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7.	Make a plot of the vertical lines $x = a$, for $a =- 1, - \frac{1}{2}, \frac{1}{2}, 1$ and the horizontal lines y= b, for b=$- 1, - \frac{1}{2}, \frac{1}{2}, 1$. Find the plot of this grid under the mapping $w = f(z) = \frac{1}{z}$.	33- 35
8.	Find a parametrization of the polygonal path $C = C_1 + C_2 + C_3$ from $- 1 + i$ to $3 - i$, where C_1 is the line from: $- 1 + i$ to $- 1$, C_2 is the line from: $- 1$ to $1 + i$ and C_3 is the line from $1 + i$ to $3 - i$. Make a plot of this path.	36 - 37
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12.	Use ML inequality to show that $\left\lvert \int_C \frac{1}{z^2+1} dz \right\rvert \leq \frac{1}{2\sqrt{5}}$ where C is the straight line segment from 2 to $2+i$. While solving, represent the distance from the point z to the points i and $-i$, respectively, i.e. $\lvert z-i\rvert$ and $\lvert z+i\rvert$ on the complex plane C.	42
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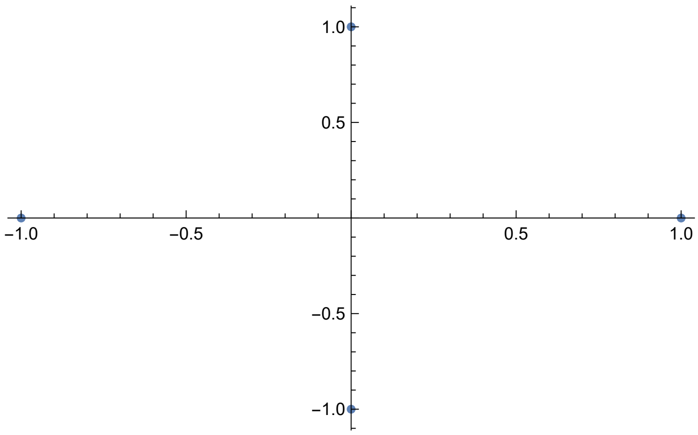
Practical 1

Make a geometric plot to show that the nth roots of unity are equally spaced points that lie on the unit circle $C_1(0) = \{z : |z| = 1\}$ and form the vertices of a regular polygon with n sides, for n = 4, 5, 6, 7, 8.

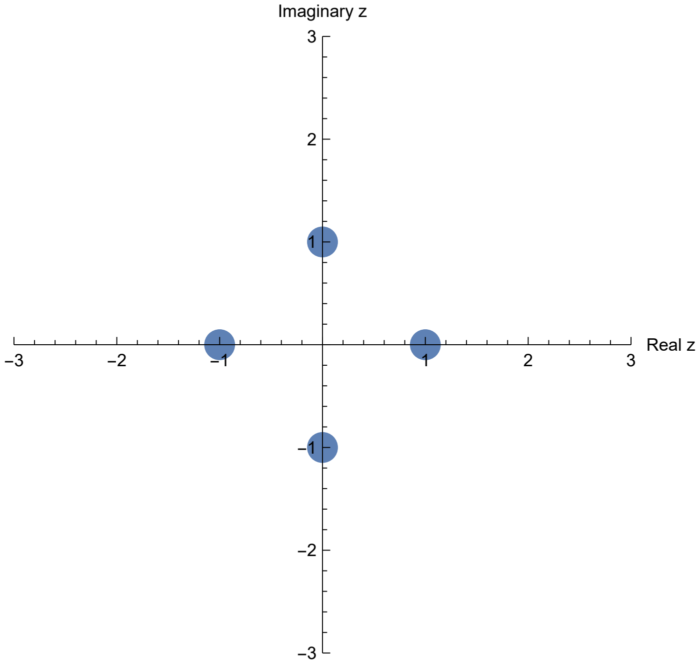
n = 4

In[1]:=
```
Solve[x + 1 == 0, x]
```
Out[1]=
```
{{x → -1}}
```
In[2]:=
```
Solve[z^4 == 1, z]
```
Out[2]=
```
{{z → -1}, {z → -i}, {z → i}, {z → 1}}
```
In[3]:=
```
root = ComplexExpand[Solve[z^4 == 1, z]]
```
Out[3]=
```
{{z → -1}, {z → -i}, {z → i}, {z → 1}}
```
In[4]:=
```
z /. root
```
Out[4]=
```
{-1, -i, i, 1}
```
In[5]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root]
```
Out[5]=



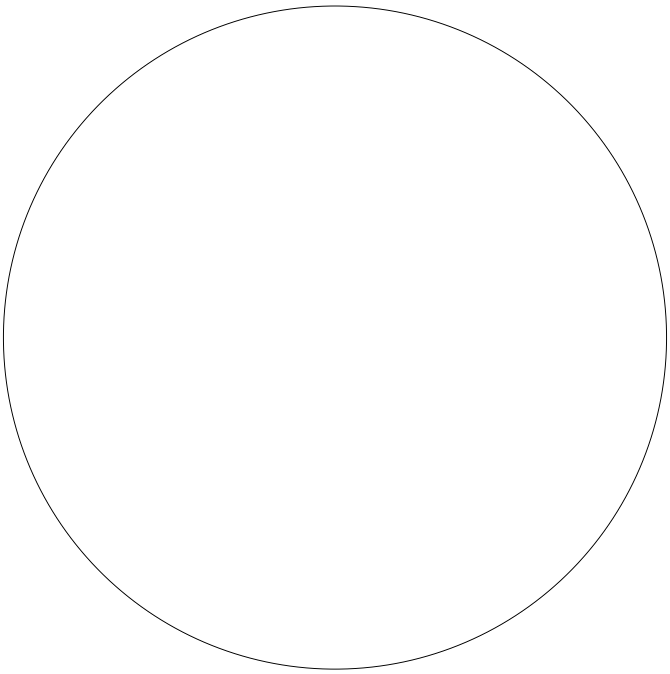
In[6]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root,
  PlotRange → {{-3, 3}, {-3, 3}}, AxesLabel → {"Real z", "Imaginary z"},
  PlotStyle → PointSize[0.05], AspectRatio → Automatic]
```

Out[6]=



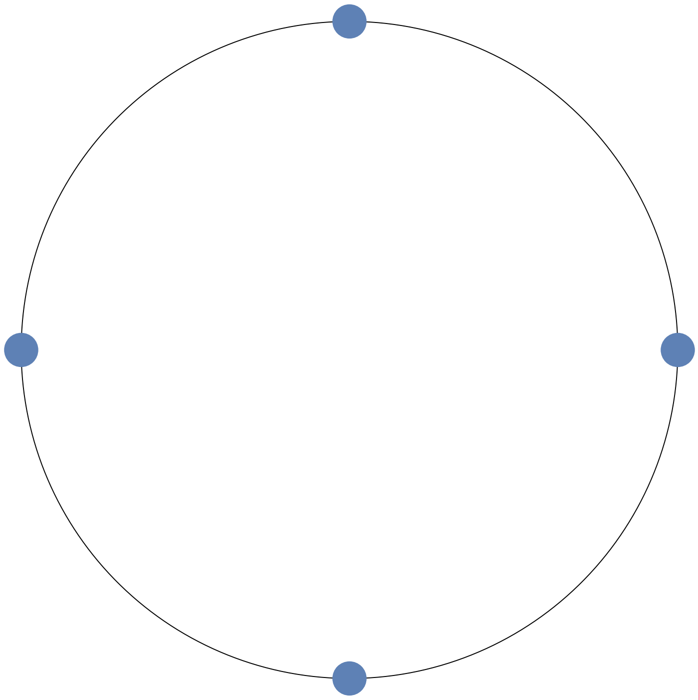
In[7]:=
```
rootstruct = Graphics[{Black, Circle[{0, 0}, 1]}]
```

Out[7]=



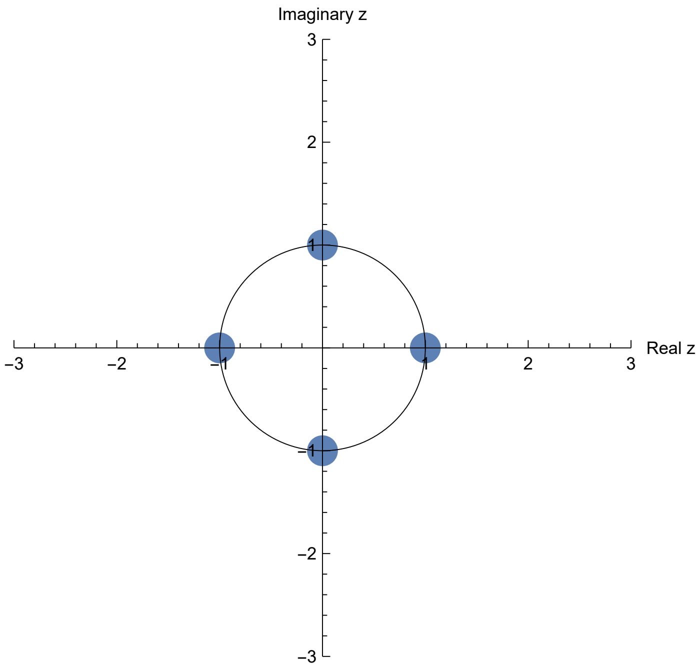
In[8]:= `Show[rootstruct, rootplot]`

Out[8]=



In[9]:= `Show[rootplot, rootstruct]`

Out[9]=



n = 5

In[1]:= `Solve[z^5 == 1, z]`

Out[1]= $\{\{z \to 1\}, \{z \to -(-1)^{1/5}\}, \{z \to (-1)^{2/5}\}, \{z \to -(-1)^{3/5}\}, \{z \to (-1)^{4/5}\}\}$

In[2]:= `root = ComplexExpand[Solve[z^5 == 1, z]]`

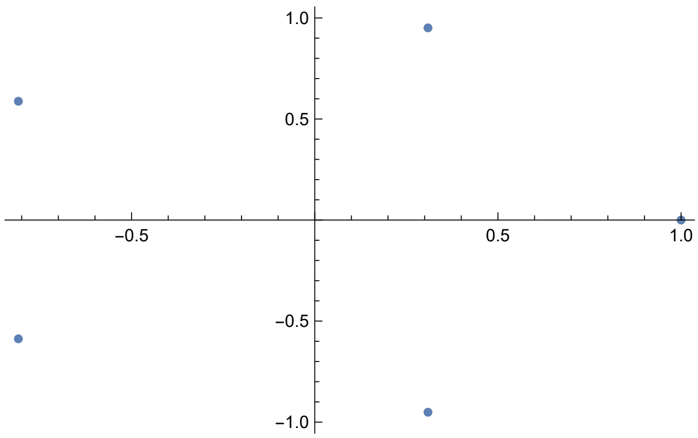
Out[2]= $$\left\{\{z \to 1\}, \left\{z \to -\frac{1}{4} - \frac{\sqrt{5}}{4} - i\sqrt{\frac{5}{8} - \frac{\sqrt{5}}{8}}\right\}, \left\{z \to -\frac{1}{4} + \frac{\sqrt{5}}{4} + i\sqrt{\frac{5}{8} + \frac{\sqrt{5}}{8}}\right\}, \left\{z \to -\frac{1}{4} + \frac{\sqrt{5}}{4} - i\sqrt{\frac{5}{8} + \frac{\sqrt{5}}{8}}\right\}, \left\{z \to -\frac{1}{4} - \frac{\sqrt{5}}{4} + i\sqrt{\frac{5}{8} - \frac{\sqrt{5}}{8}}\right\}\right\}$$

In[3]:= `z /. root`

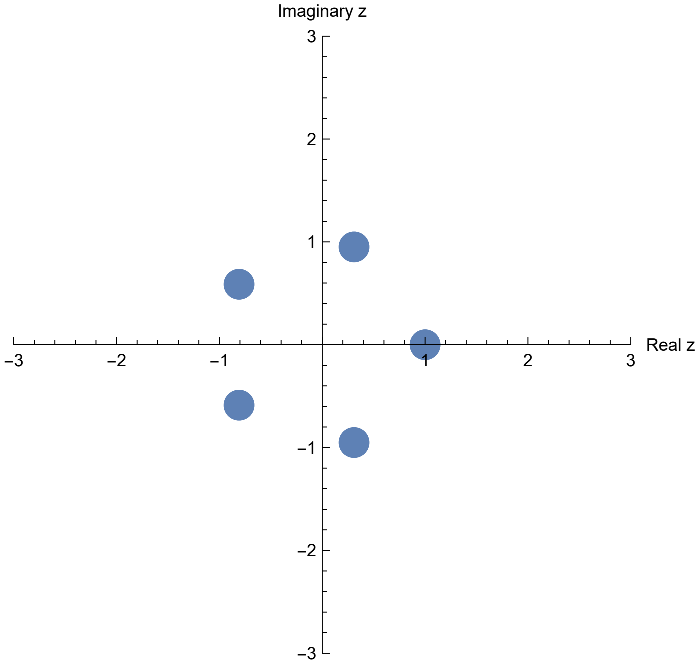
Out[3]= $$\left\{1, -\frac{1}{4} - \frac{\sqrt{5}}{4} - i\sqrt{\frac{5}{8} - \frac{\sqrt{5}}{8}}, -\frac{1}{4} + \frac{\sqrt{5}}{4} + i\sqrt{\frac{5}{8} + \frac{\sqrt{5}}{8}}, -\frac{1}{4} + \frac{\sqrt{5}}{4} - i\sqrt{\frac{5}{8} + \frac{\sqrt{5}}{8}}, -\frac{1}{4} - \frac{\sqrt{5}}{4} + i\sqrt{\frac{5}{8} - \frac{\sqrt{5}}{8}}\right\}$$

