W7 4th Hour with TA (Opal)

How to get started with Python? A guide to installation and setting up a virtual enviroment: https://www.python.org/about/gettingstarted/

In [1]:
```
# import essential Python packages 'numpy' and 'scipy'
import numpy as np
import scipy
```

In [2]:
```
# import plotting package 'matplotlib' in Python
import matplotlib.pyplot as plt
import matplotlib

# set up preferred fontsize etc...
font = {'family' : 'serif',
        'size'   : 12}

matplotlib.rc('font', **font)
matplotlib.rc('xtick', labelsize=12)
matplotlib.rc('ytick', labelsize=12)
```

Fourier Series and Fourier Transform

Week 5-7 lecture notes discussed the **continuous** $x(t)$ formulation of the Fourier series and Fourier transform. In weeks 8-10, we will discuss the **discrete** $x[n]$ analog. As expected, the discrete and continuous formulations match at the *limit of infinitely fine data* resolution.

The two-dimensional continuous Fourier transform of $x(t_1, t_2)$ is defined as

$$X(\omega_1, \omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t_1, t_2) e^{-i(\omega_1 t_1 + \omega_2 t_2)} \mathrm{d}t_1 \mathrm{d}t_2$$

The discrete Fourier transform (DFT) of $x[n, m]$ is defined as

$$X[\omega_1, \omega_2] = \frac{1}{M}\frac{1}{N}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x[n, m] e^{-2\pi i (\frac{m\omega_1}{M} + \frac{\omega_2 n}{N})}$$

which is the discrete version of the Fourier series approximation (since it also assumes the periodicity of the signal). More on this in the proceeding weeks!

Discussion questions:

(1) What is more memory intensive (a) the original image or (b) its discrete Fourier transform? Why?

(2) What is the computational complexity of computing the discrete Fourier transform? Why?

Problem (HW6-P5)

Read into Python/MATLAB the two grayscale images from the canvas page **marilyn.png** and **robert.png**. Before proceeding, please normalize the two images such that the matrix values range from 0 to 1.

(a) Plot the magnitude and phase of the discrete Fourier transform of the two images.

In [3]:
```
# read in the images
marilyn= plt.imread("marilyn.png")
robert = plt.imread("robert.png")
```

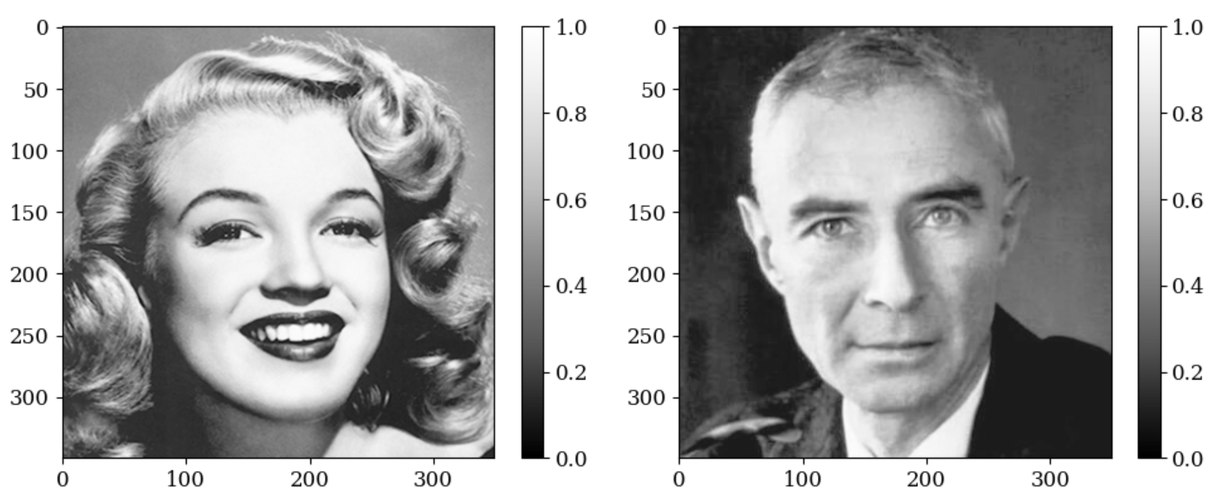
In [4]:
```
# normalize both images to be in the range of [0, 1]
marilyn = marilyn / np.max(marilyn)
robert =  robert / np.max(robert)
```

