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T2 ________		F2 ________
T3 ________	Problem Chosen	F3 ________
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2019 Interdisciplinary Contest in Modeling (ICM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Escape the Louvre

Summary

On average, more than twenty thousand visitors come to the Louvre Museum each day to admire the precious works of art within. Unfortunately, the reputation of the Louvre also makes it a potential target for terrorist attacks [4]. Under emergency situations like this, all visitors in the museum have to be evacuated as soon as possible. Thus the optimal evacuation routes need to be identified and applied in different scenarios.

To solve this problem, we start from the details of specific structures in the Louvre, including stairs, rooms, and doors, constructing several microscopic models to examine the basic properties of these building blocks with professional software. We then represent these structures as nodes and connect them with arcs, forming a macroscopic network model in accordance with the floor plans of the Louvre [6].

Based on the network, we carry out computer simulations to explore the possible routes of evacuation and their efficiency. Our prime target is to minimize the evacuation time, but we first use the Floyd-Warshall Algorithm (Dynamic Programming) to find a shortest-path solution as a basic feasible solution. Based on this solution, we use the Genetic Algorithm to optimize the route by trial and error. After about 26 rounds of evolutions (recursions), the 'chromosomes' converges to the optimal solution and is presented with visual tools. We also record the real-time number of visitors on each node throughout the simulation and diagnose the results to identify potential bottlenecks.

We then repeat the procedures above to explore different possible scenarios which may change the optimal solutions. We consider three policies where additional potential exits are opened and one specific situation that a section of the network is cut off. We find that the policy balancing evacuation time and the level of security is to open two additional exits (Porte des Art and Sully 1W). We also estimate by calculation that the cut-off of one specific section (the room presenting Mona Lisa) will lead to a slightly longer optimal time of evacuation.

Due to the limitation of information, some of the problems, such as the handicapped visitors, are not examined in detail in our work; we do, however, discuss how we could solve these problems given additional information. We also give some related suggestions about how the inner structure of the museum could be modified to facilitate the evacuation of the disabled.

Contents

1 Introduction

1.1 Problem Background

The Louvre is one of the largest and most visited museums in the world, attracting tourists around the globe. However, the complicated structure of the Louvre also makes it difficult for visitors to evacuate the museum should any emergency occurs. Thus the museum desires to improve its evacuation capability by model constructing and route finding [3].

Three major problems are discussed in this paper, which are:

- Using available information to convert the inner structure of the Louvre into a mathematical model.
- Identify the optimal evacuation route and analyze the potential bottlenecks.
- Consider different scenarios and explore several available policies.

1.2 Our work

Our work constructs a network model based on the floor plan of the Louvre [6]. Detailed structures the Louvre are simplified into nodes and arcs in a network. We then carry out computer simulation and use Genetic Algorithm to find the routes with minimum evacuation times. Finally, we diagnose the results to identify potential bottlenecks and other limitations. A structure of our work is shown in the chart below.

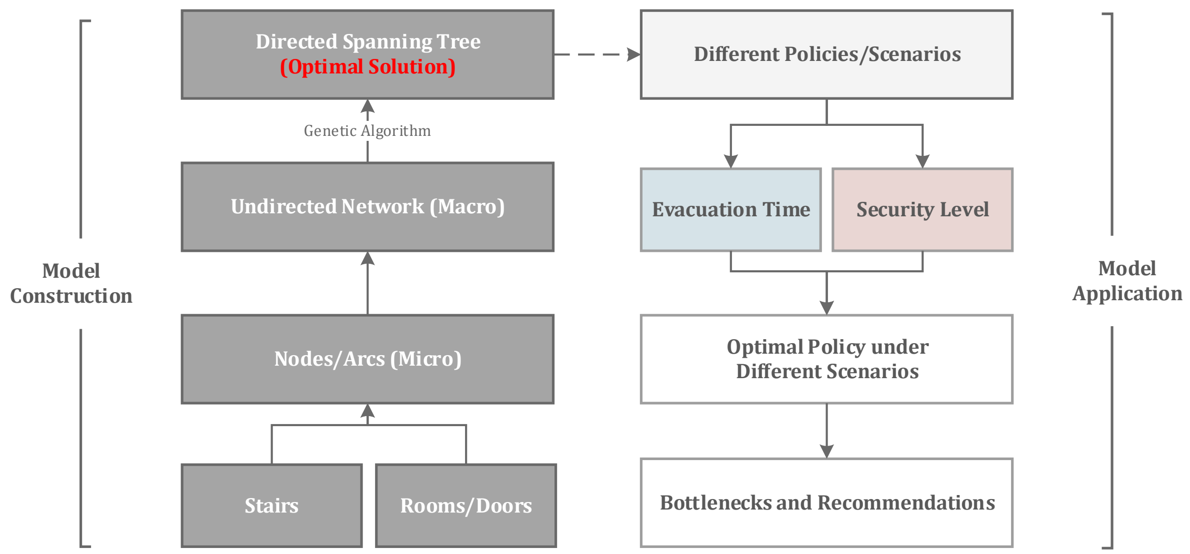

Figure 1: The structure of our work

2 Preparation of the Models

2.1 Basic Assumptions

- All rooms in the Louvre are **square**, **vacant** rooms with **level** grounds. Drawings, statues, pillars, or any other objects placed in any of these rooms have **no** effect on the behavior of the visitors.
- During the emergency, all the lifts (elevators) and escalators are **shut down** due to safety concerns. Visitors are only allowed to use stairs to go to another floor.
- In the evacuation process, all visitors will follow the **designated route** (designed by us) under the guidance of emergency personnel. They are not allowed to choose any alternative evacuation routes or use any unopened exit points.
- A visitors will get out immediately on reaching any of the museum exits. Their behavior after evacuation is not in the scope of our study.
- All 'visitors' refers to people who can walk on themselves, unless otherwise stated.

2.2 Notations

The primary notations used in this paper are listed in **Table 3**.

Table 1: Notations

Symbol	Definition
$O_{(i)}$	the set of all exit nodes, i = 1,2,3.
$S_{(i)}$	the set of all non-exit nodes, i = 1,2,3...
$E_{(n)}$	the set of all nodes in the Louvre; $E = O \bigcup S$.
$A_{(k)}$	the set of all available arcs between two nodes, k = 1,2,3.
$V = \{S, C\}$	a directed graph with sources S and targets C
$H_{(n)}$	the set of maximum visitor capacities of the Node n.
$K_{(n)}$	the set of real-time number of visitors on the Node n.
μ_{ij}	the entering rate from Node i to Node j
t_0	the beginning of the evacuation. $t_0 = 0$
t	time elapsed from the beginning of the evacuation.
n_{in}	the total number of visitors needing evacuation $n_{in} = \Sigma K_i$.
n_{out}	the number of visitors evacuated at time t.
T	the value of t when $n_{out}/n_{in} = 100\%$ for the first time

3 The Microscopic Models - Properties of the Nodes and Arcs

3.1 Simplified Models of Staircases

As is assumed above, all the escalators and lifts will be unavailable in emergency situations. Consequently, the staircases will become the only connections between different floors of the palace, making them crucial parts in the evacuation model.