In[4]:= `rootplot = ListPlot[{Re[z], Im[z]} /. root]`

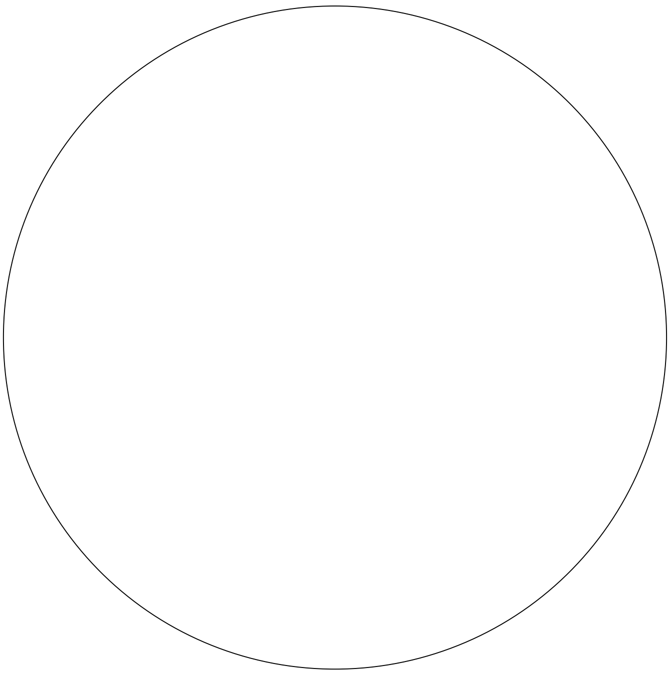
Out[4]=



In[5]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root,
  PlotRange → {{-3, 3}, {-3, 3}}, AxesLabel → {"Real z", "Imaginary z"},
  PlotStyle → PointSize[0.05], AspectRatio → Automatic]
```

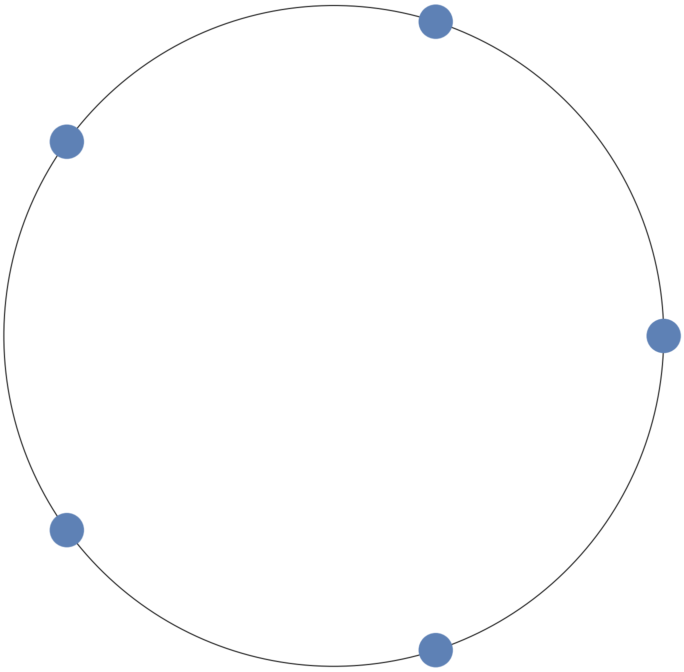
Out[5]=


In[6]:=
```
rootstruct = Graphics[{Black, Circle[{0, 0}, 1]}]
```

Out[6]=


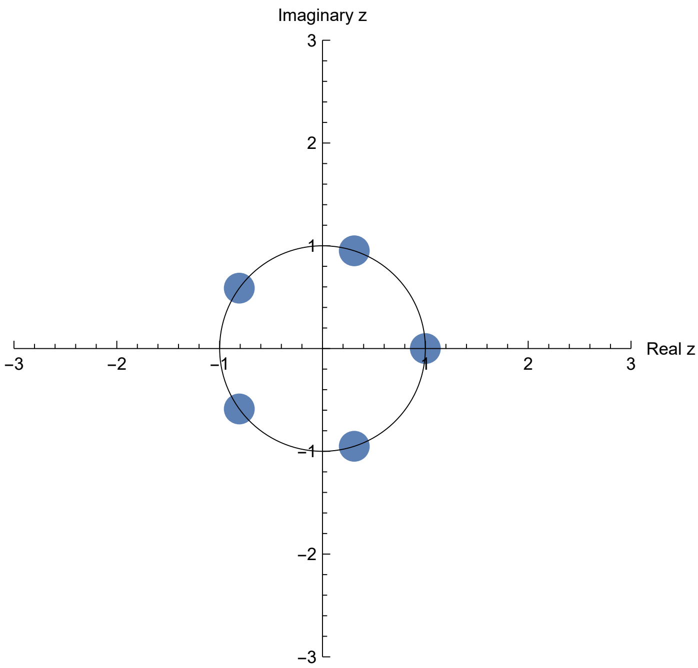
In[7]:= `Show[rootstruct, rootplot]`

Out[7]=



In[8]:= `Show[rootplot, rootstruct]`

Out[8]=



n = 6

In[1]:=
```
Solve[z^6 == 1, z]
```

Out[1]= $\{\{z \to -1\}, \{z \to 1\}, \{z \to -(-1)^{1/3}\}, \{z \to (-1)^{1/3}\}, \{z \to -(-1)^{2/3}\}, \{z \to (-1)^{2/3}\}\}$

In[2]:=
```
root = ComplexExpand[Solve[z^6 == 1, z]]
```

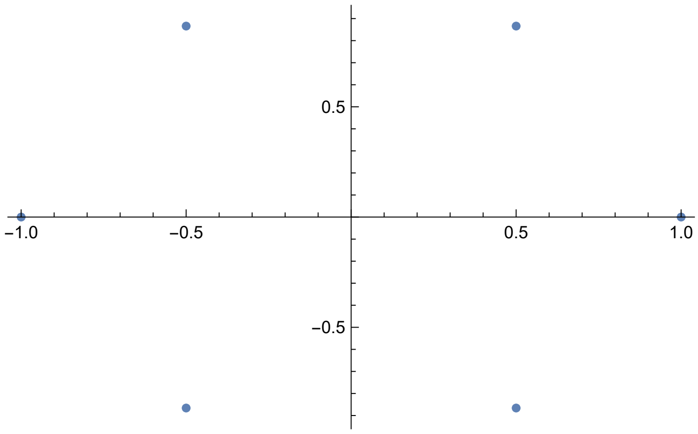
Out[2]= $\left\{\{z \to -1\}, \{z \to 1\}, \left\{z \to -\frac{1}{2} - \frac{i\sqrt{3}}{2}\right\}, \left\{z \to \frac{1}{2} + \frac{i\sqrt{3}}{2}\right\}, \left\{z \to \frac{1}{2} - \frac{i\sqrt{3}}{2}\right\}, \left\{z \to -\frac{1}{2} + \frac{i\sqrt{3}}{2}\right\}\right\}$

In[3]:=
```
z /. root
```

Out[3]= $\left\{-1, 1, -\frac{1}{2} - \frac{i\sqrt{3}}{2}, \frac{1}{2} + \frac{i\sqrt{3}}{2}, \frac{1}{2} - \frac{i\sqrt{3}}{2}, -\frac{1}{2} + \frac{i\sqrt{3}}{2}\right\}$

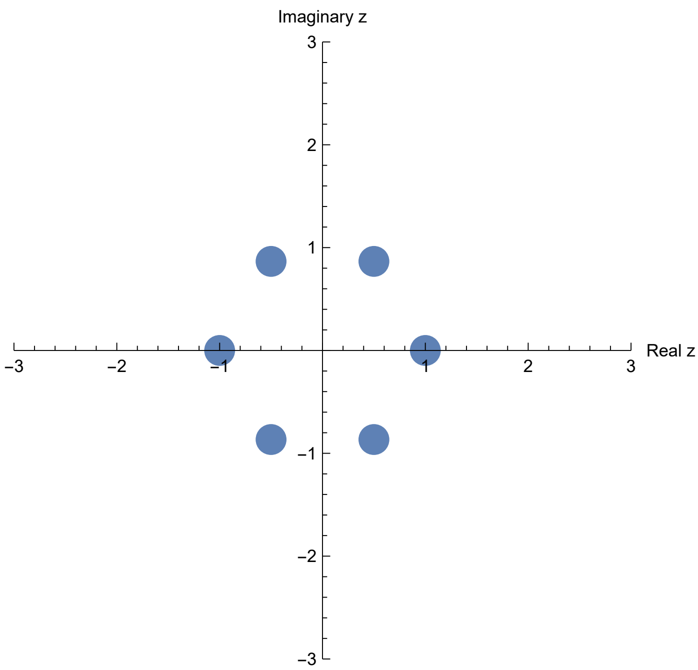
In[4]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root]
```

Out[4]=



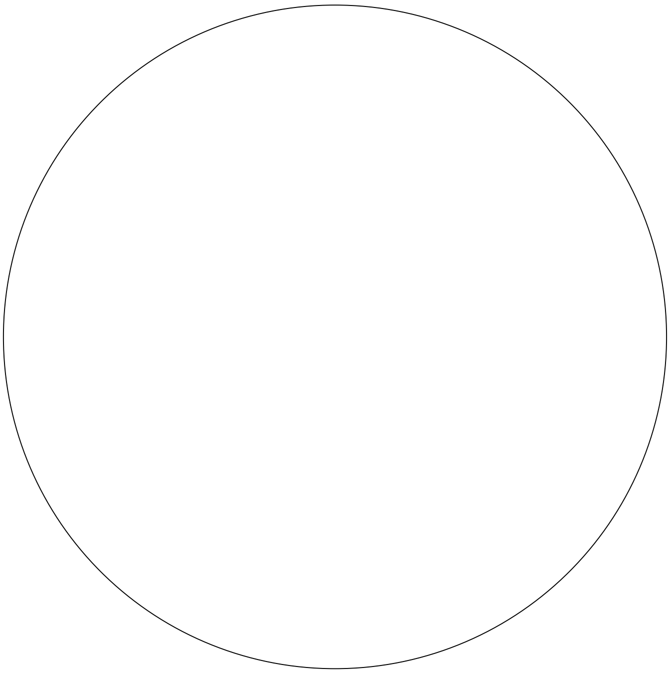
In[5]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root,
  PlotRange → {{-3, 3}, {-3, 3}}, AxesLabel → {"Real z", "Imaginary z"},
  PlotStyle → PointSize[0.05], AspectRatio → Automatic]
```

Out[5]=



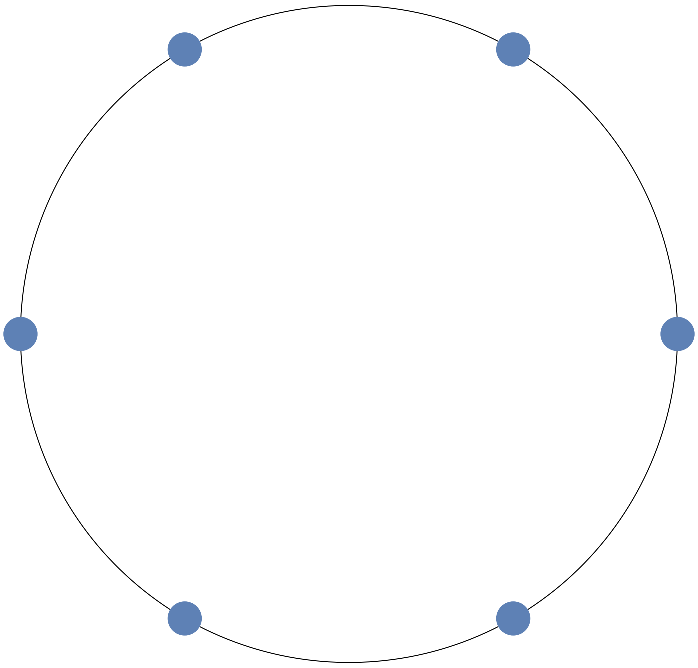
In[6]:=
```
rootstruct = Graphics[{Black, Circle[{0, 0}, 1]}]
```

Out[6]=



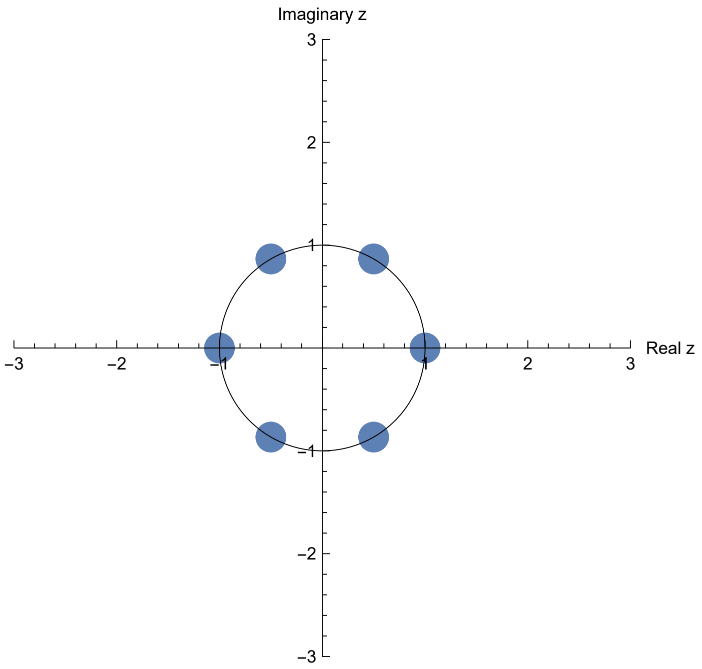
In[7]:=
```
Show[rootstruct, rootplot]
```

Out[7]=



In[8]:=
```
Show[rootplot, rootstruct]
```

Out[8]=



n = 7

In[1]:= `Solve[z^7 == 1, z]`

Out[1]= $\{\{z \to 1\}, \{z \to -(-1)^{1/7}\}, \{z \to (-1)^{2/7}\}, \{z \to -(-1)^{3/7}\}, \{z \to (-1)^{4/7}\}, \{z \to -(-1)^{5/7}\}, \{z \to (-1)^{6/7}\}\}$

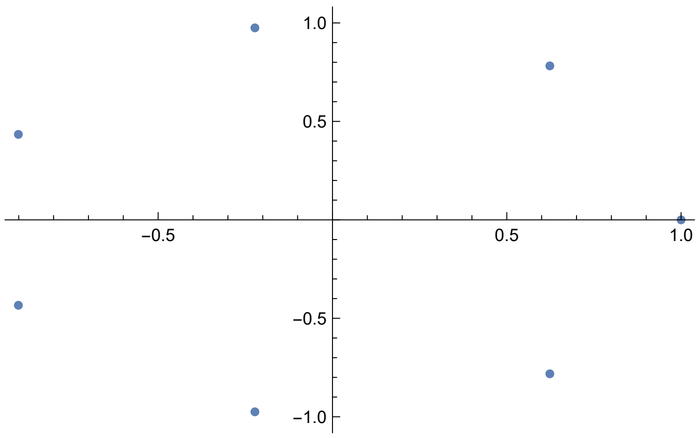
In[2]:= `root = ComplexExpand[Solve[z^7 == 1, z]]`

Out[2]= $\left\{\{z \to 1\}, \left\{z \to -\mathrm{Cos}\left[\frac{\pi}{7}\right] - i\,\mathrm{Sin}\left[\frac{\pi}{7}\right]\right\}, \left\{z \to i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right] + \mathrm{Sin}\left[\frac{3\pi}{14}\right]\right\}, \left\{z \to -i\,\mathrm{Cos}\left[\frac{\pi}{14}\right] - \mathrm{Sin}\left[\frac{\pi}{14}\right]\right\}, \left\{z \to i\,\mathrm{Cos}\left[\frac{\pi}{14}\right] - \mathrm{Sin}\left[\frac{\pi}{14}\right]\right\}, \left\{z \to -i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right] + \mathrm{Sin}\left[\frac{3\pi}{14}\right]\right\}, \left\{z \to -\mathrm{Cos}\left[\frac{\pi}{7}\right] + i\,\mathrm{Sin}\left[\frac{\pi}{7}\right]\right\}\right\}$

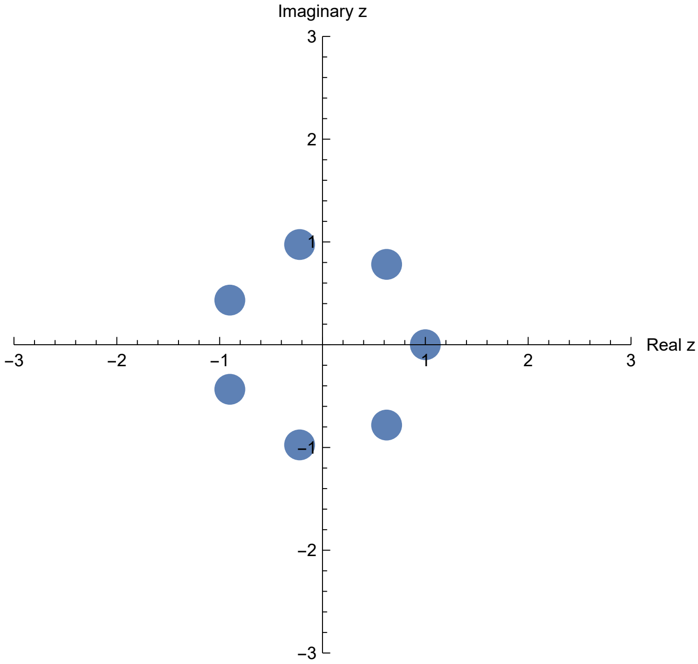
In[3]:= `z /. root`

Out[3]= $\left\{1, -\mathrm{Cos}\left[\frac{\pi}{7}\right] - i\,\mathrm{Sin}\left[\frac{\pi}{7}\right], i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right] + \mathrm{Sin}\left[\frac{3\pi}{14}\right], -i\,\mathrm{Cos}\left[\frac{\pi}{14}\right] - \mathrm{Sin}\left[\frac{\pi}{14}\right], i\,\mathrm{Cos}\left[\frac{\pi}{14}\right] - \mathrm{Sin}\left[\frac{\pi}{14}\right], -i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right] + \mathrm{Sin}\left[\frac{3\pi}{14}\right], -\mathrm{Cos}\left[\frac{\pi}{7}\right] + i\,\mathrm{Sin}\left[\frac{\pi}{7}\right]\right\}$

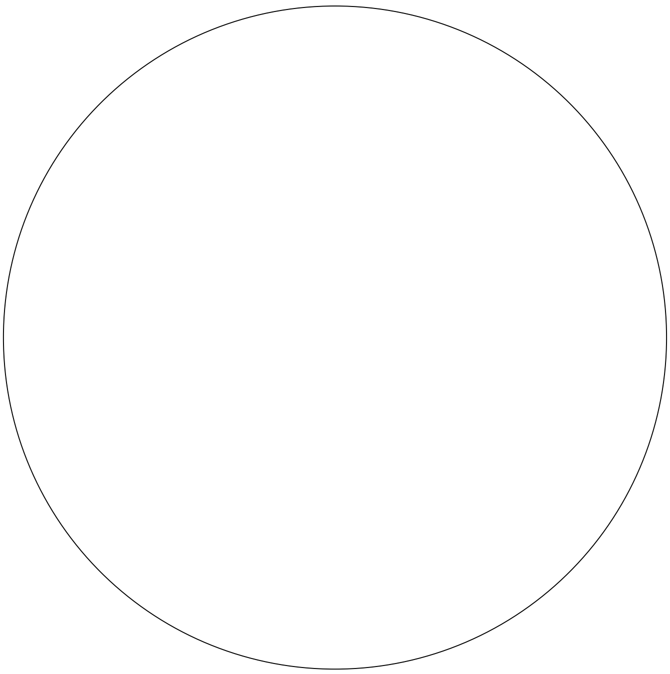
In[4]:= `rootplot = ListPlot[{Re[z], Im[z]} /. root]`

Out[4]=


In[5]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root,
  PlotRange → {{-3, 3}, {-3, 3}}, AxesLabel → {"Real z", "Imaginary z"},
  PlotStyle → PointSize[0.05], AspectRatio → Automatic]
```

