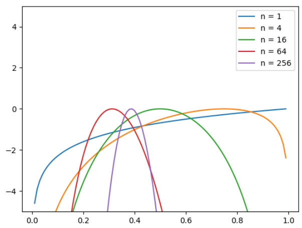

Figure 3: gauss, very far from true value

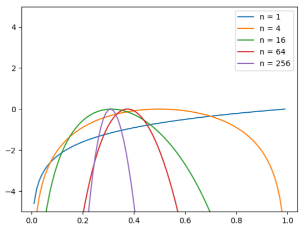

Figure 4: uniform

We can see that the closer the prior is to the true value, the tighter the peaks are. As the number of trials increase, we get closer to the true value, which is what we would expect.

The curves for $n = 1$ and $n = 2$ are always haphazard, since there are so few trials.

true probability H = 0.8:

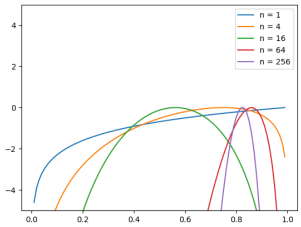

Figure 5: gauss, within 1 sigma

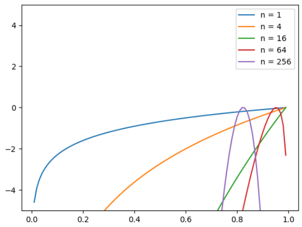

Figure 6: gauss, within 3 sigma

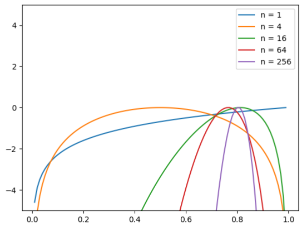

Figure 7: gauss, very far from true value

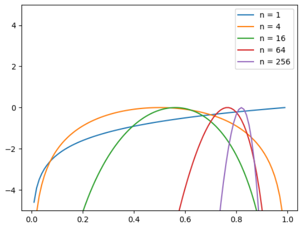

Figure 8: uniform

We can clearly see that the peak-cluster-center has been shifted over to 0.8, especially for the larger trial numbers.

2 Lighthouse: beta known

We can use trigonometric relations to find the number of hits, as depending on alpha and beta.

Letting our random variable θ be randomly and uniformly distributed, we count a flash as a hit h if in n values of θ, any θ_n is between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

The location where a successful flash is received is then $btan(\theta) + a$.

Using this and a prior for a, we get results as follows:

true alpha = 1.0

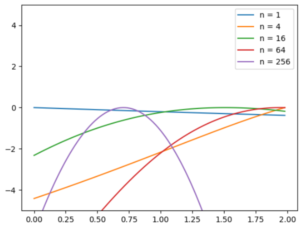

Figure 9: gauss, within 1 sigma

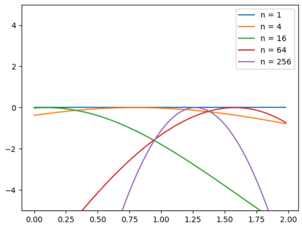

Figure 10: gauss, within 3 sigma

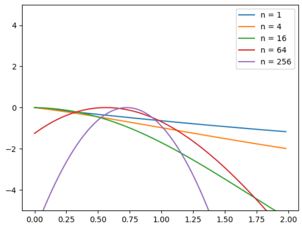

Figure 11: gauss, very far from true value

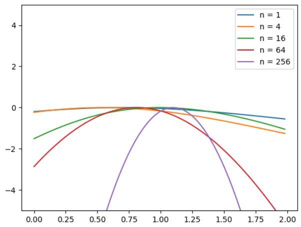

Figure 12: uniform

This time, with a more complicated situation, the true value is much harder to pin down with the same number of trials. The peaks are generally much wider; we still see them narrow as n increases.

The mean is not a good estimator since the Central Limit Theorem doesn't apply to Cauchy distributions.

3 Lighthouse, neither alpha nor beta known: updated

When we know neither a nor b, we can still use this technique to generate a heatmap of likelihoods by using the same prior twice for a and b; here $a = 1.0$, $b = 1.5$, and a is on the x axis, and b is on the y axis.

Results for different n:

n=4

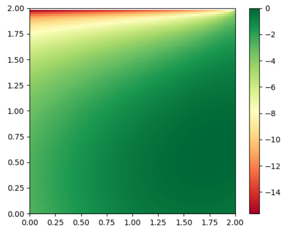

Figure 13: gauss, within 1 sigma

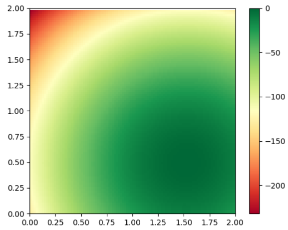

Figure 14: gauss, within 3 sigma

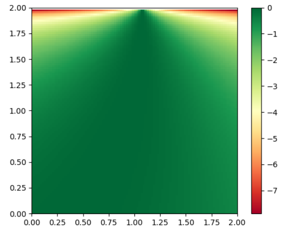

Figure 15: uniform

n=16

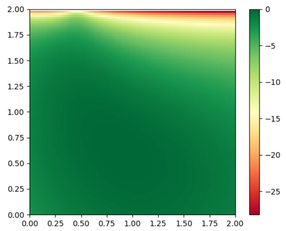

Figure 16: gauss, within 1 sigma

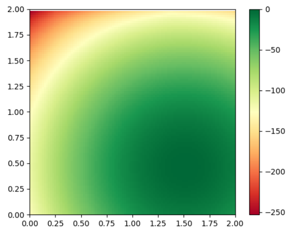

Figure 17: gauss, within 3 sigma

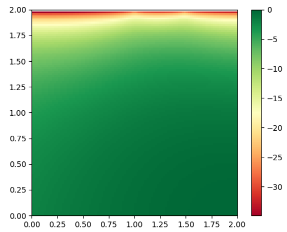

Figure 18: uniform

n=64

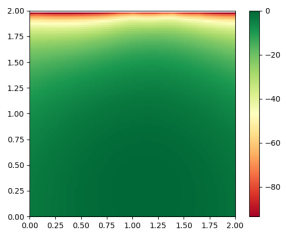

Figure 19: gauss, within 1 sigma

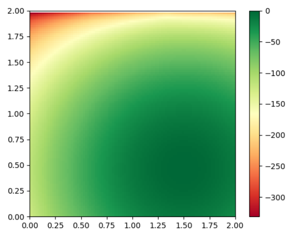

Figure 20: gauss, within 3 sigma

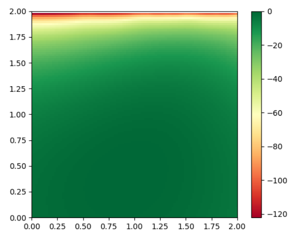

Figure 21: uniform

n=256

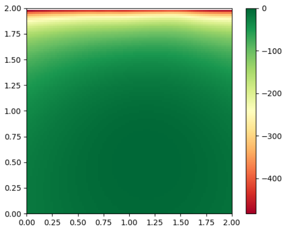

Figure 22: gauss, within 1 sigma

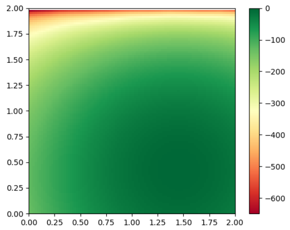

Figure 23: gauss, within 3 sigma

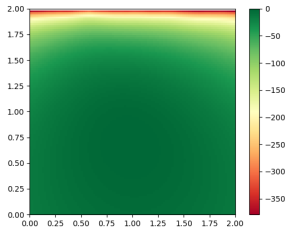

Figure 24: uniform

Updates: I only included one prior in the calculation instead of both. After fixing that and changing the extent of the plot as well as the colormap, we can see that the tight gaussian prior has narrow peaks (rings), and as we increase the sample size, the peak approaches our true $(alpha, beta)$ location.

The wider gaussian did better than the uniform, but both were too broad to be very useful.