In [5]:
```
# verify the two photos are of the same size
print("Marilyn photo is shape = ", marilyn.shape)
print("Robert photo is shape = ", robert.shape)
```

```
Marilyn photo is shape =  (350, 350)
Robert photo is shape =  (350, 350)
```

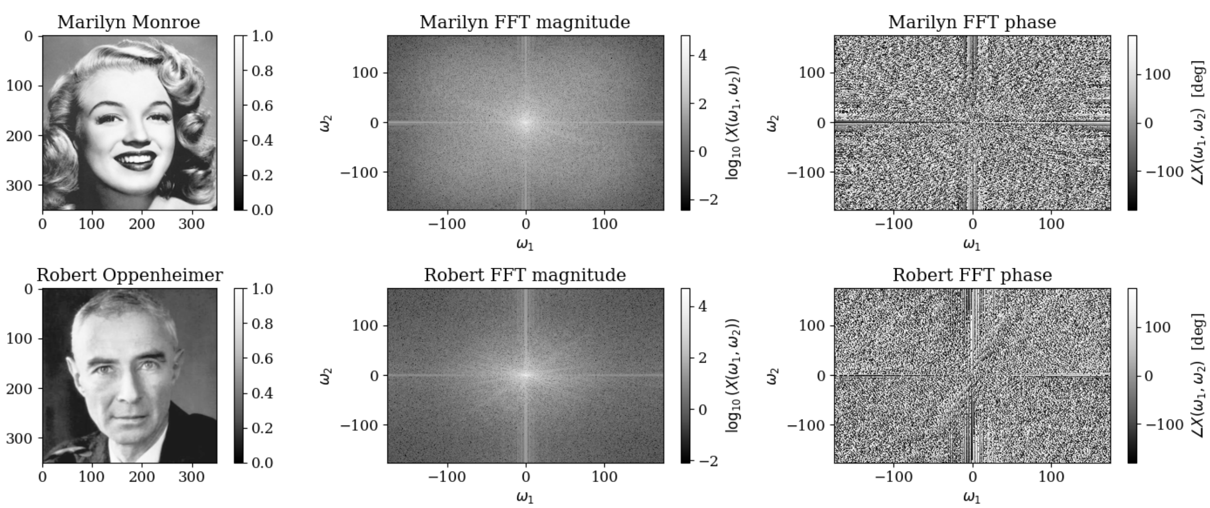
In [6]:
```
# dimension of the squared images
N = robert.shape[0]
```

In [7]:
```
# plot the two images side by side
fig, ax = plt.subplots(ncols=2, figsize=(10, 4))
pos = ax[0].imshow(marilyn, cmap="Greys_r", vmax=1, vmin=0)
fig.colorbar(pos, ax=ax[0])
pos = ax[1].imshow(robert, cmap="Greys_r", vmax=1, vmin=0)
fig.colorbar(pos, ax=ax[1])
plt.tight_layout()
```



In [8]:
```
# compute the 2D DFT of both images using the Fast Fourier Transform (FFT)
FFT_marilyn = np.fft.fft2(marilyn)
FFT_robert = np.fft.fft2(robert)
```