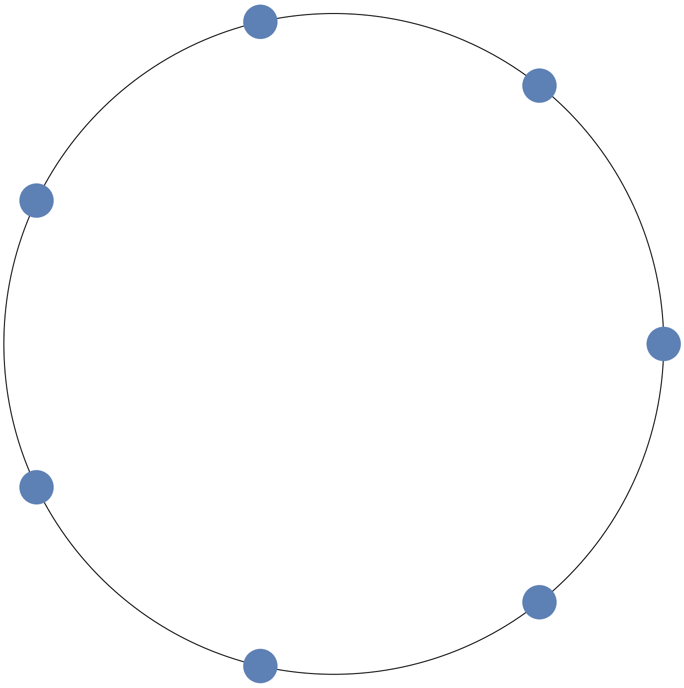
Out[5]=


In[6]:=
```
rootstruct = Graphics[{Black, Circle[{0, 0}, 1]}]
```

Out[6]=


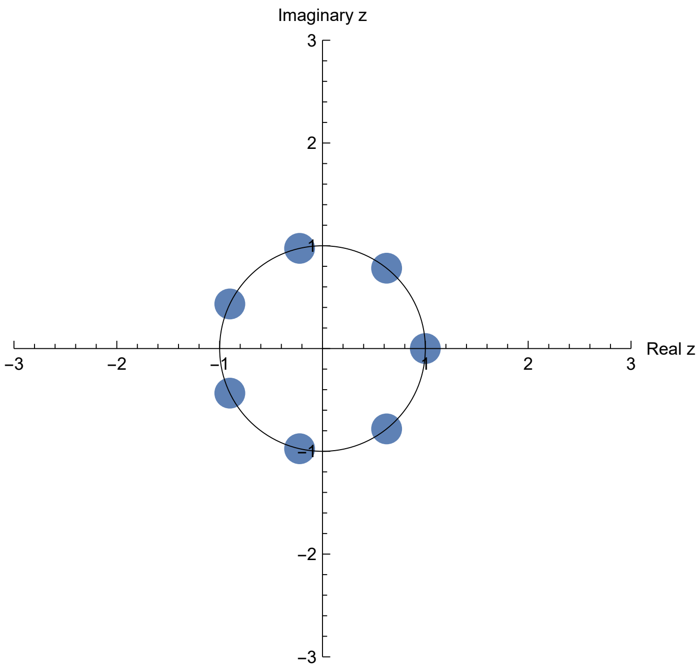
In[7]:=
```
Show[rootstruct, rootplot]
```

Out[7]=



In[8]:=
```
Show[rootplot, rootstruct]
```

Out[8]=



n = 8

In[1]:= `Solve[z^8 == 1, z]`

Out[1]= $\{\{z \to -1\}, \{z \to -i\}, \{z \to i\}, \{z \to 1\}, \{z \to -(-1)^{1/4}\}, \{z \to (-1)^{1/4}\}, \{z \to -(-1)^{3/4}\}, \{z \to (-1)^{3/4}\}\}$

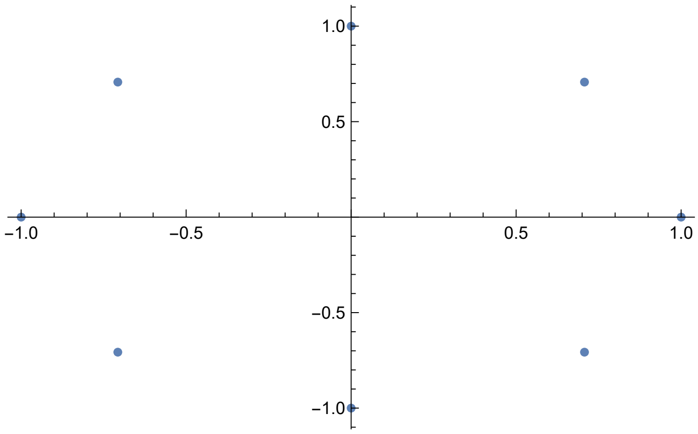
In[2]:= `root = ComplexExpand[Solve[z^8 == 1, z]]`

Out[2]= $\left\{\{z \to -1\}, \{z \to -i\}, \{z \to i\}, \{z \to 1\}, \left\{z \to -\frac{1+i}{\sqrt{2}}\right\}, \left\{z \to \frac{1+i}{\sqrt{2}}\right\}, \left\{z \to \frac{1-i}{\sqrt{2}}\right\}, \left\{z \to -\frac{1-i}{\sqrt{2}}\right\}\right\}$

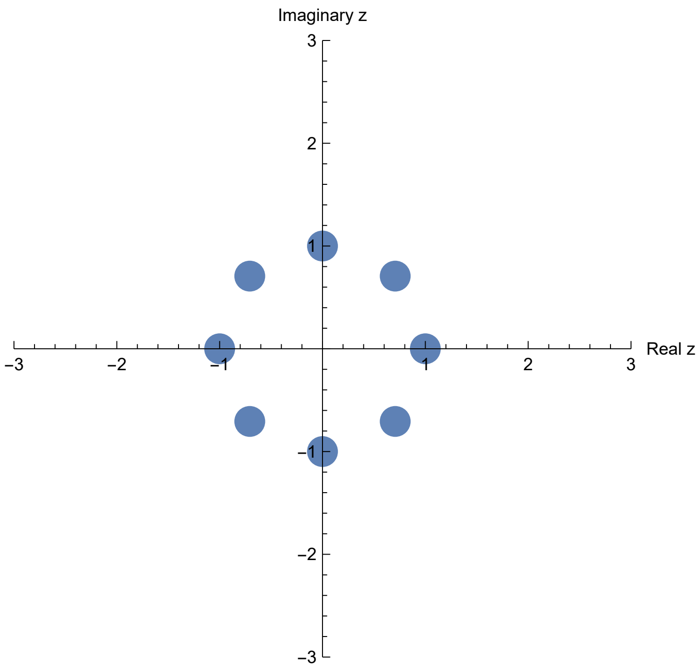
In[3]:= `z /. root`

Out[3]= $\left\{-1, -i, i, 1, -\frac{1+i}{\sqrt{2}}, \frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, -\frac{1-i}{\sqrt{2}}\right\}$

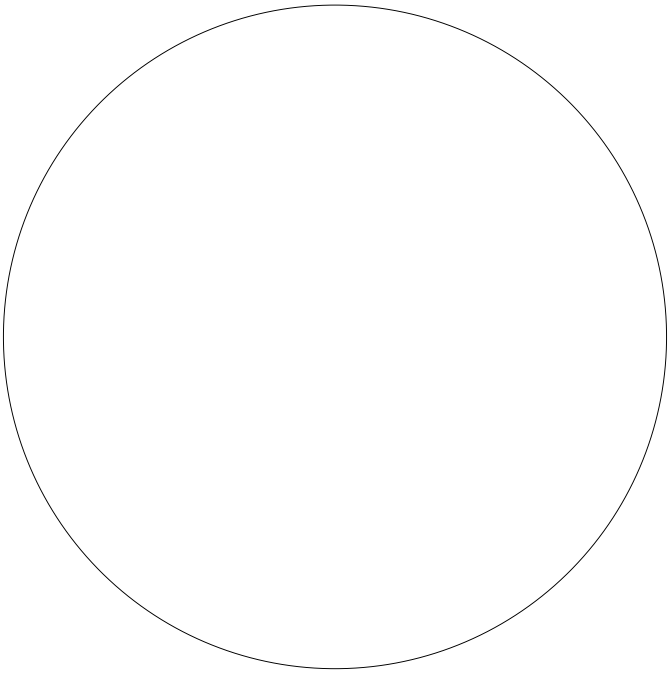
In[4]:= `rootplot = ListPlot[{Re[z], Im[z]} /. root]`

Out[4]= 

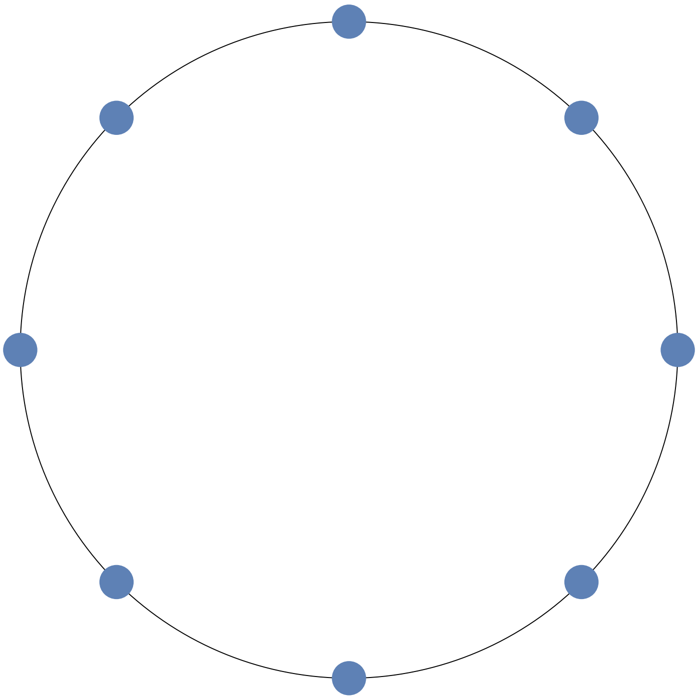
In[5]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root,
  PlotRange → {{-3, 3}, {-3, 3}}, AxesLabel → {"Real z", "Imaginary z"},
  PlotStyle → PointSize[0.05], AspectRatio → Automatic]
```

Out[5]= 

In[6]:=
```
rootstruct = Graphics[{Black, Circle[{0, 0}, 1]}]
```

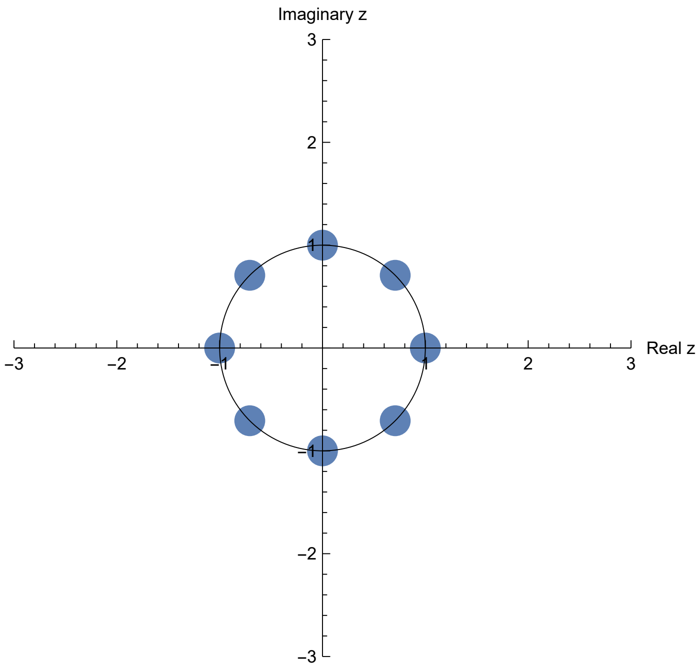
Out[6]= 

In[7]:=
```
Show[rootstruct, rootplot]
```

Out[7]= 

In[8]:= `Show[rootplot, rootstruct]`

Out[8]=



Practical - 2

Find all the solutions of the equation $z^3 = 8\,i$ and represent these geometrically.

In[1]:=
```
Solve[z^3 == 8 i, z]
```

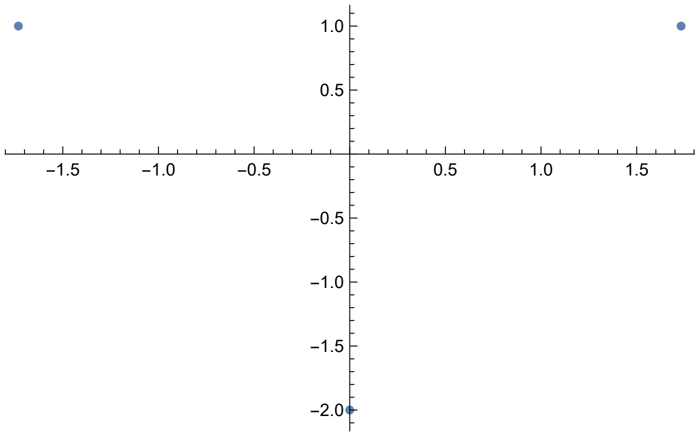
Out[1]= $\{\{z \to -2\,i\}, \{z \to 2\,(-1)^{1/6}\}, \{z \to 2\,(-1)^{5/6}\}\}$

In[2]:=
```
root = ComplexExpand[Solve[z^3 == 8 i, z]]
```

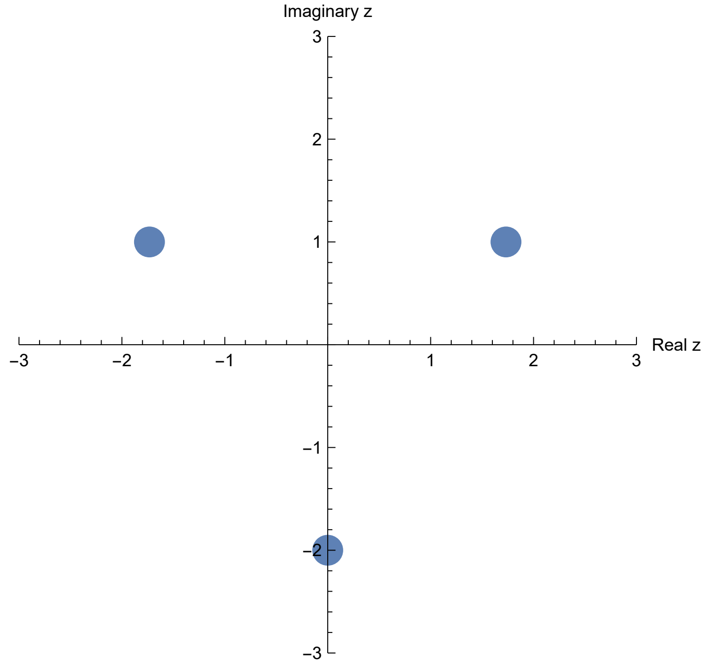
Out[2]= $\{\{z \to -2\,i\}, \{z \to i + \sqrt{3}\}, \{z \to i - \sqrt{3}\}\}$

In[3]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root]
```

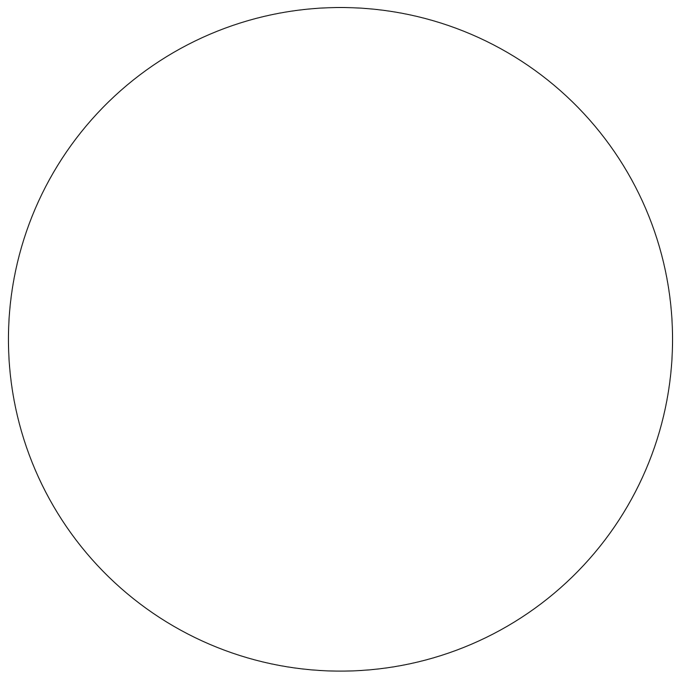
Out[3]=



In[4]:=
```
rootplot = ListPlot[{Re[z], Im[z]} /. root,
  PlotRange → {{-3, 3}, {-3, 3}}, AxesLabel → {"Real z", "Imaginary z"},
  PlotStyle → PointSize[0.05], AspectRatio → Automatic]
```

Out[4]=


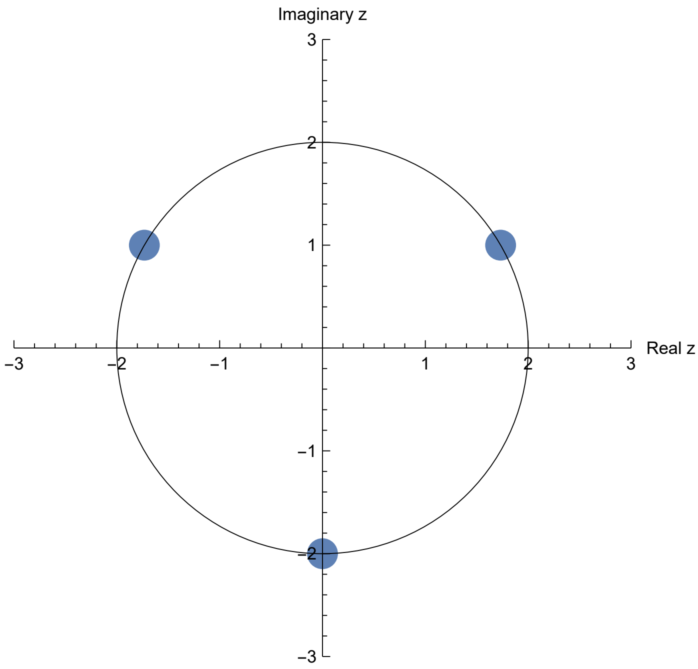
In[5]:=
```
rootstruct = Graphics[{Black, Circle[{0, 0}, 2]}]
```

Out[5]=


In[6]:=

```
Show[rootplot, rootstruct]
```

Out[6]=

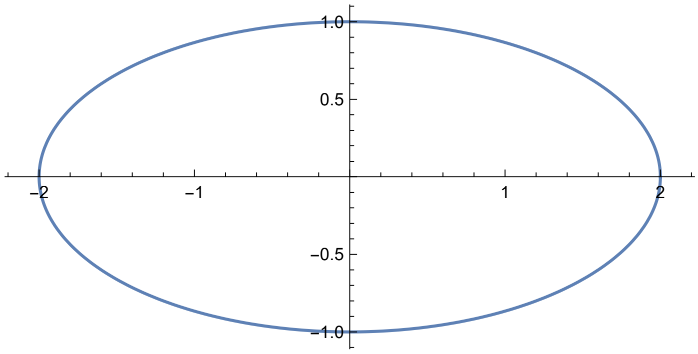


Practical 3

Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units . Show the effect of rotation of this ellipse by an angle of $\frac{\pi}{6}$ radians and shifting of the centre from (0, 0) to (2, 1), by making a parametric plot .

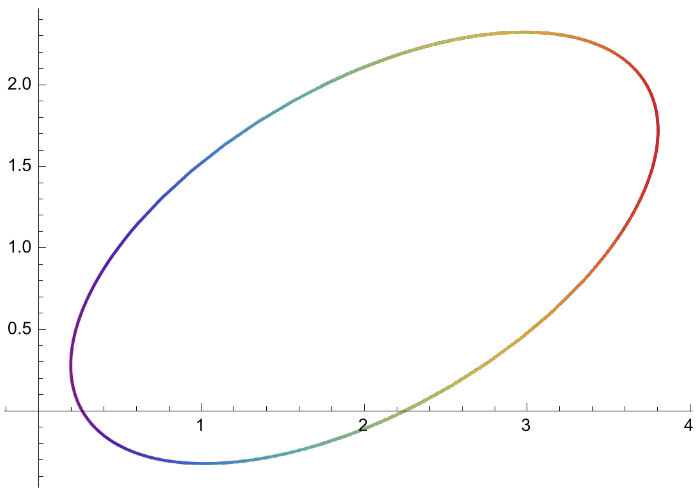
In[1]:= `a1 = ParametricPlot[{2 * Cos[t], Sin[t]}, {t, 0, 2 π}]`

Out[1]=



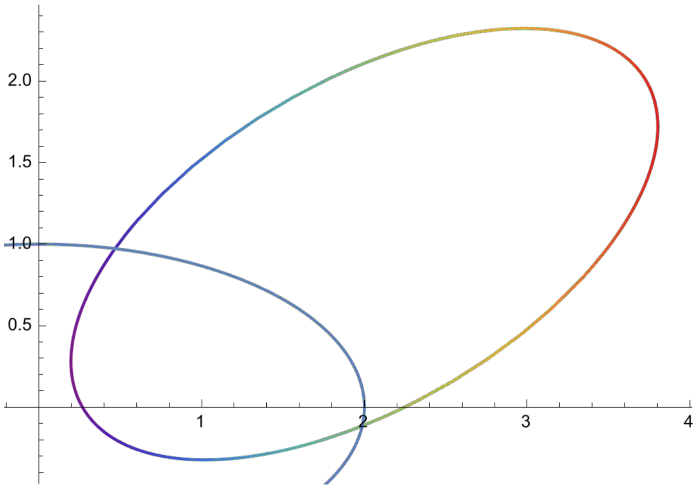
In[2]:= $\texttt{a2 = ParametricPlot}\left[\left\{\sqrt{3}\ \texttt{Cos[t]} - \frac{1}{2}\ \texttt{Sin[t]} + 2,\ \frac{\sqrt{3}}{2}\ \texttt{Sin[t]} + \texttt{Cos[t]} + 1\right\},\right.$ $\left.\texttt{\{t, 0, 2}\,\pi\texttt{\}, ColorFunction} \to \texttt{"Rainbow"}\right]$

Out[2]=



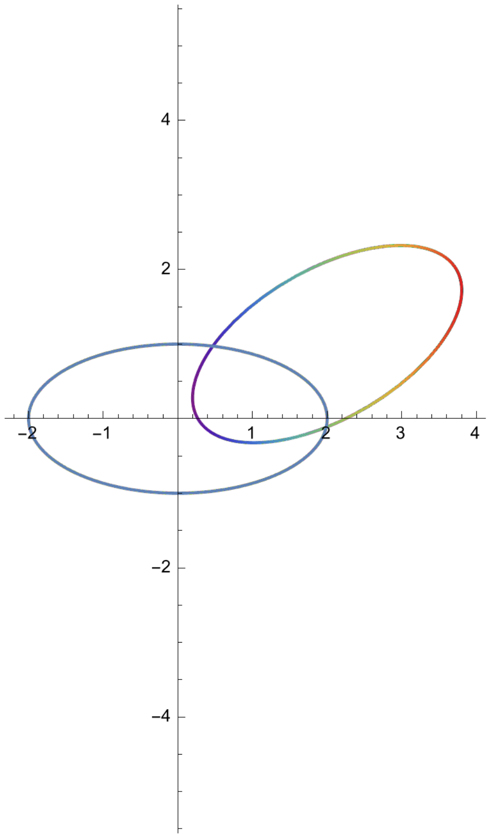
In[3]:= `Show[a2, a1]`

Out[3]=



In[4]:= `Show[a2, a1, PlotRange → {-5, 5}]`

Out[4]=

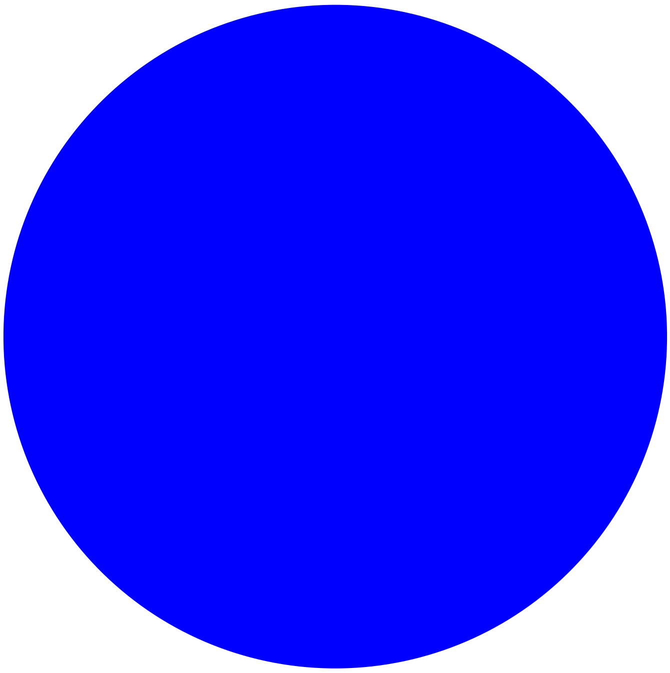


Practical - 4

Show that the image of the open disk $D_1(-1-i) = \{z: |z+1+i| < 1\}$ under the linear transformation $w = f(z) = (3-4i) + 6 + 2i$ is the open disk: $D_5(-1+3i) = \{w: |w+1-3i| < 5\}$.

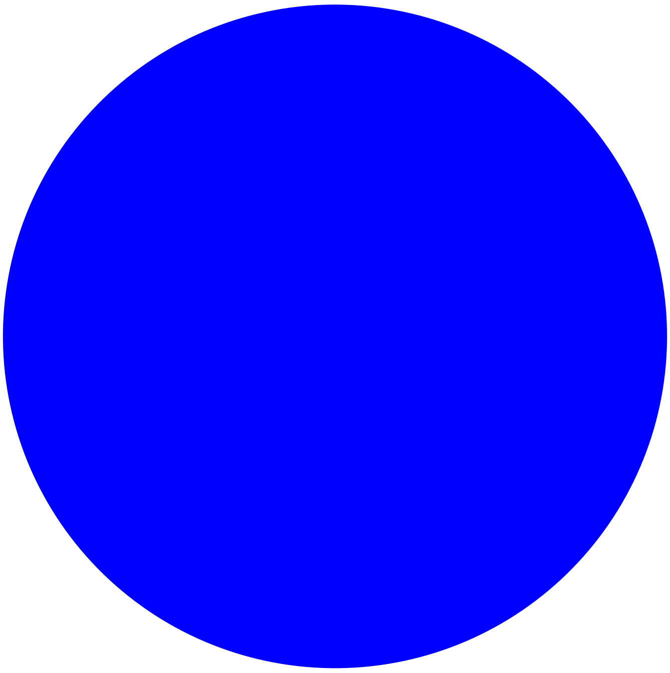
In[1]:=
```
Graphics[{Blue, Disk[{0, 0}, 1]}]
```

Out[1]=



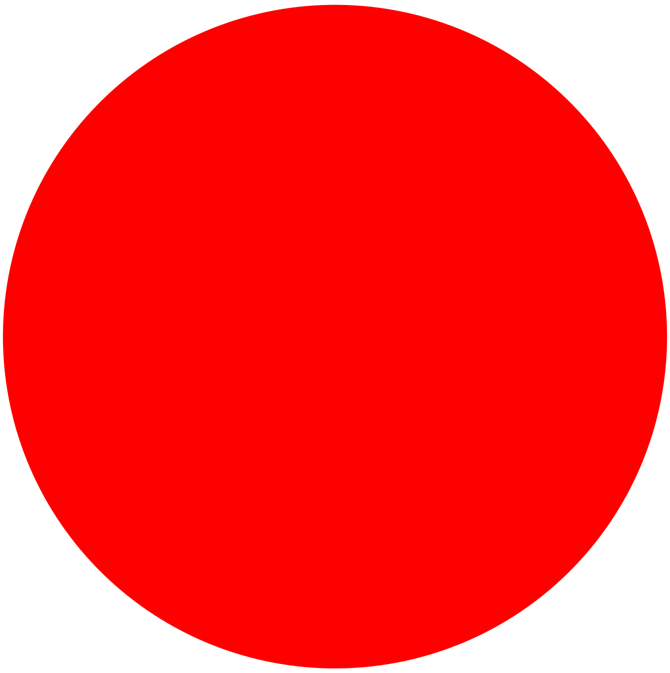
In[2]:=
```
a1 = Graphics[{Blue, Disk[{-1, -1}, 1]}]
```

Out[2]=



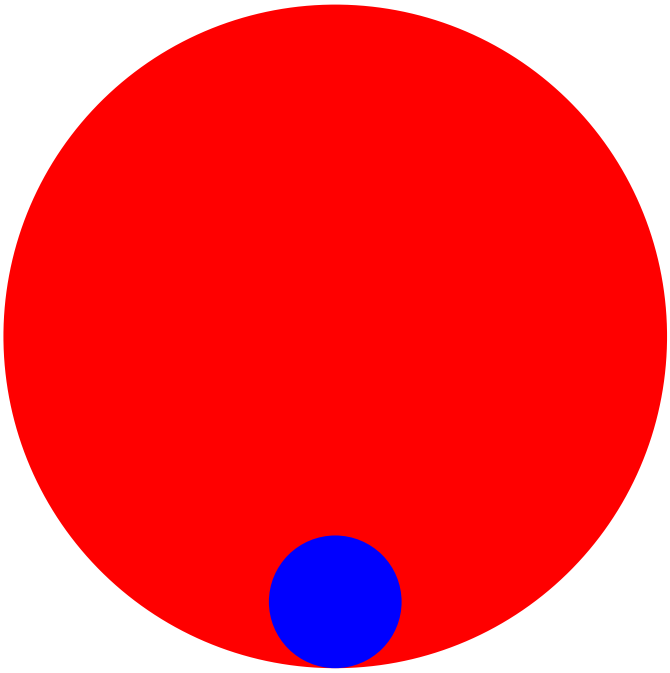
In[3]:=
```
a2 = Graphics[{Red, Disk[{-1, 3}, 5]}]
```

Out[3]=



In[4]:=
```
Show[a2, a1]
```

Out[4]=

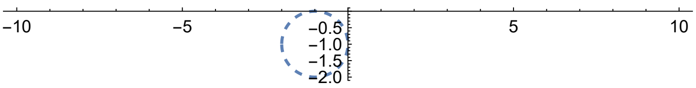


In[5]:=
```
sol1 = Solve[(x + 1)^2 + (y + 1)^2 == 1, y]
```

Out[5]= $\left\{\left\{y \to -1 - \sqrt{-2x - x^2}\right\}, \left\{y \to -1 + \sqrt{-2x - x^2}\right\}\right\}$

In[6]:=
```
p1 = Plot[y /. sol1, {x, -10, 10}, AspectRatio → Automatic, PlotStyle → Dashed]
```

