Predicting solar wind streams from the inner-heliosphere to Earth via shifted operator inference[1]

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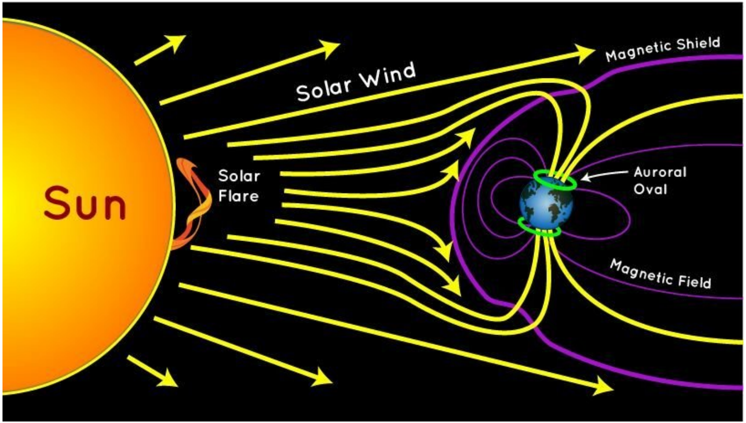
What is the solar wind?

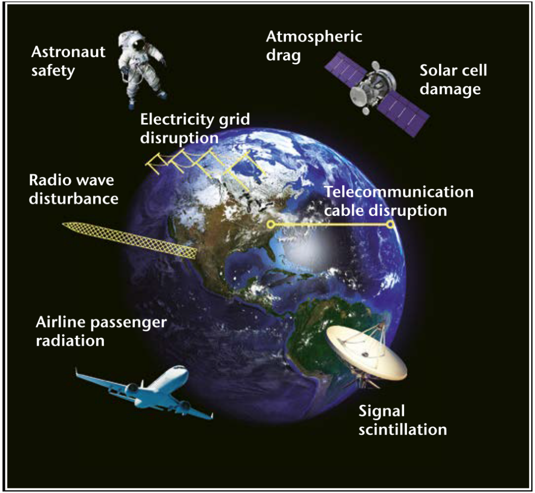
The solar wind consists of a continuous flow of charged particles that can escape the Sun's gravity due to high temperatures in the Sun's corona (2 million degrees Fahrenheit).



Credit: NASA.

Impacts of high-speed solar wind streams on Earth

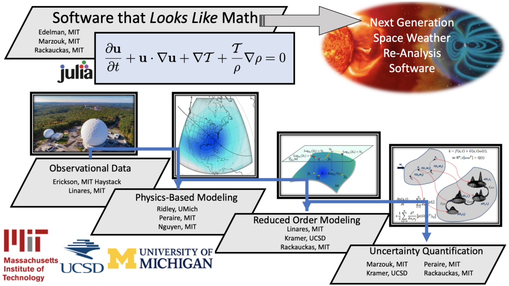
- Expose astronauts to an extreme amount of radiation
- Increase risk of low Earth orbit satellite collisions
- Disrupt telecommunication: GPS and aviation navigation
- Electric power outage via geomagnetically induced currents
- Economical impacts: Up to 2 trillion USD in damage and 4–10 years for the society to recover [Council, 2008]



Credit: NASA.

Challenges of solar wind modeling

- High-fidelity models, e.g., Magnetohydrodynamic (MHD), require **computationally intensive** simulations.
- **Reduced-order models (ROMs)** are computationally efficient approximations that can speed up MHD by several orders of magnitude.
- ROMs can be utilized for **real-time predictions** and for enabling ensemble methods used in **uncertainty quantification**.
- **Today:** New method to learn a computationally efficient ROM from steady-state MHD Around a Sphere (MAS) simulated data [Linker et al., 1999].