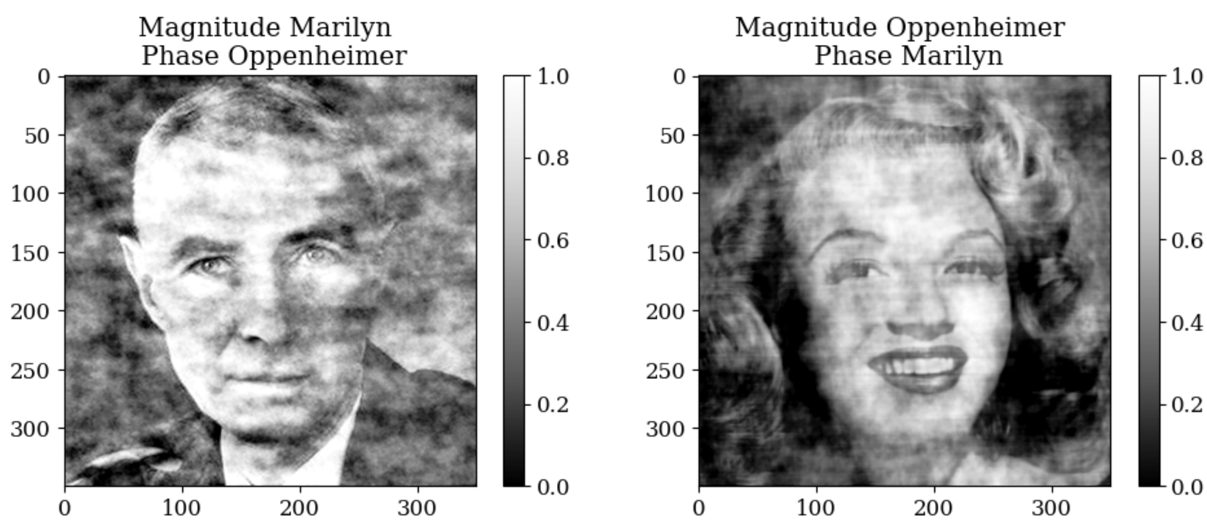
In [9]:
```
# plot the magnitude and phase of the FFT results
fig, ax = plt.subplots(ncols=3, nrows=2, figsize=(15, 6))
pos = ax[0, 0].imshow(marilyn, cmap="Greys_r", vmin=0, vmax=1)
fig.colorbar(pos, ax=ax[0, 0])
ax[0, 0].set_title("Marilyn Monroe")
pos = ax[0, 1].pcolormesh(np.arange(-N//2, N//2), np.arange(-N//2, N//2), np
fig.colorbar(pos, ax=ax[0, 1], label=r"$\log_{10}(X(\omega_{1}, \omega_{2}))
ax[0, 1].set_xlabel("$\omega_{1}$")
ax[0, 1].set_ylabel("$\omega_{2}$")
ax[0, 1].set_title("Marilyn FFT magnitude")
pos = ax[0, 2].pcolormesh(np.arange(-N//2, N//2), np.arange(-N//2, N//2), np
fig.colorbar(pos, ax=ax[0, 2], label=r"$\angle X(\omega_{1}, \omega_{2})$  [
ax[0, 2].set_title("Marilyn FFT phase")
ax[0, 2].set_xlabel("$\omega_{1}$")
ax[0, 2].set_ylabel("$\omega_{2}$")

pos = ax[1, 0].imshow(robert, cmap="Greys_r", vmin=0, vmax=1)
fig.colorbar(pos, ax=ax[1, 0])
ax[1, 0].set_title("Robert Oppenheimer")
pos = ax[1, 1].pcolormesh(np.arange(-N//2, N//2), np.arange(-N//2, N//2), np
fig.colorbar(pos, ax=ax[1, 1], label="$\log_{10}(X(\omega_{1}, \omega_{2}))$
ax[1, 1].set_title("Robert FFT magnitude")
ax[1, 1].set_xlabel("$\omega_{1}$")
ax[1, 1].set_ylabel("$\omega_{2}$")

pos = ax[1, 2].pcolormesh(np.arange(-N//2, N//2), np.arange(-N//2, N//2), np
fig.colorbar(pos, ax=ax[1, 2], label=r"$\angle X(\omega_{1}, \omega_{2})$  [
ax[1, 2].set_title("Robert FFT phase")
ax[1, 2].set_xlabel("$\omega_{1}$")
ax[1, 2].set_ylabel("$\omega_{2}$")

plt.tight_layout()
plt.savefig("fig_a.png")
```



(b) Mix the two images in the following manner: (1) plot the magnitude of **marilyn.png** with the phase of **robert.png** and (2) vice versa. What can you learn from your observations?