Out[6]=

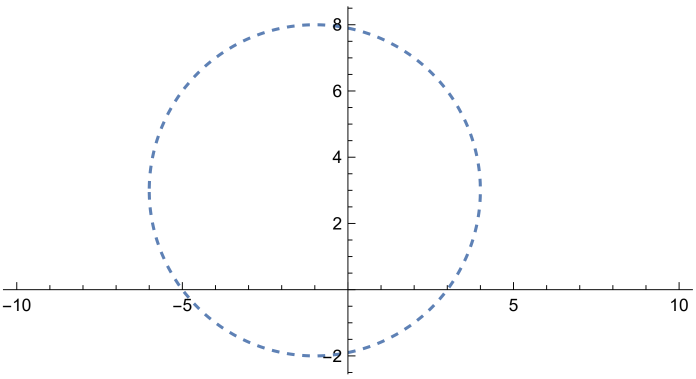


In[7]:=
```
sol2 = Solve[(u + 1)^2 + (v - 3)^2 == 25, v]
```

Out[7]= $\left\{\left\{v \to 3 - \sqrt{24 - 2u - u^2}\right\}, \left\{v \to 3 + \sqrt{24 - 2u - u^2}\right\}\right\}$

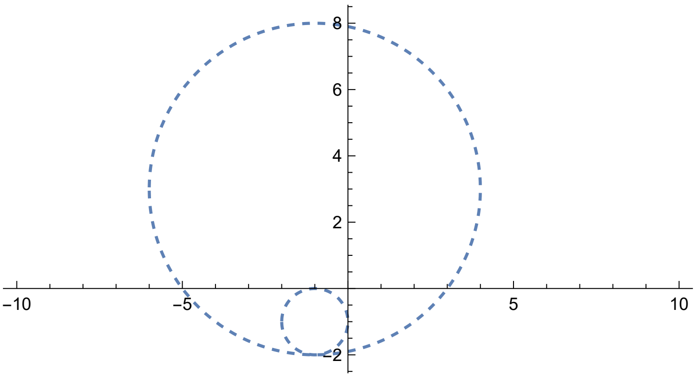
In[8]:=
```
p2 = Plot[v /. sol2, {u, -10, 10}, AspectRatio → Automatic, PlotStyle → Dashed]
```

Out[8]=



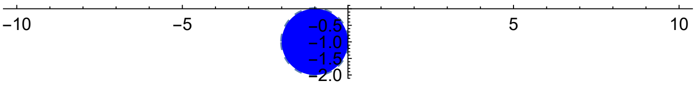
In[9]:= `Show[p2, p1]`

Out[9]=



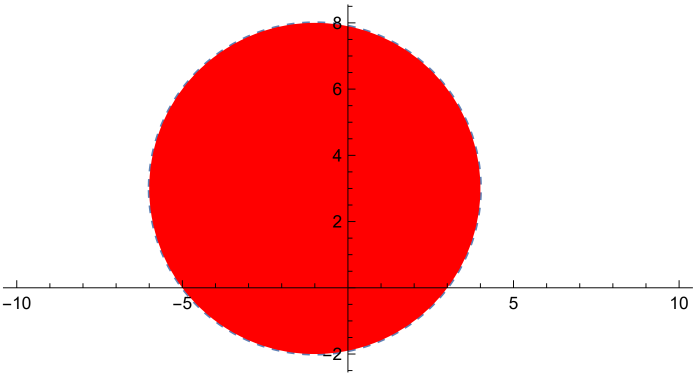
In[10]:= `v1 = Show[p1, a1]`

Out[10]=



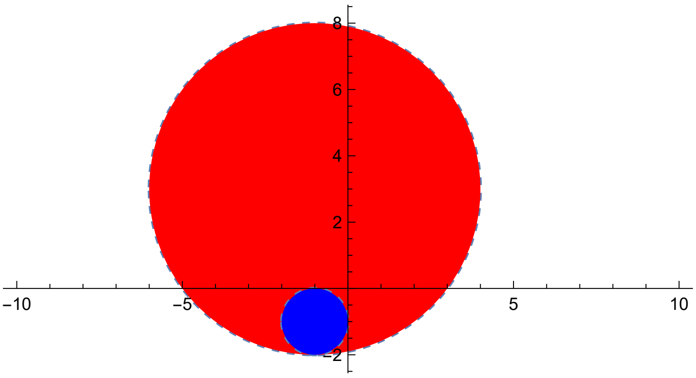
In[11]:= `v2 = Show[p2, a2]`

Out[11]=



In[12]:= `Show[v2, v1]`

Out[12]=

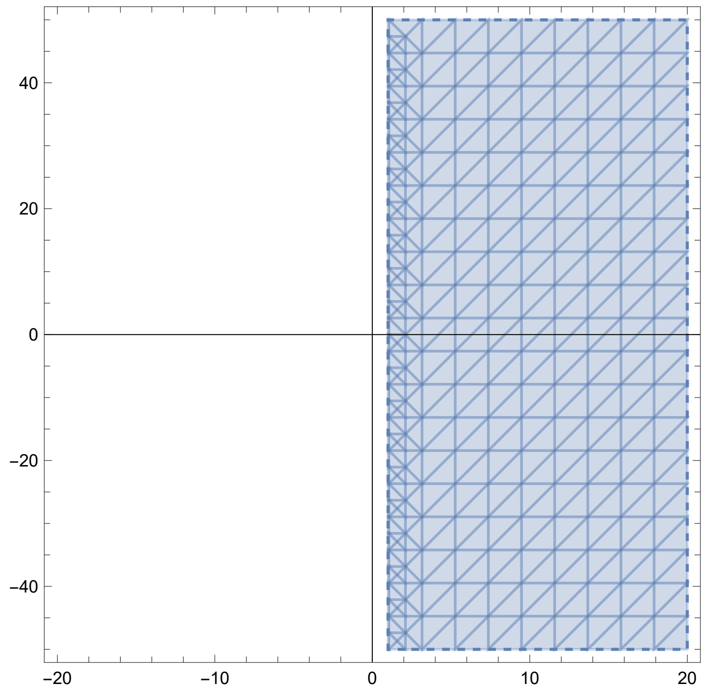


Practical - 5

Show that the image of the right half plane Re(z) = x > 1 under the linear transformation w = f(z) = (−1 + ⅈ) − 2 + 3 ⅈ is the half plane v > u + 7, where u = Re(w) etc. Plot the map.

In[1]:=
```
a1 = RegionPlot[x > 1, {x, -20, 20}, {y, -50, 50}, Axes → True, BoundaryStyle → Dashed]
```

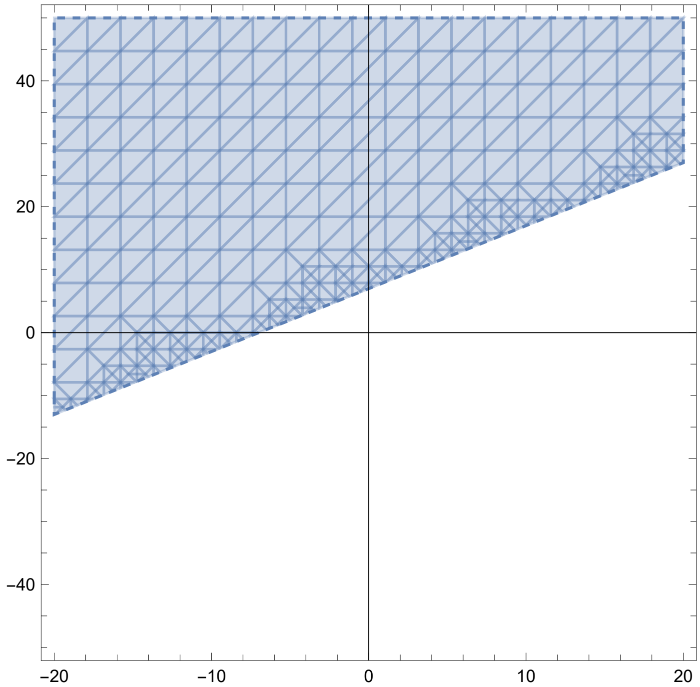
Out[1]=



In[2]:=

```
a2 = RegionPlot[v > u + 7, {u, -20, 20}, {v, -50, 50}, Axes → True, BoundaryStyle → Dashed]
```

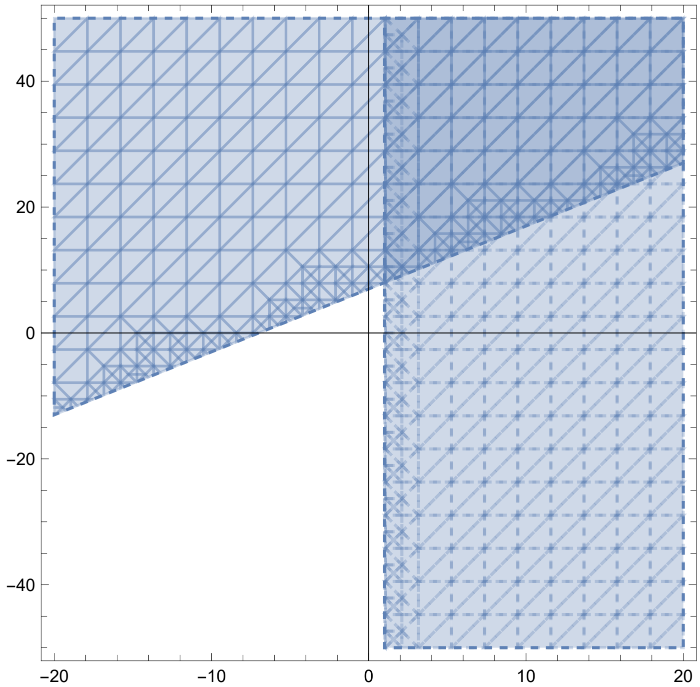
Out[2]=



In[3]:=

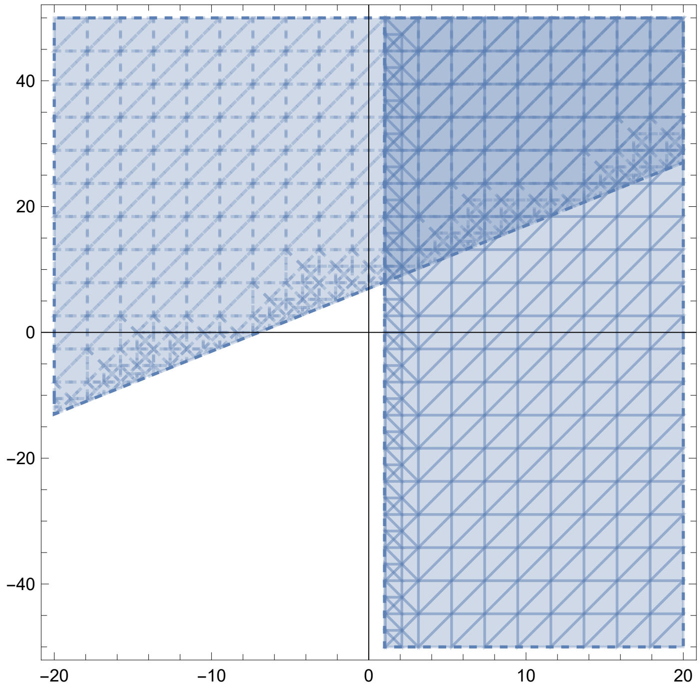
```
Show[a2, a1]
```

Out[3]=



In[4]:= **Show[a1, a2]**

Out[4]=

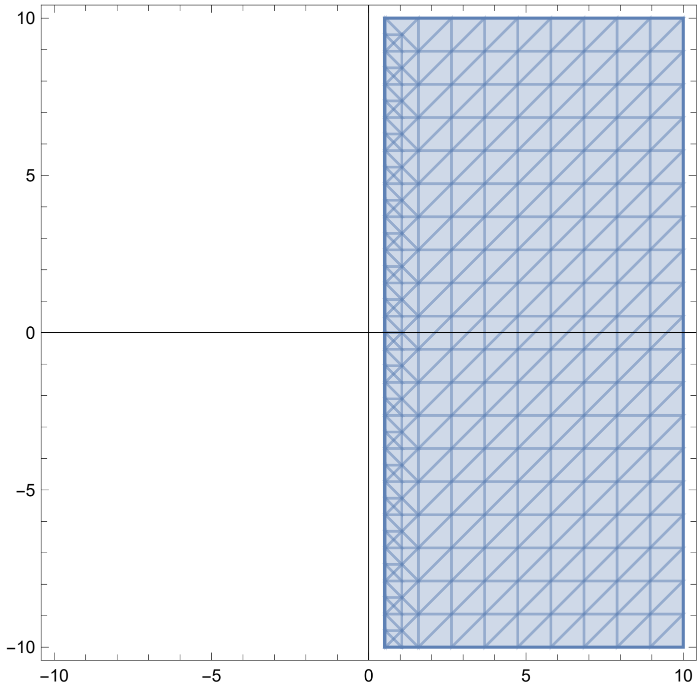


Practical - 6

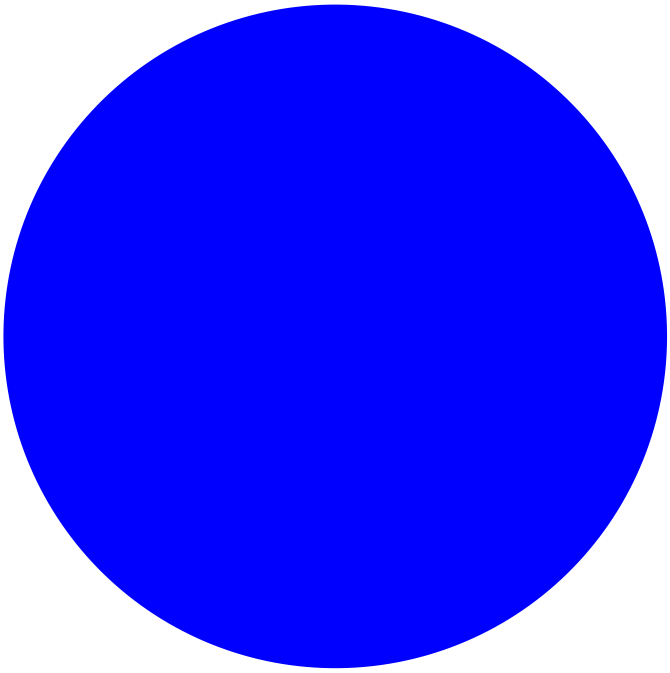
Show that the image of the right half-plane A = {z: Re z $\geq \frac{1}{2}$} under the mapping w = f(z) = $\frac{1}{2}$ is the closed disk $D_1(1)$ = {w: |w -1|≤1} in the w-plane.

In[1]:=
```
a1 = RegionPlot[x ≥ 1/2, {x, -10, 10}, {y, -10, 10}, Axes → True]
```

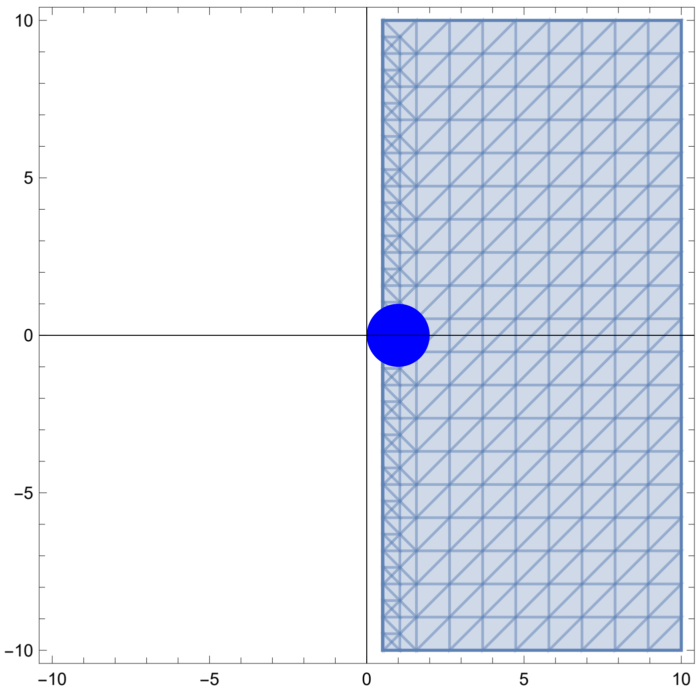
Out[1]=



In[2]:=
```
a2 = Graphics[{Blue, Disk[{1, 0}, 1]}]
```

Out[2]= 

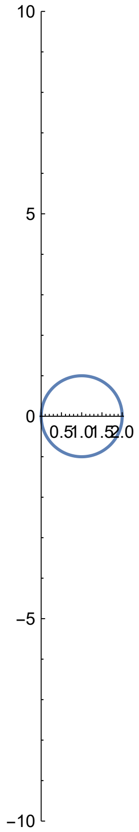
```
Show[a1, a2]
```

Out[ ]= 

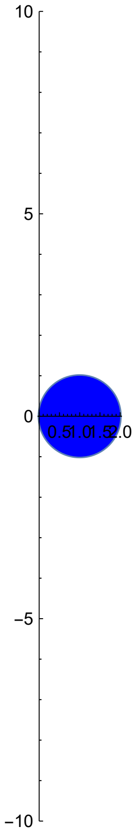
In[3]:=
$$a3 = \text{Solve}\left[(u-1)^2 + v^2 == 1,\ v\right]$$

Out[3]= $$\left\{\left\{v \to -\sqrt{2\,u - u^2}\right\},\ \left\{v \to \sqrt{2\,u - u^2}\right\}\right\}$$

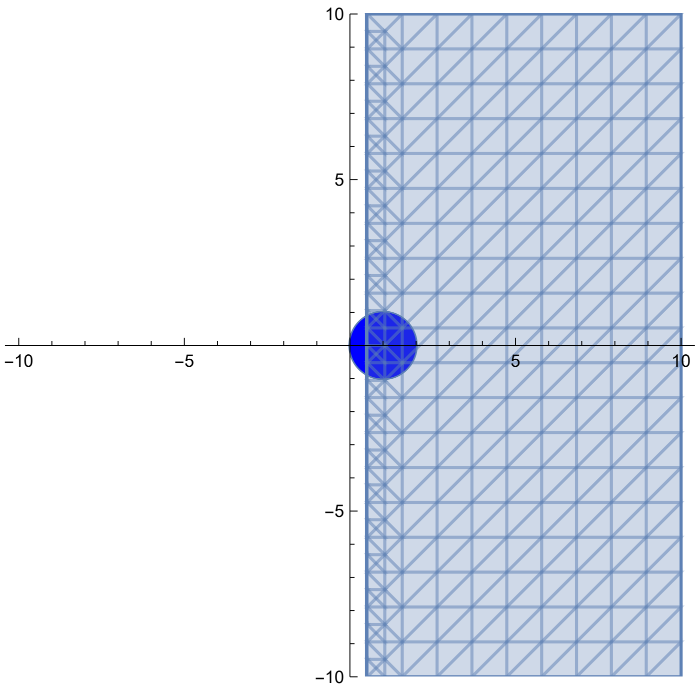
In[4]:=
```
a4 = Plot[v /. a3, {u, -10, 10},
  AspectRatio → Automatic, Axes → True, PlotRange → {-10, 10}]
```

Out[4]=


In[6]:= a5 = Show[a4, a2]

Out[6]=


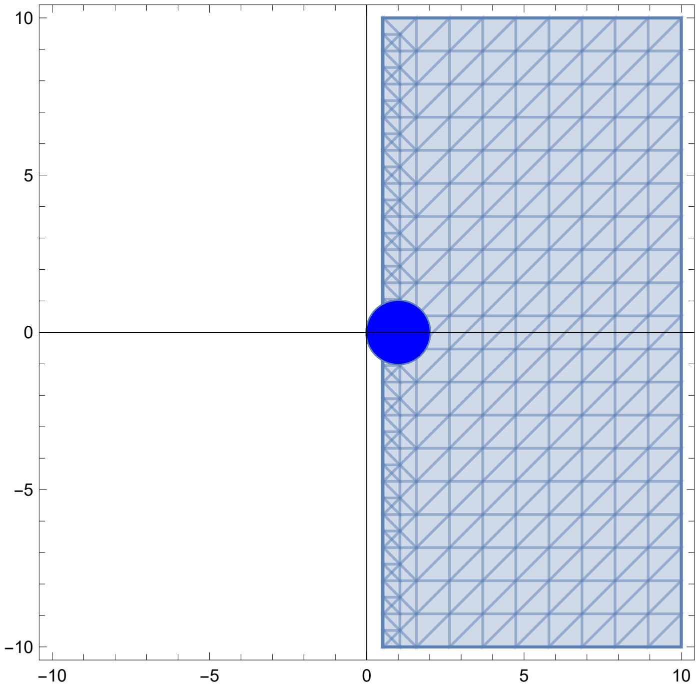
In[7]:= Show[a5, a1, PlotRange → {{-10, 10}, {-10, 10}}]

Out[7]=


In[9]:=

```
Show[a1, a5]
```

Out[9]=



Practical 7

Make a plot of the vertical lines x = a, for a $= -1, -\frac{1}{2}, \frac{1}{2}$, 1 and the

horizontal lines y= b,for b= -1, $-\frac{1}{2}, \frac{1}{2}$, 1. Find the plot of this grid under the mapping $w = f(z) = \frac{1}{z}$

In[24]:= `z = x + i * y`

Out[24]= $x + i\,y$

In[25]:= `Abs[1 / z - 1 / 2]`

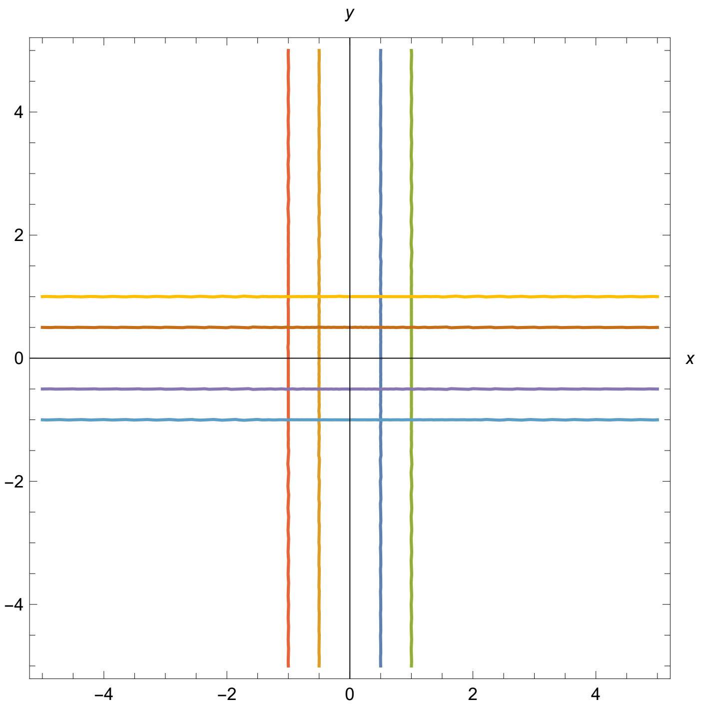
Out[25]= $\text{Abs}\left[-\frac{1}{2} + \frac{1}{x + i\,y}\right]$

In[26]:= $\text{Abs}\left[\frac{2}{(z - 1)}\right]$

Out[26]= $\frac{2}{\text{Abs}[-1 + x + i\,y]}$

In[27]:=
```
a1 = ContourPlot[
  {Abs[1 / z - 1] == 1, Abs[1 / z + 1] == 1, Abs[1 / z - 1 / 2] == 1/2, Abs[1 / z + 1 / 2] == 1/2,
   Abs[1 / z - I] == 1, Abs[1 / z + I] == 1, Abs[1 / z - I / 2] == 1/2, Abs[1 / z + I / 2] == 1/2},
  {x, -5, 5}, {y, -5, 5}, Axes → True, AxesLabel → {x, y}, AxesOrigin → {0, 0}]
```

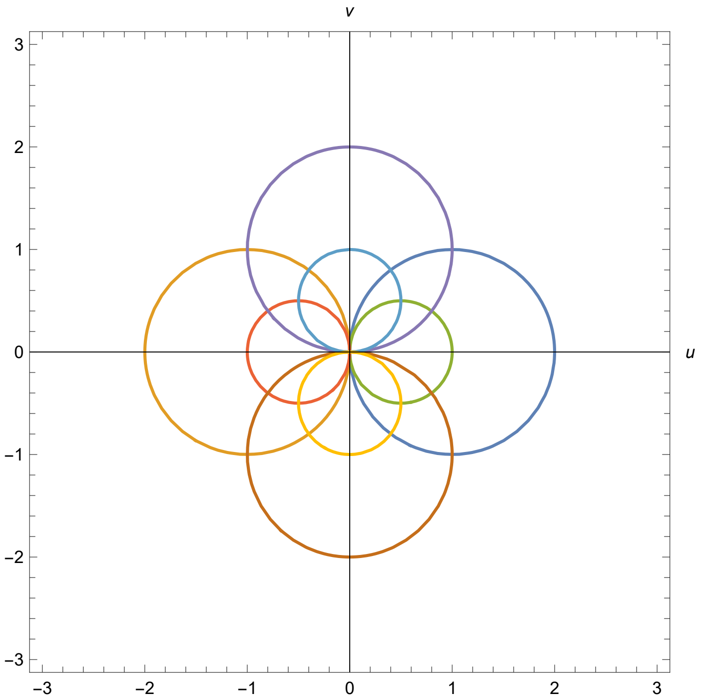
Out[27]=



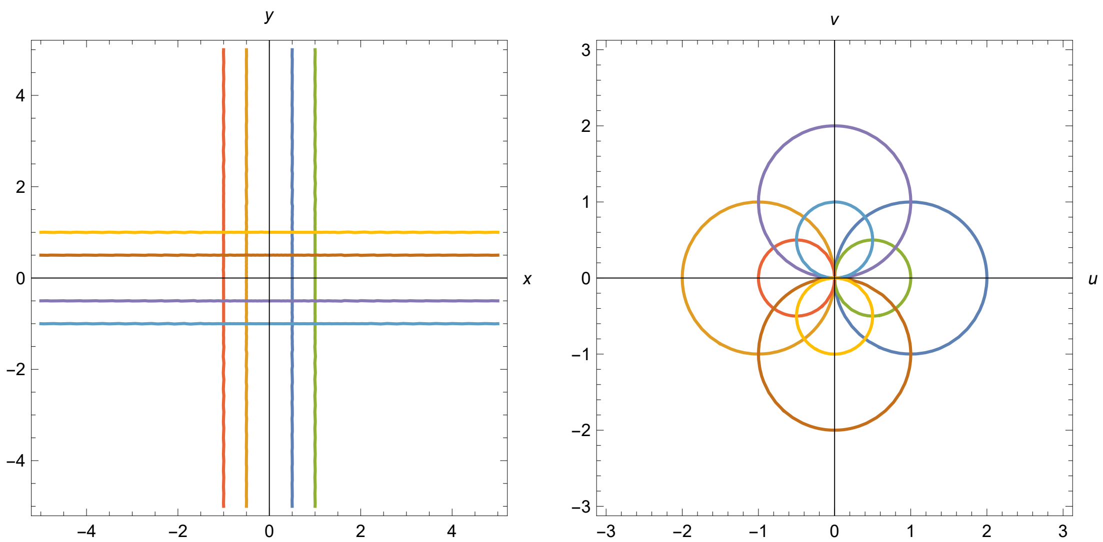
In[28]:=
```
w = u + ⅈ * v
```

Out[28]= u + ⅈ v

In[29]:=
```
a2 = ContourPlot[{Abs[w - 1] == 1, Abs[w + 1] == 1, Abs[w - 1/2] == 1/2,
   Abs[w + 1/2] == 1/2, Abs[w - I] == 1, Abs[w + I] == 1, Abs[w - I/2] == 1/2, Abs[w + I/2] == 1/2},
  {u, -3, 3}, {v, -3, 3}, Axes → True, AxesLabel → {u, v}, AxesOrigin → {0, 0}]
```

Out[29]=


In[30]:=
```
GraphicsRow[{a1, a2}]
```

Out[30]=


Practical 8

Find a parametrization of the polygonal path C = C1 + C2 + C3 from $-1 + i$ to $3 - i$, where C1 is the line from: $-1 + i$ to -1 , C2 is the line from: -1 to $1 + i$ and C3 is the line from $1 + i$ to $3 - i$. Make a plot of this path.

In[1]:=
```
z1[t_] := -1 + I (1 - t)
```

In[2]:=
```
z2[t_] := -1 + t (2 + I)
```

In[3]:=
```
z3[t_] := 1 + 2 * t + I (1 - 2 * t)
```

In[4]:=
```
c1 = ComplexExpand[{Re[z1[t]], Im[z1[t]]}]
```

Out[4]= {-1, 1 - t}

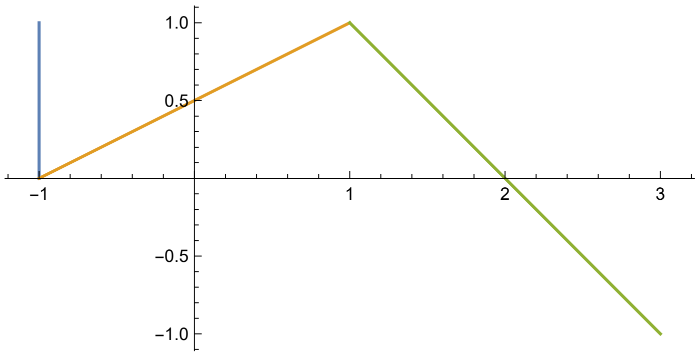
In[5]:=
```
c2 = ComplexExpand[{Re[z2[t]], Im[z2[t]]}]
```

Out[5]= {-1 + 2 t, t}

In[6]:=
```
c3 = ComplexExpand[{Re[z3[t]], Im[z3[t]]}]
```

Out[6]= {1 + 2 t, 1 - 2 t}

In[7]:=
```
ParametricPlot[Evaluate[{c1, c2, c3}], {t, 0, 1}]
```

Out[7]=


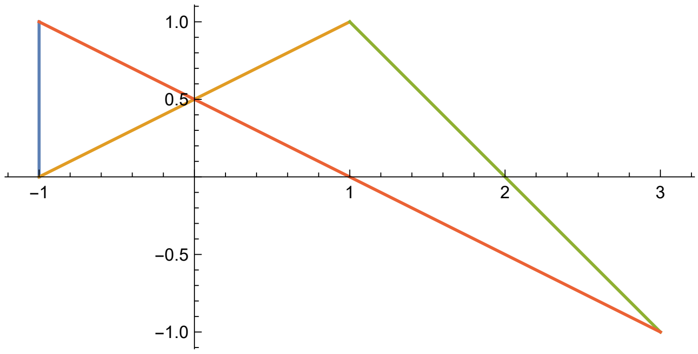
In[8]:=
```
z4[t_] := 4 * t - 1 + I (1 - 2 * t)
```

In[9]:=
```
c4 = ComplexExpand[{Re[z4[t]], Im[z4[t]]}]
```

Out[9]= {-1 + 4 t, 1 - 2 t}

In[10]:=
```
ParametricPlot[Evaluate[{c1, c2, c3, c4}], {t, 0, 1}]
```

Out[10]=



PRACTICAL – 09

Plot the line segment ‘L’ joining the point A = 0 to B = 2 + $\pi 4 i$ and give an exact calculation of $\int_L e^z \, dz$

In[1]:=
```
f[z_] := Exp[z]
z[t_] := 2 t + I * Pi/4 * t
f[z[t]]
z'[t]
```

Out[3]= $e^{2t+\frac{i\pi t}{4}}$

Out[4]= $2 + \frac{i\pi}{4}$

In[5]:=
```
int = f[z[t]] × z'[t]
```

Out[5]= $e^{2t+\frac{i\pi t}{4}}\left(2 + \frac{i\pi}{4}\right)$

In[6]:=
```
val = Integrate[int, {t, 0, 1}]
```

Out[6]= $-1 + (-1)^{1/4} e^2$

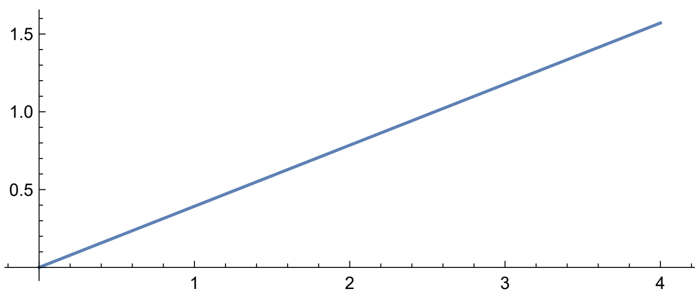
In[7]:=
```
ComplexExpand[val]
```

Out[7]= $-1 + \frac{(1+i)\, e^2}{\sqrt{2}}$

In[8]:=
```
v[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}]
ParametricPlot[Evaluate[v[t]], {t, 0, 2}]
```

Out[8]= $\left\{2t, \frac{\pi t}{4}\right\}$

Out[9]=



In[10]:=
```
sol = N[val]
```

Out[10]= $4.22485 + 5.22485\, i$

PRACTICAL – 10

Plot the semicircle ‘C with radius 1 centered at z = 2 and evaluate the contour
integral $\int_C \frac{1}{z-2}\, dz$.