NSF Award for "SWQU: Next Generation Software Framework for Space Weather Data Assimilation and Uncertainty Quantification"

Magnetohydrodynamics Around a Sphere (MAS) model

MAS model developed by [Riley et al., 2012] solves a system of 3D time-dependent resistive MHD equations in spherical coordinates (r, θ, ϕ):

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\mathbf{E} + \frac{1}{c}\boldsymbol{v}\times\mathbf{B} = \eta\mathbf{J} \tag{3}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \tag{4}$$

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v}\right) = \frac{1}{c}\mathbf{J}\times\mathbf{B} - \nabla(p + p_w) + \rho\mathbf{g} + \nabla\cdot(\nu\rho\nabla\boldsymbol{v}) \tag{5}$$

$$\frac{1}{\gamma - 1}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}\cdot\nabla T\right) = -T\nabla\cdot\boldsymbol{v} + S(T) \tag{6}$$

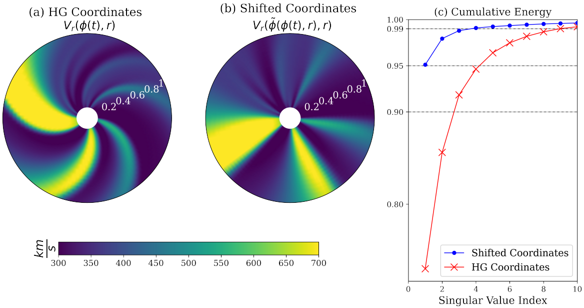
Variable	Physical Quantity
$\mathbf{B} = [B_r, B_\theta, B_\phi]$	Magnetic Field
$\mathbf{J} = [J_r, J_\theta, J_\phi]$	Current Density
$\mathbf{E} = [E_r, E_\theta, E_\phi]$	Electric Field
ρ	Plasma Density
$\boldsymbol{v} = [v_r, v_\theta, v_\phi]$	Plasma Velocity
T	Plasma Temperature
p	Plasma Pressure
p_w	Wave Pressure

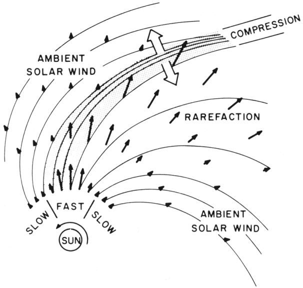
$\Rightarrow$ **MAS model requires computationally intensive high-dimensional simulations with dof =** $n_v \times n_\phi \times n_\theta \times n_r = 16 \times 128 \times 111 \times 140 \approx 31 \times 10^6$**.**

Specific challenges for solar wind ROMs

Standard ROMs via projection onto low-dimensional linear subspaces require new ROM approaches for advection-dominated systems since

1. require a high number of modes/interpolation points
2. low predictive capabilities



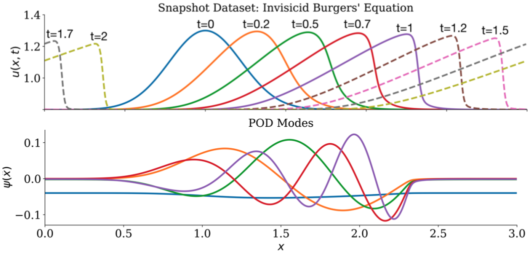


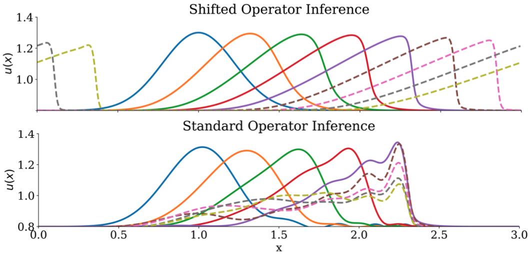
Credit: [Pizzo, 1978].

Challenges of advection-dominated projection-based ROMs

Low predictive performance & requires large amount of modes

- Linear projection-based methods require the solutions at each time-step to be comprised by a linear combination of the POD modes, i.e. $u(x,t) = \sum_{i=1}^{\ell} a_i(t)\psi_i(x)$
- For advection-dominated systems, the snapshots are linearly independent $\Rightarrow$ slow decay of singular values.





Previous Related Work

Advection-dominated ROMs

- Lagrangian-based approaches [Mojgani and Balajewicz, 2017, Lu and Tartakovsky, 2020]
- Nonlinear manifolds [Geelen et al., 2022, Wan et al., 2023]
- Online basis updates [Peherstorfer, 2020]
- Transport-invariant coordinate frame:
  - Shifted proper orthogonal Decomposition (sPOD) [Reiss et al., 2018].
  - Artificial neural network approach [Papapicco et al., 2022]
  - Ridge detection and sparse identification [Mendible et al., 2020]
  - **Shifted operator inference [Issan and Kramer, 2023a]** $\Rightarrow$ today's talk!