We deem there to be two main types of staircases in the Louvre: the normal stairs and the "double" stairs. These two types of staircases are modeled respectively as follows.

3.1.1 Normal Stairs

The "normal" stairs refers to the typical staircases, with two endpoints on two adjacent floors and a number of steps connecting them. Although there are varies shapes of normal stairs in the Louvre - rectangular ones, curved ones, spiral ones, etc. - the underlying model is more or less the same: the escaping visitors using them will have to go down stairs from the upper endpoint to the lower one, simultaneously making a right or left turn.

Figure 2: Pictures of a rectangular staircase and a spiral staircase in the Louvre

Source: https://www.louvre.fr/

To simplify our model, here we model a typical rectangular staircase based on the specifications of the Louvre's inner structure, and set it as the default staircase.

Then we use the Pathfinder software to simulate the speed and capacity of one such staircase. After several test simulations, we find out that one such staircase can sustain

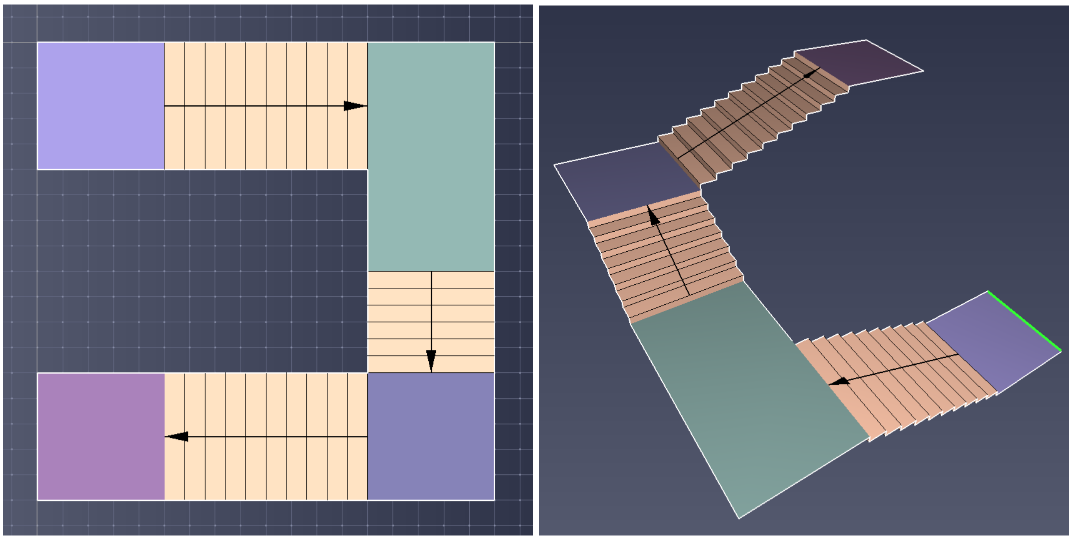

Figure 3: A top view and a perspective view of the normal stairs model

up to 43 people without being too crowded.

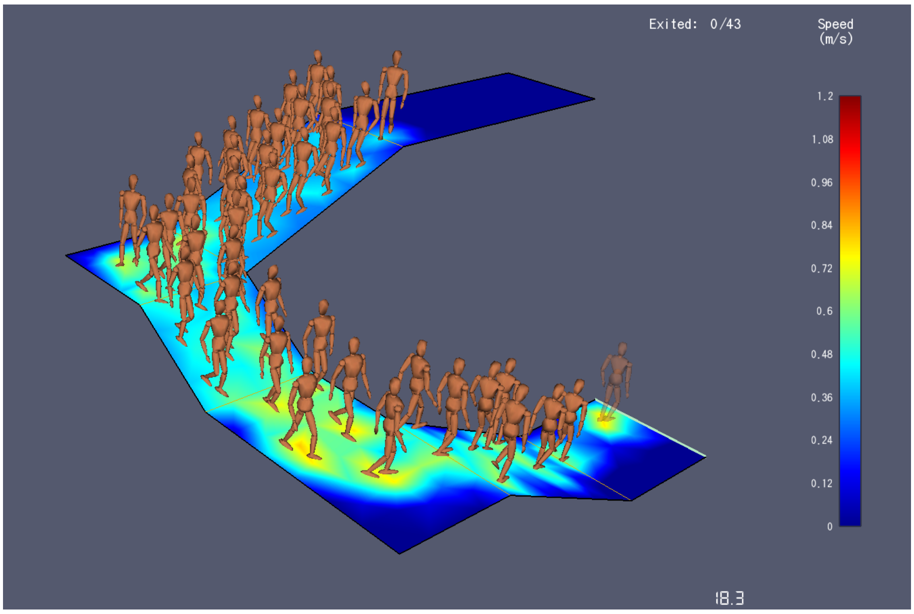

Figure 4: A simulation of normal stairs

$$\mu = \frac{\Delta K_i}{\Delta t} \tag{1}$$

Where ΔK_i represents the change in real-time population. Using an extracted period from the Pathfinder simulation, the rate is calculated to be 1.69 persons per second.

3.1.2 Double Stairs

The "double" stairs refers to a staircase with two endpoints on the upper floor converging to one endpoint on the lower floor. The structure of this type of staircase resembles a bird: two narrow "wings" and one broad "body".

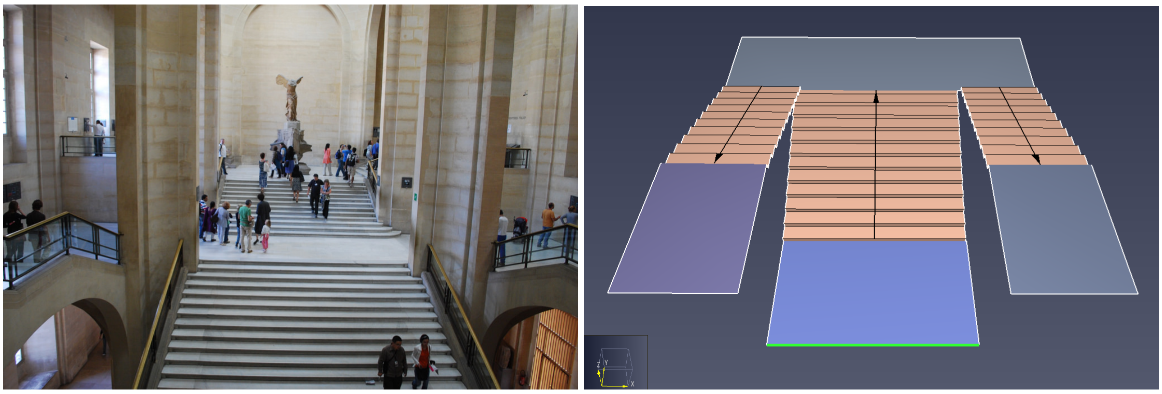

Figure 5: A picture of the Daru staircase in the Louvre, and a perspective view of our model
`http://albertis-window.com/2013/05/museum-shrines-and-performative-rituals/`

Again, we model a typical staircase with CAD tools and simulate the model with pathfinder. We find that the maximum capacity of one such staircase is around 80. The average passing rate is calculated to be 3.54 visitors per second.