In[15]:=
```
z[t_] := Exp[I * t] + 2
f[z_] := 1/(z - 2)
f[z[t]]
```
Out[17]= $e^{-i t}$

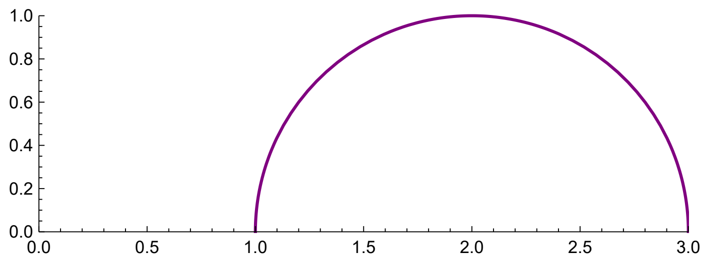
In[18]:=
```
z'[t]
```
Out[18]= $i\, e^{i t}$

In[19]:=
```
int = f[z[t]] × z'[t]
```
Out[19]= i

In[20]:=
```
val = Integrate[int, {t, 0, Pi}]
```
Out[20]= $i\, \pi$

In[21]:=
```
v[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}]
```
Out[21]= {2 + Cos[t], Sin[t]}

In[23]:=
```
ParametricPlot[Evaluate[v[t]], {t, 0, Pi},
 PlotRange → {{0, 3}, {0, 1}}, PlotStyle → Purple]
```
Out[23]=



PRACTICAL – 11

Show that $\int_{C1} z\, dz = \int_{C2} z\, dz = 4 + 2\,i$ where C1 is the line segment from–1–i to $3 + i$ and C2 is the portion of the parabola $x = y2 + 2\,y$ joining–1–i to $3 + i$.Make plots of two contours C1 and C2 joining–1–i to $3 + i$.

In[30]:=
```
f[z_] = z;
z[t_] = 2 * t + 1 + t * I
int = Integrate[f[z[t] × z'[t]], t]
val = Integrate[f[z[t]] × z'[t], {t, -1, 1}]
v[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}]
a1 = ParametricPlot[Evaluate[v[t]], {t, -1, 1}, PlotRange → Automatic,
  AxesLabel → {"x", "y"}, PlotStyle → Purple, AxesOrigin → Automatic]
```

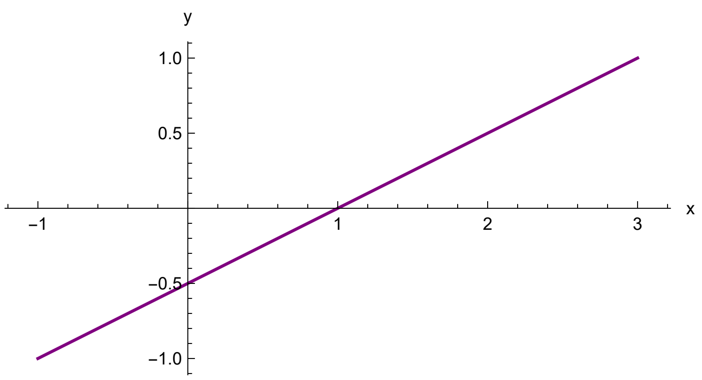
Out[31]= $1 + (2 + i)\, t$

Out[32]= $(2 + i)\, t + \left(\frac{3}{2} + 2\, i\right) t^2$

Out[33]= $4 + 2\, i$

Out[34]= $\{1 + 2\, t,\ t\}$

Out[35]=



```
In[42]:= f[z_] = z;
z[t_] = t^2 + 2 * t + t * I
int = Integrate[f[z[t]] × z'[t]], t]
val = Integrate[f[z[t]] × z'[t], {t, -1, 1}]
v[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}]
a2 = ParametricPlot[Evaluate[v[t]], {t, -1, 1}, PlotRange → Automatic,
  AxesLabel → {"x", "y"}, PlotStyle → Orange, AxesOrigin → Automatic]
```

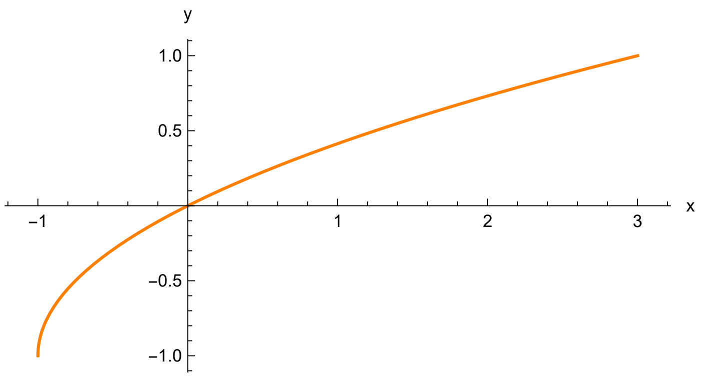
Out[43]= $(2 + i)\, t + t^2$

Out[44]= $\left(\frac{3}{2} + 2\, i\right) t^2 + (2 + i)\, t^3 + \frac{t^4}{2}$

Out[45]= $4 + 2\, i$

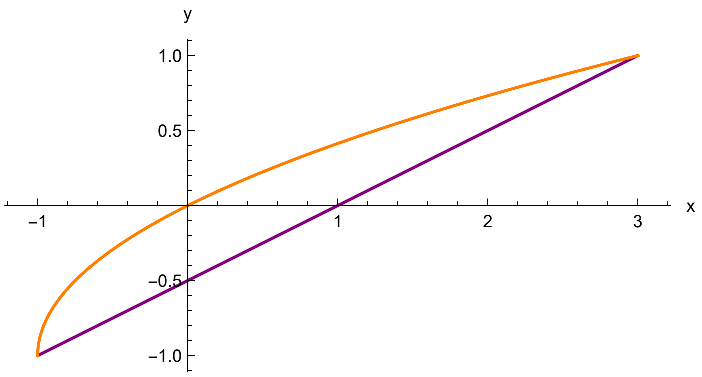
Out[46]= $\{2\, t + t^2,\ t\}$

Out[47]=




```
In[48]:= Show[a1, a2]
```

Out[48]=



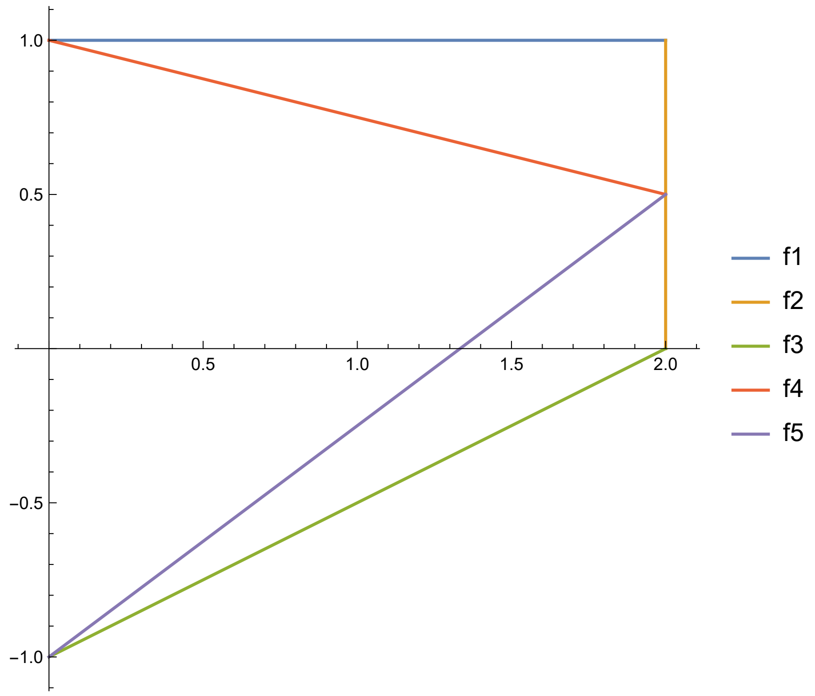
Practical 12

Use ML inequality to show that $\left| \int_C \frac{1}{z^2+1} \, dz \right| \leq \frac{1}{2\sqrt{5}}$,

where C is the straight line segment from 2 to 2 + i.While solving represent the distance from the point z to the points i and − i respectively, i.e. $|z - i|$ and $|z + i|$ on the complex plane.

In[ ]:=
```
f1[t_] := 2 * t + I
F1[t_] := ComplexExpand[{Re[f1[t]], Im[f1[t]]}]
f2[t_] := 2 + I * t
F2[t_] := ComplexExpand[{Re[f2[t]], Im[f2[t]]}]
f3[t_] := 2 t + I * (-1 + t)
F3[t_] := ComplexExpand[{Re[f3[t]], Im[f3[t]]}]
f4[t_] := 2 t + I * (1 - t / 2)
F4[t_] := ComplexExpand[{Re[f4[t]], Im[f4[t]]}]
f5[t_] := 2 t + I * (-1 + 3 t / 2)
F5[t_] := ComplexExpand[{Re[f5[t]], Im[f5[t]]}]
ParametricPlot[Evaluate[{F1[t], F2[t], F3[t], F4[t], F5[t]}],
 {t, 0, 1}, PlotLegends → {"f1", "f2", "f3", "f4", "f5"}]
```

Out[ ]=



Practical 13

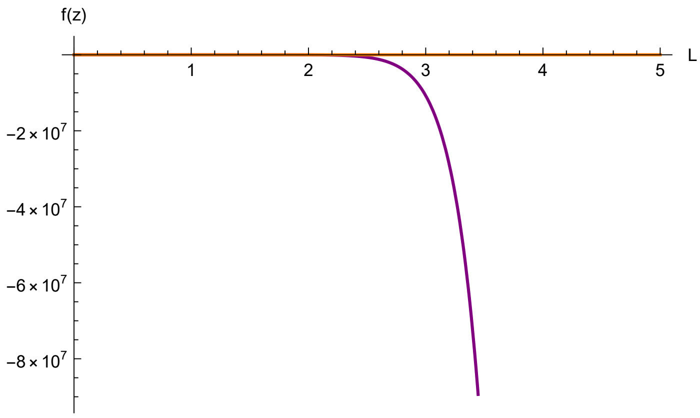
In[8]:=
```
f[z_] := 3 / (2 + z - z^2);
P[z] = Apart[f[z]];
L = Normal[Series[f[z], {z, 0, 15}]];
Print["f[z]=", f[z]]
Print["Partial Fraction of f[z]=", P[z]]
Print["Laurent Series of f[z]=", L, "+..."]
Evaluate[
 Plot[{L, f[z]}, {z, 0, 5}, PlotStyle → {Purple, Orange}, AxesLabel → {"L", "f(z)"}]]
```

f[z]= $\frac{3}{2+z-z^2}$

Partial Fraction of f[z]= $-\frac{1}{-2+z}+\frac{1}{1+z}$

Laurent Series of f[z]= $\frac{3}{2}-\frac{3z}{4}+\frac{9z^2}{8}-\frac{15z^3}{16}+\frac{33z^4}{32}-\frac{63z^5}{64}+\frac{129z^6}{128}-\frac{255z^7}{256}+\frac{513z^8}{512}-\frac{1023z^9}{1024}+\frac{2049z^{10}}{2048}-\frac{4095z^{11}}{4096}+\frac{8193z^{12}}{8192}-\frac{16383z^{13}}{16384}+\frac{32769z^{14}}{32768}-\frac{65535z^{15}}{65536}+...$

Out[14]=



In[15]:=
```
f1[z_] := 1 / (1 + z);
f2[z_] := (1 / 2) / (1 - z / 2)
L1 = Normal[Series[f1[z], {z, Infinity, 15}]];
L2 = Normal[Series[f2[z], {z, 0, 15}]];
Print["f1[z]=", f1[z]]
Print["f2[z]=", f2[z]]
Print["Series of f1[z]=", "...+", L1]
Print["Series of f2[z]=", L2, "+..."]
Print["Laurent Series of f[z]=", L2 + L1, "+..."]
Evaluate[Plot[{L1 + L2, f1[z] + f2[z]}, {z, 0, 5},
  PlotStyle → {Purple, Orange}, AxesLabel → {"L", "f(z)"}]]
```

$$f1[z] = \frac{1}{1+z}$$

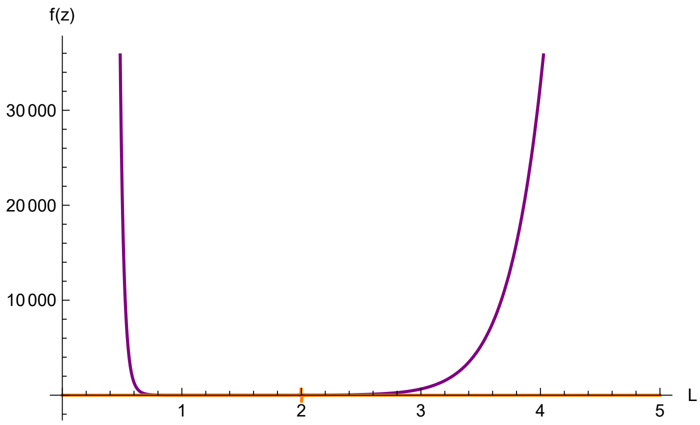
$$f2[z] = \frac{1}{2 \times \left(1 - \frac{z}{2}\right)}$$

$$\text{Series of } f1[z] = \ldots + \frac{1}{z^{15}} - \frac{1}{z^{14}} + \frac{1}{z^{13}} - \frac{1}{z^{12}} + \frac{1}{z^{11}} - \frac{1}{z^{10}} + \frac{1}{z^{9}} - \frac{1}{z^{8}} + \frac{1}{z^{7}} - \frac{1}{z^{6}} + \frac{1}{z^{5}} - \frac{1}{z^{4}} + \frac{1}{z^{3}} - \frac{1}{z^{2}} + \frac{1}{z}$$

$$\text{Series of } f2[z] = \frac{1}{2} + \frac{z}{4} + \frac{z^2}{8} + \frac{z^3}{16} + \frac{z^4}{32} + \frac{z^5}{64} + \frac{z^6}{128} + \frac{z^7}{256} + \frac{z^8}{512} + \frac{z^9}{1024} + \frac{z^{10}}{2048} + \frac{z^{11}}{4096} + \frac{z^{12}}{8192} + \frac{z^{13}}{16384} + \frac{z^{14}}{32768} + \frac{z^{15}}{65536} + \ldots$$

$$\text{Laurent Series of } f[z] = \frac{1}{2} + \frac{1}{z^{15}} - \frac{1}{z^{14}} + \frac{1}{z^{13}} - \frac{1}{z^{12}} + \frac{1}{z^{11}} - \frac{1}{z^{10}} + \frac{1}{z^{9}} - \frac{1}{z^{8}} + \frac{1}{z^{7}} - \frac{1}{z^{6}} + \frac{1}{z^{5}} - \frac{1}{z^{4}} + \frac{1}{z^{3}} - \frac{1}{z^{2}} + \frac{1}{z} + \frac{z}{4} + \frac{z^2}{8} + \frac{z^3}{16} + \frac{z^4}{32} + \frac{z^5}{64} + \frac{z^6}{128} + \frac{z^7}{256} + \frac{z^8}{512} + \frac{z^9}{1024} + \frac{z^{10}}{2048} + \frac{z^{11}}{4096} + \frac{z^{12}}{8192} + \frac{z^{13}}{16384} + \frac{z^{14}}{32768} + \frac{z^{15}}{65536} + \ldots$$