Shifted operator inference (sOpInf)

- **Non-intrusive** method based on standard operator inference [Peherstorfer and Willcox, 2016].
- While developed for solar wind models, sOpInf generalizes to a wide class of **advection-dominated** phenomena on a periodic domain (i.e. spherical/cylindrical coordinates).

Shifted Operator Inference Framework [Issan and Kramer, 2023b]

- Step (I): Data collection and translation
- Step (II): Data reduction via projection
- Step (III): Model learning and prediction via operator inference
- Step (IV): Re-shifting predicted ROM data

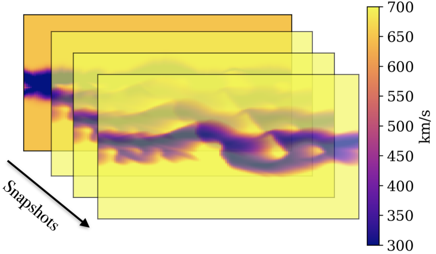
Step (I): Data collection and translation

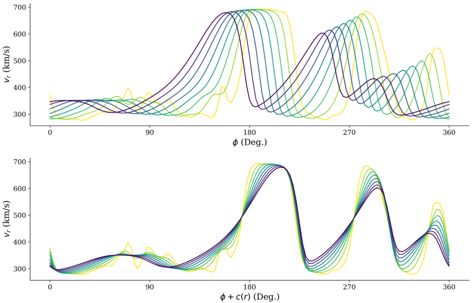
We collect the data snapshots and shift each snapshot to a **advection-invariant coordinate**

$$\mathbf{u}_i \approx u(\phi(t), r_i) \mapsto \tilde{u}(\tilde{\phi}(\phi(t), r_i), r_i) \approx \tilde{\mathbf{u}}_i \qquad \text{with} \qquad \tilde{\phi}(\phi(t), r) = \phi(t) + \mathbf{c}(r) \tag{7}$$

where $\tilde{\phi}(\phi(t), r_i)$ denotes the advection-invariant coordinate frame and $\mathbf{c}(r) \in \mathbb{R}^n$ represents the shift function. We evaluate $\tilde{\mathbf{u}}_i$ via interpolation, i.e.,

$$\tilde{\mathbf{u}}_i = \mathcal{P}_i^k \left[\mathbf{u}_i, \phi(t), \tilde{\phi}(\phi(t), r_i) \right] \tag{8}$$





Determination of spatial shift velocity $c(r)$

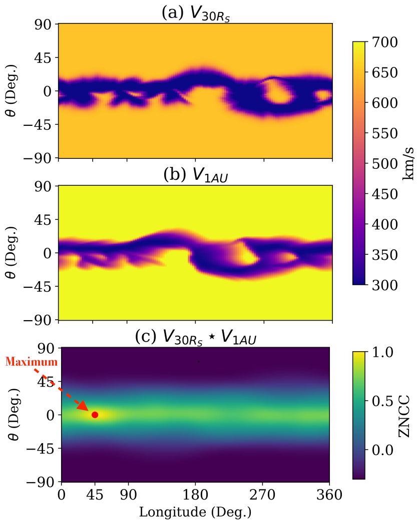
Method of characteristics (analytic)

- Requires access to the underlying equation.
- This method is applicable for first-order quasi-linear partial differential equations.
- $\mathbf{c}(r)$ is the mean characteristic curve,

$$c(r) = \begin{cases} \frac{1}{q-p}\sum_{j=p}^{q} \frac{-\Omega_{\text{rot}}(0)}{\mathbf{v}(\phi_j, r_0)}(r - r_0) & \text{if } r_0 < r < r_s \\ s(r) - s(r_s) + a & \text{if } r > r_s \end{cases}$$

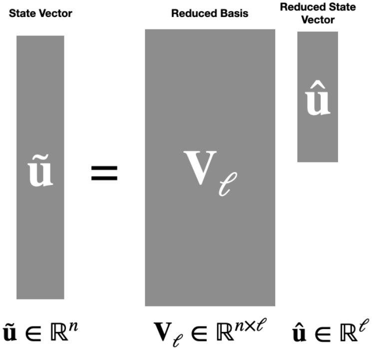
Cross-correlation extrapolation method (data-driven)