In [10]:
```
# reconstruct mixing between the photos
# magntitude Marilyn Monroe and phase of J. Robert Oppenheimer
mm_pr = np.fft.ifft2(np.abs(FFT_marilyn) * np.exp(1j * np.angle(FFT_robert)))
# magnitude of J. Robert Oppenheimer and phase Marilyn Monroe
mr_pm = np.fft.ifft2(np.abs(FFT_robert) * np.exp(1j * np.angle(FFT_marilyn)))
```

In [11]:
```
# plot the mixed images
fig, ax = plt.subplots(ncols=2, figsize=(10, 4))
pos = ax[0].imshow(mm_pr.real, cmap="Greys_r", vmin=0, vmax=1)
fig.colorbar(pos, ax=ax[0])
ax[0].set_title("Magnitude Marilyn \n Phase Oppenheimer")
pos = ax[1].imshow(mr_pm.real, cmap="Greys_r", vmin=0, vmax=1)
fig.colorbar(pos, ax=ax[1])
ax[1].set_title("Magnitude Oppenheimer \n Phase Marilyn")
plt.tight_layout()
```

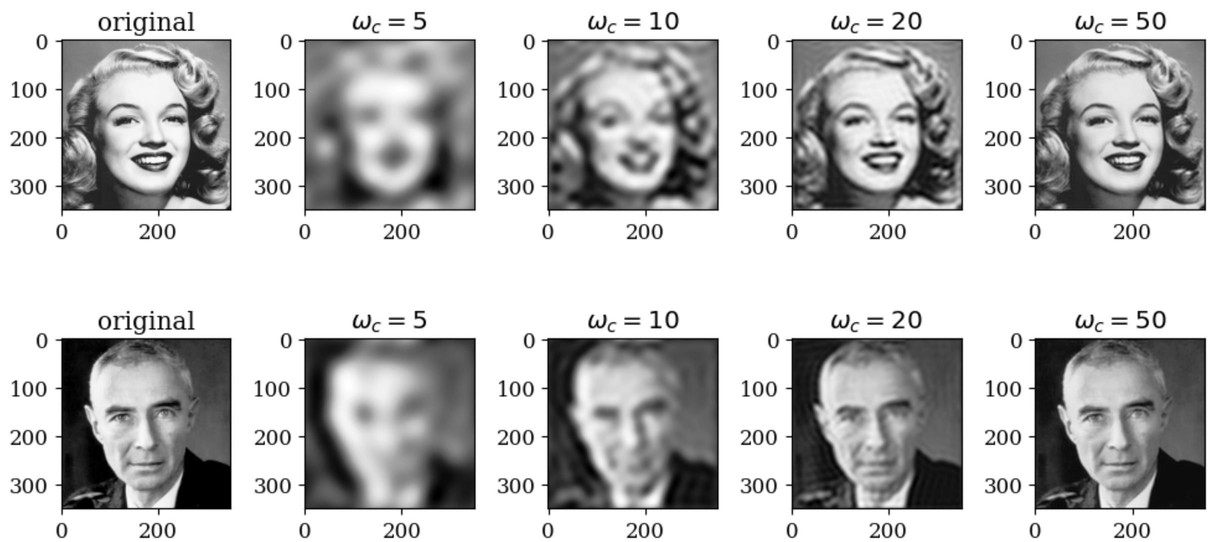


(c) Use an ideal low-pass filter with ωc = 5, 10, 20, 50. Plot the filtered images for both **marilyn.png** and **robert.png**. Analyze/discuss your results.

In [12]:
```
# define a function that is an 'ideal' low-pass filter
def image_low_pass_filter(image_fft, n):
    # Apply a low-pass filter
    N = np.shape(image_fft)[0]
    FFT_lp = np.zeros((N, N), dtype="complex128")
    FFT_lp[N//2-n: N//2+n, N//2-n:N//2+n] = np.fft.fftshift(image_fft)[N//2-
    return np.fft.ifft2(np.fft.ifftshift(FFT_lp))
```