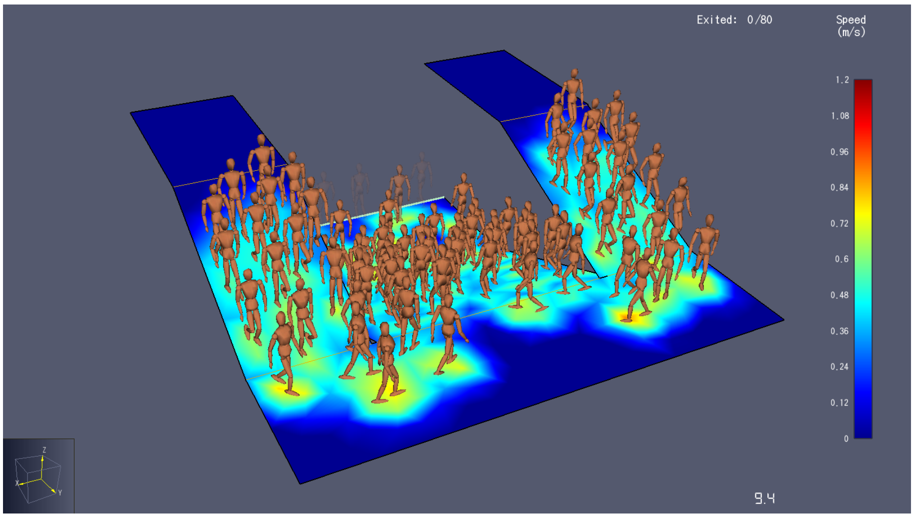

Figure 6: A simulation of double stairs

3.1.3 Representation of stairs models in networks

Based on 3.1.1 and 3.1.2, we then convert the stairs models above into nodes and arcs that can be used to construct a network:

1. The endpoints of a staircase is represented by two nodes, one on the upper floor and the other on then lower floor.
2. The stairs are represented by an arc connecting these two nodes
3. Each node has a maximum capacity of H_i. Arcs has no capacities. When the real-time number of visitors on the staircase node reaches H_i ($K_i = H_i$), visitors from the adjacent nodes could not enter this node ($\mu_{xi} = 0$).
4. The visitor on this node i can move to any other node S_y at a constant rate of unless that node has reached its maximum capacity ($K_y = H_y$)

According the simulations and calculations above, values of H and μ are given as below

Table 2: Properties of Stairs

Type of Staircase	Max.Capacity ($H/persons$)	Passing Rate ($\mu/person/second$)
$Normal$	43	1.69
$Double$	80	3.54

A detailed visual representation of stairs will be demonstrated in Section 4.

3.2 Simplified Models of Doors and Rooms

As there are more than 400 rooms and halls in the Louvre, it is virtually impossible and also unnecessary to simulate the situation in every single room, for most of the rooms will have little effect on the plan of evacuation.

Figure 7: A picture of a typical room in the Louvre

Source: `https://www.louvre.fr/`

3.2.1 Simulation of the door model

To demonstrate our idea, we create a model with three rooms. Room 1 (on the left side) has only one door (Denoted Door 1); Room 2 has two doors (Door 1 and Door 2); and Room 3 has one door and one exit (Door 2 and Door 3). All people are randomly distributed in the first room, initially. All people have to escape the series of rooms from the door on the right of the third room. All the doors are designed to be 96 centimeters in width, the most typical spec used in European buildings.

We still use Pathfinder to simulate the evacuation process. The behavior of 100 people are demonstrated as follows.

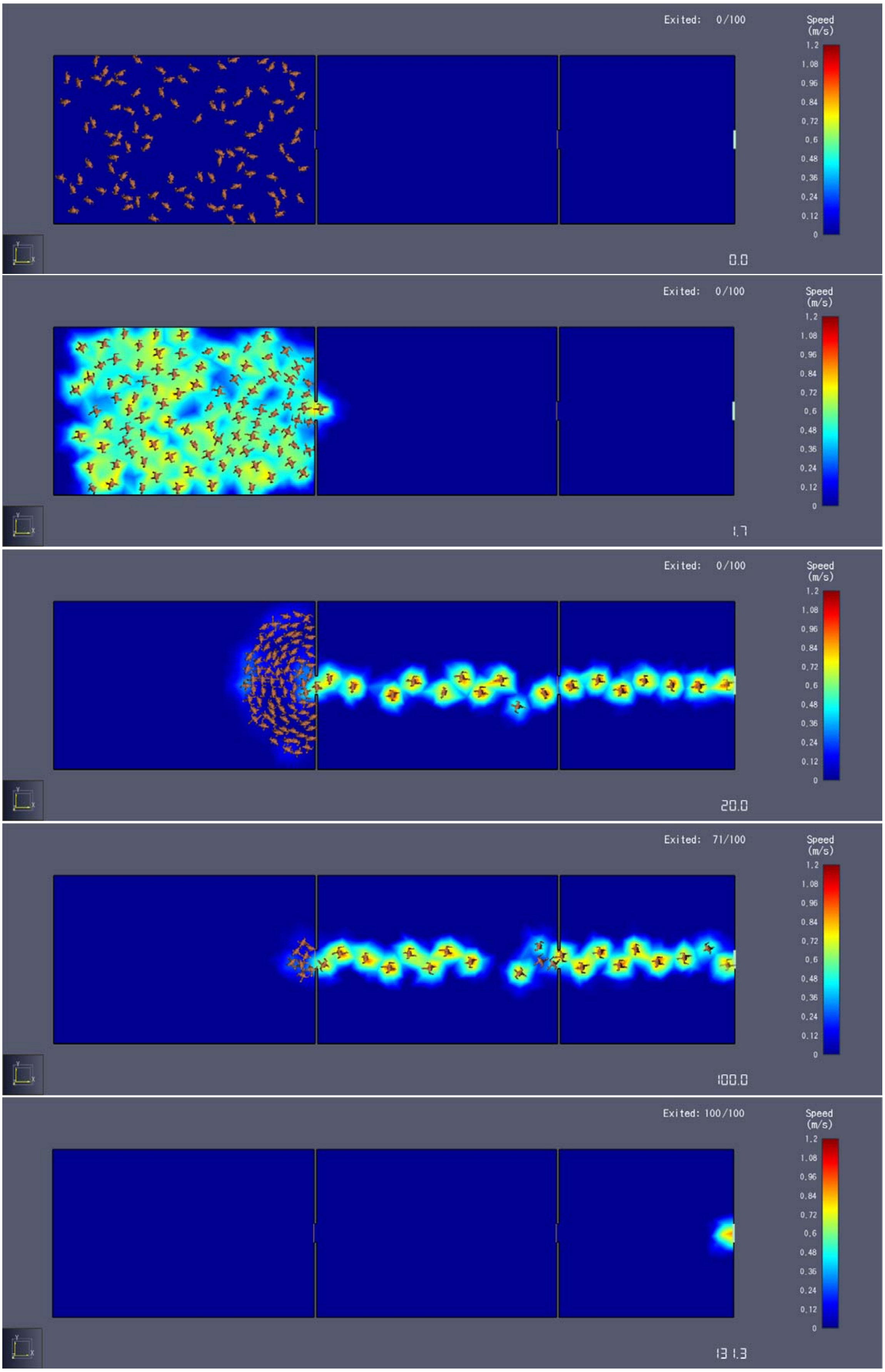

Figure 8: Extracted sections from the simulation of the room model (at time t = 0.0, 1.7, 20.0, 100.0, 131.3, respectively)