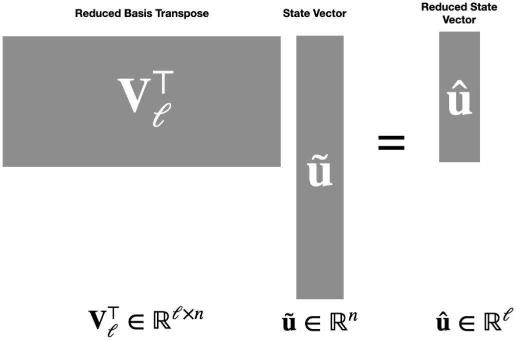
- Maximizing the circular cross-correlation between the two snapshots
- $\mathbf{c}(r)$ is found via least squares polynomial curve fitting to time increments and corresponding spatial location of the shift.



Step (II): Data Reduction via projection

The low dimensional basis is constructed by the left singular vectors of the data matrix $\tilde{\mathbf{U}} = \mathbf{V}\boldsymbol{\Sigma}\mathbf{W}^\top$, where $\mathbf{V} \in \mathbb{R}^{n \times K}$, $\boldsymbol{\Sigma} \in \mathbb{R}^{K \times K}$ and $\mathbf{W} \in \mathbb{R}^{K \times K}$. The $\ell \ll n$ dimensional POD basis, $\mathbf{V}_\ell = [\mathbf{v}_1, \ldots, \mathbf{v}_\ell]$, is given by the first ℓ columns of the left singular vectors $\mathbf{V}$.





Step (III): Model learning and prediction via operator inference

Full-order model (FOM)

$$\frac{\mathrm{d}\tilde{\mathbf{u}}}{\mathrm{d}r} = \mathbf{A}\tilde{\mathbf{u}} + \mathbf{H}(\tilde{\mathbf{u}} \otimes \tilde{\mathbf{u}}) + \mathbf{B} + \mathrm{HOT}$$

- Shifted state vector $\tilde{\mathbf{u}} \in \mathbb{R}^n$, constant vector $\mathbf{B} \in \mathbb{R}^n$, linear operator $\mathbf{A} \in \mathbb{R}^{n \times n}$, quadratic operator $\mathbf{H} \in \mathbb{R}^{n \times n^2}$.

Reduced-order model (ROM)

$$\frac{\mathrm{d}\hat{\mathbf{u}}}{\mathrm{d}r} = \hat{\mathbf{A}}\hat{\mathbf{u}} + \hat{\mathbf{H}}(\hat{\mathbf{u}} \otimes \hat{\mathbf{u}}) + \hat{\mathbf{B}} + \mathrm{HOT}$$

- Low dimensional basis $\mathbf{V}_\ell \in \mathbb{R}^{n \times \ell}$, $\ell \ll n$.
- Reduced state vector $\hat{\mathbf{u}} \in \mathbb{R}^\ell$, $\mathbf{u} \approx \mathbf{V}_\ell \hat{\mathbf{u}}$, reduced operators $\hat{\mathbf{B}} = \mathbf{V}_\ell^\top \mathbf{B} \in \mathbb{R}^\ell$, $\hat{\mathbf{A}} = \mathbf{V}_\ell^\top \mathbf{A} \mathbf{V}_\ell \in \mathbb{R}^{\ell \times \ell}$, $\hat{\mathbf{H}} = \mathbf{V}_\ell^\top \mathbf{H}(\mathbf{V}_\ell \otimes \mathbf{V}_\ell) \in \mathbb{R}^{\ell \times \ell^2}$.