In [13]:
```
fig, ax = plt.subplots(ncols=5, nrows=2, figsize=(10, 5))
ax[0, 0].imshow(marilyn, cmap="Greys_r")
ax[0, 0].set_title("original")
ax[0, 1].imshow(image_low_pass_filter(FFT_marilyn, n=5).real, cmap="Greys_r"
ax[0, 1].set_title("$\omega_{c}=5$")
ax[0, 2].imshow(image_low_pass_filter(FFT_marilyn, n=10).real, cmap="Greys_r
```

```
ax[0, 2].set_title("$\omega_{c}=10$")
ax[0, 3].imshow(image_low_pass_filter(FFT_marilyn, n=20).real, cmap="Greys_r
ax[0, 3].set_title("$\omega_{c}=20$")
ax[0, 4].imshow(image_low_pass_filter(FFT_marilyn, n=50).real, cmap="Greys_r
ax[0, 4].set_title("$\omega_{c}=50$")

ax[1, 0].imshow(robert, cmap="Greys_r")
ax[1, 0].set_title("original")
ax[1, 1].imshow(image_low_pass_filter(FFT_robert, n=5).real, cmap="Greys_r")
ax[1, 1].set_title("$\omega_{c}=5$")
ax[1, 2].imshow(image_low_pass_filter(FFT_robert, n=10).real, cmap="Greys_r"
ax[1, 2].set_title("$\omega_{c}=10$")
ax[1, 3].imshow(image_low_pass_filter(FFT_robert, n=20).real, cmap="Greys_r"
ax[1, 3].set_title("$\omega_{c}=20$")
ax[1, 4].imshow(image_low_pass_filter(FFT_robert, n=50).real, cmap="Greys_r"
ax[1, 4].set_title("$\omega_{c}=50$")
plt.tight_layout()
```

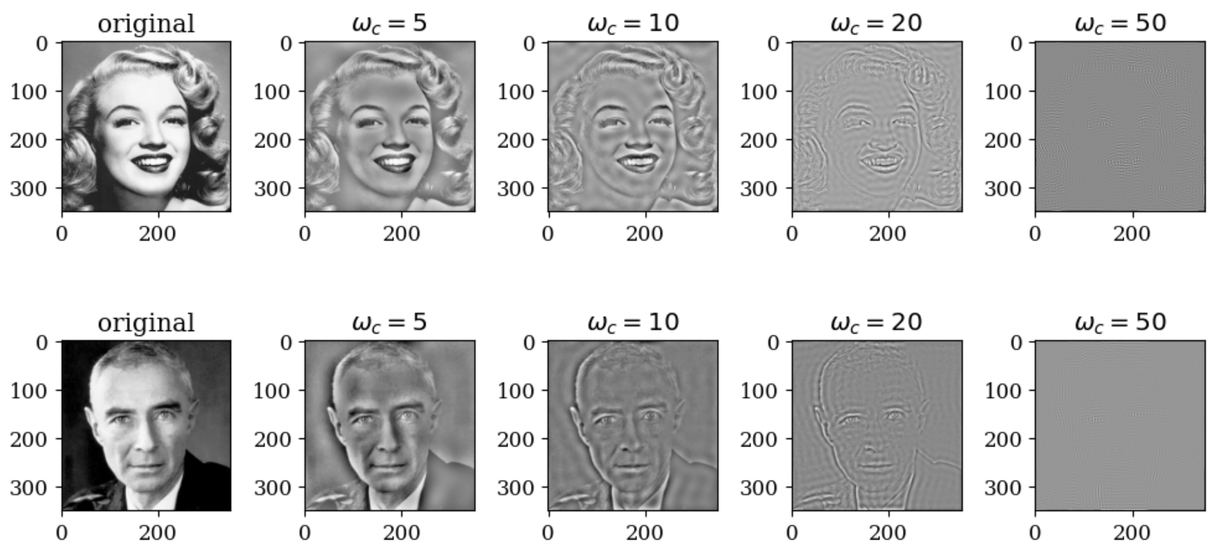


(d) Use an ideal high-pass filter with ωc = 160, 165, 170, 172. Plot the filtered images for both **marilyn.png** and **robert.png**. Analyze/discuss your results

In [14]:
```
# define a function that is an 'ideal' high-pass filter
def image_high_pass_filter(image_fft, n):
    # Apply a high-pass filter
    N = np.shape(image_fft)[0]
    FFT_hp = np.copy(np.fft.fftshift(image_fft))
    FFT_hp[N//2-n: N//2+n, N//2-n:N//2+n] = np.zeros((2*n, 2*n), dtype="comp
    return np.fft.ifft2(np.fft.ifftshift(FFT_hp))
```