We can observe from the simulation that:

- All 100 people, initially randomly distributed in Room 1, almost immediately clusters to the door.
- At time t = 1.7, which is quite soon, one of the visitors has already thoroughly gone through Door 1, Entering Room 2.
- The flow of people in Room 2 (a room with two doors) is continuous and approximately uniform (keeps the same speed from the beginning to the last exit). This substantiates our assumption that the flow speed is unaffected by the density in the source room.
- Room 3, which can also be seen as a room with two doors, demonstrates a flow pattern identical to the second room. Also, There is little queuing nor congestion at Door 2 (the door between Room 2 and Room 3). As a result, We deem it acceptable to ignore Door 2 and consider Room 2 and Room 3 as a single, longer room.
- The size a room and the population in the room (or the population density in a room) has no or little effect on the speed of going through doors and rooms - We have tried cramming up to 300 people in Room 1, and the flowing pattern in Room 2 and 3 are almost the same.

3.2.2 Simplification of the door model in networks

Based on the observations above, we can make the following simplifications to the inner structure of the Louvre. We assume that all the doors in the Louvre are identical, 96 centimeters in width, and that there can be at most one door connecting two rooms:

- The doors are the main limitations of visitor flowing speed. The rooms are simply the space between the doors.
- The transferring rate from one door to another is solely determined by the property of the door, unless the visitors are trying to enter a room that is already full.

3.2.3 Representation of doors and rooms in networks

1. A room with only one door is represented with a node connected to the nearest other room.
2. A series of rooms, each with two doors, can be seen as a single, longer room (or corridor). It is then represented by a single arc.
3. A room with three or more doors is represented as a node. However, it is worth noticing that only two or less of its doors will ever be used for evacuation purposes (one in and another one out, at most).

From the simulation above, we have the following properties

Table 3: Properties of Doors and Rooms

Max.Capacity-Room ($H/persons$)	Passing Rate-Door ($\mu/person/second$)
200	0.88

4 The Macroscopic Model - A Network of the Louvre

4.1 Network Construction

With the rules introduced in the previous section, we can simplify the whole inner structure of the Louvre using nodes and arcs. A network is built based on the official floor plan of the Louvre [6]. The nodes are named from 1 to 58. **Note that** the -2F consists only of one node, so it is combined into the graph of -1F.

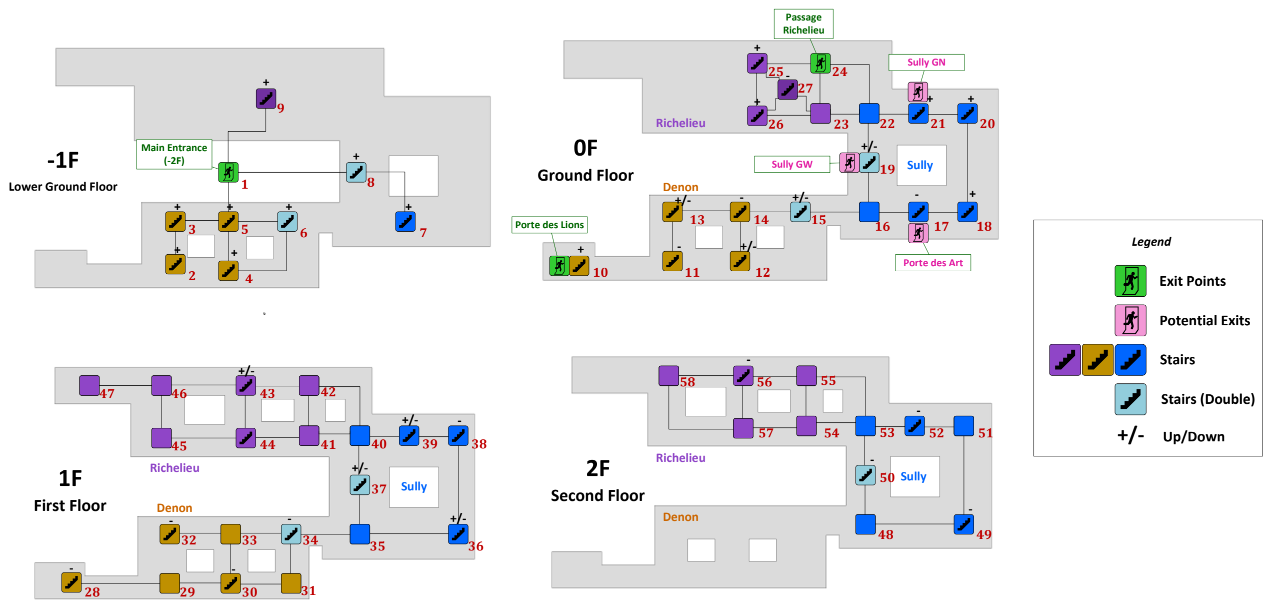

Figure 9: The network representation of the inner structure of the Louvre

4.2 Network Properties

We shall reiterate some of the most important properties of the network

1. Each node has a maximum capacity of H_i. Arcs has no capacities. When the real-time number of visitors on the staircase node reaches H_i ($K_i = H_i$), visitors from the adjacent nodes could not enter this node ($\mu_{xi} = 0$).

2. The visitor on this node i can move to any other node S_y at a constant rate of μ unless that node has reached its maximum capacity ($K_y = H_y$) I

3. The capacity of an exit is defined to be infinity. it has no transferring rate because no one will try to leave that node.

This network can also be seen as a graph with **undirected** edges, which can be stored as in MATLAB for future uses.

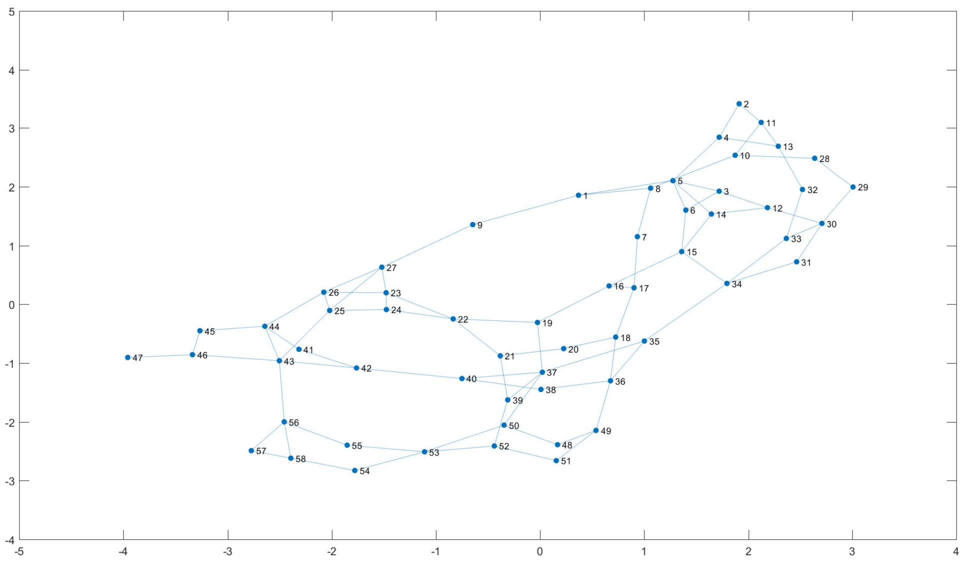

Figure 10: An undirected graph representing our network, genenrated with MATLAB

5 The Model of the Number of Visitors

According to the data provided by the museum website, the number of visitors to the Louvre in 2017 is 8.6 million. Since the regular opening time of the Louvre is from 9 am to 18 pm, and that the extended hours for special dates are ignored in this model, it is estimated that the average number of visitors 23562 per day or 2618 per hour, rounding to the nearest integer. It is further distributed with the opening time interval considering peak hours [5]. We assume that terror attacks are most likely to happen during the peak hours due to the motivation of terrorists and the inter-arrival time of visitors follow an exponential distribution [1]. Therefore, it can be inferred that the number of visitors per hour follow a Poisson distribution [2].

The generation for a random number of real-time visitors could be achieved by the following steps:

1. Since we assume that inter-arrival time is exponential with $1/\lambda$, the random number of time can be produced by the method of inverse transformation method and random number uniformed distributed from 0 to 1.

$$t = -\frac{1}{\lambda} \ln u_i, u_i \sim U(0, 1) \tag{2}$$

2. For the Poisson distribution, n visitors could appear when time t satisfies $t_n \leq t < t_{n+1}$. Then we produce random number with

$$\begin{cases} \prod_{i=1}^{n} u_i \geq e^{-\lambda t} > \prod_{i=1}^{n+1} u_i, n > 0 \\ 1 \geq e^{-\lambda t} > u_1, n = 0 \end{cases} \tag{3}$$

3. Thus getting

.

$$n = \min \left\{ n : \prod_{i=1}^{n} u_i < e^{-\lambda t} \right\} - 1 \tag{4}$$

With that, we generate the initial number of visitors in each room K_{i0}, with $\lambda = 2618/50 = 52.36$

6 The Optimal Solution of Evacuation Routes

After importing our parameters into MATLAB, we set out to find the optimal route.

6.1 Definition of a Feasible Solution

One feasible solution, or a route, is a directed graph with 58 nodes and 55 directed arcs. Each of the 55 non-exit node is the source of one and only one directed arc, while the exit nodes are not the source of any arcs.

In the mathematical language, a route V is defined by

$$V = \{S, C\}, S = [2, 3, 4, 5, ..., 58], (S_i, C_i) \in G$$

where **G** is the undirected graph of the network.

6.2 A Basic Solution with the Floyd-Warshall (DP) Algorithm

To set a benchmark and make further simulations faster to converge, we first find a basic feasible solution to the route problem, looking for a directed graph converging to the exit nodes.

We use the Floyd-Warshall Algorithm, a dynamic programming method, to compare the shortest route of each node to any of the three exit, eventually getting a shortest path tree.

This result almost certainly not the optimal solution, for the shorter path is not equivalent to faster evacuation. It can, however, set a benchmark for further simulations. As we find that the basic feasible solution takes 748.6 seconds to finish evacuation, we decide that **all** the following simulations lasts at most 1000 seconds: any simulation failing to finish evacuation within 1000 second are considered 'infeasible' routes. It also provides many parameters that can be later used in the probability matrices of the Genetic Algorithm.

As it is not the final solution we are looking for, the result is not visually presented.

6.3 Optimization based on the Genetic Algorithm

To perform the Genetic Algorithm (GA), we make the following definitions:

Chromosome - C , the target vector

Fitness Function - Defined by $F = (1000 - T)$, T is the evacuation time, and $T = 1000$ if an algorithm is 'infeasible'.

Probability of Crossover - a matrix based on the importance of the nodes

Probability of Mutation - a matrix based on a node's distance from the nearest exit node. Specifically, C_i adjacent to an exit node is not allowed to mutate if node S_i is adjacent to an exit node.

Options of Mutation - $\{C_i \| (S_i, C_i) \in G\}$

Heuristics - Similarity is not punished in our model.

The GA typically start with random generated genes as the first generation. On applying this strategy to this problem, however, we find that the algorithm converges rather slowly: most of the randomly generated genes are unable to evacuate within 1000 seconds, making the genetic population very difficult to evolve. As a result, we decided to import the basic solution and its mutations as the first generation. We generate 50 chromosomes in each recursion, and perform 40 recursions. Below is a result showing the evacuation time of each chromosome generated.

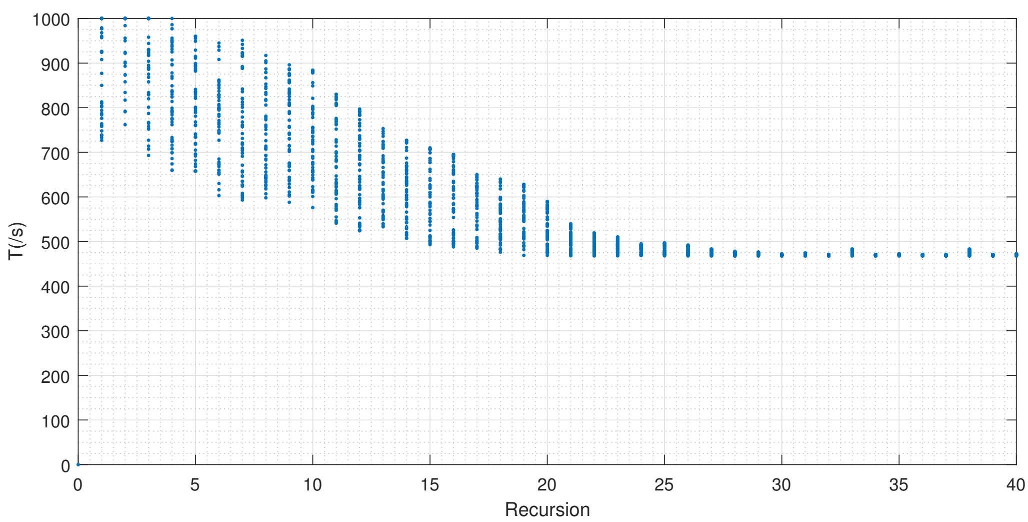

Figure 11: The result of 40 recursions

We conclude that the optimal evacuation time is 468.2 second

The optimized route is demonstrated as below

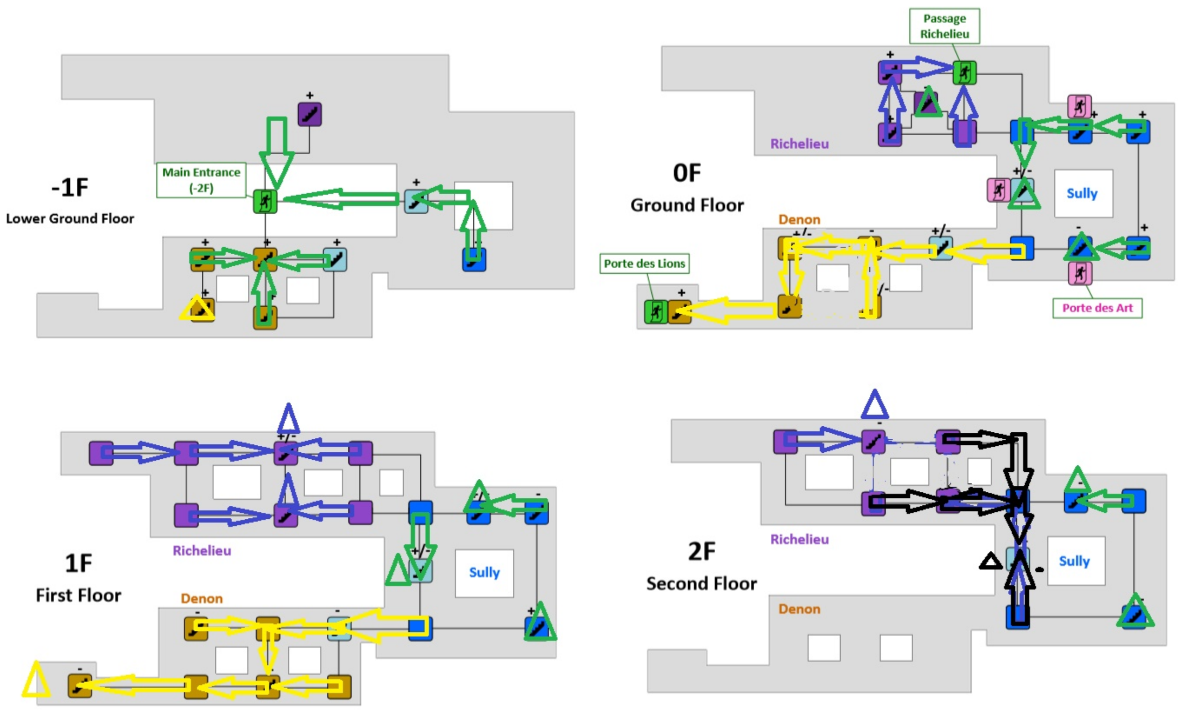

Figure 12: The optimized evacuation route when opening three main exits

6.4 Results and Diagnostics (Bottlenecks)

Campared with the basic feasible solution, the optimized route is apparently much more efficient in evacuating visitors. As a result, our algorithm is validated.

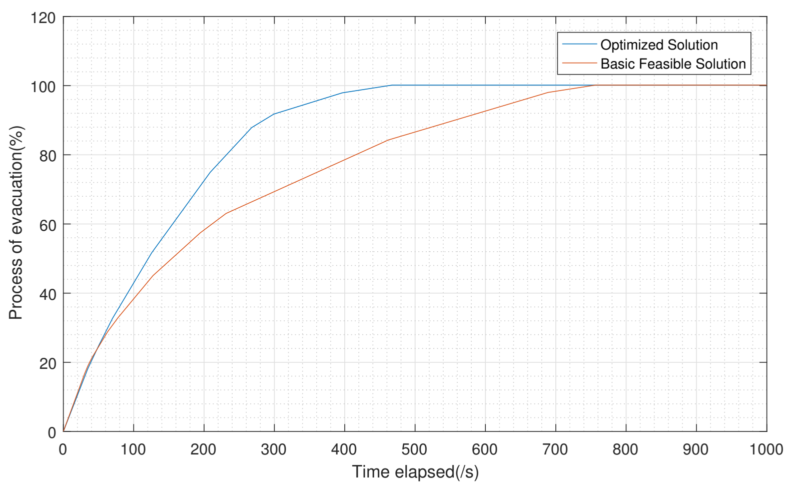

Figure 13: The optimized evacuation route when opening three main exits

For the identification of bottlenecks, we store and plot the real-time numbers on each node throughout the simulation:

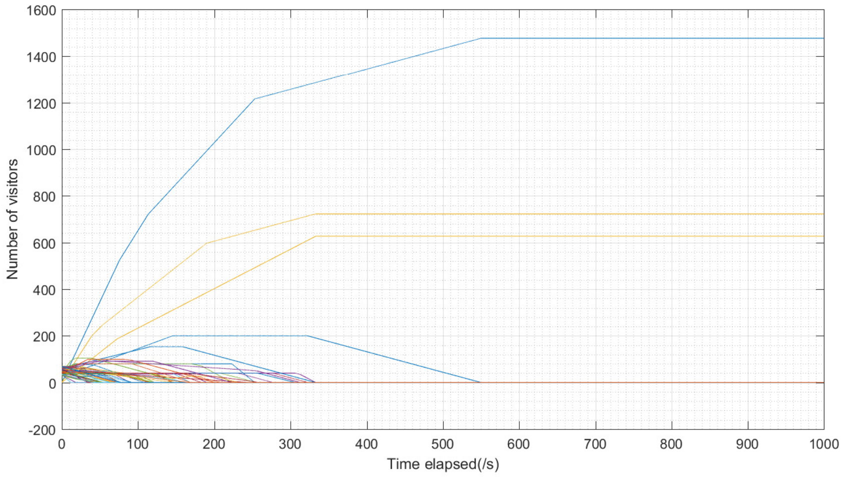

Figure 14: The optimized evacuation route when opening three main exits

According to the figure, the only room node ever reaching its capacity limit is $node22$ (The intersection between the Sully Wing and the Richelieu Wing on Ground Floor). It is full for around 3 minutes throughout the evacuation, making it a bottleneck in out model.

If the museum would like to keep the evacuation process based on this route in order, it is recommended that deploy more emergency personnel on that node to prevent any potential danger caused by crowding.

7 Application of the Model in Different Scenarios

7.1 Additional Doors

So far we have only used the three main exits: The Main Entrance on -2F (Directing to Carrousel or the Pyramid), Porte des Lions, and Passage Richelieu. However, three additional exits - Porte des Art and two more (here named Sully GW and Sully GN) can be opened if necessary. We now inspect the policies of opening additional doors.

The three scenario are as follows:

1. Policy 1: Open one additional exit - Porte Des Art (Node 17)
2. Policy 2: Open two additional exits - Porte Des Art and Sully GW (Node 17, Node 19)
3. Policy 3: Open three additional exits - Porte Des Art, Sully GW, and Sully GN (Node 17, Node 19, Node 21)

For each of the scenarios, we repeat the procedure in Section 6, using the Genetic Algorithm to find the corresponding route with the fastest time of evacuation.

The comparison between evacuation times are shown as below

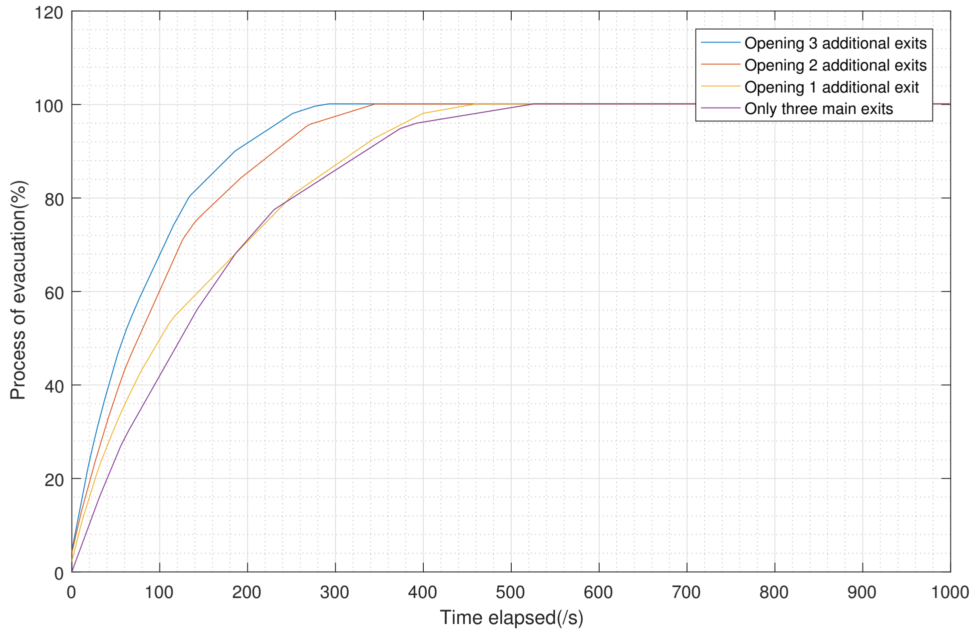

Figure 15: Comparisons between the optimal speeds when additional exits are opened

As can be seen from the figure above, opening two more doors is a balance point between the evacuation efficiency and the level of security. It much faster than opening one or no additional doors, while not so susceptible to art theft compared to the policy of opening three additional doors, for the third door (Sully GN, Node 21) leads directly to the street of Rue de Rivoli, making it easier for art thieves to get away. So we decide that opening two doors is an optimal policy if the museum find it necessary to open more exits.

A detailed evacuation route is given below.

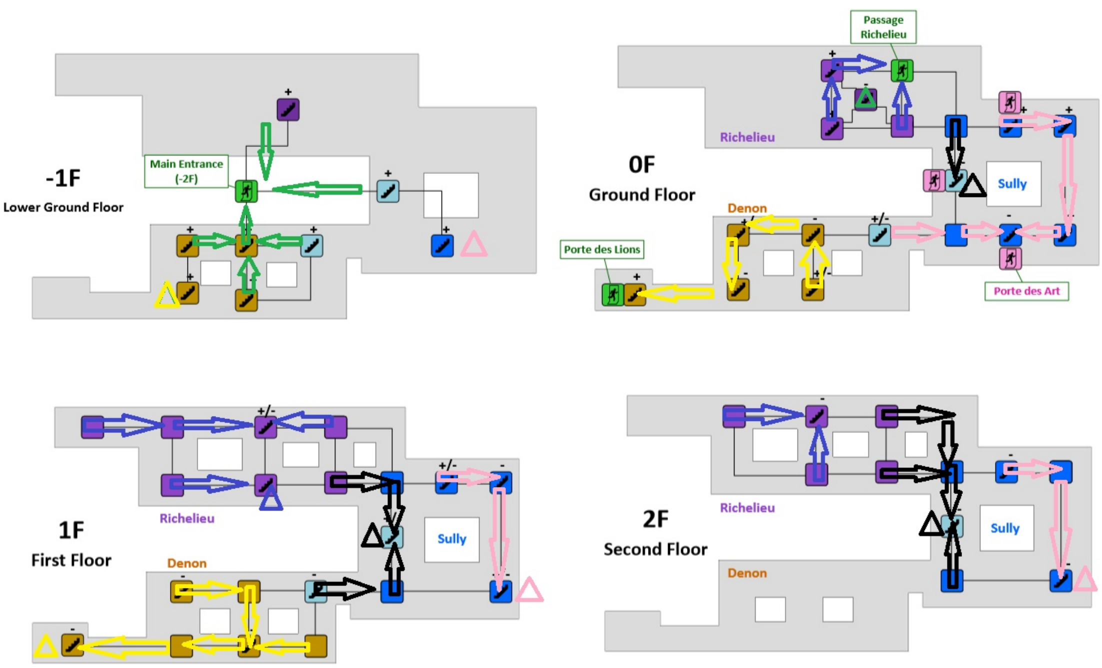

Figure 16: Comparisons between the optimal speeds when additional exits are opened

Now we diagnose the room usage situation under this route.

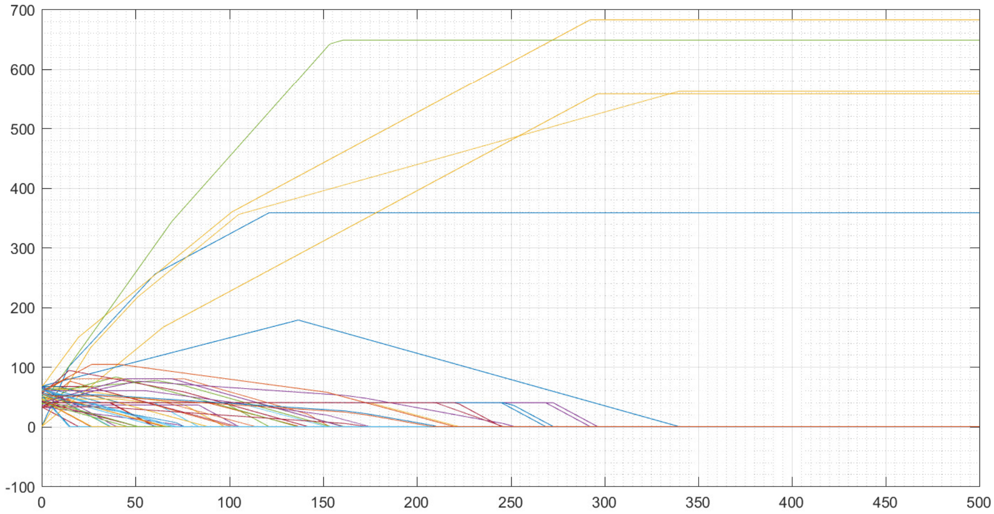

Figure 17: Room usage when two additional exits are opened

The bottlenecks are largely alleviated under this policy, and no room is ever fully occupied. This is the main reason why the evacuation is so much more efficient. For most of the situations, we will recommend the museum to open two additional doors because it is much faster than just opening main exits. A shorter evacuation time means a higher possibility saving invaluable lives. However, if the emergency is not very serious, the museum may only open main exits with higher security levels to prevent art theft.

7.2 One Section is Cut off

Now let us consider the following scenario.

A terrorist has successfully fooled the securities and carries a bomb into the Louvre. For he wants to make a sensation, he decided that the best place to set off the bomb is in front of Mona Lisa, the famous painting, located in 1F, Denon wing. The blast destroys Node 30, killing all visitors on the node and making the node inaccessible for evacuation purposes. The evacuation of people in other rooms begins immediately for fear of further attacks.

Under this circumstance, we consider the policies of only opening main exits and opening two additional exits, respectively. We create new routes under different policies, and compare them to the normal ones (those planned without explosion)

As we can see from the graph (Figure 18),

1. Under the same policy, the cut-off of Node 30 slows down the evacuation process; this effect, however, is not very insignificant.
2. Opening two more exits can still significantly reduce the evacuation time so we recommend this plan.

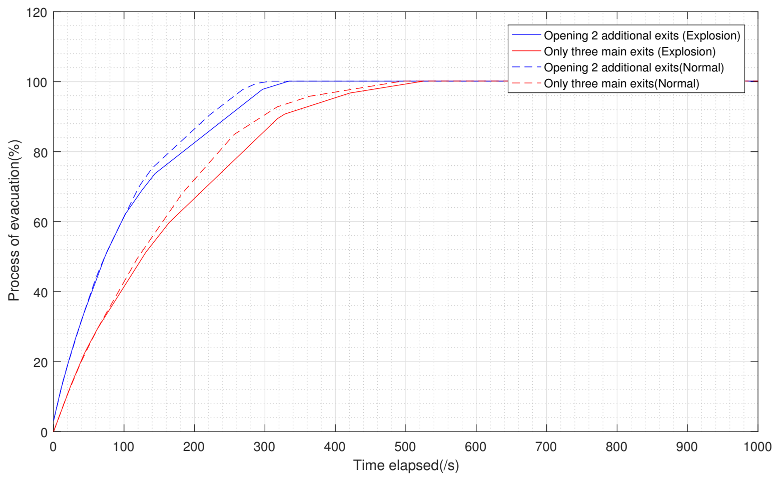

Figure 18: Comparion between different policies and different scenarios

The optimal route after the 'Mona Lisa' explosion, with the policy of opening two additional exits is given as below.

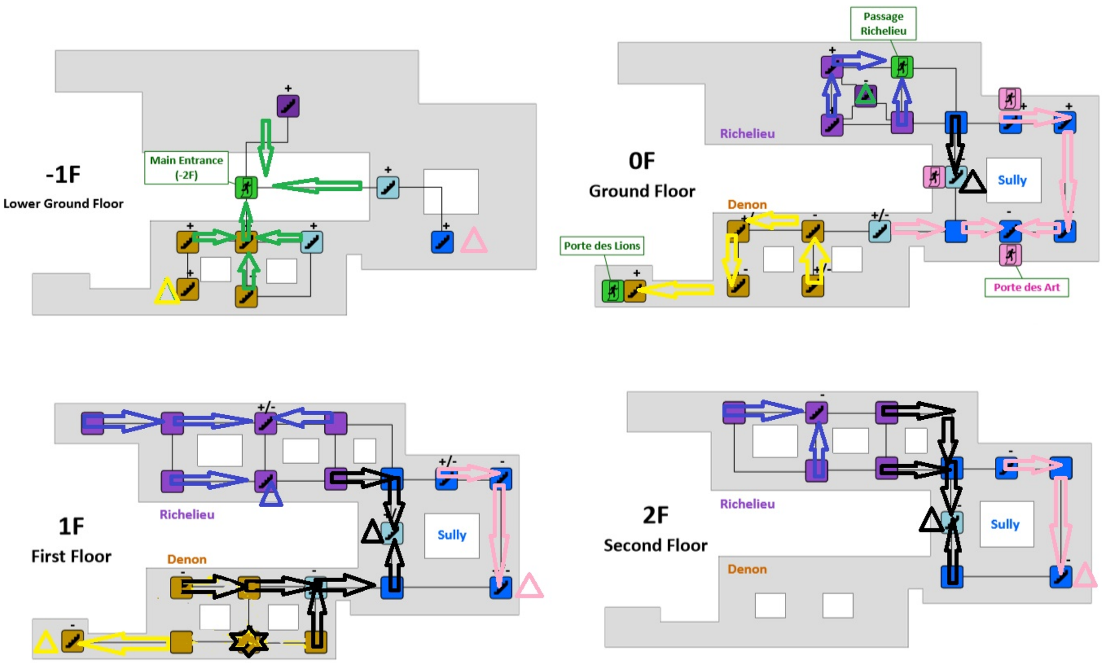

Figure 19: Evacuation Route after 'Mona Lisa' explosion, two additional exits

8 Implementation of our model

Because the optimal evacuation routes changes in different scenarios, we recommend the Louvre officials to place several LED plates in each room of the museum. Each LED plate should be able display patterns of different directions. These plates will light up only during evacuation, indicating the desired moving directions for the visitors. It is important that these plates be powered with backup batteries so that they can work even if the regular power supply is cut off.

9 Brief Discussions about Several Other Concerns

9.1 Handicapped Visitors

We fail to take the handicapped visitors into account due to a contradiction to our assumption. As far as we can infer from the map, people on wheel chairs can only move from one floor to another using disabled lifts. However, as we assume that all lifts are shut down due to safety concerns, it is impossible for the disabled on higher floors to evacuate. It is unreasonable that a public museum like this do not offer slopes for wheelchairs - maybe they are just not marked on the map. If there really are no wheelchair slopes, we highly recommend the museum to add such slopes among 0F, 1F, and 2F so that the disabled visitors on higher floors will be able to evacuate.

9.2 Deployment of emergency personnel

As a number of emergency personnel are regularly deployed in the museum, more emergency personnel should only be deployed while not affecting the speed of evacuation. We can work out a plan based our analysis of room usage - rooms and exits without many visitors can be used to deploy emergency personnel.

9.3 Implementing our model to other buidings

Our model is quite versatile. The rules of simplification can easily be applied to other large, crowded structures if detailed maps were given. However, the properties of nodes and arcs should be modified based on the specifications of stairs and doors in other buildings.

10 Strengths and Weaknesses

10.1 Strengths

- Our model is versatile and able to adjust to different scenarios based on the real situation.
- The running speed of our programs are fast (taking tens of seconds to calculate with a microcomputer), so the emergency personnel can quickly work out the optimal route on occurrence of an emergency.

10.2 Weaknesses

- The models of stairs and doors may be oversimplified. they may not reflect the actual properties of the structures.

- Some of the model parameters are based on approximation and not very accurate.
- Our main model fails to take handicapped visitors into account.

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