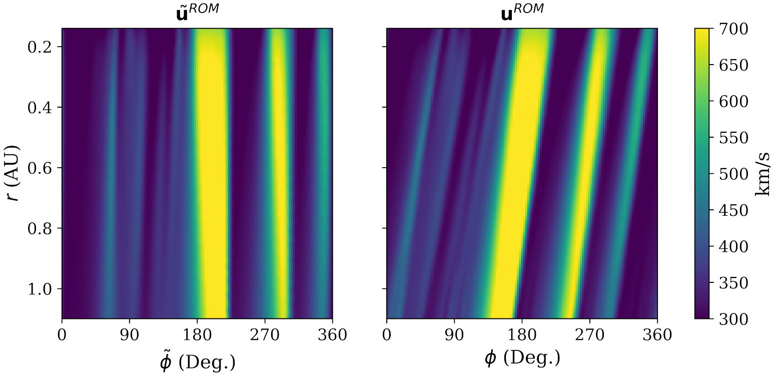
Non-intrusive approach to compute the reduced operators

$$\min_{\hat{\mathbf{A}} \in \mathbb{R}^{\ell \times \ell}, \hat{\mathbf{H}} \in \mathbb{R}^{\ell \times \ell^2}, \hat{\mathbf{B}} \in \mathbb{R}^\ell} \left\| \left[\hat{\mathbf{A}} \hat{\mathbf{U}} + \hat{\mathbf{H}}(\hat{\mathbf{U}} \otimes \hat{\mathbf{U}}) + \hat{\mathbf{B}} \, \mathbf{1}_K - \dot{\hat{\mathbf{U}}} \right]^\top \right\|_2^2 \tag{9}$$

Step (IV): Re-shifting predicted ROM data

We shift the ROM-predicted solutions back to the original coordinate system via interpolation

$$\mathbf{u}_i^{\text{ROM}}(\phi(t), r_i) = \mathcal{P}_i^k \left[\tilde{\mathbf{u}}_i^{\text{ROM}}, \tilde{\phi}(\phi(t), r_i), \phi\right]$$



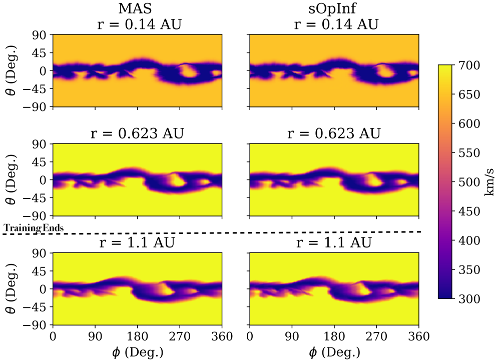
The sOpInf predicted snapshots are shifted back to the HG coordinate frame.

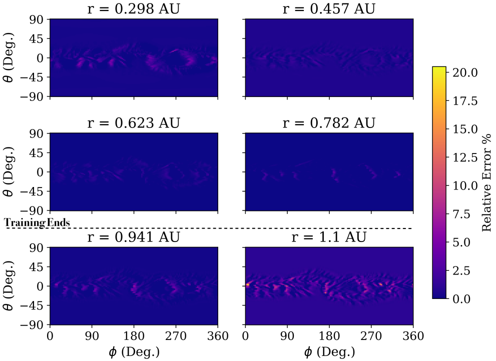
Case study: Carrington Rotation 2210

- Time interval: Oct 26–Nov 23, 2018 $\Rightarrow$ solar minimum (during PSP first perihelion).
- sOpInf ROM model form

$$\frac{\mathrm{d}\hat{\mathbf{u}}}{\mathrm{d}r} = \hat{\mathbf{A}}\hat{\mathbf{u}} + \hat{\mathbf{H}}(\hat{\mathbf{u}} \otimes \hat{\mathbf{u}}) + \hat{\mathbf{B}}$$

- sOpInf successfully reproduced the high fidelity 3D full-Sun MAS dataset, where $n_x = n_\phi \times n_\theta = 14,208$ with only $\ell = 8$ reduced dimensions, leading to a substantial reduction in the model's dimensionality.





Conclusions

- We propose a new strategy for efficient heliospheric solar wind modeling via data-driven reduced-order modeling.
- While developed for solar wind models, the method generalizes to a wide class of advection-dominated phenomena on a periodic domain.

Preprint Link and Code Accessibility

- This work *"Predicting Solar Wind Streams from the Inner-Heliosphere to Earth via Shifted Operator Inference"* is published at the Journal of Computational Physics (JCP).
- The code used to generate these results is available at `https://github.com/opaliss/Space-Weather-ROM`.

Credit: NASA.

Thank you! Questions?

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