In [15]:
```
fig, ax = plt.subplots(ncols=5, nrows=2, figsize=(10, 5))
ax[0, 0].imshow(marilyn, cmap="Greys_r")
ax[0, 0].set_title("original")
ax[0, 1].imshow(image_high_pass_filter(FFT_marilyn, n=5).real, cmap="Greys_r
ax[0, 1].set_title("$\omega_{c}=5$")
ax[0, 2].imshow(image_high_pass_filter(FFT_marilyn, n=10).real, cmap="Greys_
ax[0, 2].set_title("$\omega_{c}=10$")
ax[0, 3].imshow(image_high_pass_filter(FFT_marilyn, n=20).real, cmap="Greys_
ax[0, 3].set_title("$\omega_{c}=20$")
```

```
ax[0, 4].imshow(image_high_pass_filter(FFT_marilyn, n=50).real, cmap="Greys_
ax[0, 4].set_title("$\omega_{c}=50$")

ax[1, 0].imshow(robert, cmap="Greys_r")
ax[1, 0].set_title("original")
ax[1, 1].imshow(image_high_pass_filter(FFT_robert, n=5).real, cmap="Greys_r"
ax[1, 1].set_title("$\omega_{c}=5$")
ax[1, 2].imshow(image_high_pass_filter(FFT_robert, n=10).real, cmap="Greys_r
ax[1, 2].set_title("$\omega_{c}=10$")
ax[1, 3].imshow(image_high_pass_filter(FFT_robert, n=20).real, cmap="Greys_r
ax[1, 3].set_title("$\omega_{c}=20$")
ax[1, 4].imshow(image_high_pass_filter(FFT_robert, n=50).real, cmap="Greys_r
ax[1, 4].set_title("$\omega_{c}=50$")
plt.tight_layout()
```



(e) Convolve using conv2(A, B, 'same') in MATLAB or scipy.signal.convolve2d(in1, in2, mode="same") in Python the figures **marilyn.png** and **robert.png** with the following kernels:

$$K_1 = \text{ones}((10, 10)), K_2 = K_2 = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, K_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

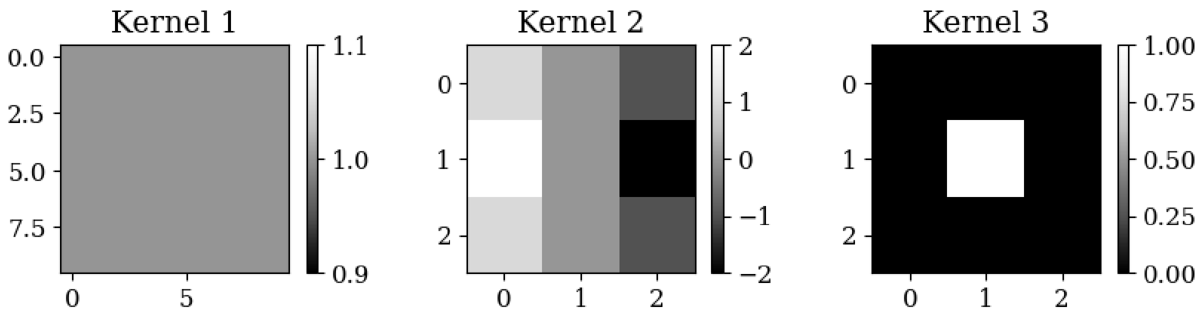
In [16]:

```
# define kernels
kernel_1 = np.ones((10, 10))
kernel_2 = np.array(([1, 0, -1], [2, 0, -2], [1, 0, -1]))
kernel_3 = np.array(([0, 0, 0], [0, 1, 0], [0, 0, 0]))
```

In [17]:

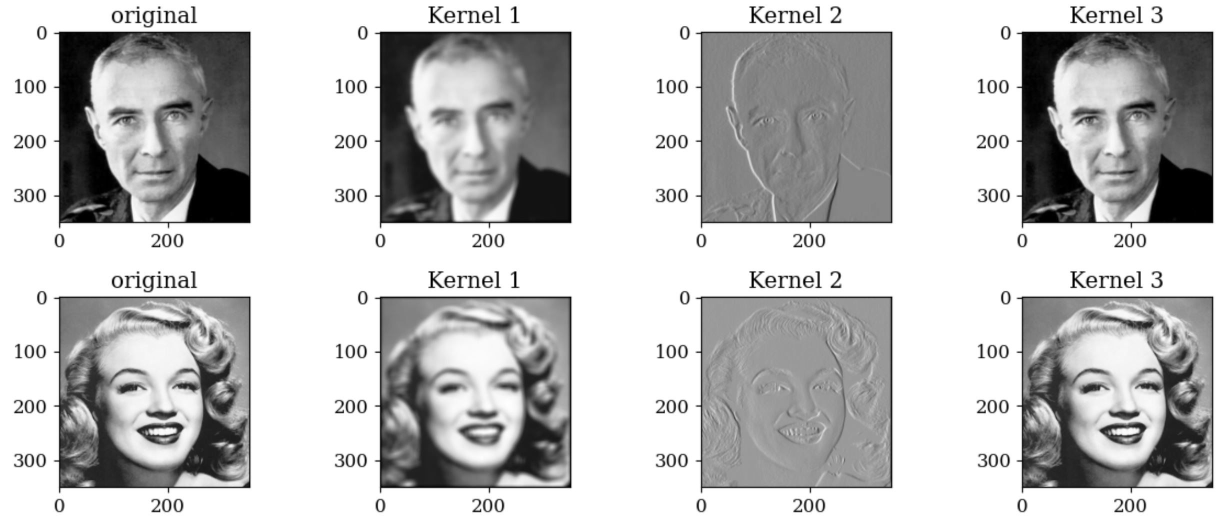
```
fig, ax = plt.subplots(ncols =3, figsize=(10, 2))
pos = ax[0].imshow(kernel_1, cmap="Greys_r")
fig.colorbar(pos, ax=ax[0])
ax[0].set_title("Kernel 1")

pos = ax[1].imshow(kernel_2, cmap="Greys_r")
fig.colorbar(pos, ax=ax[1])
ax[1].set_title("Kernel 2")
```

```
pos = ax[2].imshow(kernel_3, cmap="Greys_r", vmin=0, vmax=1)
fig.colorbar(pos, ax=ax[2])
_ = ax[2].set_title("Kernel 3")
#plt.tight_layout()
```



In [18]:

```
fig, ax = plt.subplots(ncols=4, nrows=2, figsize=(12, 5))

pos = ax[0, 0].imshow(robert, cmap="Greys_r")
ax[0, 0].set_title(r"original")

pos = ax[0, 1].imshow(scipy.signal.convolve2d(robert, kernel_1, mode="same")
ax[0, 1].set_title(r"Kernel 1")

pos = ax[0, 2].imshow(scipy.signal.convolve2d(robert, kernel_2, mode="same")
ax[0, 2].set_title(r"Kernel 2")

pos = ax[0, 3].imshow(scipy.signal.convolve2d(robert, kernel_3, mode="same")
ax[0, 3].set_title(r"Kernel 3")

pos = ax[1, 0].imshow(marilyn, cmap="Greys_r")
ax[1, 0].set_title(r"original")

pos = ax[1, 1].imshow(scipy.signal.convolve2d(marilyn, kernel_1, mode="same"
ax[1, 1].set_title(r"Kernel 1")

pos = ax[1, 2].imshow(scipy.signal.convolve2d(marilyn, kernel_2, mode="same"
ax[1, 2].set_title(r"Kernel 2")

pos = ax[1, 3].imshow(scipy.signal.convolve2d(marilyn, kernel_3, mode="same"
ax[1, 3].set_title(r"Kernel 3")
plt.tight_layout()
```


original
Kernel 1
Kernel 2
Kernel 3
0
100
200
300
0
200
original
Kernel 1
Kernel 2
Kernel 3

In [ ]:

In [ ]:

In [ ]: