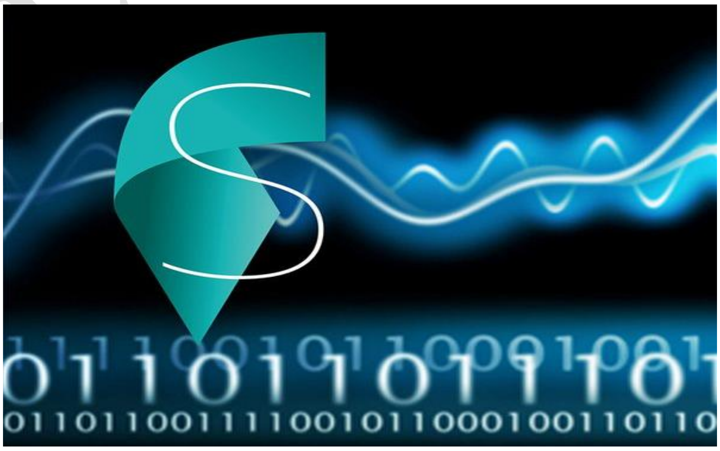
CYBER SECURITY TECHNOLOGY ENGINEERING DEPARTMENT

DIGITAL SIGNAL PROCESSING

THIRD STAGE

Lect.5 Discrete Time Systems



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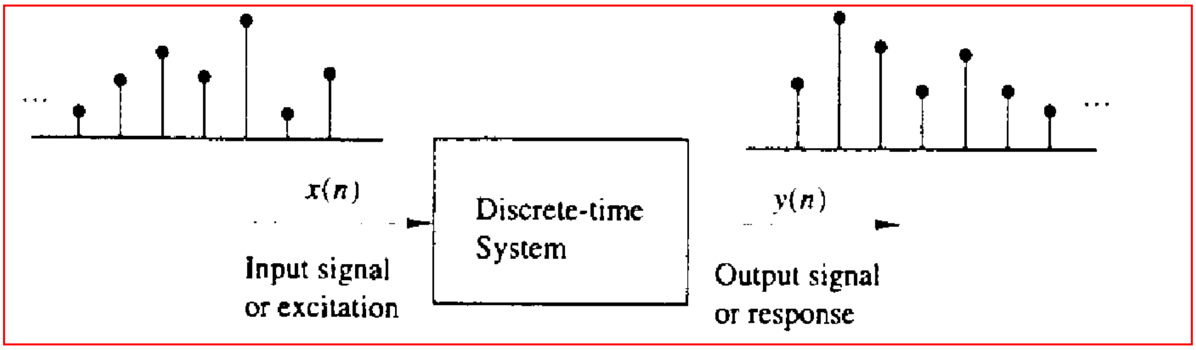
Discrete – Time Systems

In many applications of digital signal processing we wish to design a device or an algorithm that performs some prescribed operation on discrete – time signal. Such a device or algorithm is called a discrete – time system. More specifically, a discrete – time system is a device or algorithm that operates on a discrete – time signal called the input or excitation, according to some well – defined rule, to produce another discrete – time signal called the output or response of the system.

Input – output Description of Systems

The input – output description of a discrete – time system consists of a mathematical expression or a rule, which explicitly defines the relation between the input and output signals.



Block diagram representation of a discrete – time system

Example

Determine the response of the following systems to the input signal

$$x(n) = \begin{cases} |n|, & -3 \le n \le 3 \\ 0, & otherwise \end{cases}$$

a) $y(n) = x(n)$
b) $y(n) = x(n-1)$
c) $y(n) = x(n+1)$

Solution First, we determine explicitly the sample values of the input signal

$$x(n) = \{\dots\dots 0,3,2,1,0,1,2,3,0 \dots\dots\}$$

Next, we determine the output of each system using its input – output relationship.

a) In this case the output is exactly the same as the input signal. Such a system is known as the identity system.

b) This system simply delays the input by one sample

$x(n)$	3	2	1	0	1	2	3
$x(n-1)$	0	3	2	1	0	1	2

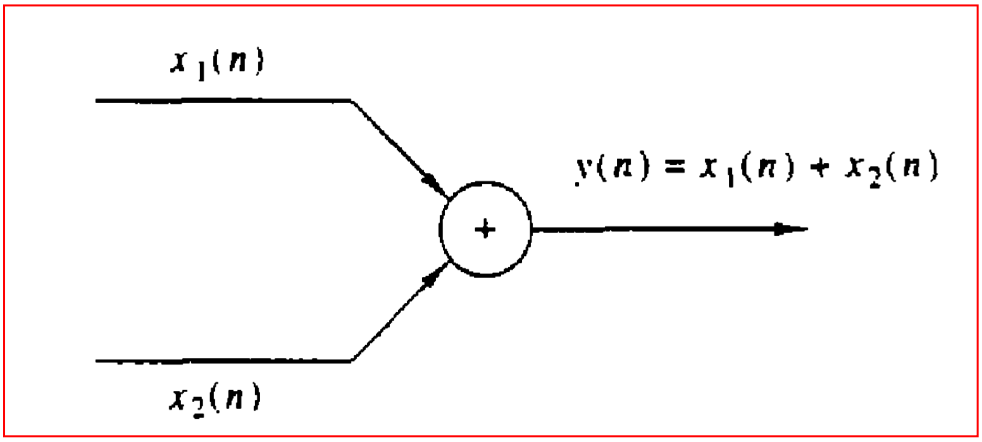
c) In this case the system "advances" the input one sample into the future.

