Olivia Pomerenk
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Worksheet 1

We review methods for finding roots of nonlinear functions.

Both this worksheet and its solution (writeup and code) may be found at https://github.com/op554/Numerical_Analysis_2023.

1 Fixed point methods

Let's find roots of the nonlinear equation

$$f(x) := 2x - \tan(x) = 0$$

using the fixed point method.

1.1

Plot $f(x)$ in MATLAB (or Python or Julia, if you prefer) on the domain $[-0.5, 1.3]$.

1. Remember to add a title and axis labels to your plot.
2. Try to "generalize" your code. For example, if we want to plot another function, is it straightforward to edit your code to adapt to the new task?

1.2

1. $f(x)$ has two roots on this domain. What are they (just approximate by eyeballing your plot)?
2. Let's try to find the positive root ξ using fixed point iteration, i.e. $x_{k+1} = g(x_k)$. First, verify that ξ is also a fixed point, i.e. $g(\xi) = \xi$, of these three functions g_i:
   - $g_1(x) = \frac{1}{2}\tan(x)$
   - $g_2(x) = \text{atan}(2x)$
   - $g_3(x) = x - \frac{2x - \tan(x)}{1 - \tan(x)^2}$
3. Implement the fixed point method for these three functions with starting value $x_0 = 1$. What stopping criterion should you use? Record the number of iterations taken for each g_i. Use this to get a "feel" for how each g_i behaves.
4. Based on the above, use one of the g_i to compute a "super accurate" value for ξ. How accurate is your solution (i.e. what's the L1 error, $|2\xi - \tan(\xi)|$)?
5. Discuss the stability of ξ for each choice of g_i using the contraction mapping theorem. Do your results make sense given your numerical experiments?
6. Derive Newton's method for the solution of $f(x) = 0$ and compare with the fixed point method corresponding to g_3. What do you observe?