Out[24]=



In[25]:=

```
f3[z_] := 1 / (1 + z);
f4[z_] := (1 / 2) / (1 - z / 2)
L3 = Normal[Series[f3[z], {z, Infinity, 15}]];
L4 = Normal[Series[f4[z], {z, Infinity, 15}]];
Print["f1[z]=", f3[z]]
Print["f2[z]=", f4[z]]
Print["Series of f3[z]=", "...+", L3]
Print["Series of f4[z]=", "...+", L4]
Print["Laurent Series of f[z]=", L3 + L4, "+..."]
Evaluate[Plot[{L3 + L4, f3[z] + f4[z]}, {z, 0, 5},
  PlotStyle → {Purple, Orange}, AxesLabel → {"L", "f(z)"}]]
```

$$f1[z] = \frac{1}{1+z}$$

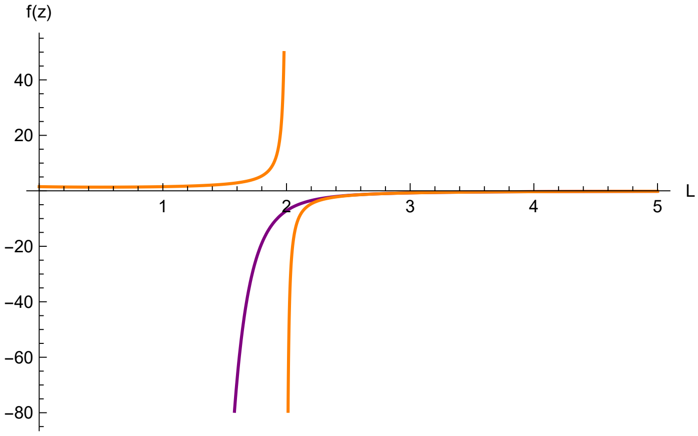
$$f2[z] = \frac{1}{2 \times \left(1 - \frac{z}{2}\right)}$$

$$\text{Series of } f3[z] = \ldots + \frac{1}{z^{15}} - \frac{1}{z^{14}} + \frac{1}{z^{13}} - \frac{1}{z^{12}} + \frac{1}{z^{11}} - \frac{1}{z^{10}} + \frac{1}{z^{9}} - \frac{1}{z^{8}} + \frac{1}{z^{7}} - \frac{1}{z^{6}} + \frac{1}{z^{5}} - \frac{1}{z^{4}} + \frac{1}{z^{3}} - \frac{1}{z^{2}} + \frac{1}{z}$$

$$\text{Series of } f4[z] = \ldots + -\frac{16384}{z^{15}} - \frac{8192}{z^{14}} - \frac{4096}{z^{13}} - \frac{2048}{z^{12}} - \frac{1024}{z^{11}} - \frac{512}{z^{10}} - \frac{256}{z^{9}} - \frac{128}{z^{8}} - \frac{64}{z^{7}} - \frac{32}{z^{6}} - \frac{16}{z^{5}} - \frac{8}{z^{4}} - \frac{4}{z^{3}} - \frac{2}{z^{2}} - \frac{1}{z}$$

$$\text{Laurent Series of } f[z] = -\frac{16383}{z^{15}} - \frac{8193}{z^{14}} - \frac{4095}{z^{13}} - \frac{2049}{z^{12}} - \frac{1023}{z^{11}} - \frac{513}{z^{10}} - \frac{255}{z^{9}} - \frac{129}{z^{8}} - \frac{63}{z^{7}} - \frac{33}{z^{6}} - \frac{15}{z^{5}} - \frac{9}{z^{4}} - \frac{3}{z^{3}} - \frac{3}{z^{2}} + \ldots$$

Out[34]=



Practical 14

In[35]:=
```
pole = ComplexExpand[Solve[5 z^4 + 26 z^2 + 5 == 0, z]]
```

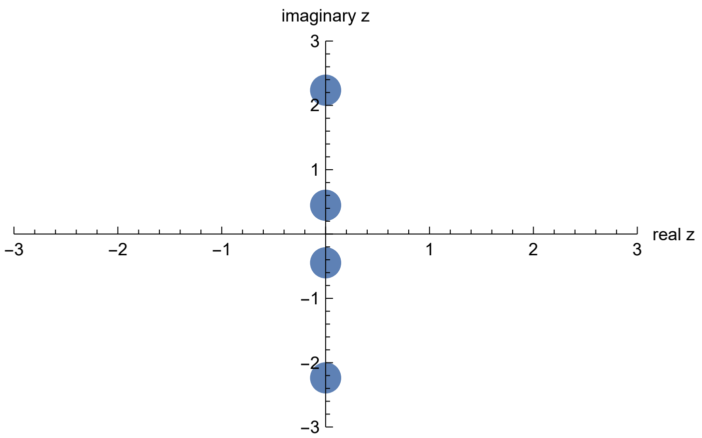
Out[35]= $\left\{\left\{z \to -\frac{i}{\sqrt{5}}\right\}, \left\{z \to \frac{i}{\sqrt{5}}\right\}, \left\{z \to -i\sqrt{5}\right\}, \left\{z \to i\sqrt{5}\right\}\right\}$

In[36]:=
```
z /. pole
```

Out[36]= $\left\{-\frac{i}{\sqrt{5}}, \frac{i}{\sqrt{5}}, -i\sqrt{5}, i\sqrt{5}\right\}$

In[37]:=
```
locatepole = ListPlot[{Re[z], Im[z]} /. pole, PlotRange → {{-3, 3}, {-3, 3}},
  AxesLabel → {"real z", "imaginary z"}, PlotStyle → PointSize[0.05]]
```

Out[37]=



Practical 15

In[38]:=
```
f[z_] := (Pi Cot[Pi z]) / (z^2);
pole = ComplexExpand[Reduce[z^2 Sin[Pi z] == 0, z]]
```

Out[39]= $(c_1 \in \mathbb{Z} \,\&\&\, (z == 2 c_1 \,||\, z == 1 + 2 c_1)) \,||\, z == 0$

In[40]:=
```
f1[z_] := z^2 Sin[Pi z];
f1'[0]
```

Out[41]= 0

In[42]:=
```
f1''[0]
```

Out[42]= 0

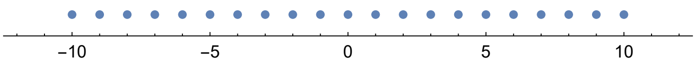
In[43]:=
```
f1'''[0]
```

Out[43]= 6π

f(z) has a pole of order 3 at the point z = 0

In[44]:=
```
NumberLinePlot[Range[-10, 10]]
```

Out[44]=



In[45]:=
```
zeros = ComplexExpand[Reduce[Pi Cos[Pi z] == 0, z]]
```

Out[45]= $c_1 \in \mathbb{Z} \,\&\&\, \left(z == -\frac{1}{2} + 2 c_1 \,||\, z == \frac{1}{2} + 2 c_1\right)$

In[46]:=
```
f2[z_] := Pi Cos[Pi z];
f2'[0]
```

Out[47]= 0

In[48]:=
```
f2''[0]
```

Out[48]= $-\pi^3$

f(z) has a zeros of order 2 at the point z = 0 and we get simple pole at the points z= ±1/2, ±3/2, ±5/2,...

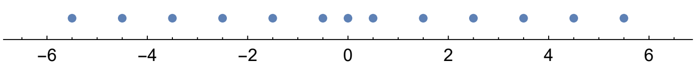
In[49]:=
```
m = Normal[Series[Pi Cos[Pi z], {z, 0, 15}]];
Print["Series expansion of f2[z]=", m, "+..."]
```

Series expansion of f2[z]=

$$\pi - \frac{\pi^3 z^2}{2} + \frac{\pi^5 z^4}{24} - \frac{\pi^7 z^6}{720} + \frac{\pi^9 z^8}{40320} - \frac{\pi^{11} z^{10}}{3628800} + \frac{\pi^{13} z^{12}}{479001600} - \frac{\pi^{15} z^{14}}{87178291200} + \ldots$$

In[51]:=
```
NumberLinePlot[Insert[Table[i, {i, -11, 11, 2}] / 2, 0, (11 + 1) / 2]]
```

Out[51]=



In[52]:=
```
f[z_] := (Pi Cot[Pi z]) / (z^2);
S[z_] := Normal[Series[Pi Cot[Pi z], {z, 0, 15}]];
P = Apart[f[z]];
Print["f[z]=", f[z]]
L = Normal[Series[f[z], {z, 0, 15}]];
Print["Partial Fraction of f[z]=", P]
Plot3D[{Re[f[x + i x y]], Im[f[x + i x y]]}, {x, -2, 2}, {y, -2, 2}]
Print["S[z]= (", S[z], "...)/(", z^2, ")"]
Print["L[z]=", L, "+..."]
```

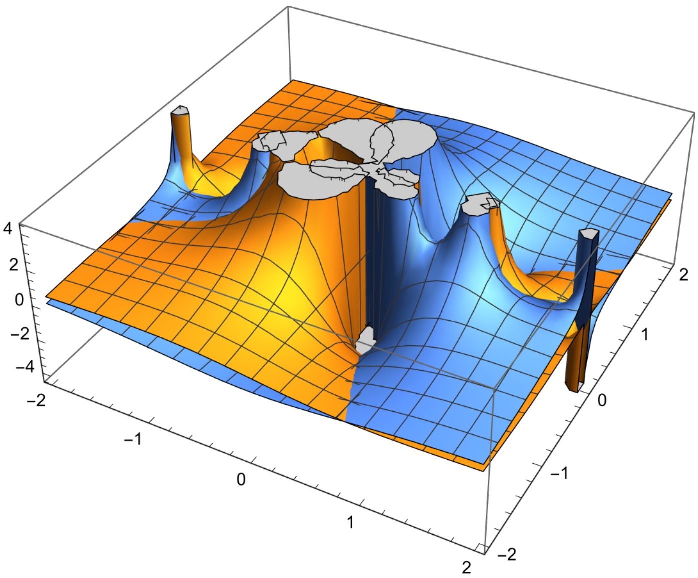
$$\text{f[z]} = \pi \frac{\text{Cot}[\pi z]}{z^2}$$

$$\text{Partial Fraction of f[z]} = \pi \frac{\text{Cot}[\pi z]}{z^2}$$

$$\text{f[z]} = \frac{\pi\,\text{Cot}[\pi z]}{z^2}$$

$$\text{Partial Fraction of f[z]} = \frac{\pi\,\text{Cot}[\pi z]}{z^2}$$

Out[58]=



$$\text{S[z]} = \left(\frac{1}{z} - \frac{\pi^2 z}{3} - \frac{\pi^4 z^3}{45} - \frac{2\pi^6 z^5}{945} - \frac{\pi^8 z^7}{4725} - \frac{2\pi^{10} z^9}{93\,555} - \frac{1382\pi^{12} z^{11}}{638\,512\,875} - \frac{4\pi^{14} z^{13}}{18\,243\,225} - \frac{3617\pi^{16} z^{15}}{162\,820\,783\,125} \ldots\right) / (z^2)$$

$$\text{L[z]} = \frac{1}{z^3} - \frac{\pi^2}{3z} - \frac{\pi^4 z}{45} - \frac{2\pi^6 z^3}{945} - \frac{\pi^8 z^5}{4725} - \frac{2\pi^{10} z^7}{93\,555} - \frac{1382\pi^{12} z^9}{638\,512\,875} - \frac{4\pi^{14} z^{11}}{18\,243\,225} - \frac{3617\pi^{16} z^{13}}{162\,820\,783\,125} - \frac{87\,734\pi^{18} z^{15}}{38\,979\,295\,480\,125} + \ldots$$

Out[61]= $\frac{\pi\,\text{Cot}[\pi z]}{z^2}$

Set: Tag Times in $\frac{\text{Fraction of Partial } (\pi\,\text{Cot}[\pi z])}{z^2}$ is Protected.

Out[62]= $\frac{\pi\,\text{Cot}[\pi z]}{z^2}$

We know that coefficient of 1/z is called the residue of f(z); b_1= $-\pi^2/3$

Another Method:

In[63]:=
```
Residue[f[z], {z, 0}]
```

Out[63]= $-\frac{\pi^2}{3}$

Practical 16

In[64]:=
```
f[z_] = Exp[2 / z];
z[t_] = Exp[I t];
int = ∫ f[z[t]] × z'[t] ⅆt
val = N[∫
0
2 Pi
f[z[t]] × z'[t] ⅆt]

v[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}]
ParametricPlot[Evaluate[v[t]], {t, 0, 2 Pi},
 PlotRange → {{-2, 2}, {-2, 2}}, AxesLabel → {"x", "y"}, PlotStyle → Orange]
```

Out[66]= $i \int e^{2 e^{-i t}+i t} \, dt$

Out[67]= N[∫

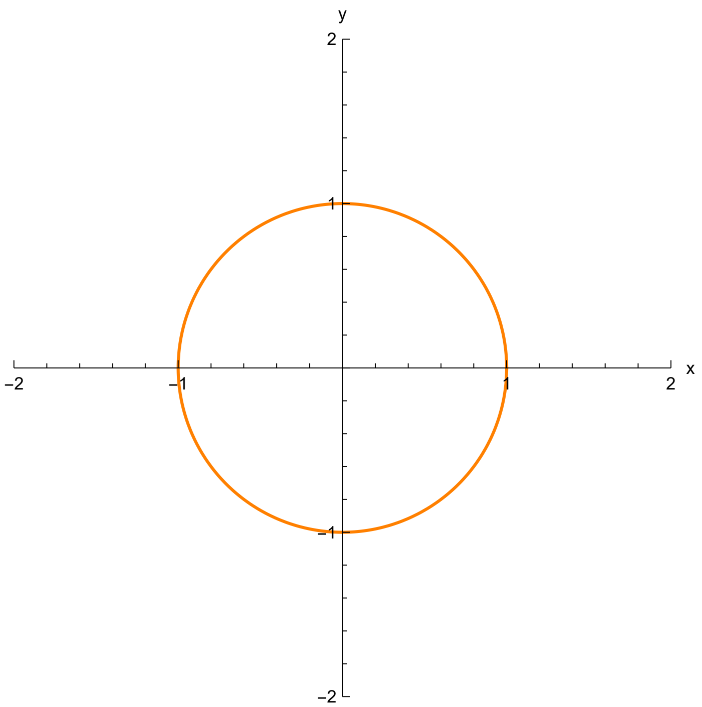
Out[68]= 0

Out[69]= 2π

Out[70]= $i\, e^{2 e^{-i t}+i t}\, dt$]

Out[71]= {Cos[t], Sin[t]}

Out[72]=



Another Method;

In[73]:=
```
f1[z_] = Exp[2 / z];
P[z] = Apart[f1[z]];
L = Normal[Series[f1[z], {z, Infinity, 15}]];
Print["Partial Fraction of f1[z]=", P[z]]
Print["Laurent Series of f1[z]=", "...+", L]
```

••• Set: Tag Times in $\frac{\pi \operatorname{Cot}[\pi z]}{z^2}$[z] is Protected.

Partial Fraction of f1[z]= $\frac{\pi \operatorname{Cot}[\pi z]}{z^2}$[z]

Laurent Series of f1[z]=...+ $1+\frac{16}{638512875 z^{15}}+\frac{8}{42567525 z^{14}}+\frac{8}{6081075 z^{13}}+\frac{4}{467775 z^{12}}+\frac{8}{155925 z^{11}}+\frac{4}{14175 z^{10}}+\frac{4}{2835 z^9}+\frac{2}{315 z^8}+\frac{8}{315 z^7}+\frac{4}{45 z^6}+\frac{4}{15 z^5}+\frac{2}{3 z^4}+\frac{4}{3 z^3}+\frac{2}{z^2}+\frac{2}{z}$

In[78]:=
```
g[z_] := (1 / z^2) f1[1 / z]
Print["g[z]=", g[z]]
Residue[g[z], {z, 0}]
```

g[z] = $\frac{e^{2z}}{z^2}$

g[z]= $\frac{e^{2z}}{z^2}$

Out[80]= 2

Out[81]= $\frac{e^{2z}}{z^2}$

In[82]:= `int1 = 2 Pi i × (2)`

Out[82]= $4 i \pi$

In[83]:= `val = N[4 Pi i ]`

Out[83]= 0. + 12.5664 i

Part 2;

In[84]:=
```
f2[z_] = 1 / (z^4 + z^3 - 2 × z^2);
Print["f2[z]=", f2[z]]
Factor[z^4 + z^3 - 2 × z^2]
```

$$f2[z] = \frac{1}{-2 z^2 + z^3 + z^4}$$

$$f2[z] = \frac{1}{-2 z^2 + z^3 + z^4}$$

Out[86]= $(-1 + z)\, z^2\, (2 + z)$

Out[87]= $\frac{1}{-2 z^2 + z^3 + z^4}$

In[88]:=
```
Int2 = 2 Pi ⅈ × (Residue[f8[z], {z, 0}] + Residue[f8[z], {z, 1}] + Residue[f8[z], {z, -2}])
```

Out[88]= 0