$x(n)$	3	2	1	0	1	2	3
$x(n+1)$	2	1	0	1	2	3	0

Block Diagram representation of Discrete – Time Systems

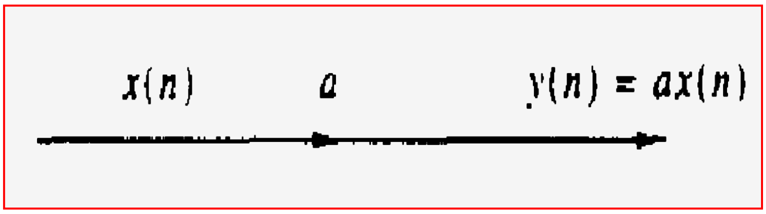
It is useful at this point to introduce a block diagram representation of discrete – time systems. For this purpose we need to define some basic building blocks that can be interconnected to form complex systems.

An adder. The figure below illustrate a system (adder) that performs the addition of two signal sequences to form another (the sum) sequence, which we denote as $y(n)$.

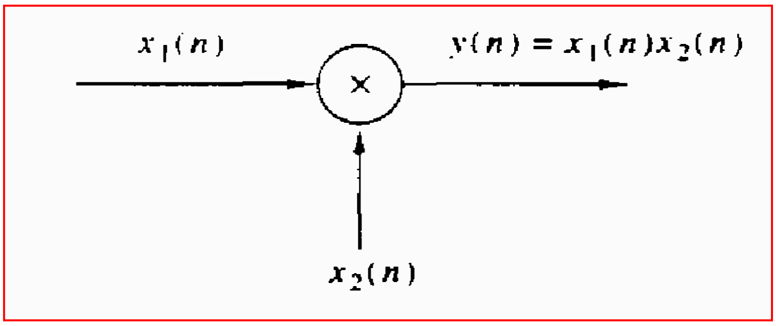


Note that it is not necessary to store either one of the sequences in order to perform the addition. In other words, the addition operation is memoryless.

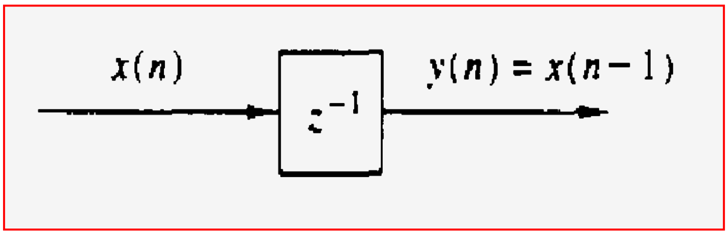
A constant multiplier. This operation is depicted by figure below, and simply represent applying a scale factor on the input $x(n)$. Note that this operation is also memoryless.



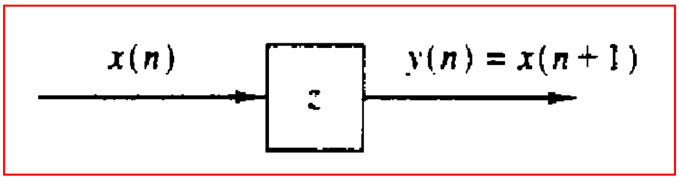
A signal multiplier. Figure below illustrates the multiplication of two signal sequences, the multiplication operation is memoryless.



A unit delay element. The unit delay is a special system that simply .delays the signal passing through it by one sample



A unit advance element. In contrast to the unit delay, a unit advance moves the input a head by one sample.

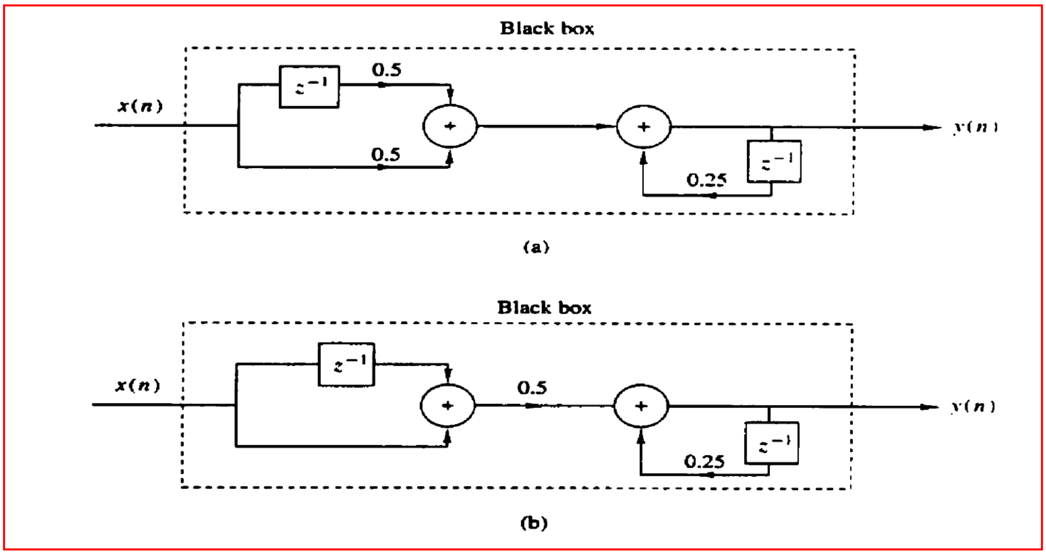


Example:-

Using basic building blocks introduced above, sketch the block diagram representation of the discrete – time system described by the input – output relation.

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

Solution:-



Classification of Discrete – Time Systems:-

a) **Static versus dynamic systems.**

Static systems	**Dynamic systems**
$\boldsymbol{y(n) = ax(n)}$ $\boldsymbol{y(n) = nx(n) + bx^3(n)}$	$\boldsymbol{y(n) = x(n) + 3x(n-1)}$ $\boldsymbol{y(n) = \sum_{k=0}^{n} x(n-k)}$

b) **Time – Invariant versus Time – Variant systems**

A relaxed system T is time invariant or shift invariant if and only if

$$x(n) \xrightarrow{T} y(n)$$

Implies that

$$x(n-k) \xrightarrow{T} y(n-k)$$

For every input signal $x(n)$ and every time shift k.

In general, we can write the output as

$$y(n,k) = T[x(n-k)]$$

Now if this output $y(n,k) = y(n-k)$, for all possible values of k, the system is time invariant. On the other hand, if the output $y(n,k) \neq y(n-k)$, even for one value of k, the system is time variant.

Example:-

The system described by the input – output equation

$$y(n) = T[x(n)] = x(n) - x(n-1)$$

Now if the input is delayed by k units in time and applied to the system

$$y(n,k) = x(n-k) - x(n-k-1)$$

On the other hand, if we delay $y(n)$ by k units in time, we obtain

$$y(n-k) = x(n-k) - x(n-k-1)$$

Therefore, $y(n,k) = y(n-k)$ and the system is time invariant.

Example:-

The system described by the input – output equation

$$y(n) = T[x(n)] = x(-n)$$

The response of this system to $x(n-k)$ is

$$y(n,k) = T[x(n-k)] = x(-n-k)$$

Now, if we delay the output $y(n)$ by k units in time, the result will be

$$y(n-k) = x(-n+k)$$

Since $y(n,k) \neq y(n-k)$, the system is time variant.

c) **Linear versus nonlinear systems.**

A relaxed T system is linear if and only if

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$

For any arbitrary input sequences $x_1(n)$ and $x_2(n)$, and any arbitrary constants a_1 and a_2.

Example:-

Determine if the systems described by the following input – output equations are linear or nonlinear.

a) $y(n) = nx(n)$
b) $y(n) = x^2(n)$

Solution:-

a) For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$y_1(n) = nx_1(n)$$
$$y_2(n) = nx_2(n)$$

A linear combination of the two input sequences results in the output

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)] = n[a_1x_1(n) + a_2x_2(n)]$$

A linear combination of the $y_1(n)$ and $y_2(n)$ results in

$$a_1y_1(n) + a_2y_2(n) = n[a_1x_1(n) + a_2x_2(n)]$$

Since $y_3(n) \equiv a_1y_1(n) + a_2y_2(n)$ the system is linear.

b) The responses of the system to two separate input signals are

$$y_1(n) = {x_1}^2(n)$$
$$y_2(n) = {x_2}^2(n)$$

The response of the system to a linear combination of these two input signals is

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$$
$$= [a_1x_1(n) + a_2x_2(n)]^2$$
$$= {a_1}^2{x_1}^2(n) + 2a_1a_2x_1(n)x_2(n) + {a_2}^2{x_2}^2(n)$$

On the other hand, if the system is linear, it would produce a linear combination of the two outputs

$$a_1y_1(n) + a_2y_2(n) = a_1{x_1}^2(n) + a_2{x_2}^2(n)$$

Since the actual output of the system, is not equal to the above equation, the system is nonlinear.

d) **Causal versus noncausal systems**

In mathematical terms, the output of a causal system satisfies an equation of the form

$$y(n) = F[x(n), x(n-1), x(n-2), \dots]$$

Where F[.] is some arbitrary function.

If a system does not satisfy this definition, it is called noncausal. Such a system has an output that depends not only on present and past inputs but also on future inputs.

Example:-

Determine if the systems described by the following input – output equations are causal or noncausal.

a) $y(n) = x(n) - x(n-1)$

b) $y(n) = \sum_{k=-\infty}^{n} x(k)$

c) $y(n) = ax(n)$

d) $y(n) = x(n) + 3x(n+4)$

d) $y(n) = x(n^2)$

f) $y(n) = x(2n)$

g) $y(n) = x(-n)$

Solution:-

The systems described by parts (a), (b), and (c) are causal.

The systems described by rest parts are noncausal.